



## CHAPTER - 4

# **SIMPLE AND COMPOUND INTEREST INCLUDING ANNUITY- APPLICATIONS**



## LEARNING OBJECTIVES

After studying this chapter students will be able to understand:-

- ◆ The concept of interest, related terms and computation thereof;
- ◆ Difference between simple and compound interest;
- ◆ The concept of annuity;
- ◆ The concept of present value and future value;
- ◆ Use of present value concept in Leasing, Capital expenditure and Valuation of Bond.

## 4.1 INTRODUCTION

People earn money for spending it on housing, food, clothing, education, entertainment etc. Sometimes extra expenditures have also to be met with. For example there might be a marriage in the family; one may want to buy house, one may want to set up his or her business, one may want to buy a car and so on. Some people can manage to put aside some money for such expected and unexpected expenditures. But most people have to borrow money for such contingencies. From where they can borrow money?

Money can be borrowed from friends or money lenders or Banks. If you can arrange a loan from your friend it might be interest free but if you borrow money from lenders or Banks you will have to pay some charge periodically for using money of money lenders or Banks. This charge is called interest.

Let us take another view. People earn money for satisfying their various needs as discussed above. After satisfying those needs some people may have some savings. People may invest their savings in debentures or lend to other person or simply deposit it into bank. In this way they can earn interest on their investment.

Most of you are very much aware of the term interest. Interest can be defined as the price paid by a borrower for the use of a lender's money.

We will know more about interest and other related terms later.

## 4.2 WHY IS INTEREST PAID?

Now question arises why lenders charge interest for the use of their money. There are a variety of reasons. We will now discuss those reasons.

1. **Time value of money:** Time value of money means that the value of a unity of money is different in different time periods. The sum of money received in future is less valuable than it is today. In other words the present worth of rupees received after some time will be less than a rupee received today. Since a rupee received today has more value rational investors would prefer current receipts to future receipts. If they postpone their receipts they will certainly charge some money i.e. interest.
2. **Opportunity Cost:** The lender has a choice between using his money in different investments. If he chooses one he forgoes the return from all others. In other words lending incurs an opportunity cost due to the possible alternative uses of the lent money.



3. **Inflation:** Most economies generally exhibit inflation. Inflation is a fall in the purchasing power of money. Due to inflation a given amount of money buys fewer goods in the future than it will now. The borrower needs to compensate the lender for this.
4. **Liquidity Preference:** People prefer to have their resources available in a form that can immediately be converted into cash rather than a form that takes time or money to realize.
5. **Risk Factor:** There is always a risk that the borrower will go bankrupt or otherwise default on the loan. Risk is a determinable factor in fixing rate of interest.

A lender generally charges more interest rate (risk premium) for taking more risk.

### 4.3 DEFINITION OF INTEREST AND SOME OTHER RELATED TERMS

Now we can define interest and some other related terms.

**4.3.1 Interest:** Interest is the price paid by a borrower for the use of a lender's money. If you borrow (or lend) some money from (or to) a person for a particular period you would pay (or receive) more money than your initial borrowing (or lending). This excess money paid (or received) is called interest. Suppose you borrow (or lend) Rs.50000 for a year and you pay (or receive) Rs.55000 after one year the difference between initial borrowing (or lending) Rs.50000 and end payment (or receipts) Rs.55000 i.e. Rs.5000 is the amount of interest you paid (or earned).

**4.3.2 Principal:** Principal is initial value of lending (or borrowing). If you invest your money the value of initial investment is also called principal. Suppose you borrow (or lend) Rs.50000 from a person for one year. Rs.50000 in this example is the 'Principal.' Take another example suppose you deposit Rs.20000 in your bank account for one year. In this example Rs.20000 is the principal.

**4.3.3 Rate of Interest:** The rate at which the interest is charged for a defined length of time for use of principal generally on a yearly basis is known to be the rate of interest. Rate of interest is usually expressed as percentages. Suppose you invest Rs.20000 in your bank account for one year with the interest rate of 5% per annum. It means you would earn Rs.5 as interest every Rs.100 of principal amount in a year.

Per annum means for a year.

**4.3.4 Accumulated amount (or Balance):** Accumulated amount is the final value of an investment. It is the sum total of principal and interest earned. Suppose you deposit Rs.50000 in your bank for one year with a interest rate of 5% p.a. you would earn interest of Rs.2500 after one year. (method of computing interest will be illustrated later). After one year you will get Rs.52500 (principal+ interest), Rs.52 500 is amount here.

Amount is also known as the balance.

### 4.4 SIMPLE INTEREST AND COMPOUND INTEREST

Now we can discuss the method of computing interest. Interest accrues as either simple interest or compound interest. We will discuss simple interest and compound interest in the following paragraphs:



**4.4.1 Simple Interest:** Now we would know what is simple interest and the methodology of computing simple interest and accumulated amount for an investment (principal) with a simple rate over a period of time. As you already know the money that you borrow is known as principal and the additional money that you pay for using somebody else's money is known as interest. The interest paid for keeping Rs.100 for one year is known as the rate percent per annum. Thus if money is borrowed at the rate of 8% per annum then the interest paid for keeping Rs.100 for one year is Rs.8. The sum of principal and interest is known as the amount.

Clearly the interest you pay is proportionate to the money that you borrow and also to the period of time for which you keep the money; the more the money and the time, the more the interest. Interest is also proportionate to the rate of interest agreed upon by the lending and the borrowing parties. Thus interest varies directly with principal, time and rate.

Simple interest is the interest computed on the principal for the entire period of borrowing. It is calculated on the outstanding principal balance and not on interest previously earned. It means no interest is paid on interest earned during the term of loan.

Simple interest can be computed by applying following formulas:

$$I = Pit$$

$$A = P + I$$

$$= P + Pit$$

$$= P(1 + it)$$

$$I = A - P$$

Here,

A = Accumulated amount (final value of an investment)

P = Principal (initial value of an investment)

i = Annual interest rate in decimal.

I = Amount of Interest

t = Time in years

Let us consider the following examples in order to see how exactly are these quantities related.

**Example 1:** How much interest will be earned on Rs.2000 at 6% simple interest for 2 years?

**Solution:** Required interest amount is given by

$$I = P \times i \times t$$

$$= 2000 \times \frac{6}{100} \times 2$$

$$= \text{Rs. } 240$$



**Example 2:** Sania deposited Rs.50000 in a bank for two years with the interest rate of 5.5% p.a. How much interest would she earn?

**Solution:** Required interest amount is given by

$$\begin{aligned} I &= P \times i \times t \\ &= \text{Rs. } 50000 \times \frac{5.5}{100} \times 2 \\ &= \text{Rs. } 5500 \end{aligned}$$

**Example 3:** In example 2 what will be the final value of investment?

**Solution:** Final value of investment is given by

$$\begin{aligned} A &= P(1 + it) \\ &= \text{Rs. } 50000 \left( 1 + \frac{5.5}{100} \times 2 \right) \\ &= \text{Rs. } 50000 \left( 1 + \frac{11}{100} \right) \\ &= \text{Rs. } \frac{50000 \times 111}{100} \\ &= \text{Rs. } 55500 \\ &\text{Or} \\ A &= P + I \\ &= \text{Rs. } (50000 + 5500) \\ &= \text{Rs. } 55500 \end{aligned}$$

**Example 4:** Sachin deposited Rs.100000 in his bank for 2 years at simple interest rate of 6%. How much interest would he earn? How much would be the final value of deposit?

**Solution:** (a) Required interest amount is given by

$$\begin{aligned} I &= P \times it \\ &= \text{Rs. } 100000 \times \frac{6}{100} \times 2 \\ &= \text{Rs. } 12000 \end{aligned}$$

(b) Final value of deposit is given by

$$\begin{aligned} A &= P + I \\ &= \text{Rs. } (100000 + 12000) \\ &= \text{Rs. } 112000 \end{aligned}$$



**Example 5:** Find the rate of interest if the amount owed after 6 months is Rs.1050, borrowed amount being Rs.1000.

**Solution:** We know  $A = P + Pit$

$$\text{i.e. } 1050 = 1000 + 1000 \times i \times 6/12$$

$$\text{➤ } 50 = 500 i$$

$$\text{➤ } i = 1/10 = 10\%$$

**Example 6:** Rahul invested Rs.70000 in a bank at the rate of 6.5% p.a. simple interest rate. He received Rs.85925 after the end of term. Find out the period for which sum was invested by Rahul.

**Solution:** We know  $A = P (1+it)$

$$\text{i.e. } 85925 = 70000 \left( 1 + \frac{6.5}{100} \times t \right)$$

$$\text{➤ } 85925/70000 = \frac{100+6.5t}{100}$$

$$\text{➤ } \frac{85925 \times 100}{70000} - 100 = 6.5t$$

$$\text{➤ } 22.75 = 6.5t$$

$$\text{➤ } t = 3.5$$

$$\therefore \text{ time} = 3.5 \text{ years}$$

**Example 7:** Kapil deposited some amount in a bank for  $7\frac{1}{2}$  years at the rate of 6% p.a. simple interest. Kapil received Rs.101500 at the end of the term. Compute initial deposit of Kapil.

**Solution:** We know  $A = P(1+ it)$

$$\text{i.e. } 101500 = P \left( 1 + \frac{6}{100} \times \frac{15}{2} \right)$$

$$\text{➤ } 101500 = P \left( 1 + \frac{45}{100} \right)$$

$$\text{➤ } 101500 = P \left( \frac{145}{100} \right)$$

$$\text{➤ } P = \frac{101500 \times 100}{145}$$

$$= \text{Rs. } 70000$$

$$\therefore \text{ Initial deposit of Kapil} = \text{Rs. } 70000$$



**Example 8:** A sum of Rs.46875 was lent out at simple interest and at the end of 1 year 8 months the total amount was Rs.50000. Find the rate of interest percent per annum.

**Solution:** We know  $A = P(1 + it)$

$$\text{i.e. } 50000 = 46875 \left( 1 + i \times 1 \frac{8}{12} \right)$$

$$\text{➤ } 50000/46875 = 1 + \frac{5}{3} i$$

$$\text{➤ } (1.067 - 1) \times 3/5 = i$$

$$\text{➤ } i = 0.04$$

$$\text{➤ } \text{rate} = 4\%$$

**Example 9:** What sum of money will produce Rs.28600 as an interest in 3 years and 3 months at 2.5% p.a. simple interest?

**Solution:** We know  $I = P \times it$

$$\text{i.e. } 28600 = P \times \frac{2.5}{100} \times 3 \frac{3}{12}$$

$$\text{➤ } 28600 = \frac{2.5}{100} P \times \frac{13}{4}$$

$$\text{➤ } 28600 = \frac{32.5}{400} P$$

$$\text{➤ } P = \frac{28600 \times 400}{32.5}$$

$$= \text{Rs. } 352000$$

$\therefore$  Rs.352000 will produce Rs.28600 interest in 3 years and 3 months at 2.5% p.a. simple interest

**Example 10:** In what time will Rs.85000 amount to Rs.157675 at 4.5 % p.a. ?

**Solution:** We know

$$A = P(1 + it)$$

$$\text{➤ } 157675 = 85000 \left( 1 + \frac{4.5}{100} \times t \right)$$

$$\text{➤ } \frac{157675}{85000} = \frac{100 + 4.5 t}{100}$$

$$\text{➤ } 4.5t = \left[ \frac{157675}{85000} \times 100 \right] - 100$$



$$\text{> } t = \frac{85.5}{4.5} = 19$$

∴ In 19 years Rs.85000 will amount to Rs.157675 at 4.5% p.a. simple interest rate.

#### Exercise 4 (A)

#### Choose the most appropriate option (a) (b) (c) (d)

- S.I on Rs. 3500 for 3 years at 12% per annum is  
(a) Rs. 1200      (b) 1260      (c) 2260      (d) none of these
- $P = 5000, R = 15, T = 4\frac{1}{2}$  using  $I = \frac{PRT}{100}$ , I will be  
(a) Rs. 3375      (b) Rs. 3300      (c) Rs. 3735      (d) none of these
- If  $P = 5000, T = 1, I = \text{Rs. } 300$ , R will be  
(a) 5%      (b) 4%      (c) 6%      (d) none of these
- If  $P = \text{Rs. } 4500, A = \text{Rs. } 7200$ , than Simple interest i.e. I will be  
(a) Rs. 2000      (b) Rs. 3000      (c) Rs. 2500      (d) 2700
- $P = \text{Rs. } 12000, A = \text{Rs. } 16500, T = 2\frac{1}{2}$  years. Rate percent per annum simple interest will be  
(a) 15%      (b) 12%      (c) 10%      (d) none of these
- $P = \text{Rs. } 10000, I = \text{Rs. } 2500, R = 12\frac{1}{2}\%$  SI. The number of years T will be  
(a)  $1\frac{1}{2}$  years      (b) 2 years      (c) 3 years      (d) none of these
- $P = \text{Rs. } 8500, A = \text{Rs. } 10200, R = 12\frac{1}{2}\%$  SI, t will be.  
(a) 1 yr. 7 mth.      (b) 2 yrs.      (c)  $1\frac{1}{2}$  yr.      (d) none of these
- The sum required to earn a monthly interest of Rs 1200 at 18% per annum SI is  
(a) Rs. 50000      (b) Rs. 60000      (c) Rs. 80000      (d) none of these
- A sum of money amount to Rs. 6200 in 2 years and Rs. 7400 in 3 years. The principal and rate of interest are  
(a) Rs. 3800, 31.57%      (b) Rs. 3000, 20%      (c) Rs. 3500, 15%      (d) none of these
- A sum of money doubles itself in 10 years. The number of years it would triple itself is  
(a) 25 years.      (b) 15 years.      (c) 20 years      (d) none of these

**4.4.2 Compound Interest:** We have learnt about the simple interest. We know that if the principal remains the same for the entire period or time then interest is called as simple interest. However in practice the method according to which banks, insurance corporations and other money lending and deposit taking companies calculate interest is different. To understand this method we consider an example :



Suppose you deposit Rs.50000 in ICICI bank for 2 years at 7% p.a. compounded annually. Interest will be calculated in the following way:

### INTEREST FOR FIRST YEAR

$$\begin{aligned} I &= Pit \\ &= \text{Rs. } 50000 \times \frac{7}{100} \times 1 = \text{Rs. } 3500 \end{aligned}$$

### INTEREST FOR SECOND YEAR

For calculating interest for second year principal would not be the initial deposit. Principal for calculating interest for second year will be the initial deposit plus interest for the first year. Therefore principal for calculating interest for second year would be

$$\begin{aligned} &= \text{Rs. } 50000 + \text{Rs. } 3500 \\ &= \text{Rs. } 53500 \end{aligned}$$

$$\begin{aligned} \text{Interest for the second year} &= \text{Rs. } 53500 \times \frac{7}{100} \times 1 \\ &= \text{Rs. } 3745 \end{aligned}$$

Total interest = Interest for first year + Interest for second year

$$\begin{aligned} &= \text{Rs. } (3500+3745) \\ &= \text{Rs. } 7245 \end{aligned}$$

This interest is Rs. 245 more than the simple interest on Rs. 50000 for two years at 7% p.a. As you must have noticed this excess in interest is due to the fact that the principal for the second year was more than the principal for first year. The interest calculated in this manner is called compound interest.

Thus we can define the compound interest as the interest that accrues when earnings for each specified period of time added to the principal thus increasing the principal base on which subsequent interest is computed.

**Example 11:** Saina deposited Rs. 100000 in a nationalized bank for three years. If the rate of interest is 7% p.a., calculate the interest that bank has to pay to Saina after three years if interest is compounded annually. Also calculate the amount at the end of third year.

**Solution:** Principal for first year Rs. 100000

$$\begin{aligned} \text{Interest for first year} &= Pit \\ &= 100000 \times \frac{7}{100} \times 1 \\ &= \text{Rs. } 7000 \end{aligned}$$

$$\begin{aligned} \text{Principal for the second year} &= \text{Principal for first year} + \text{Interest for first year} \\ &= \text{Rs. } 100000 + \text{Rs. } 7000 \end{aligned}$$



$$= \text{Rs. } 107000$$

$$\text{Interest for second year} = 107000 \times \frac{7}{100} \times 1$$

$$= \text{Rs. } 7490$$

$$\text{Principal for the third year} = \text{Principal for second year} + \text{Interest for second year}$$

$$= 107000 + 7490$$

$$= 114490$$

$$\text{Interest for the third year} = \text{Rs. } 114490 \times \frac{7}{100} \times 1$$

$$= \text{Rs. } 8014.30$$

$$\text{Compound interest at the end of third year}$$

$$= \text{Rs. } (7000 + 7490 + 8014.30)$$

$$= \text{Rs. } 22504.30$$

$$\text{Amount at the end of third year}$$

$$= \text{Principal (initial deposit)} + \text{compound interest}$$

$$= \text{Rs. } (100000 + 22504.30)$$

$$= \text{Rs. } 122504.30$$

Now we can summarize the main difference between simple interest and compound interest. The main difference between simple interest and compound interest is that in simple interest the principal remains constant throughout whereas in the case of compound interest principal goes on changing at the end of specified period. For a given principal, rate and time the compound interest is generally more than the simple interest.

**4.4.3 Conversion period:** In the example discussed above the interest was calculated on yearly basis i.e. the interest was compounded annually. However in practice it is not necessary that the interest be compounded annually. For example in banks the interest is often compounded twice a year (half yearly or semi annually) i.e. interest is calculated and added to the principal after every six months. In some financial institutions interest is compounded quarterly i.e. four times a year. The period at the end of which the interest is compounded is called conversion period. When the interest is calculated and added to the principal every six months the conversion period is six months. In this case number of conversion periods per year would be two. If the loan or deposit was for five years then the number of conversion period would be ten.



Typical conversion periods are given below:

Conversion period	Description	Number of conversion period in a year
1 day	Compounded daily	365
1 month	Compounded monthly	12
3 months	Compounded quarterly	4
6 months	Compounded semi annually	2
12 months	Compounded annually	1

**4.4.4 Formula for compound interest:** Taking the principal as  $P$ , the rate of interest per conversion period as  $i$  (in decimal), the number of conversion period as  $n$ , the accrued amount after  $n$  payment periods as  $A_n$  we have accrued amount at the end of first payment period

$$A_1 = P + P i = P (1 + i) ;$$

at the end of second payment period

$$A_2 = A_1 + A_1 i = A_1 (1 + i)$$

$$= P (1 + i) (1 + i)$$

$$= P (1 + i)^2 ;$$

at the end of third payment period

$$A_3 = A_2 + A_2 i$$

$$= A_2 (1+i)$$

$$= P(1+i)^2 (1+i)$$

$$= P(1+i)^3$$

$$A_n = A_{n-1} + A_{n-1} i$$

$$= A_{n-1} (1 + i)$$

$$= P (1 + i)^{n-1} (1 + i)$$

$$= P(1+i)^n$$

Thus the accrued amount  $A_n$  on a principal  $P$  after  $n$  conversion periods at  $i$  (in decimal) rate of interest per conversion period is given by

$$A_n = P (1 + i)^n$$

where,  $i = \frac{\text{Annual rate of interest}}{\text{Number of conversion periods per year}}$

$$\text{Interest} = A_n - P = P (1 + i)^n - P$$



$$= P [(1+i)^n - 1]$$

n is total conversions i.e. t x no. of conversions per year

Computation of A shall be quite simple with a calculator. However compound interest table and tables for at various rates per annum with (a) annual compounding ; (b) monthly compounding and (c) daily compounding are available.

**Example 12:** Rs. 2000 is invested at annual rate of interest of 10%. What is the amount after two years if compounding is done (a) Annually (b) Semi-annually (c) Quarterly (d) monthly.

**Solution:** (a) Compounding is done annually

Here principal P = Rs. 2000; since the interest is compounded yearly the number of conversion periods n in 2 years are 2. Also the rate of interest per conversion period (1 year) i is 0.10

$$A_n = P (1 + i)^n$$

$$\begin{aligned} A_2 &= \text{Rs. } 2000 (1 + 0.1)^2 \\ &= \text{Rs. } 2000 \times (1.1)^2 \\ &= \text{Rs. } 2000 \times 1.21 \\ &= \text{Rs. } 2420 \end{aligned}$$

(b) For semiannual compounding

$$\begin{aligned} n &= 2 \times 2 = 4 \\ i &= \frac{0.1}{2} = 0.05 \end{aligned}$$

$$\begin{aligned} A_4 &= 2000 (1+0.05)^4 \\ &= 2000 \times 1.2155 \\ &= \text{Rs. } 2431 \end{aligned}$$

(c) For quarterly compounding

$$\begin{aligned} n &= 4 \times 2 = 8 \\ i &= \frac{0.1}{4} = 0.025 \end{aligned}$$

$$\begin{aligned} A_8 &= 2000 (1+ 0.025)^8 \\ &= 2000 \times 1.2184 \\ &= \text{Rs. } 2436.80 \end{aligned}$$

(d) For monthly compounding

$$\begin{aligned} n &= 12 \times 2 = 24, i = 0.1/12 = 0.00833 \\ A_{24} &= 2000 (1 + 0.00833)^{24} \\ &= 2000 \times 1.22029 \\ &= \text{Rs. } 2440.58 \end{aligned}$$



**Example 13:** Determine the compound amount and compound interest on Rs.1000 at 6% compounded semi-annually for 6 years. Given that  $(1 + i)^n = 1.42576$  for  $i = 3\%$  and  $n = 12$ .

**Solution:**  $i = \frac{0.06}{2} = 0.03$ ;  $n = 6 \times 2 = 12$

$$P = 1000$$

$$\begin{aligned}\text{Compound Amount (A}_{12}\text{)} &= P (1 + i)^n \\ &= \text{Rs. } 1000(1 + 0.03)^{12} \\ &= 1000 \times 1.42576 \\ &= \text{Rs. } 1425.76 \\ \text{Compound Interest} &= \text{Rs. } (1425.76 - 1000) \\ &= \text{Rs. } 425.76\end{aligned}$$

**Example 14:** Compute the compound interest on Rs. 4000 for  $1\frac{1}{2}$  years at 10% per annum compounded half- yearly.

**Solution:** Here principal  $P = \text{Rs. } 4000$ . Since the interest is compounded half-yearly the number of conversion periods in  $1\frac{1}{2}$  years are 3. Also the rate of interest per conversion period (6 months) is  $10\% \times \frac{1}{2} = 5\%$  (0.05 in decimal).

Thus the amount  $A_n$  ( in Rs.) is given by

$$\begin{aligned}A_n &= P (1 + i)^n \\ A_3 &= 4000(1 + 0.05)^3 \\ &= 4630.50\end{aligned}$$

The compound interest is therefore  $\text{Rs.}(4630.50 - 4000)$   
 $= \text{Rs.}630.50$

### To find the Principal/Time/Rate

The Formula  $A_n = P(1 + i)^n$  connects four variables  $A_n$ ,  $P$ ,  $i$  and  $n$ .

Similarly,  $\text{C.I. (Compound Interest)} = P[(1 + i)^n - 1]$  connects C.I.,  $P$ ,  $i$  and  $n$ . Whenever three out of these four variables are given the fourth can be found out by simple calculations.

**Examples 15:** On what sum will the compound interest at 5% per annum for two years compounded annually be Rs.1640?

**Solution:** Here the interest is compounded annually the number of conversion periods in two years are 2. Also the rate of interest per conversion period (1 year) is 5%.

$$n = 2 \quad i = 0.05$$

We know

$$\text{C.I.} = P[(1+i)^n - 1]$$



$$\begin{aligned}
 \text{➤ } 1640 &= P[(1+0.05)^2-1] \\
 \text{➤ } 1640 &= P(1.1025 - 1) \\
 \text{➤ } P &= \frac{1640}{0.1025} \\
 &= 16000
 \end{aligned}$$

Hence the required sum is Rs.16000.

**Example 16:** What annual rate of interest compounded annually doubles an investment in 7 years? Given that  $2^{1/7} = 1.104090$

**Solution:** If the principal be P then  $A_n = 2P$ .

$$\begin{aligned}
 \text{Since } A_n &= P(1+i)^n \\
 \text{➤ } 2P &= P(1+i)^7 \\
 \text{➤ } 2^{1/7} &= (1+i) \\
 \text{➤ } 1.104090 &= 1+i \\
 \text{➤ } i &= 0.10409
 \end{aligned}$$

∴ Required rate of interest = 10.41% per annum

**Example 17:** In what time will Rs.8000 amount to Rs.8820 at 10% per annum interest compounded half-yearly?

**Solution:** Here interest rate per conversion period

$$\begin{aligned}
 (i) &= \frac{10}{2} \% \\
 &= 5\% (= 0.05 \text{ in decimal})
 \end{aligned}$$

$$\begin{aligned}
 \text{Principal (P)} &= \text{Rs. } 8000 \\
 \text{Amount (A}_n) &= \text{Rs. } 8820
 \end{aligned}$$

We know

$$\begin{aligned}
 A_n &= P(1+i)^n \\
 \text{➤ } 8820 &= 8000(1+0.05)^n \\
 \text{➤ } \frac{8820}{8000} &= (1.05)^n \\
 \text{➤ } 1.1025 &= (1.05)^n \\
 \text{➤ } (1.05)^2 &= (1.05)^n \\
 \text{➤ } n &= 2
 \end{aligned}$$

Hence number of conversion period is 2 and the required time =  $n/2 = 2/2 = 1$  year

**Example 18:** Find the rate percent per annum if Rs.200000 amount to Rs.231525 in  $1\frac{1}{2}$  year interest being compounded half-yearly.



**Solution:** Here  $P = \text{Rs. } 200000$

Number of conversion period  $(n) = 1\frac{1}{2} \times 2 = 3$

Amount  $(A_3) = \text{Rs. } 231525$

We know that

$$A_3 = P(1+i)^3$$

- $231525 = 200000(1+i)^3$
- $\frac{231525}{200000} = (1+i)^3$
- $1.157625 = (1+i)^3$
- $(1.05)^3 = (1+i)^3$
- $i = 0.05$

$i$  is the Interest rate per conversion period (six months)  $= 0.05 = 5\%$  &

Interest rate per annum  $= 5\% \times 2 = 10\%$

**Example 19:** A certain sum invested at 4% per annum compounded semi-annually amounts to Rs.78030 at the end of one year. Find the sum.

**Solution:** Here  $A_n = 78030$

$$n = 2 \times 1 = 2$$

$$i = 4 \times 1/2 \% = 2\% = 0.02$$

$P(\text{in Rs.}) = ?$

We have

$$A_n = P(1+i)^n$$

- $A_2 = P(1+0.02)^2$
- $78030 = P(1.02)^2$
- $P = \frac{78030}{(1.02)^2}$
- $= 75000$

Thus the sum invested is Rs.75000 at the beginning of 1 year.

**Example 20:** Rs.16000 invested at 10% p.a. compounded semi-annually amounts to Rs.18522. Find the time period of investment.

**Solution:** Here  $P = \text{Rs. } 16000$

$$A_n = \text{Rs. } 18522$$



$$i = 10 \times 1/2 \% = 5\% = 0.05$$

$$n = ?$$

We have  $A_n = P(1 + i)^n$

$$\text{➤ } 18522 = 16000(1+0.05)^n$$

$$\text{➤ } \frac{18522}{16000} = (1.05)^n$$

$$\text{➤ } (1.157625) = (1.05)^n$$

$$\text{➤ } (1.05)^3 = (1.05)^n$$

$$\text{➤ } n = 3$$

Therefore time period of investment is three half years i.e.  $1\frac{1}{2}$  years.

**Example 21:** A person opened an account on April, 2001 with a deposit of Rs.800. The account paid 6% interest compounded quarterly. On October 1 2001 he closed the account and added enough additional money to invest in a 6 month time-deposit for Rs. 1000, earning 6% compounded monthly.

- How much additional amount did the person invest on October 1?
- What was the maturity value of his time deposit on April 1 2002?
- How much total interest was earned?

Given that  $(1 + i)^n$  is 1.03022500 for  $i=1\frac{1}{2}\%$   $n=2$  and  $(1 + i)^n$  is 1.03037751 for  $i = \frac{1}{2}\%$  and  $n = 6$ .

**Solution:** (a) The initial investment earned interest for April-June and July- September quarter

i.e. for two quarters. In this case  $i = 6/4 = 1\frac{1}{2}\% = 0.015$ ,  $n \left[ n = \frac{6}{12} \times 4 \right] = 2$

$$\begin{aligned} \text{and the compounded amount} &= 800(1 + 0.015)^2 \\ &= 800 \times 1.03022500 \\ &= \text{Rs. } 824.18 \end{aligned}$$

$$\begin{aligned} \text{The additional amount invested} &= \text{Rs. } (1000 - 824.18) \\ &= \text{Rs. } 175.82 \end{aligned}$$

- In this case the time-deposit earned interest compounded monthly for six months.

$$\begin{aligned} \text{Here } i &= \frac{6}{12} = 1/2 \% = (0.005) \quad n = 6 \quad \text{and} \quad P = \text{Rs. } 1000 \\ &= \frac{6}{12} \times 12 \end{aligned}$$

$$\text{Maturity value} = 1000(1+0.005)^6$$



$$\begin{aligned} &= 1000 \times 1.03037751 \\ &= \text{Rs. } 1030.38 \end{aligned}$$

(c) Total interest earned = Rs. (24.18+30.38) = Rs. 54.56

## 4.5 EFFECTIVE RATE OF INTEREST

If interest is compounded more than once a year the effective interest rate for a year exceeds the per annum interest rate. Suppose you invest Rs.10000 for a year at the rate of 6% per annum compounded semi annually. Effective interest rate for a year will be more than 6% per annum since interest is being compounded more than once in a year. For computing effective rate of interest first we have to compute the interest. Let us compute the interest.

$$\begin{aligned} \text{Interest for first six months} &= \text{Rs. } 10000 \times 6/100 \times 6/12 \\ &= \text{Rs. } 300 \end{aligned}$$

$$\begin{aligned} \text{Principal for calculation of interest for next six months} \\ &= \text{Principal for first period one} + \text{Interest for first period} \\ &= \text{Rs. } (10000 + 300) \\ &= \text{Rs. } 10300 \end{aligned}$$

$$\text{Interest for next six months} = \text{Rs. } 10300 \times 6/100 \times 6/12 = \text{Rs. } 309$$

$$\begin{aligned} \text{Total interest earned during the current year} \\ &= \text{Interest for first six months} + \text{Interest for next six months} \\ &= \text{Rs. } (300 + 309) = \text{Rs. } 609 \end{aligned}$$

Interest of Rs. 609 can also be computed directly from the formula of compound interest.

We can compute effective rate of interest by following formula

$$I = PEt$$

Where I = Amount of interest

E = Effective rate of interest in decimal

t = Time period

P = Principal amount

Putting the values we have

$$\begin{aligned} 609 &= 10000 \times E \times 1 \\ \Rightarrow E &= \frac{609}{10000} \\ &= 0.0609 \text{ or} \\ &= 6.09\% \end{aligned}$$



Thus if we compound the interest more than once a year effective interest rate for the year will be more than actual interest rate per annum. But if interest is compounded annually effective interest rate for the year will be equal to actual interest rate per annum.

So effective interest rate can be defined as the equivalent annual rate of interest compounded annually if interest is compounded more than once a year.

The effective interest rate can be computed directly by following formula:

$$E = (1 + i)^n - 1$$

Where E is the effective interest rate

i = actual interest rate in decimal

n = number of conversion period

**Example 22:** Rs. 5000 is invested in a Term Deposit Scheme that fetches interest 6% per annum compounded quarterly. What will be the interest after one year? What is effective rate of interest?

**Solution:** We know that

$$I = P [(1+i)^n - 1]$$

Here P = Rs. 5000

i = 6% p.a. = 0.06 p.a. or 0.015 per quarter

n = 4

and I = amount of compound interest

putting the values we have

$$\begin{aligned} I &= \text{Rs. } 5000 [(1+0.015)^4 - 1] \\ &= \text{Rs. } 5000 \times 0.06136355 \\ &= \text{Rs. } 306.82 \end{aligned}$$

For effective rate of interest using  $I = PEt$  we find

$$306.82 = 5000 \times E \times 1.$$

$$\begin{aligned} \text{➤ } E &= \frac{306.82}{5000} \\ &= 0.0613 \text{ or } 6.13\% \end{aligned}$$

**Note:** We may arrive at the same result by using

$$\begin{aligned} E &= (1+i)^n - 1 \\ \text{➤ } E &= (1 + 0.015)^4 - 1 \\ &= 1.0613 - 1 \\ &= .0613 \text{ or } 6.13\% \end{aligned}$$

We may also note that effective rate of interest is not related to the amount of principal. It is related to the interest rate and frequency of compounding the interest.



**Example 23:** Find the amount of compound interest and effective rate of interest if an amount of Rs.20000 is deposited in a bank for one year at the rate of 8% per annum compounded semi annually.

**Solution:** We know that

$$I = P [(1+i)^n - 1]$$

here  $P = \text{Rs. } 20000$

$$i = 8\% \text{ p.a.} = 8/2 \% \text{ semi annually} = 0.04$$

$$n = 2$$

$$I = \text{Rs. } 20000 [(1+0.04)^2 - 1]$$

$$= \text{Rs. } 20000 \times 0.0816$$

$$= \text{Rs. } 1632$$

**Effective rate of interest:**

We know that

$$I = PEt$$

$$\text{➤ } 1632 = 20000 \times E \times 1$$

$$\text{➤ } E = \frac{1632}{20000} = 0.0816$$

$$= 8.16\%$$

Effective rate of interest can also be computed by following formula

$$E = (1 + i)^n - 1$$

$$= (1 + 0.04)^2 - 1$$

$$= 0.0816 \quad \text{or } 8.16\%$$

**Example 24:** Which is a better investment 3% per year compounded monthly or 3.2% per year simple interest? Given that  $(1+0.0025)^{12} = 1.0304$ .

**Solution:**  $i = 3/12 = 0.25\% = 0.0025$

$$n = 12$$

$$E = (1 + i)^n - 1$$

$$= (1 + 0.0025)^{12} - 1$$

$$= 1.0304 - 1 = 0.0304$$

$$= 3.04\%$$

Effective rate of interest (E) being less than 3.2%, the simple interest 3.2% per year is the better investment.



**Exercise 4 (B)**

**Choose the most appropriate option (a) (b) (c) (d)**

1. If  $P = \text{Rs. } 1000$ ,  $R = 5\% \text{ p.a.}$ ,  $n = 4$ ; What is Amount and C.I. is  
(a) Rs. 1215.50, Rs. 215.50                      (b) Rs. 1125, Rs. 125  
(c) Rs. 2115, Rs. 115                              (d) none of these
2. Rs. 100 will become after 20 years at 5% p.a compound interest amount  
(a) Rs. 250              (b) Rs. 205              (c) Rs. 265.50              (d) none of these
3. The effective rate of interest corresponding to a nominal rate 3% p.a payable half yearly is  
(a) 3.2% p.a              (b) 3.25% p.a              (c) 3.0225% p.a              (d) none of these
4. A machine is depreciated at the rate of 20% on reducing balance. The original cost of the machine was Rs. 100000 and its ultimate scrap value was Rs. 30000. The effective life of the machine is  
(a) 4.5 years (appx.)                              (b) 5.4 years (appx.)  
(c) 5 years (appx.)                                  (d) none of these
5. If  $A = \text{Rs. } 1000$ ,  $n = 2 \text{ years}$ ,  $R = 6\% \text{ p.a}$  compound interest payable half-yearly, then principal ( P ) is  
(a) Rs. 888.80              (b) Rs. 885              (c) 800                              (d) none of these
6. The population of a town increases every year by 2% of the population at the beginning of that year. The number of years by which the total increase of population be 40% is  
(a) 7 years                      (b) 10 years                      (c) 17 years (app)              (d) none of these
7. The difference between C.I and S.I on a certain sum of money invested for 3 years at 6% p.a is Rs. 110.16. The sum is  
(a) Rs. 3000              (b) Rs. 3700                      (c) Rs. 12000                      (d) Rs. 10000
8. The useful life of a machine is estimated to be 10 years and cost Rs. 10000. Rate of depreciation is 10% p.a. The scrap value at the end of its life is  
(a) Rs. 3486              (b) Rs. 4383                      (c) Rs. 3400                      (d) none of these
9. The effective rate of interest corresponding a nominal rate of 7% p.a convertible quarterly is  
(a) 7%                              (b) 7.5%                              (c) 5%                              (d) 7.18%
10. The C.I on Rs. 16000 for 1 ½ years at 10% p.a payable half -yearly is  
(a) Rs. 2222                      (b) Rs. 2522                      (c) Rs. 2500                      (d) none of these
11. The C.I on Rs. 40000 at 10% p.a for 1 year when the interest is payable quarterly is  
(a) Rs. 4000                      (b) Rs. 4100                      (c) Rs. 4152.51                      (d) none of these



12. The difference between the S.I and the C.I on Rs. 2400 for 2 years at 5% p.a is  
(a) Rs. 5                      (b) Rs. 10                      (c) Rs. 16                      (d) Rs. 6
13. The annual birth and death rates per 1000 are 39.4 and 19.4 respectively. The number of years in which the population will be doubled assuming there is no immigration or emigration is  
(a) 35 yrs.                      (b) 30 yrs.                      (c) 25 yrs                      (d) none of these
14. The C.I on Rs. 4000 for 6 months at 12% p.a payable quarterly is  
(a) Rs. 243.60                      (b) Rs. 240                      (c) 243                      (d) none of these

## 4.6 ANNUITY

In many cases you must have noted that your parents have to pay an equal amount of money regularly like every month or every year. For example payment of life insurance premium, rent of your house (if you stay in a rented house), payment of housing loan, vehicle loan etc. In all these cases they pay a constant amount of money regularly. Time period between two consecutive payments may be one month, one quarter or one year.

Sometimes some people received a fixed amount of money regularly like pension rent of house etc. In all these cases annuity comes into the picture. When we pay (or receive) a fixed amount of money periodically over a specified time period we create an annuity.

Thus annuity can be defined as a sequence of periodic payments (or receipts) regularly over a specified period of time.

There is a special kind of annuity also that is called Perpetuity. It is one where the receipt or payment takes place forever. Since the payment is forever we cannot compute a future value of perpetuity. However we can compute the present value of the perpetuity. We will discuss later about future value and present value of annuity.

To be called annuity a series of payments (or receipts) must have following features:

- (1) Amount paid (or received) must be constant over the period of annuity and
- (2) Time interval between two consecutive payments (or receipts) must be the same.

Consider following tables. Can payments/receipts shown in the table for five years be called annuity?

TABLE- 4.1		TABLE- 4.2	
Year end	Payments/Receipts(Rs.)	Year end	Payments/Receipts (Rs.)
I	5000	I	5000
II	6000	II	5000
III	4000	III	–
IV	5000	IV	5000
V	7000	V	5000



TABLE- 4.3

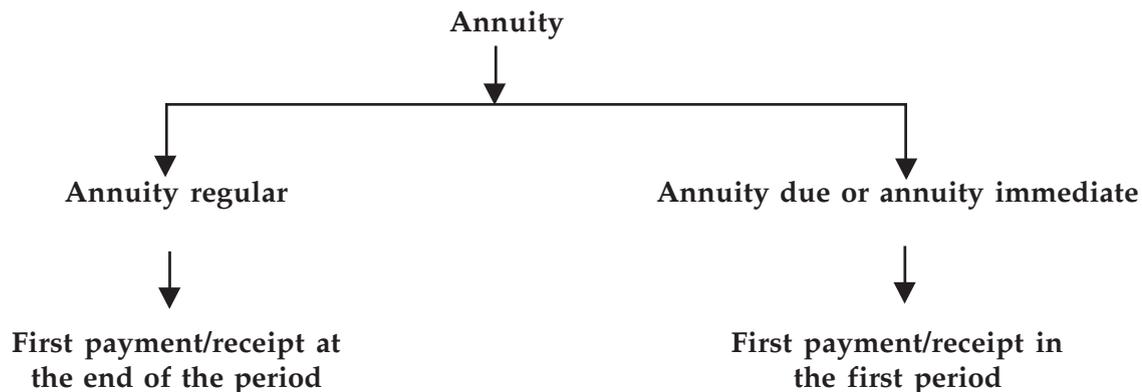
Year end	Payments/Receipts(Rs.)
I	5000
II	5000
III	5000
IV	5000
V	5000

Payments/Receipts shown in table 4.1 cannot be called annuity. Payments/Receipts though have been made at regular intervals but amount paid are not constant over the period of five years.

Payments/receipts shown in table 4.2 cannot also be called annuity. Though amounts paid/received are same in every year but time interval between different payments/receipts is not equal. You may note that time interval between second and third payment/receipt is two year and time interval between other consecutive payments/receipts (first and second third and fourth and fourth and fifth) is only one year. You may also note that for first two year the payments/receipts can be called annuity.

Now consider table 4.3. You may note that all payments/receipts over the period of 5 years are constant and time interval between two consecutive payments/receipts is also same i.e. one year. Therefore payments/receipts as shown in table-4.3 can be called annuity.

#### 4.6.1 Annuity regular and Annuity due/immediate



Annuity may be of two types:

- (1) **Annuity regular:** In annuity regular first payment/receipt takes place at the end of first period. Consider following table:



**TABLE- 4.4**

Year end	Payments/Receipts(Rs.)
I	5000
II	5000
III	5000
IV	5000
V	5000

We can see that first payment/receipts takes place at the end of first year therefore it is an annuity regular.

- (2) **Annuity Due or Annuity Immediate:** When the first receipt or payment is made today (at the beginning of the annuity) it is called annuity due or annuity immediate. Consider following table:

**TABLE- 4.5**

In the beginning of	Payment/Receipt(Rs.)
I year	5000
II year	5000
III year	5000
IV year	5000
V year	5000

We can see that first receipt or payment is made in the beginning of the first year. This type of annuity is called annuity due or annuity immediate.

## 4.7 FUTURE VALUE

Future value is the cash value of an investment at some time in the future. It is tomorrow's value of today's money compounded at the rate of interest. Suppose you invest Rs.1000 in a fixed deposit that pays you 7% per annum as interest. At the end of first year you will have Rs.1070. This consist of the original principal of Rs.1000 and the interest earned of Rs.70. Rs.1070 is the future value of Rs.1000 invested for one year at 7%. We can say that Rs.1000 today is worth Rs.1070 in one year's time if the interest rate is 7%.

Now suppose you invested Rs.1000 for two years. How much would you have at the end of the second year. You had Rs.1070 at the end of the first year. If you reinvest it you end up having  $\text{Rs.}1070(1+0.07)=\text{Rs.}1144.90$  at the end of the second year. Thus Rs.1144.90 is the future value of Rs.1000 invested for two years at 7%. We can compute the future value of a single cash flow by applying the formula of compound interest.



We know that

$$A_n = P(1+i)^n$$

Where A = Accumulated amount

n = number of conversion period

i = rate of interest per conversion period in decimal

P = principal

Future value of a single cash flow can be computed by above formula. Replace A by future value (F) and P by single cash flow (C.F.) therefore

$$F = \text{C.F.} (1 + i)^n$$

**Example 25:** You invest Rs. 3000 in a two year investment that pays you 12% per annum. Calculate the future value of the investment.

**Solution:** We know

$$F = \text{C.F.} (1 + i)^n$$

where F = Future value

$$\text{C.F.} = \text{Cash flow} = \text{Rs.}3000$$

$$i = \text{rate of interest} = 0.12$$

$$n = \text{time period} = 2$$

$$F = \text{Rs. } 3000(1+0.12)^2$$

$$= \text{Rs. } 3000 \times 1.2544$$

$$= \text{Rs. } 3763.20$$

**4.7.1 Future value of an annuity regular :** Now we can discuss how do we calculate future value of an annuity.

Suppose a constant sum of Re. 1 is deposited in a savings account at the end of each year for four years at 6% interest. This implies that Re.1 deposited at the end of the first year will grow for three years, Re. 1 at the end of second year for 2 years, Re.1 at the end of the third year for one year and Re.1 at the end of the fourth year will not yield any interest. Using the concept of compound interest we can compute the future value of annuity. The compound value (compound amount) of Re.1 deposited in the first year will be

$$A_3 = \text{Rs. } 1 (1 + 0.06)^3$$

$$= \text{Rs. } 1.191$$

The compound value of Re.1 deposited in the second year will be

$$A_2 = \text{Rs. } 1 (1 + 0.06)^2$$

$$= \text{Rs. } 1.124$$



The compound value of Re.1 deposited in the third year will be

$$A_1 = \text{Rs. } 1 (1 + 0.06)^1$$

$$= \text{Rs. } 1.06$$

and the compound value of Re. 1 deposited at the end of fourth year will remain Re. 1.

The aggregate compound value of Re. 1 deposited at the end of each year for four years would be:

$$\text{Rs. } (1.191 + 1.124 + 1.060 + 1.00) = \text{Rs. } 4.375$$

This is the compound value of an annuity of Re.1 for four years at 6% rate of interest.

The above computation is summarized in the following table:

Table 4.6		
End of year	Amount Deposit (Re.)	Future value at the end of fourth year(Re.)
0	–	–
1	1	$1 (1 + 0.06)^3 = 1.191$
2	1	$1 (1 + 0.06)^2 = 1.124$
3	1	$1 (1 + 0.06)^1 = 1.060$
4	1	$1 (1 + 0.06)^0 = 1$
	Future Value	4.375

The computation shown in the table can be expressed as follows:

$$A (4, i) = A (1 + i)^0 + A (1 + i) + A(1 + i)^2 + A (1 + i)^3$$

$$\text{i.e. } A (4, i) = A [1+(1+i) + (1+i)^2 + (1 + i)^3]$$

In above equation A is annuity, A (4, i) is future value at the end of year four, i is the rate of interest shown in decimal.

We can extend above equation for n periods and rewrite as follows:

$$A (n, i) = A (1 + i)^0 + A (1 + i)^1 + \dots + A (1 + i)^{n-2} + A (1 + i)^{n-1}$$

$$\text{Here } A = \text{Re.1}$$

Therefore

$$A (n, i) = 1 (1 + i)^0 + 1 (1 + i)^1 + \dots + 1 (1 + i)^{n-2} + 1 (1 + i)^{n-1}$$

$$= 1 + (1 + i)^1 + \dots + (1 + i)^{n-2} + (1 + i)^{n-1}$$

[a geometric series with first term 1 and common ratio (1 + i)]

$$= \frac{1 \cdot [1 - (1+i)^n]}{1 - (1+i)}$$



$$\begin{aligned} &= \frac{1-(1+i)^n}{-i} \\ &= \frac{(1+i)^n - 1}{i} \end{aligned}$$

If A be the periodic payments, the future value A(n, i) of the annuity is given by

$$A(n, i) = A \left[ \frac{(1+i)^n - 1}{i} \right]$$

**Example 26:** Find the future value of an annuity of Rs.500 made annually for 7 years at interest rate of 14% compounded annually. Given that  $(1.14)^7 = 2.5023$ .

**Solution:** Here annual payment  $A = \text{Rs.}500$   
 $n = 7$   
 $i = 14\% = 0.14$

Future value of the annuity

$$\begin{aligned} A(7, 0.14) &= 500 \left[ \frac{(1+0.14)^7 - 1}{(0.14)} \right] \\ &= \frac{500 \times (2.5023 - 1)}{0.14} \\ &= \text{Rs. } 5365.35 \end{aligned}$$

**Example 27:** Rs. 200 is invested at the end of each month in an account paying interest 6% per year compounded monthly. What is the future value of this annuity after 10<sup>th</sup> payment? Given that  $(1.005)^{10} = 1.0511$

**Solution:** Here  $A = \text{Rs.}200$   
 $n = 10$   
 $i = 6\% \text{ per annum} = 6/12 \% \text{ per month} = 0.005$

Future value of annuity after 10 months is given by

$$\begin{aligned} A(n, i) &= A \left[ \frac{(1+i)^n - 1}{i} \right] \\ A(10, 0.005) &= 200 \left[ \frac{(1+0.005)^{10} - 1}{0.005} \right] \\ &= 200 \left[ \frac{1.0511 - 1}{0.005} \right] \end{aligned}$$



$$\begin{aligned} &= 200 \times 10.22 \\ &= \text{Rs. } 2044 \end{aligned}$$

**4.7.2 Future value of Annuity due or Annuity Immediate:** As we know that in Annuity due or Annuity immediate first receipt or payment is made today. Annuity regular assumes that the first receipt or the first payment is made at the end of first period. The relationship between the value of an annuity due and an ordinary annuity in case of future value is:

Future value of an Annuity due/Annuity immediate = Future value of annuity regular  $\times (1+i)$  where  $i$  is the interest rate in decimal.

Calculating the future value of the annuity due involves two steps.

**Step-1** Calculate the future value as though it is an ordinary annuity.

**Step-2** Multiply the result by  $(1+i)$

**Example 28:** Z invests Rs. 10000 every year starting from today for next 10 years. Suppose interest rate is 8% per annum compounded annually. Calculate future value of the annuity. Given that  $(1 + 0.08)^{10} = 2.15892500$ .

**Solution:** Step-1: Calculate future value as though it is an ordinary annuity.

Future value of the annuity as if it is an ordinary annuity

$$\begin{aligned} &= \text{Rs. } 10000 \left[ \frac{(1+0.08)^{10} - 1}{0.08} \right] \\ &= \text{Rs. } 10000 \times 14.4865625 \\ &= \text{Rs. } 144865.625 \end{aligned}$$

Step-2: Multiply the result by  $(1+i)$

$$\begin{aligned} &= \text{Rs. } 144865.625 \times (1+0.08) \\ &= \text{Rs. } 156454.875 \end{aligned}$$

## 4.8 PRESENT VALUE

We have read that future value is tomorrow's value of today's money compounded at some interest rate. We can say present value is today's value of tomorrow's money discounted at the interest rate. Future value and present value are related to each other in fact they are the reciprocal of each other. Let's go back to our fixed deposit example. You invested Rs. 1000 at 7% and get Rs. 1070 at the end of the year. If Rs. 1070 is the future value of today's Rs. 1000 at 7% then Rs. 1000 is present value of tomorrow's Rs. 1070 at 7%. We have also seen that if we invest Rs. 1000 for two years at 7% per annum we will get Rs. 1144.90 after two years. It means Rs. 1144.90 is the future value of today's Rs. 1000 at 7% and Rs. 1000 is the present value of Rs. 1144.90 where time period is two years and rate of interest is 7% per annum. We can get the present value of a cash flow (inflow or outflow) by applying compound interest formula.



The present value P of the amount  $A_n$  due at the end of n period at the rate of i per interest period may be obtained by solving for P the below given equation

$$A_n = P(1 + i)^n$$

i.e.  $P = \frac{A_n}{(1+i)^n}$

Computation of P may be simple if we make use of either the calculator or the present value

table showing values of  $\frac{1}{(1+i)^n}$  for various time periods/per annum interest rates. For positive

i the factor  $\frac{1}{(1+i)^n}$  is always less than 1 indicating thereby future amount has smaller present value.

**Example 29:** What is the present value of Re.1 to be received after two years compounded annually at 10% interest rate?

**Solution:** Here  $A_n = \text{Re.1}$   
 $i = 10\% = 0.1$   
 $n = 2$

$$\begin{aligned} \text{Required present value} &= \frac{A_n}{(1+i)^n} \\ &= \frac{1}{(1+0.1)^2} \\ &= \frac{1}{1.21} = 0.8264 \\ &= \text{Re. } 0.83 \end{aligned}$$

Thus Re. 0.83 shall grow to Re. 1 after 2 years at 10% interest rate compounded annually.

**Example 30:** Find the present value of Rs. 10000 to be required after 5 years if the interest rate be 9%. Given that  $(1.09)^5 = 1.5386$ .

**Solution:** Here  $i = 0.09 = 9\%$   
 $n = 5$   
 $A_n = 10000$



$$\begin{aligned} \text{Required present value} &= \frac{A_n}{(1+i)^n} \\ &= \frac{10000}{(1+0.09)^5} \\ &= \frac{10000}{1.5386} = \text{Rs. } 6499.42 \end{aligned}$$

**4.8.1 Present value of an Annuity regular:** We have seen how compound interest technique can be used for computing the future value of an Annuity. We will now see how we compute present value of an annuity. We take an example, Suppose your mom promise you to give you Rs.1000 on every 31<sup>st</sup> December for the next five years. Suppose today is 1<sup>st</sup> January. How much money will you have after five years from now if you invest this gift of the next five years at 10%? For getting answer we will have to compute future value of this annuity.

But you don't want Rs. 1000 to be given to you each year. You instead want a lump sum figure today. Will you get Rs. 5000. The answer is no. The amount that she will give you today will be less than Rs. 5000. For getting the answer we will have to compute the present value of this annuity. For getting present value of this annuity we will compute the present value of these amounts and then aggregate them. Consider following table:

Table 4.7		
Year End	Gift Amount (Rs.)	Present Value $[A_n / (1 + i)^n]$
I	1000	$1000 / (1 + 0.1) = 909.091$
II	1000	$1000 / (1 + 0.1) = 826.446$
III	1000	$1000 / (1 + 0.1) = 751.315$
IV	1000	$1000 / (1 + 0.1) = 683.013$
V	1000	$1000 / (1 + 0.1) = \underline{620.921}$
<b>Present Value</b>		<b>= <u>3790.86</u></b>

Thus the present value of annuity of Rs. 1000 for 5 years at 10% is Rs. 3790.79

It means if you want lump sum payment today instead of Rs.1000 every year you will get Rs. 3790.79.

The above computation can be written in formula form as below.

The present value (V) of an annuity (A) is the sum of the present values of the payments.

$$\therefore V = \frac{A}{(1+i)^1} + \frac{A}{(1+i)^2} + \frac{A}{(1+i)^3} + \frac{A}{(1+i)^4} + \frac{A}{(1+i)^5}$$

We can extend above equation for n periods and rewrite as follows:

$$V = \frac{A}{(1+i)^1} + \frac{A}{(1+i)^2} + \dots + \frac{A}{(1+i)^{n-1}} + \frac{A}{(1+i)^n} \dots \dots \dots (1)$$



multiplying throughout by  $\frac{1}{(1+i)}$  we get

$$\frac{V}{(1+i)} = \frac{A}{(1+i)^2} + \frac{A}{(1+i)^3} + \dots + \frac{A}{(1+i)^n} + \frac{A}{(1+i)^{n+1}} \dots \dots \dots (2)$$

subtracting (2) from (1) we get

$$V - \frac{V}{(1+i)} = \frac{A}{(1+i)^1} - \frac{A}{(1+i)^{n+1}}$$

Or  $V(1+i) - V = A - \frac{A}{(1+i)^n}$

Or  $Vi = A \left[ 1 - \frac{1}{(1+i)^n} \right]$

$$\therefore V = A \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right] = A.P(n, i)$$

Where,  $P(n, i) = \frac{(1+i)^n - 1}{i(1+i)^n}$

Consequently  $A = \frac{V}{P(n, i)}$  which is useful in problems of amortization.

A loan with fixed rate of interest is said to be amortized if entire principal and interest are paid over equal periods of time by way of sequence of equal payment.

$A = \frac{V}{P(n, i)}$  can be used to compute the amount of annuity if we have present value (V), n the number of time period and the rate of interest in decimal.

Suppose your dad purchases a car for Rs. 550000. He gets a loan of Rs. 500000 at 15% p.a. from a Bank and balance 50000 he pays at the time of purchase. Your dad has to pay whole amount of loan in 12 equal monthly instalments with interest starting from the end of first month.

Now we have to calculate how much money has to be paid at the end of every month. We can compute equal instalment by following formula

$$A = \frac{V}{P(n, i)}$$

Here  $V = \text{Rs. } 500000$

$n = 12$



$$i = \frac{0.15}{12} = 0.0125$$

$$P(n, i) = \frac{(1+i)^n - 1}{i(1+i)^n}$$

$$\begin{aligned} P(12, 0.0125) &= \frac{(1+0.0125)^{12} - 1}{0.0125(1+0.0125)^{12}} \\ &= \frac{1.16075452 - 1}{0.0125 \times 1.16075452} \\ &= \frac{0.16075452}{0.01450943} = 11.079 \end{aligned}$$

$$\therefore A = \frac{500000}{11.079} = \text{Rs. } 45130.43$$

Therefore your dad will have to pay 12 monthly instalments of Rs. 45130.43.

**Example 31:** S borrows Rs. 500000 to buy a house. If he pays equal instalments for 20 years and 10% interest on outstanding balance what will be the equal annual instalment?

**Solution:** We know

$$A = \frac{V}{P(n, i)}$$

Here  $V = \text{Rs. } 500000$

$n = 20$

$i = 10\% \text{ p.a.} = 0.10$

$$\begin{aligned} \therefore A &= \frac{V}{P(n, i)} = \text{Rs. } \frac{500000}{P(20, 0.10)} \\ &= \text{Rs. } \frac{500000}{8.51356} [P(20, 0.10) = 8.51356 \text{ from table 2(a)}] \\ &= \text{Rs. } 58729.84 \end{aligned}$$

**Example 32:** Rs. 5000 is paid every year for ten years to pay off a loan. What is the loan amount if interest rate be 14% per annum compounded annually?

**Solution:**  $V = A.P.(n, i)$

Here  $A = \text{Rs. } 5000$

$n = 10$



$$\begin{aligned}i &= 0.14 \\V &= 5000 \times P(10, 0.14) \\&= 5000 \times 5.21611 = \text{Rs. } 26080.55\end{aligned}$$

Therefore the loan amount is Rs. 26080.55

**Note:** Value of  $P(10, 0.14)$  can be seen from table 2(a) or it can be computed by formula derived in preceding paragraph.

**Example 33:** Y bought a TV costing Rs. 13000 by making a down payment of Rs. 3000 and agreeing to make equal annual payment for four years. How much would be each payment if the interest on unpaid amount be 14% compounded annually?

**Solution:** In the present case we have present value of the annuity i.e. Rs. 10000 (13000-3000) and we have to calculate equal annual payment over the period of four years.

We know that

$$\begin{aligned}V &= A.P(n, i) \\ \text{Here } n &= 4 \text{ and } i = 0.14 \\ A &= \frac{V}{P(n, i)} \\ &= \frac{10000}{P(4, 0.14)} \\ &= \frac{10000}{2.91371} \text{ [from table 2(a)]} \\ &= \text{Rs. } 3432.05\end{aligned}$$

Therefore each payment would be Rs. 3432.05

**4.8.2 Present value of annuity due or annuity immediate:** Present value of annuity due/immediate for  $n$  years is the same as an annuity regular for  $(n-1)$  years plus an initial receipt or payment in beginning of the period. Calculating the present value of annuity due involves two steps.

**Step 1:** Compute the present value of annuity as if it were a annuity regular for one period short.

**Step 2:** Add initial cash payment/receipt to the step 1 value.

**Example 34:** Suppose your mom decides to gift you Rs. 10000 every year starting from today for the next five years. You deposit this amount in a bank as and when you receive and get 10% per annum interest rate compounded annually. What is the present value of this annuity?

**Solution:** It is an annuity immediate. For calculating value of the annuity immediate following steps will be followed:



**Step 1:** Present value of the annuity as if it were a regular annuity for one year less i.e. for four years

$$\begin{aligned} &= \text{Rs. } 10000 \times P(4, 0.10) \\ &= \text{Rs. } 10000 \times 3.16987 \\ &= \text{Rs. } 31698.70 \end{aligned}$$

**Step 2:** Add initial cash deposit to the step 1 value

$$\text{Rs. } (31698.70 + 10000) = \text{Rs. } 41698.70$$

## 4.9 SINKING FUND

It is the fund credited for a specified purpose by way of sequence of periodic payments over a time period at a specified interest rate. Interest is compounded at the end of every period. Size of the sinking fund deposit is computed from  $A = P.A(n, i)$  where  $A$  is the amount to be saved,  $P$  the periodic payment,  $n$  the payment period.

**Example 35:** How much amount is required to be invested every year so as to accumulate Rs. 300000 at the end of 10 years if interest is compounded annually at 10%?

**Solution:** Here  $A = 300000$

$$n = 10$$

$$i = 0.1$$

Since  $A = P.A(n, i)$

$$\begin{aligned} 300000 &= P.A(10, 0.1) \\ &= P \times 15.9374248 \end{aligned}$$

$$\therefore P = \frac{300000}{15.9374248} = \text{Rs. } 18823.62$$

This value can also be calculated by the formula of future value of annuity regular.

We know that

$$\begin{aligned} A(n, i) &= A \left[ \frac{(1+i)^n - 1}{i} \right] \\ 300000 &= A \left[ \frac{(1+0.1)^{10} - 1}{0.1} \right] \\ 300000 &= A \times 15.9374248 \\ A &= \frac{300000}{15.9374248} \\ &= \text{Rs. } 18823.62 \end{aligned}$$



## 4.10 APPLICATIONS

**4.10.1 Leasing:** Leasing is a financial arrangement under which the owner of the asset (lessor) allows the user of the asset (lessee) to use the asset for a defined period of time (lease period) for a consideration (lease rental) payable over a given period of time. This is a kind of taking an asset on rent. How can we decide whether a lease agreement is favourable to lessor or lessee, it can be seen by following example.

**Example 36:** ABC Ltd. wants to lease out an asset costing Rs. 360000 for a five year period. It has fixed a rental of Rs. 105000 per annum payable annually starting from the end of first year. Suppose rate of interest is 14% per annum compounded annually on which money can be invested by the company. Is this agreement favourable to the company?

**Solution:** First we have to compute the present value of the annuity of Rs. 105000 for five years at the interest rate of 14% p.a. compounded annually.

The present value  $V$  of the annuity is given by

$$\begin{aligned}V &= A.P (n, i) \\ &= 105000 \times P(5, 0.14) \\ &= 105000 \times 3.43308 = \text{Rs. } 360473.40\end{aligned}$$

which is greater than the initial cost of the asset and consequently leasing is favourable to the lessor.

**Example 37:** A company is considering proposal of purchasing a machine either by making full payment of Rs.4000 or by leasing it for four years at an annual rate of Rs.1250. Which course of action is preferable if the company can borrow money at 14% compounded annually?

**Solution:** The present value  $V$  of annuity is given by

$$\begin{aligned}V &= A.P (n, i) \\ &= 1250 \times P (4, 0.14) \\ &= 1250 \times 2.91371 = \text{Rs.}3642.11\end{aligned}$$

which is less than the purchase price and consequently leasing is preferable.

**4.10.2 Capital Expenditure (investment decision):** Capital expenditure means purchasing an asset (which results in outflows of money) today in anticipation of benefits (cash inflow) which would flow across the life of the investment. For taking investment decision we compare the present value of cash outflow and present value of cash inflows. If present value of cash inflows is greater than present value of cash outflows decision should be in the favour of investment. Let us see how do we take capital expenditure (investment) decision.

**Example 38:** A machine can be purchased for Rs.50000. Machine will contribute Rs.12000 per year for the next five years. Assume borrowing cost is 10% per annum compounded annually. Determine whether machine should be purchased or not.

**Solution:** The present value of annual contribution

$$V = A.P(n, i)$$



$$\begin{aligned} &= 12000 \times P(5, 0.10) \\ &= 12000 \times 3.79079 \\ &= \text{Rs. } 45489.48 \end{aligned}$$

which is less than the initial cost of the machine. Therefore machine must not be purchased.

**Example 39:** A machine with useful life of seven years costs Rs. 10000 while another machine with useful life of five years costs Rs. 8000. The first machine saves labour expenses of Rs. 1900 annually and the second one saves labour expenses of Rs. 2200 annually. Determine the preferred course of action. Assume cost of borrowing as 10% compounded per annum.

**Solution:** The present value of annual cost savings for the first machine

$$\begin{aligned} &= \text{Rs. } 1900 \times P(7, 0.10) \\ &= \text{Rs. } 1900 \times 4.86842 \\ &= \text{Rs. } 9249.99 \\ &= \text{Rs. } 9250 \end{aligned}$$

Cost of machine being Rs. 10000 it costs more by Rs. 750 than it saves in terms of labour cost.

The present value of annual cost savings of the second machine

$$\begin{aligned} &= \text{Rs. } 2200 \times P(5, 0.10) \\ &= \text{Rs. } 2200 \times 3.79079 \\ &= \text{Rs. } 8339.74 \end{aligned}$$

Cost of the second machine being Rs. 8000 effective savings in labour cost is Rs. 339.74. Hence the second machine is preferable.

**4.10.3 Valuation of Bond:** A bond is a debt security in which the issuer owes the holder a debt and is obliged to repay the principal and interest. Bonds are generally issued for a fixed term longer than one year.

**Example 40:** An investor intends purchasing a three year Rs. 1000 par value bond having nominal interest rate of 10%. At what price the bond may be purchased now if it matures at par and the investor requires a rate of return of 14%?

**Solution:** Present value of the bond

$$\begin{aligned} &= \frac{100}{(1+0.14)^1} + \frac{100}{(1+0.14)^2} + \frac{100}{(1+0.14)^3} + \frac{1000}{(1+0.14)^3} \\ &= 100 \times 0.87719 + 100 \times 0.769467 + 100 \times 0.674972 + 1000 \times 0.674972 \\ &= 87.719 + 76.947 + 67.497 + 674.972 \\ &= 907.125 \end{aligned}$$

Thus the purchase value of the bond is Rs.907.125

**Exercise 4 (C)****Choose the most appropriate option (a) (b) (c) (d)**

1. The present value of an annuity of Rs. 3000 for 15 years at 4.5% p.a CI is  
(a) Rs. 23809.41 (b) Rs. 32218.63 (c) Rs. 32908.41 (d) none of these
2. The amount of an annuity certain of Rs. 150 for 12 years at 3.5% p.a C.I is  
(a) Rs. 2190.28 (b) Rs. 1290.28 (c) Rs. 2180.28 (d) none of these
3. A loan of Rs. 10,000 is to be paid back in 30 equal instalments. The amount of each installment to cover the principal and at 4% p.a CI is  
(a) Rs. 587.87 (b) Rs. 587 (c) Rs. 578.87 (d) none of these
4.  $A = \text{Rs. } 1200$   $n = 12$  yrs  $i = 0.08$   $v = ?$

Using the formula  $V = \frac{A}{i} \left[ 1 - \frac{1}{(1+i)^n} \right]$  value of  $v$  will be

- (a) Rs. 3039 (b) Rs. 3990 (c) Rs. 9930 (d) none of these
5.  $a = \text{Rs. } 100$   $n = 10$ ,  $i = 5\%$  find the FV of annuity  
Using the formula  $FV = a / \{1 + i\}^n - 1$ , FV is equal to  
(a) Rs. 1258 (b) Rs. 2581 (c) Rs. 1528 (d) none of these
6. If the amount of an annuity after 25 years at 5% p.a C.I is Rs. 50000 the annuity will be  
(a) Rs. 1406.90 (b) Rs. 1046.90 (c) Rs. 1146.90 (d) none of these
7. Given annuity of Rs. 100 amounts to Rs. 3137.12 at 4.5% p.a C. I. The number of years will be  
(a) 25yrs. (appx.) (b) 20 yrs. (appx.) (c) 22 yrs. (d) none of these
8. A company borrows Rs. 10000 on condition to repay it with compound interest at 5% p.a by annual installments of Rs. 1000 each. The number of years by which the debt will be clear is  
(a) 14.2 yrs. (b) 10 yrs. (c) 12 yrs. (d) none of these
9. Mr. X borrowed Rs. 5120 at  $12 \frac{1}{2} \%$  p.a C.I. At the end of 3 yrs, the money was repaid along with the interest accrued. The amount of interest paid by him is  
(a) Rs. 2100 (b) Rs. 2170 (c) Rs. 2000 (d) none of these
10. Mr. Paul borrows Rs. 20000 on condition to repay it with C.I. at 5% p.a in annual installments of Rs. 2000 each. The number of years for the debt to be paid off is  
(a) 10 yrs. (b) 12 yrs. (c) 11 yrs. (d) none of these
11. A person invests Rs. 500 at the end of each year with a bank which pays interest at 10% p. a C.I. annually. The amount standing to his credit one year after he has made his yearly



investment for the 12<sup>th</sup> time is.

- (a) Rs. 11764.50 (b) Rs. 10000 (c) Rs. 12000 (d) none of these
12. The present value of annuity of Rs. 5000 per annum for 12 years at 4% p.a C.I. annually is  
(a) Rs. 46000 (b) Rs. 46850 (c) RS. 15000 (d) none of these
13. A person desires to create a fund to be invested at 10% CI per annum to provide for a prize of Rs. 300 every year. Using  $V = a/I$  find V and V will be  
(a) Rs. 2000 (b) 2500 (c) Rs. 3000 (d) none of these

## MISCELLANEOUS PROBLEMS

### Exercise 4 (D)

Choose the most appropriate option (a) (b) (c) (d)

1.  $A = \text{Rs. } 5200$ ,  $R = 5\%$  p.a.,  $T = 6$  years, P will be  
(a) Rs. 2000 (b) Rs. 3880 (c) Rs. 3000 (d) none of these
2. If  $P = 1000$ ,  $n = 4$  yrs.,  $R = 5\%$  p.a then C. I will be  
(a) Rs. 215.50 (b) Rs. 210 (c) Rs. 220 (d) none of these
3. The time in which a sum of money will be double at 5% p.a C.I is  
(a) Rs. 10 years (b) 12 yrs. (c) 14.2 years (d) none of these
4. If  $A = \text{Rs. } 10000$ ,  $n = 18$  yrs.,  $R = 4\%$  p.a C.I, P will be  
(a) Rs. 4000 (b) Rs. 4900 (c) Rs. 4500 (d) none of these
5. The time by which a sum of money would treble it self at 8% p. a C. I is  
(a) 14.28 yrs. (b) 14 yrs. (c) 12 yrs. (d) none of these
6. The present value of an annuity of Rs. 80 a years for 20 years at 5% p.a is  
(a) Rs. 997 (appx.) (b) Rs. 900 (c) Rs. 1000 (d) none of these
7. A person bought a house paying Rs. 20000 cash down and Rs. 4000 at the end of each year for 25 yrs. at 5% p.a. C.I. The cash down price is  
(a) Rs. 75000 (b) Rs. 76000 (c) Rs. 76392 (d) none of these.
8. A man purchased a house valued at Rs. 300000. He paid Rs. 200000 at the time of purchase and agreed to pay the balance with interest at 12% per annum compounded half yearly in 20 equal half yearly instalments. If the first instalment is paid after six months from the date of purchase then the amount of each instalment is  
[Given  $\log 10.6 = 1.0253$  and  $\log 31.19 = 1.494$ ]  
(a) Rs. 8719.66 (b) Rs. 8769.21 (c) Rs. 7893.13 (d) none of these.



**ANSWERS**

**Exercise 4(A)**

1. b	2. a	3. c	4. d	5. a	6. b	7. a	8. c
9. a	10. c						

**Exercise 4(B)**

1. a	2. c	3. c	4. b	5. a	6. c	7. d	8. a
9. d	10. b	11. c	12. d	13. a	14. a		

**Exercise 4(C)**

1. b	2. a	3. c	4. d	5. a	6. b	7. b	8. a
9. b	10. d	11. a	12. d	13. c			

**Exercise 4(D)**

1. b	2. a	3. c	4. d	5. a	6. a	7. c	8. a
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## ADDITIONAL QUESTION BANK

- The difference between compound and simple interest at 5% per annum for 4 years on Rs. 20000 is Rs. \_\_\_\_\_  
(A) 250 (B) 277 (C) 300 (D) 310
- The compound interest on half-yearly rests on Rs.10000 the rate for the first and second years being 6% and for the third year 9% p.a. is Rs.\_\_\_\_\_.  
(A) 2200 (B) 2287 (C) 2285 (D) None
- The present value of Rs.10000 due in 2 years at 5% p.a. compound interest when the interest is paid on yearly basis is Rs.\_\_\_\_\_.  
(A) 9070 (B) 9000 (C) 9061 (D) None
- The present value of Rs.10000 due in 2 years at 5% p.a. compound interest when the interest is paid on half-yearly basis is Rs.\_\_\_\_\_.  
(A) 9070 (B) 9069 (C) 9061 (D) None
- Johnson left Rs. 100000 with the direction that it should be divided in such a way that his minor sons Tom, Dick and Harry aged 9, 12 and 15 years should each receive equally after attaining the age 25 years. The rate of interest being 3.5%, how much each son receive after getting 25 years old?  
(A) 50000 (B) 51994 (C) 52000 (D) None
- In how many years will a sum of money double at 5% p.a. compound interest?  
(A) 15 years 3 months (B) 14 years 2 months  
(C) 14 years 3 months (D) 15 years 2 months
- In how many years a sum of money trebles at 5% p.a. compound interest payable on half-yearly basis?  
(A) 18 years 7 months (B) 18 years 6 months  
(C) 18 years 8 months (D) 22 years 3 months
- A machine depreciates at 10% of its value at the beginning of a year. The cost and scrap value realized at the time of sale being Rs. 23240 and Rs. 9000 respectively. For how many years the machine was put to use?  
(A) 7 years (B) 8 years (C) 9 years (D) 10 years
- A machine worth Rs. 490740 is depreciated at 15% on its opening value each year. When its value would reduce to Rs. 200000?  
(A) 4 years 6 months (B) 4 years 7 months  
(C) 4 years 5 months (D) 5 years 7 months approximately



## SIMPLE AND COMPOUND INTEREST INCLUDING ANNUITY- APPLICATIONS

10. A machine worth Rs. 490740 is depreciated at 15% of its opening value each year. When its value would reduce by 90%?
- (A) 11 years 6 months (B) 11 years 7 months  
(C) 11 years 8 months (D) 14 years 2 months approximately
11. Alibaba borrows Rs. 6 lakhs Housing Loan at 6% repayable in 20 annual installments commencing at the end of the first year. How much annual payment is necessary.
- (A) 52420 (B) 52419 (C) 52310 (D) 52320
12. A sinking fund is created for redeeming debentures worth Rs. 5 lakhs at the end of 25 years. How much provision needs to be made out of profits each year provided sinking fund investments can earn interest at 4% p.a.?
- (A) 12006 (B) 12040 (C) 12039 (D) 12035
13. A machine costs Rs. 520000 with an estimated life of 25 years. A sinking fund is created to replace it by a new model at 25% higher cost after 25 years with a scrap value realization of Rs. 25000. what amount should be set aside every year if the sinking fund investments accumulate at 3.5% compound interest p.a.?
- (A) 16000 (B) 16500 (C) 16050 (D) 16005
14. Raja aged 40 wishes his wife Rani to have Rs.40 lakhs at his death. If his expectation of life is another 30 years and he starts making equal annual investments commencing now at 3% compound interest p.a. how much should he invest annually?
- (A) 84448 (B) 84450 (C) 84449 (D) 84077
15. Appu retires at 60 years receiving a pension of 14400 a year paid in half-yearly installments for rest of his life after reckoning his life expectation to be 13 years and that interest at 4% p.a. is payable half-yearly. What single sum is equivalent to his pension?
- (A) 145000 (B) 144900 (C) 144800 (D) 144700

## ANSWERS

- |     |   |     |   |     |   |     |   |     |   |     |   |
|-----|---|-----|---|-----|---|-----|---|-----|---|-----|---|
| 1)  | D | 2)  | D | 3)  | A | 4)  | C | 5)  | D | 6)  | B |
| 7)  | D | 8)  | C | 9)  | D | 10) | D | 11) | C | 12) | A |
| 13) | C | 14) | D | 15) | B |     |   |     |   |     |   |