

Notes for valuers

- Give full marks for any correct alternative method. For 4- and 8-mark questions, award step marks as shown in brackets.
- If a student makes an arithmetic slip but the method is correct, deduct only the marks for that step.

SECTION – I**12 × 1 = 12**

1. C) 420 ($2^2 \times 3 \times 5 \times 7$)
2. 3
3. -7 ($\alpha + \beta = -b/a$)
4. B) parallel ($1/2 = 2/4 \neq 4/12$)
5. Real and equal roots ($D = 16 - 16 = 0$)
6. $4/3$
7. A) Both A and R are true and R is the correct explanation of A.
8. 0 (zero)
9. B) 1
10. True
11. 25 ($((20 + 30)/2)$)
12. B) -1.5

SECTION – II**8 × 2 = 16**

13. $6^n = (2 \times 3)^n = 2^n \times 3^n$. A number ends with 0 only if its prime factorisation contains both 2 and 5.
 6^n has no factor 5, so 6^n can never end with the digit zero.
14. $x^2 - (\text{sum})x + \text{product} = x^2 - (-3)x + 2 = x^2 + 3x + 2$
15. Adding: $2x = 18 \rightarrow x = 9$; $y = 14 - 9 = 5$
Solution: $x = 9, y = 5$
16. $D = b^2 - 4ac = (-4)^2 - 4(2)(3) = 16 - 24 = -8$
 $D < 0 \rightarrow$ no real roots.
17. $\triangle ABC \sim \triangle DEF \rightarrow AB/DE = BC/EF \rightarrow 4/6 = BC/9$
 $BC = (4 \times 9)/6 = 6 \text{ cm}$
18. $d = \sqrt{[(4 - 2)^2 + (1 - 3)^2]} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$ units
19. $2(1)^2 + (\sqrt{3}/2)^2 - (\sqrt{3}/2)^2 = 2 + 3/4 - 3/4 = 2$
20. Median = $l + [(n/2 - cf)/f] \times h$, where [formula 1, terms 1]
 l = lower limit of the median class, $n = \Sigma f$, cf = cumulative frequency of the class before the median class, f = frequency of the median class, h = class size.

SECTION – III

8 × 4 = 32

21. Suppose $5 - \sqrt{3}$ is rational: $5 - \sqrt{3} = a/b$ (a, b integers, $b \neq 0$).

$$\text{Then } \sqrt{3} = 5 - a/b = (5b - a)/b.$$

Since a, b are integers, $(5b - a)/b$ is rational, so $\sqrt{3}$ would be rational.

This contradicts the fact that $\sqrt{3}$ is irrational. $\therefore 5 - \sqrt{3}$ is irrational.

22. $a_{11} = a + 10d = 38$; $a_{16} = a + 15d = 73$

[equations 1, a & d 2, answer 1]

$$\text{Subtracting: } 5d = 35 \rightarrow d = 7; a = 38 - 70 = -32$$

$$a_{31} = a + 30d = -32 + 210 = 178$$

23. $m : n = 2 : 3$, $(x_1, y_1) = (-1, 7)$, $(x_2, y_2) = (4, -3)$

$$x = (2 \times 4 + 3 \times (-1))/5 = (8 - 3)/5 = 1$$

$$y = (2 \times (-3) + 3 \times 7)/5 = (-6 + 21)/5 = 3$$

Required point = $(1, 3)$

24. LHS = $\sec A (1 - \sin A)(\sec A + \tan A) = (1/\cos A)(1 - \sin A)(1 + \sin A)/\cos A$

$$= (1 - \sin^2 A)/\cos^2 A = \cos^2 A/\cos^2 A = 1 = \text{RHS}$$

25. Rough figure: string AC as hypotenuse, height of kite BC = 60 m, $\angle A = 60^\circ$.

[figure 1, ratio 1, answer 2]

$$\sin 60^\circ = 60/AC \rightarrow AC = 60 \times 2/\sqrt{3} = 40\sqrt{3} \text{ m} = 40 \times 1.732 = 69.28 \text{ m}$$

26. Assumed mean $a = 47.5$:

Class	x_i	f_i	$d_i = x_i - 47.5$	$f_i d_i$
10–25	17.5	2	–30	–60
25–40	32.5	3	–15	–45
40–55	47.5	7	0	0
55–70	62.5	6	15	90
70–85	77.5	6	30	180
85–100	92.5	6	45	270
Total		30		435

$$\text{Mean} = a + \Sigma f_i d_i / \Sigma f_i = 47.5 + 435/30 = 47.5 + 14.5 = 62$$

(Any other assumed mean gives the same answer, 62.)

27. Total outcomes = 36

[1 each]

(i) Sum 8: $(2,6), (3,5), (4,4), (5,3), (6,2) \rightarrow 5/36$

(ii) Doublet: 6 outcomes $\rightarrow 6/36 = 1/6$

(iii) Sum ≥ 11 : $(5,6), (6,5), (6,6) \rightarrow 3/36 = 1/12$

(iv) Sum 13: impossible $\rightarrow 0$

28. Equal roots $\rightarrow D = 0 \rightarrow k^2 - 4(2)(3) = 0 \rightarrow k^2 = 24$

$$k = \pm\sqrt{24} = \pm 2\sqrt{6}$$

SECTION – IV

5 × 8 = 40

29. a) Assume $\sqrt{2}$ is rational: $\sqrt{2} = a/b$, a and b co-prime, $b \neq 0$.

[assumption 2, steps 4, conclusion 2]

$$2b^2 = a^2 \rightarrow 2 \text{ divides } a^2 \rightarrow 2 \text{ divides } a. \text{ Let } a = 2c.$$

$$2b^2 = 4c^2 \rightarrow b^2 = 2c^2 \rightarrow 2 \text{ divides } b^2 \rightarrow 2 \text{ divides } b.$$

2 is a common factor of a and b — contradiction.

$\therefore \sqrt{2}$ is irrational.

29. b) $404 = 2^2 \times 101$; $96 = 2^5 \times 3$

[factors 2, HCF 2, LCM 2, verification 2]

$$\text{HCF} = 2^2 = 4; \text{LCM} = 2^5 \times 3 \times 101 = 9696$$

$$\text{HCF} \times \text{LCM} = 4 \times 9696 = 38784; 404 \times 96 = 38784 \checkmark \text{ (verified)}$$

30. a) Tables of values:

$2x + y = 6$	$x: 0, 1, 3$	$y: 6, 4, 0$
$2x - y + 2 = 0$	$x: -1, 0, 1$	$y: 0, 2, 4$

The lines meet at (1, 4). The first line cuts the x-axis at (3, 0), the second at (-1, 0).

Vertices of the triangle: (1, 4), (3, 0) and (-1, 0).

(Area = $\frac{1}{2} \times 4 \times 4 = 8$ sq. units — not required, may be given credit as extra.)

Marks: tables 2, graph 4, vertices 2.

30. b) Let the integers be x and $x + 1$. $x^2 + (x + 1)^2 = 365$

$$2x^2 + 2x - 364 = 0 \rightarrow x^2 + x - 182 = 0 \rightarrow (x + 14)(x - 13) = 0$$

$x = 13$ (positive). The integers are 13 and 14. ($13^2 + 14^2 = 169 + 196 = 365$ ✓)

31. a) $a = 24$, $d = -3$. $S_n = n/2 [48 + (n - 1)(-3)] = 78$

[equation 3, solving 3, explanation 2]

$$n(51 - 3n) = 156 \rightarrow 3n^2 - 51n + 156 = 0 \rightarrow n^2 - 17n + 52 = 0$$

$$(n - 4)(n - 13) = 0 \rightarrow n = 4 \text{ or } n = 13$$

Double answer: a_5 to a_{13} are 12, 9, 6, 3, 0, -3, -6, -9, -12, whose sum is 0. So the sum of the first 4 terms and of the first 13 terms are both 78.

31. b) $AB = \sqrt{[(4-1)^2 + (2-7)^2]} = \sqrt{(9 + 25)} = \sqrt{34}$

[sides 4, diagonals 2, conclusion 2]

$$BC = \sqrt{[(-1-4)^2 + (-1-2)^2]} = \sqrt{(25 + 9)} = \sqrt{34}$$

$$CD = \sqrt{[(-4+1)^2 + (4+1)^2]} = \sqrt{(9 + 25)} = \sqrt{34}$$

$$DA = \sqrt{[(1+4)^2 + (7-4)^2]} = \sqrt{(25 + 9)} = \sqrt{34}$$

$$\text{Diagonals: } AC = \sqrt{[(-2)^2 + (-8)^2]} = \sqrt{68}; \quad BD = \sqrt{[(-8)^2 + 2^2]} = \sqrt{68}$$

All sides equal and diagonals equal $\rightarrow$ ABCD is a square.

32. a) Converse of BPT. Given: $\triangle ABC$, line DE meets AB at D and AC at E with $AD/DB = AE/EC$. To prove: $DE \parallel BC$.

[given/figure 2, proof 6]

Suppose DE is not parallel to BC. Draw $DE' \parallel BC$, meeting AC at E' .

By BPT: $AD/DB = AE'/E'C$. Given $AD/DB = AE/EC \rightarrow AE/EC = AE'/E'C$.

Adding 1 to both sides: $AC/EC = AC/E'C \rightarrow EC = E'C \rightarrow E$ and E' coincide.

$\therefore DE \parallel BC$. Hence proved.

32. b) Let $AB = h$ be the tower, C and D points at 4 m and 9 m from B. $\angle ACB = \theta$, $\angle ADB = 90^\circ - \theta$. [figure 2, ratios 4, answer 2]

$$\text{In } \triangle ABC: \tan \theta = h/4 \dots (1)$$

$$\text{In } \triangle ABD: \tan(90^\circ - \theta) = \cot \theta = h/9 \dots (2)$$

$$(1) \times (2): \tan \theta \cdot \cot \theta = h^2/36 \rightarrow 1 = h^2/36 \rightarrow h = 6 \text{ m. Hence proved.}$$

33. a) Step-deviation method ($a = 150$, $h = 20$):

Wages	x_i	f_i	$u_i = (x_i - 150)/20$	$f_i u_i$
100–120	110	12	-2	-24
120–140	130	14	-1	-14
140–160	150	8	0	0
160–180	170	6	1	6
180–200	190	10	2	20
Total		50		-12

$$\text{Mean} = a + (\sum f_i u_i / \sum f_i) \times h = 150 + (-12/50) \times 20 = 150 - 4.8 = ₹ 145.20$$

33. b) Total outcomes = 90

[2 each]

(i) Two-digit numbers 10 to 90 $\rightarrow 81 \rightarrow 81/90 = 9/10$

(ii) Perfect squares 1, 4, 9, 16, 25, 36, 49, 64, 81 $\rightarrow 9 \rightarrow 9/90 = 1/10$

(iii) Divisible by 5: 5, 10, ..., 90 $\rightarrow$ 18 $\rightarrow$ $18/90 = 1/5$

(iv) Divisible by both 2 and 3 (i.e. by 6): 6, 12, ..., 90 $\rightarrow$ 15 $\rightarrow$ $15/90 = 1/6$

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