CHAPTER – 1

RATIO AND PROPORTION, INDICES, LOGARITHMS

LEARNING OBJECTIVES

After reading this unit a student will learn –

- How to compute and compare two ratios;
- Effect of increase or decrease of a quantity on the ratio;
- The concept and application of inverse ratio.

We use ratio in many ways in practical fields. For example, it is given that a certain sum of money is divided into three parts in the given ratio. If first part is given then we can find out total amount and the other two parts.

In the case when ratio of boys and girls in a school is given and the total no. of student is also given, then if we know the no. of boys in the school, we can find out the no. of girls of that school by using ratios.

1.1 RATIO

A ratio is a comparison of the sizes of two or more quantities of the same kind by division.

If a and b are two quantities of the same kind (in same units), then the fraction a/b is called the ratio of a to b. It is written as a : b. Thus, the ratio of a to b = a/b or a : b. The quantities a and b are called the **terms** of the ratio, a is called the **first term or antecedent** and b is called the **second term or consequent**.

For example, in the ratio 5 : 6, 5 & 6 are called terms of the ratio. 5 is called first term and 6 is called second term.

1.1.2 REMARKS

- **Both terms of a ratio can be multiplied or divided by the same (non–zero) number. Usually a ratio is expressed in lowest terms (or simplest form).**

Illustration I:

$12 : 16 = 12/16 = (3 \times 4)/(4 \times 4) = 3/4 = 3 : 4$

- **The order of the terms in a ratio is important**.

Illustration II:

3 : 4 is not same as 4 : 3.

- **Ratio exists only between quantities of the same kind.**

Illustration III:

(i) There is no ratio between no. of students in a class and the salary of a teacher.

(ii) There is no ratio between the weight of one child and the age of another child.

- **Quantities to be compared (by division) must be in the same units**.

Illustration IV:

(i) Ratio between 150 gm and 2 kg = Ratio between 150 gm and 2000 gm

= 150/2000 = 3/40 = 3 : 40

(ii) Ratio between 25 minutes and 45 seconds. = Ratio between (25 × 60) sec and 45 sec.

= 1500/45 = 100/3 = 100 : 3

Illustration V:

(i) Ratio between 3 kg & 5 kg. = 3/5

- **To compare two ratios, convert them into equivalent like fractions.**

Illustration VI: To find which ratio is greater ——

$$2\frac{1}{3} : 3\frac{1}{3} \quad ; \; 3.6 : 4.8$$

Solution: $2\frac{1}{3} : 3\frac{1}{3}$ = 7/3 : 10/3 = 7 : 10 = 7/10

3.6 : 4.8 = 3.6/4.8 = 36/48 = 3/4

L.C.M of 10 and 4 is 20.

So, 7/10 = (7 × 2)/(10 × 2) = 14/20

And 3/4 = (3 × 5)/(4 × 5) = 15/20

As 15 > 14 so, 15/20 > 14/20 i. e. 3/4 > 7/10

Hence, 3.6 : 4.8 is greater ratio.

- **If a quantity increases or decreases in the ratio a : b then new quantity = b of the original quantity/a**

The fraction by which the original quantity is multiplied to get a new quantity is called the factor multiplying ratio.

Illustration VII: Rounaq weighs 56.7 kg. If he reduces his weight in the ratio 7 : 6, find his new weight.

Solution: Original weight of Rounaq = 56.7 kg.

He reduces his weight in the ratio 7 : 6

His new weight = (6 × 56.7)/7 = 6 × 8.1 = 48.6 kg.

Example 1: Simplify the ratio 1/3 : 1/8 : 1/6

Solution: L.C.M. of 3, 8 and 6 is 24.

1/3 : 1/8 : 1/6 = 1 × 24/3 : 1 × 24/8 : 1 × 24/6

= 8 : 3 : 4

Example 2: The ratio of the no. of boys to the no. of girls in a school of 720 students is 3 : 5. If 18 new girls are admitted in the school, find how many new boys may be admitted so that the ratio of the no. of boys to the no. of girls may change to 2 : 3.

Solution: The ratio of the no. of boys to the no. of girls = 3 : 5

Sum of the ratios = 3 + 5 = 8

So, the no. of boys in the school = (3 × 720)/8 = 270

And the no. of girls in the school = (5 × 720)/8 = 450

Let the no. of new boys admitted be x, then the no. of boys become (270 + x).

After admitting 18 new girls, the no. of girls become 450 + 18 = 468

According to given description of the problem, (270 + x)/468 = 2/3

or, 3 (270 + x) = 2 x 468

or, 810 + 3x = 936 or, 3x = 126 or, x = 42.

Hence the no. of new boys admitted = 42.

1.1.3 INVERSE RATIO

One ratio is the inverse of another if their product is 1. Thus a : b is the inverse of b : a and vice–versa.

1. A ratio a : b is said to be of greater inequality if a>b and of less inequality if a<b.
2. The ratio compounded of the two ratios a : b and c : d is ac : bd.

   For example compound ratio of 3 : 4 and 5 : 7 is 15 : 28.

   Compound ratio of 2 : 3, 5 : 7 and 4 : 9 is 40 : 189.
3. A ratio compounded of itself is called its duplicate ratio.

   Thus $a^2 : b^2$ is the duplicate ratio of a : b. Similarly, the triplicate ratio of a : b is $a^3 : b^3$.

   For example, duplicate ratio of 2 : 3 is 4 : 9. Triplicate ratio of 2 : 3 is 8 : 27.
4. The sub–duplicate ratio of a : b is $\sqrt{a} : \sqrt{b}$ and the sub triplicate ratio of a : b is $\sqrt[3]{a} : \sqrt[3]{b}$.

   For example sub duplicate ratio of 4 : 9 is $\sqrt{4} : \sqrt{9}$ = 2 : 3

   And sub triplicate ratio of 8 : 27 is $\sqrt[3]{8} : \sqrt[3]{27}$ = 2 : 3.
5. If the ratio of two similar quantities can be expressed as a ratio of two integers, the quantities are said to be commensurable; otherwise, they are said to be incommensurable. $\sqrt{3} : \sqrt{2}$ cannot be expressed as the ratio of two integers and therefore, $\sqrt{3}$ and $\sqrt{2}$ are incommensurable quantities.
6. Continued Ratio is the relation (or compassion) between the magnitudes of three or more quantities of the same kind. The continued ratio of three similar quantities a, b, c is written as a: b: c.

Illustration I: The continued ratio of Rs. 200, Rs. 400 and Rs. 600 is Rs. 200 : Rs. 400 : Rs. 600 = 1 : 2 : 3.

Example 1: The monthly incomes of two persons are in the ratio 4 : 5 and their monthly expenditures are in the ratio 7 : 9. If each saves Rs. 50 per month, find their monthly incomes.

Solution: Let the monthly incomes of two persons be Rs. 4x and Rs. 5x so that the ratio is Rs. 4x : Rs. 5x = 4 : 5. If each saves Rs. 50 per month, then the expenditures of two persons are Rs. (4x – 50) and Rs. (5x – 50).

$$\frac{4x-50}{5x-50}=\frac{7}{9}, \text{ or, } 36x-450=35x-350$$

or, 36x – 35x = 450 – 350, or, x = 100

Hence, the monthly incomes of the two persons are Rs. 4 × 100 and Rs. 5 × 100 i.e. Rs. 400 and Rs. 500.

Example 2 : The ratio of the prices of two houses was 16 : 23. Two years later when the price of the first has increased by 10% and that of the second by Rs. 477, the ratio of the prices becomes 11 : 20. Find the original prices of the two houses.

Solution: Let the original prices of two houses be Rs. 16x and Rs. 23x respectively. Then by the given conditions,

$$\frac{16x+10\%\text{ of }16x}{23x+477}=\frac{11}{20}$$

or, $\frac{16x+1.6x}{23x+477}=\frac{11}{20}$, or, 320x + 32x = 253x + 5247

or, 352x – 253x = 5247, or, 99x = 5247; ∴ x = 53

Hence, the original prices of two houses are Rs. 16 × 53 and Rs. 23 × 53 i.e. Rs. 848 and Rs. 1,219.

Example 3 : Find in what ratio will the total wages of the workers of a factory be increased or decreased if there be a reduction in the number of workers in the ratio 15 : 11 and an increment in their wages in the ratio 22 : 25.

Solution: Let x be the original number of workers and Rs. y the (average) wages per workers. Then the total wages before changes = Rs. xy.

After reduction, the number of workers = (11 x)/15

After increment, the (average) wages per workers = Rs. (25 y)/22

∴ The total wages after changes = $(\frac{11}{15}x) \times (\text{Rs.}\frac{25}{22}y)$ = Rs. $\frac{5xy}{6}$

Thus, the total wages of workers get decreased from Rs. xy to Rs. 5xy/6

Hence, the required ratio in which the total wages decrease is $xy:\frac{5xy}{6}=6:5$.

Exercise 1(A)

Choose the most appropriate option (a) (b) (c) or (d)

1. The inverse ratio of 11 : 15 is
   (a) 15 : 11 (b) $\sqrt{11}:\sqrt{15}$ (c) 121 : 225 (d) none of these
2. The ratio of two quantities is 3 : 4. If the antecedent is 15, the consequent is
   (a) 16 (b) 60 (c) 22 (d) 20
3. The ratio of the quantities is 5 : 7. If the consequent of its inverse ratio is 5, the antecedent is
   (a) 5 (b) $\sqrt{5}$ (c) 7 (d) none of these
4. The ratio compounded of 2 : 3, 9 : 4, 5 : 6 and 8 : 10 is
   (a) 1 : 1 (b) 1 : 5 (c) 3 : 8 (d) none of these
5. The duplicate ratio of 3 : 4 is
   (a) $\sqrt{3}:2$ (b) 4 : 3 (c) 9 : 16 (d) none of these
6. The sub duplicate ratio of 25 : 36 is
   (a) 6 : 5 (b) 36 : 25 (c) 50 : 72 (d) 5 : 6
7. The triplicate ratio of 2 : 3 is
   (a) 8 : 27 (b) 6 : 9 (c) 3 : 2 (d) none of these
8. The sub triplicate ratio of 8 : 27 is
   (a) 27 : 8 (b) 24 : 81 (c) 2 : 3 (d) none of these
9. The ratio compounded of 4 : 9 and the duplicate ratio of 3 : 4 is
   (a) 1 : 4 (b) 1 : 3 (c) 3 : 1 (d) none of these
10. The ratio compounded of 4 : 9, the duplicate ratio of 3 : 4, the triplicate ratio of 2 : 3 and 9 : 7 is
    (a) 2 : 7 (b) 7 : 2 (c) 2 : 21 (d) none of these
11. The ratio compounded of duplicate ratio of 4 : 5, triplicate ratio of 1 : 3, sub duplicate ratio of 81 : 256 and sub triplicate ratio of 125 : 512 is
    (a) 4 : 512 (b) 3 : 32 (c) 1 : 12 (d) none of these
12. If a : b = 3 : 4, the value of (2a+3b) : (3a+4b) is
    (a) 54 : 25 (b) 8 : 25 (c) 17 : 24 (d) 18 : 25
13. Two numbers are in the ratio 2 : 3. If 4 be subtracted from each, they are in the ratio 3 : 5. The numbers are
    (a) (16,24) (b) (4,6) (c) (2,3) (d) none of these
14. The angles of a triangle are in ratio 2 : 7 : 11. The angles are
    (a) $(20°, 70°, 90°)$ (b) $(30°, 70°, 80°)$ (c) $(18°, 63°, 99°)$ (d) none of these
15. Division of Rs. 324 between X and Y is in the ratio 11 : 7. X & Y would get Rupees
    (a) (204, 120) (b) (200, 124) (c) (180, 144) (d) none of these

16. Anand earns Rs. 80 in 7 hours and Promode Rs. 90 in 12 hours. The ratio of their earnings is
(a) 32 : 21 (b) 23 : 12 (c) 8 : 9 (d) none of these

17. The ratio of two numbers is 7 : 10 and their difference is 105. The numbers are
(a) (200, 305) (b) (185, 290) (c) (245, 350) (d) none of these

18. P, Q and R are three cities. The ratio of average temperature between P and Q is 11 : 12 and that between P and R is 9 : 8. The ratio between the average temperature of Q and R is
(a) 22 : 27 (b) 27 : 22 (c) 32 : 33 (d) none of these

19. If x : y = 3 : 4, the value of $x^2y + xy^2 : x^3 + y^3$ is
(a) 13 : 12 (b) 12 : 13 (c) 21 : 31 (d) none of these

20. If p : q is the sub duplicate ratio of $p-x^2 : q-x^2$ then x^2 is
(a) $\frac{p}{p+q}$ (b) $\frac{q}{p+q}$ (c) $\frac{pq}{p+q}$ (d) none of these

21. If 2s : 3t is the duplicate ratio of 2s – p : 3t – p then
(a) $p^2 = 6st$ (b) p = 6st (c) 2p = 3st (d) none of these

22. If p : q = 2 : 3 and x : y = 4 : 5, then the value of $5px + 3qy : 10px + 4qy$ is
(a) 71 : 82 (b) 27 : 28 (c) 17 : 28 (d) none of these

23. The number which when subtracted from each of the terms of the ratio 19 : 31 reducing it to 1 : 4 is
(a) 15 (b) 5 (c) 1 (d) none of these

24. Daily earnings of two persons are in the ratio 4:5 and their daily expenses are in the ratio 7 : 9. If each saves Rs. 50 per day, their daily incomes in Rs. are
(a) (40, 50) (b) (50, 40) (c) (400, 500) (d) none of these

25. The ratio between the speeds of two trains is 7 : 8. If the second train runs 400 Kms. in 5 hours, the speed of the first train is
(a) 10 Km/hr (b) 50 Km/hr (c) 70 Km/hr (d) none of these

1.2 PROPORTION

LEARNING OBJECTIVES

After reading this unit, a student will learn –

- What is proportion?
- Properties of proportion and how to use them.

If the income of a man is increased in the given ratio and if the increase in his income is given then to find out his new income, Proportion problem is used.

Again if the ages of two men are in the given ratio and if the age of one man is given, we can find out the age of the another man by Proportion.

An equality of two ratios is called a **proportion**. Four quantities a, b, c, d are said to be in proportion if a : b = c : d (also written as a : b :: c : d) i.e. if a/b = c/d i.e. if ad = bc.

The quantities a, b, c, d are called **terms** of the proportion; a, b, c and d are called its first, second, third and fourth terms respectively. First and fourth terms are called **extremes** (or extreme terms). Second and third terms are called **means (or middle terms).**

If a : b = c : d then d is called fourth proportional.

If a : b = c : d are in proportion then a/b = c/d i.e. ad = bc

i.e. **product of extremes = product of means**.

This is called *cross product rule.*

Three quantities a, b, c of the same kind (in same units) are said to be in continuous proportion if a : b = b : c i.e. a/b = b/c i.e. $b^2 = ac$

If a, b, c are in continuous proportion, then the middle term b is called the mean proportional between a and c, a is the first proportional and c is the third proportional.

Thus, if b is mean proportional between a and c, then $b^2 = ac$ i.e. b = $\sqrt{ac}$.

When three or more numbers are so related that the ratio of the first to the second, the ratio of the second to the third, third to the fourth etc. are all equal, the numbers are said to be in continued proportion. We write it as

x/y = y/z = z/w = w/p = p/q = ... when

x, y, z, w, p and q are in continued proportion. If a ratio is equal to the reciprocal of the other, then either of them is in inverse (or reciprocal) proportion of the other. For example 5/4 is in inverse proportion of 4/5 and vice–versa.

Note: In a ratio a : b, both quantities must be of the same kind while in a proportion a : b = c : d, all the four quantities need not be of the same type. The first two quantities should be of the same kind and last two quantities should be of the same kind.

Illustration I:

Rs. 6 : Rs. 8 = 12 toffees : 16 toffees are in a proportion.

Here 1st two quantities are of same kind and last two are of same kind.

Example 1: The numbers 2.4, 3.2, 1.5, 2 are in proportion because these numbers satisfy the property the product of extremes = product of means.

Here 2.4 × 2 = 4.8   and   3.2 × 1.5 = 4.8

Example 2: Find the value of x if 10/3 : x : : 5/2 : 5/4

Solution: 10/3 : x = 5/2 : 5/4

Using cross product rule, x × 5/2 = (10/3) × 5/4

Or, x = (10/3) × (5/4) × (2/5) = 5/3

Example 3: Find the fourth proportional to 2/3, 3/7, 4

Solution: If the fourth proportional be x, then 2/3, 3/7, 4, x are in proportion.

Using cross product rule, $(2/3) \times x = (3 \times 4)/7$

or, $x = (3 \times 4 \times 3)/(7 \times 2) = 18/7$.

Example 4: Find the third proportion to 2.4 kg, 9.6 kg

Solution: Let the third proportion to 2.4 kg, 9.6 kg be x kg.

Then 2.4 kg, 9.6 kg and x kg are in continued proportion since $b^2 = ac$

So, $2.4/9.6 = 9.6/x$ or, $x = (9.6 \times 9.6)/2.4 = 38.4$

Hence the third proportional is 38.4 kg.

Example 5: Find the mean proportion between 1.25 and 1.8

Solution: Mean proportion between 1.25 and 1.8 is $\sqrt{(1.25 \times 1.8)} = \sqrt{2.25} = 1.5$.

1.2.1 PROPERTIES OF PROPORTION

1. If a : b = c : d, then ad = bc

Proof. $\frac{a}{b} = \frac{c}{d}$; $\therefore ad = bc$ (By cross-multiplication)

2. If a : b = c : d, then b : a = d : c (Invertendo)

Proof. $\frac{a}{b} = \frac{c}{d}$ or $1\Big/\frac{a}{b} = 1\Big/\frac{c}{d}$, or, $\frac{b}{a} = \frac{d}{c}$

Hence, b : a = d : c.

3. If a : b = c : d, then a : c = b : d (Alternendo)

Proof. $\frac{a}{b} = \frac{c}{d}$ or, $ad = bc$

Dividing both sides by cd, we get

$\frac{ad}{cd} = \frac{bc}{cd}$, or $\frac{a}{c} = \frac{b}{d}$, i.e. a : c = b : d.

4. If a : b = c : d, then a + b : b = c + d : d (Componendo)

Proof. $\frac{a}{b} = \frac{c}{d}$, or, $\frac{a}{b} + 1 = \frac{c}{d} + 1$

or, $\frac{a+b}{b} = \frac{c+d}{d}$, i.e. a + b : b = c + d : d.

5. If a : b = c : d, then a – b : b = c – d : d (Dividendo)

Proof. $\frac{a}{b}=\frac{c}{d}$, $\therefore \frac{a}{b}-1=\frac{c}{d}-1$

$\frac{a-b}{b}=\frac{c-d}{d}$, i.e. $a-b : b = c-d : d$.

6. If a : b = c : d, then a + b : a – b = c + d : c – d (Componendo and Dividendo)

Proof. $\frac{a}{b}=\frac{c}{d}$, or $\frac{a}{b}+1=\frac{c}{d}+1$, or $\frac{a+b}{b}=\frac{c+d}{d}$ 1

Again $\frac{a}{b}-1,=\frac{c}{d}-1$, or $\frac{a-b}{b}=\frac{c-d}{d}$ 2

Dividing (1) by (2) we get

$\frac{a+b}{a-b}=\frac{c+d}{c-d}$, i.e. $a+b : a-b = c+d : c-d$

7. If a : b = c : d = e : f = ……………..….., then each of these ratios (Addendo) is equal (a + c + e + ……..) : (b + d + f + …….)

Proof. $\frac{a}{b}=\frac{c}{d},=\frac{e}{f}=$ ....(say) k,

$\therefore a = bk, c = dk, e = fk,$ ....

Now $a+c+e.......=k(b+d+f)$............ or $\frac{a+c+e.....}{b+d+f.....}=k$

Hence, (a + c + e + ……..) : (b + d + f + …….) is equal to each ratio

Example 1: If a : b = c : d = 2.5 : 1.5, what are the values of ad : bc and a+c : b+d?

Solution: we have $\frac{a}{b}=\frac{c}{d},=\frac{2.5}{1.5}$............................(1)

From (1) ad = bc, or, $\frac{ad}{bc}=1$, i.e. ad : bc = 1:1

Again from (1) $\frac{a}{b}=\frac{c}{d} = \frac{a+c}{b+d}$

$\therefore \frac{a+c}{b+d}=\frac{2.5}{1.5}=\frac{25}{15}=\frac{5}{3}$, i.e. $a+c : b+d = 5:3$

Hence, the values of ad : bc and a + c : b + d are 1 : 1 and 5 : 3 respectively.

Example 2: If $\frac{a}{3}=\frac{b}{4}=\frac{c}{7}$, then prove that $\frac{a+b+c}{c}=2$

Solution: We have $\frac{a}{3}=\frac{b}{4}=\frac{c}{7}=\frac{a+b+c}{3+4+7}=\frac{a+b+c}{14}$

$\therefore \frac{a+b+c}{14}=\frac{c}{7}$ or $\frac{a+b+c}{c}=\frac{14}{7}=2$

Example 3: A dealer mixes tea costing Rs. 6.92 per kg. with tea costing Rs. 7.77 per kg. and sells the mixture at Rs. 8.80 per kg. and earns a profit of $17\frac{1}{2}\%$ on his sale price. In what proportion does he mix them?

Solution: Let us first find the cost price (C.P.) of the mixture. If S.P. is Rs. 100, profit is $17\frac{1}{2}$ Therefore C.P. = Rs. (100 - $17\frac{1}{2}$) = Rs. $82\frac{1}{2}$ = Rs. 165/2

If S.P. is Rs. 8.80, C.P. is (165 × 8.80)/(2 × 100) = Rs. 7.26

∴ C.P. of the mixture per kg = Rs. 7.26

2nd difference = Profit by selling 1 kg. of 2nd kind @ Rs. 7.26

= Rs. 7.77 – Rs. 7.26 = 51 paise

1st difference = Rs. 7.26 – Rs. 6.92 = 34 paise

We have to mix the two kinds in such a ratio that the amount of profit in the first case must balance the amount of loss in the second case.

Hence, the required ratio = (2nd diff) : (1st diff.) = 51 : 34 = 3 : 2.

1.2.2 LAWS ON PROPORTION AS DERIVED EARLIER

(i) p : q = r : s => q : p = s : r (Invertendo)

(p/q = r/s) => (q/p = s/r)

(ii) a : b = c : d => a : c = b : d (Alternendo)

(a/b = c/d) => (a/c = b/d)

(iii) a : b = c : d => a+b : b = c+d : d (Componendo)

(a/b = c/d) => (a+b)/b = (c+d)/d

(iv) a : b = c : d => a–b : b = c–d : d (Dividendo)

(a/b = c/d) => (a–b)/b = (c–d)/d

(v) a : b = c : d => a+b : a–b = c+d : c–d (Componendo & Dividendo)

(a+b)/(a–b) = (c+d)/(c–d)

(vi) a : b = c : d = a+c : b+d (Addendo)

(a/b = c/d = a+c/b+d)

(vii) a : b = c : d = a–c : b–d (Subtrahendo)

(a/b = c/d = a–c/b–d)

(viii) If a : b = c : d = e : f = ... then each of these ratios = (a – c – e – ....) : (b – d – f – .....)

Proof: The reader may try it as an exercise (Subtrahendo) as the proof is similar to that derival in 7 above

Exercise 1(B)

Choose the most appropriate option (a) (b) (c) or (d)

1. The fourth proportional to 4, 6, 8 is
   (a) 12 (b) 32 (c) 48 (d) none of these
2. The third proportional to 12, 18 is
   (a) 24 (b) 27 (c) 36 (d) none of these
3. The mean proportional between 25, 81 is
   (a) 40 (b) 50 (c) 45 (d) none of these
4. The number which has the same ratio to 26 that 6 has to 13 is
   (a) 11 (b) 10 (c) 21 (d) none of these
5. The fourth proportional to 2a, a^2, c is
   (a) ac/2 (b) ac (c) 2/ac (d) none of these
6. If four numbers 1/2, 1/3, 1/5, 1/x are proportional then x is
   (a) 6/5 (b) 5/6 (c) 15/2 (d) none of these
7. The mean proportional between $12x^2$ and $27y^2$ is
   (a) 18xy (b) 81xy (c) 8xy (d) none of these

   (Hint: Let z be the mean proportional and z = $\sqrt{(12x^2 \text{ x } 27y^2)}$)
8. If A = B/2 = C/5, then A : B : C is
   (a) 3 : 5 : 2 (b) 2 : 5 : 3 (c) 1 : 2 : 5 (d) none of these
9. If a/3 = b/4 = c/7, then a+b+c/c is
   (a) 1 (b) 3 (c) 2 (d) none of these
10. If p/q = r/s = 2.5/1.5, the value of ps:qr is
    (a) 3/5 (b) 1:1 (c) 5/3 (d) none of these
11. If x : y = z : w = 2.5 : 1.5, the value of (x+z)/(y+w) is
    (a) 1 (b) 3/5 (c) 5/3 (d) none of these
12. If (5x–3y)/(5y–3x) = 3/4, the value of x : y is
    (a) 2 : 9 (b) 7 : 2 (c) 7 : 9 (d) none of these
13. If A : B = 3 : 2 and B : C = 3 : 5, then A:B:C is
    (a) 9 : 6 : 10 (b) 6 : 9 : 10 (c) 10 : 9 : 6 (d) none of these

14. If $x/2 = y/3 = z/7$, then the value of $(2x–5y+4z)/2y$ is
(a) 6/23 (b) 23/6 (c) 3/2 (d) 17/6

15. If x : y = 2 : 3, y : z = 4 : 3 then x : y : z is
(a) 2 : 3 : 4 (b) 4 : 3 : 2 (c) 3 : 2 : 4 (d) none of these

16. Division of Rs. 750 into 3 parts in the ratio 4 : 5 : 6 is
(a) (200, 250, 300) (b) (250, 250, 250) (c) (350, 250, 150) (d) 8 : 12 : 9

17. The sum of the ages of 3 persons is 150 years. 10 years ago their ages were in the ratio 7 : 8 : 9. Their present ages are
(a) (45, 50, 55) (b) (40, 60, 50) (c) (35, 45, 70) (d) none of these

18. The numbers 14, 16, 35, 42 are not in proportion. The fourth term for which they will be in proportion is
(a) 45 (b) 40 (c) 32 (d) none of these

19. If $x/y = z/w$, implies $y/x = w/z$, then the process is called
(a) Dividendo (b) Componendo (c) Alternendo (d) none of these

20. If $p/q = r/s = p–r/q–s$, the process is called
(a) Subtrahendo (b) Addendo (c) Invertendo (d) none of these

21. If $a/b = c/d$, implies $(a+b)/(a–b) = (c+d)/(c–d)$, the process is called
(a) Componendo (b) Dividendo (c) Componendo and Dividendo (d) none of these

22. If $u/v = w/p$, then $(u–v)/(u+v) = (w–p)/(w+p)$. The process is called
(a) Invertendo (b) Alternendo (c) Addendo (d) none of these

23. 12, 16, *, 20 are in proportion. Then * is
(a) 25 (b) 14 (c) 15 (d) none of these

24. 4, *, 9, 13½ are in proportion. Then * is
(a) 6 (b) 8 (c) 9 (d) none of these

25. The mean proportional between 1.4 gms and 5.6 gms is
(a) 28 gms (b) 2.8 gms (c) 3.2 gms (d) none of these

26. If $\frac{a}{4}=\frac{b}{5}=\frac{c}{9}$ then $\frac{a+b+c}{c}$ is
(a) 4 (b) 2 (c) 7 (d) none of these.

27. Two numbers are in the ratio 3 : 4; if 6 be added to each terms of the ratio, then the new ratio will be 4 : 5, then the numbers are
(a) 14, 20 (b) 17, 19 (c) 18 and 24 (d) none of these

28. If $\frac{a}{4} = \frac{b}{5}$ then
(a) $\frac{a+4}{a-4} = \frac{b-5}{b+5}$ (b) $\frac{a+4}{a-4} = \frac{b+5}{b-5}$ (c) $\frac{a-4}{a+4} = \frac{b+5}{b-5}$ (d) none of these

29. If $a : b = 4 : 1$ then $\sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}}$ is

(a) 5/2 (b) 4 (c) 5 (d) none of these

30. If $\frac{x}{b+c-a} = \frac{y}{c+a-b} = \frac{z}{a+b-c}$ then

$(b - c)x + (c - a)y + (a - b)z$ is

(a) 1 (b) 0 (c) 5 (d) none of these

1.3 INDICES

LEARNING OBJECTIVES

After reading this unit, a student will learn –

- A meaning of indices and their applications;
- Laws of indices which facilitates their easy applications.

We are aware of certain operations of addition and multiplication and now we take up certain higher order operations with powers and roots under the respective heads of indices.

We know that the result of a repeated addition can be held by multiplication e.g.

$$4 + 4 + 4 + 4 + 4 = 5(4) = 20$$

$$a + a + a + a + a = 5(a) = 5a$$

Now, $4 \times 4 \times 4 \times 4 \times 4 = 4^5$;

$$a \times a \times a \times a \times a = a^5.$$

It may be noticed that in the first case 4 is multiplied 5 times and in the second case 'a' is multiplied 5 times. In all such cases a factor which multiplies is called the "**base**" and the number of times it is multiplied is called the "**power**" or the "**index**". Therefore, "4" and "a" are the bases and "5" is the index for both. Any base raised to the power zero is defined to be 1; i.e. $a^o = 1$. We also define $\sqrt[r]{a} = a^{1/r}$.

If n is a positive integer, and 'a' is a real number, i.e. $n \in N$ and $a \in R$ (where N is the set of positive integers and R is the set of real numbers), 'a' is used to denote the continued product of n factors each equal to 'a' as shown below:

$a^n = a \times a \times a$ ………….. to n factors.

Here a^n is a power of "a" whose base is "a" and the index or power is "n".

For example, in $3 \times 3 \times 3 \times 3 = 3^4$, 3 is base and 4 is index or power.

Law 1

$a^m \times a^n = a^{m+n}$, when m and n are positive integers; by the above definition, $a^m = a \times a$ ………….. to m factors and $a^n = a \times a$ ………….. to n factors.

$\therefore a^m \times a^n = (a \times a$ ………….. to m factors) × (a × a ……….. to n factors)

= a × a ………….. to (m + n) factors

$= a^{m+n}$

Now, we extend this logic to negative integers and fractions. First let us consider this for negative integer, that is m will be replaced by –n. By the definition of $a^m \times a^n = a^{m+n}$,

we get $a^{-n} \times a^n = a^{-n+n} = a^0 = 1$

For example $3^4 \times 3^5 = (3 \times 3 \times 3 \times 3) \times (3 \times 3 \times 3 \times 3 \times 3) = 3^{4+5} = 3^9$

Again, $3^{-5} = 1/3^5 = 1/(3 \times 3 \times 3 \times 3 \times 3) = 1/243$

Example 1: Simplify $2x^{1/2}3x^{-1}$ if x = 4

Solution: We have $2x^{1/2}3x^{-1}$

$= 6x^{1/2}x^{-1} = 6x^{1/2-1}$

$= 6x^{-1/2}$

$$= \frac{6}{x^{1/2}} = \frac{6}{4^{1/2}} = \frac{6}{(2^2)^{1/2}} = \frac{6}{2} = 3$$

Example 2: Simplify $6ab^2c^3 \times 4b^{-2}c^{-3}d$

Solution: $6ab^2c^3 \times 4b^{-2}c^{-3}d$

$= 24 \times a \times b^2 \times b^{-2} \times c^3 \times c^{-3} \times d$

$= 24 \times a \times b^{2+(-2)} \times c^{3+(-3)} \times d$

$= 24 \times a \times b^{2-2} \times c^{3-3} \times d$

$= 24\ a\ b^0 \times c^0 \times d$

= 24ad

Law 2

$a^m/a^n = a^{m-n}$, when m and n are positive integers and m > n.

By definition, $a^m = a \times a$ ………….. to m factors

$$\text{Therefore, } a^m \div a^n = \frac{a^m}{a^n} = \frac{a \times a \text{ ....to m factors}}{a \times a \text{ .... to n factors}}$$

= a × a ……… to m–n factors

= a^{m-n}

Now we take a numerical value for a and check the validity of this Law

$$2^7 \div 2^4 = \frac{2^7}{2^4} = \frac{2\times 2............\text{to }7\text{ factors}}{2\times 2...............\text{to }4\text{ factors}}$$

= 2 × 2 × 2 ……….. to (7–4) factors.

= 2 × 2 × 2 ……….. to 3 factors

= $2^3 = 8$

or $$2^7 \div 2^4 = \frac{2^7}{2^4} = \frac{2\times2\times2\times2\times2\times2\times2}{2\times2\times2\times2}$$

= $2 \times 2 \times 2 = 2^{1+1+1} = 2^3$

= 8

Example 3: Find the value of $\frac{4x^{-1}}{X^{-1/3}}$

Solution: $$\frac{4x^{-1}}{x^{-1/3}}$$

$= 4x^{-1-(-1/3)}$

$= 4x^{-1+1/3}$

$= 4x^{-2/3}$ or $\frac{4}{x^{2/3}}$

Example 4: Simplify $$\frac{2a^{\frac{1}{2}}\times a^{\frac{2}{3}}\times 6a^{-\frac{7}{3}}}{9a^{\frac{-5}{3}}\times a^{\frac{3}{2}}} \text{ if } a=4$$

Solution: $$\frac{2a^{\frac{1}{2}}\times a^{\frac{2}{3}}\times 6a^{-\frac{7}{3}}}{9a^{\frac{-5}{3}}\times a^{\frac{3}{2}}} \text{ if } a=4$$

$$= \frac{2.2.3.a^{\frac{1}{2}+\frac{2}{3}-\frac{7}{3}}}{3.3a^{-\frac{5}{3}+\frac{3}{2}}} = \frac{4}{3}\frac{a^{(3+4-14)/6}}{a^{(-10+9)/6}}$$

$$= \frac{4}{3} \cdot \frac{a^{-7/6}}{a^{-1/6}} = \frac{4}{3} a^{\frac{-7}{6}+\frac{1}{6}}$$

$$= \frac{4}{3} a^{-1} = \frac{4}{3} \cdot \frac{1}{a} = \frac{4}{3} \cdot \frac{1}{4} = \frac{1}{3}$$

Law 3

$(a^m)^n = a^{mn}$. where m and n are positive integers

By definition $(a^m)^n = a^m \times a^m \times a^m$ …….. to n factors

= (a × a …….. to m factors) a × a ×…….. to n factors……..to n times

= a × a …….. to mn factors

$= a^{mn}$

Following above, $(a^m)^n = (a^m)^{p/q}$

(We will keep m as it is and replace n by p/q, where p and q are positive integers)

Now the qth power of $(a^m)^{p/q}$ is $\left\{(a^m)^{p/q}\right\}^q$

$= (a^m)^{(p/q) \times q}$

$= a^{mp}$

If we take the qth root of the above we obtain

$$\left(a^{mp}\right)^{1/q} = \sqrt[q]{a^{mp}}$$

Now with the help of a numerical value for a let us verify this law.

$(2^4)^3 = 2^4 \times 2^4 \times 2^4$

$= 2^{4+4+4}$

$= 2^{12} = 4096$

Law 4

$(ab)^n = a^n \cdot b^n$ when n can take all of the values.

For example $6^3 = (2 \times 3)^3 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^3 \times 3^3$

First, we look at n when it is a positive integer. Then by the definition, we have

$(ab)^n$ = ab × ab ……………. to n factors

= (a × a ……..……. to n factors) × (b × b …………. n factors)

$= a^n \times b^n$

When n is a positive fraction, we will replace n by p/q.

Then we will have $(ab)^n = (ab)^{p/q}$

The qth power of $(ab)^{p/q} = \{(ab)^{(p/q)}\}^q = (ab)^p$

Example 5: Simplify $(x^a.y^{-b})^3 \cdot (x^3\ y^2)^{-a}$

Solution: $(x^a.y^{-b})^3 \cdot (x^3\ y^2)^{-a}$

$= (x^a)^3 \ .\ (y^{-b})^3 \cdot (x^3)^{-a} \ .\ (y^2)^{-a}$

$= x^{3a-3a} \ .\ y^{-3b-2a.}$

$= x^0 .\ y^{-3b-2a.}$

$$= \frac{1}{y^{3b+2a}}$$

Example 6: $\sqrt[6]{a^{4b}x^6}\,.(a^{2/3}x^{-1})^{-b}$

Solution: $\sqrt[6]{a^{4b}x^6}\,.(a^{2/3}x^{-1})^{-b}$

$$= (a^{4b}x^6)^{\frac{1}{6}}.(a^{\frac{2}{3}})^{-b}.(x^{-1})^{-b}$$

$$= (a^{4b})^{\frac{1}{6}}.(x^6)^{\frac{1}{6}}.a^{-\frac{2}{3}b}.x^{-1\times-b}$$

$$= a^{\frac{2}{3}b}.x.a^{-\frac{2b}{3}}.x^b$$

$$= a^{\frac{2}{3}b-\frac{2}{3}b}.x^{1+b}$$

$= a^0 \ .\ x^{1+b} = x^{1+b}$

Example 7: Find x, if $x\sqrt{x} = (x\sqrt{x})^x$

Solution: $x(x)^{1/2} = x^x.x^{x/2}$

or, $x^{1+1/2} = x^{x+x/2}$

or, $x^{3/2} = x^{3x/2}$

[If base is equal, then power is also equal]

i.e. $\frac{3}{2} = \frac{3x}{2}$ or, $x = \frac{3}{2} \times \frac{2}{3} = 1$

$\therefore\ X = 1$

Example 8: Find the value of k from $(\sqrt{9})^{-7} \times (\sqrt{3})^{-5} = 3^k$

Solution: $(\sqrt{9})^{-7} \times (\sqrt{3})^{-5} = 3^k$

or, $(3^{2 \times 1/2})^{-7} \times (3^{1/2})^{-5} = 3^k$

or, $3^{-7-5/2} = 3^k$

or, $3^{-19/2} = 3^k$ or, $k = -19/2$

1.3.1 LAWS OF INDICES

(i) $a^m \times a^n = a^{m+n}$ (base must be same)

Ex. $2^3 \times 2^2 = 2^{3+2} = 2^5$

(ii) $a^m \div a^n = a^{m-n}$

Ex. $2^5 \div 2^3 = 2^{5-3} = 2^2$

(iii) $(a^m)^n = a^{mn}$

Ex. $(2^5)^2 = 2^{5\times 2} = 2^{10}$

(iv) $a^o = 1$

Example : $2^0 = 1, 3^0 = 1$

(v) $a^{-m} = 1/a^m$ and $1/a^{-m} = a^m$

Example: $2^{-3} = 1/2^3$ and $1/2^{-5} = 2^5$

(vi) If $a^x = a^y$, then x=y

(vii) If $x^a = y^a$, then x=y

(viii) $\sqrt[m]{a} = a^{1/m}$, $\sqrt{x} = x^{1/2}$, $\sqrt{4} = \left(2^2\right)^{1/2} = 2^{1/2 \times 2} = 2$

Example: $\sqrt[3]{8} = 8^{1/3} = (2^3)^{1/3} = 2^{3\times 1/3} = 2$

Exercise 1(C)

Choose the most appropriate option (a) (b) (c) or (d)

1. $4x^{-1/4}$ is expressed as

(a) $-4x^{1/4}$ (b) x^{-1} (c) $4/x^{1/4}$ (d) none of these

2. The value of $8^{1/3}$ is

(a) $\sqrt[3]{2}$ (b) 4 (c) 2 (d) none of these

3. The value of $2 \times (32)^{1/5}$ is

(a) 2 (b) 10 (c) 4 (d) none of these

4. The value of $4/(32)^{1/5}$ is

(a) 8 (b) 2 (c) 4 (d) none of these

5. The value of $(8/27)^{1/3}$ is

(a) 2/3 (b) 3/2 (c) 2/9 (d) none of these

6. The value of $2(256)^{-1/8}$ is

(a) 1 (b) 2 (c) 1/2 (d) none of these

7. $2^{½} . 4^{¾}$ is equal to

(a) a fraction (b) a positive integer (c) a negative integer (d) none of these

8. $\frac{81x^4}{y^{-8}}^{\frac{1}{4}}$ has simplified value equal to

(a) xy^2 (b) x^2y (c) $9xy^2$ (d) none of these

9. $x^{a-b} \times x^{b-c} \times x^{c-a}$ is equal to

(a) x (b) 1 (c) 0 (d) none of these

10. The value of $\left(\frac{2p^2q^3}{3xy}\right)^0$ where p, q, x, y ≠ 0 is equal to

(a) 0 (b) 2/3 (c) 1 (d) none of these

11. $\{(3^3)^2 \times (4^2)^3 \times (5^3)^2\} / \{(3^2)^3 \times (4^3)^2 \times (5^2)^3\}$ is

(a) 3/4 (b) 4/5 (c) 4/7 (d) 1

12. Which is True ?

(a) $2^0 > (1/2)^0$ (b) $2^0 < (1/2)^0$ (c) $2^0 = (1/2)^0$ (d) none of these

13. If $x^{1/p} = y^{1/q} = z^{1/r}$ and xyz = 1, then the value of p+q+r is

(a) 1 (b) 0 (c) 1/2 (d) none of these

14. The value of $y^{a-b} \times y^{b-c} \times y^{c-a} \times y^{-a-b}$ is

(a) y^{a+b} (b) y (c) 1 (d) $1/y^{a+b}$

15. The True option is

(a) $x^{2/3} = \sqrt[3]{x^2}$ (b) $x^{2/3} = \sqrt{x^3}$ (c) $x^{2/3} > \sqrt[3]{x^2}$ (d) $x^{2/3} < \sqrt[3]{x^2}$

16. The simplified value of $16x^{-3}y^2 \times 8^{-1}x^3y^{-2}$ is

(a) 2xy (b) xy/2 (c) 2 (d) none of these

17. The value of $(8/27)^{-1/3} \times (32/243)^{-1/5}$ is

(a) 9/4 (b) 4/9 (c) 2/3 (d) none of these

18. The value of $\{(x+y)^{2/3} (x-y)^{3/2}/\sqrt{x+y} \times \sqrt{(x-y)^3}\}^6$ is

(a) $(x+y)^2$ (b) $(x-y)$ (c) $x+y$ (d) none of these

19. Simplified value of $(125)^{2/3} \times \sqrt{25} \times \sqrt[3]{5^3} \times 5^{1/2}$ is

(a) 5 (b) 1/5 (c) 1 (d) none of these

20. $[\{(2)^{1/2} \cdot (4)^{3/4} \cdot (8)^{5/6} \cdot (16)^{7/8} \cdot (32)^{9/10}\}^4]^{3/25}$ is

(a) A fraction (b) an integer (c) 1 (d) none of these

21. $[1-\{1-(1-x^2)^{-1}\}^{-1}]^{-1/2}$ is equal to

(a) x (b) 1/x (c) 1 (d) none of these

22. $\{(x^n)^{(n-1)/n}\}^{1/n+1}$ is equal to

(a) x^n (b) x^{n+1} (c) x^{n-1} (d) none of these

23. If $a^3-b^3 = (a-b)(a^2 + ab + b^2)$, then the simplified form of

$$\left[\frac{x^l}{x^m}\right]^{l^2+lm+m^2} \times \left[\frac{x^m}{x^n}\right]^{m^2+mn+n^2} \times \left[\frac{x^n}{x^l}\right]^{l^2+ln+n^2}$$

(a) 0 (b) 1 (c) x (d) none of these

24. Using $(a-b)^3 = a^3-b^3-3ab(a-b)$ tick the correct of these when $x = p^{1/3} - p^{-1/3}$

(a) $x^3+3x = p + 1/p$ (b) $x^3 + 3x = p - 1/p$ (c) $x^3 + 3x = p + 1$ (d) none of these

25. On simplification, $1/(1+a^{m-n}+a^{m-p}) + 1/(1+a^{n-m}+a^{n-p}) + 1/(1+a^{p-m}+a^{p-n})$ is equal to

(a) 0 (b) a (c) 1 (d) 1/a

26. The value of $\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a}$

(a) 1 (b) 0 (c) 2 (d) none of these

27. If $x = 3^{\frac{1}{3}} + 3^{-\frac{1}{3}}$, then $3x^3 - 9x$ is

(a) 15 (b) 10 (c) 12 (d) none of these

28. If $a^x = b$, $b^y = c$, $c^z = a$, then xyz is

(a) 1 (b) 2 (c) 3 (d) none of these

29. The value of $\left(\frac{x^a}{x^b}\right)^{(a^2+ab+b^2)} \times \left(\frac{x^b}{x^c}\right)^{(b^2+bc+c^2)} \times \left(\frac{x^c}{x^a}\right)^{(c^2+ca+a^2)}$

(a) 1 (b) 0 (c) –1 (d) none of these

30. If $2^x = 3^y = 6^{-z}$, $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$ is

(a) 1 (b) 0 (c) 2 (d) none of these

1.4 LOGARITHM

LEARNING OBJECTIVE

- After reading this unit, a student will get fundamental knowledge of logarithm and its application for solving business problems.

The logarithm of a number to a given base is the index or the power to which the base must be raised to produce the number, i.e. to make it equal to the given number. If there are three quantities indicated by say a, x and n, they are related as follows:

If $a^x = n$, where $n > 0$, $a > 0$ and $a \neq 1$

then x is said to be the logarithm of the number n to the base 'a' symbolically it can be expressed as follows:

$\log_a n = x$

i.e. the logarithm of n to the base 'a' is x. We give some illustrations below:

(i) $2^4 = 16 \Rightarrow \log_2 16 = 4$

i.e. the logarithm of 16 to the base 2 is equal to 4

(ii) $10^3 = 1000 \Rightarrow \log_{10}1000 = 3$

i.e. the logarithm of 1000 to the base 10 is 3

(iii) $5^{-3} = \frac{1}{125} \Rightarrow \log_5\left(\frac{1}{125}\right) = -3$

i.e. the logarithm of $\frac{1}{125}$ to the base 5 is –3

(iv) $2^3 = 8 \Rightarrow \log_2 8 = 3$

i.e. the logarithm of 8 to the base 2 is 3

1. The two equations $a^x = n$ and $x = \log_a n$ are only transformations of each other and should be remembered to change one form of the relation into the other.
2. The logarithm of 1 to any base is zero. This is because any number raised to the power zero is one.

   Since $a^0 = 1$, $\log_a 1 = 0$

3. The logarithm of any quantity to the same base is unity. This is because any quantity raised to the power 1 is that quantity only.

   Since $a^1 = a$, $\log_a a = 1$

Illustrations:

1. If $\log_a \sqrt{2} = \frac{1}{6}$, find the value of a.

   We have $a^{1/6} = \sqrt{2} \Rightarrow a = (\sqrt{2})^6 = 2^3 = 8$

2. Find the logarithm of 5832 to the base 3√ 2.

   Let us take $\log_{3\sqrt{2}} 5832 = x$

   We may write, $(3\sqrt{2})^x = 5832 = 8 \times 729 = 2^3 \times 3^6 = (\sqrt{2})^6 (3)^6 = (3\sqrt{2})^6$

   Hence, x = 6

Logarithms of numbers to the base 10 are known as common logarithm.

1.4.1 FUNDAMENTAL LAWS OF LOGARITHM

1. Logarithm of the product of two numbers is equal to the sum of the logarithms of the numbers to the same base, i.e.

   $\text{Log}_a mn = \log_a m + \log_a n$

   Proof:

   Let $\log_a m = x$ so that $a^x = m$ – (I)

   $\text{Log}_a n = y$ so that $a^y = n$ – (II)

   Multiplying (I) and (II), we get

   $m \times n = a^x \times a^y = a^{x+y}$

   $\log_a mn = x + y$ (by definition)

   $\therefore \log_a mn = \log_a m + \log_a n$

2. The logarithm of the quotient of two numbers is equal to the difference of their logarithms to the same base, i.e.

   $\log_a \frac{m}{n} = \log_a m - \log_a n$

Proof:

   Let $\log_a m = x$ so that $a^x = m$ ————(I)

   $\log_a n = y$ so that $a^y = n$ ——————(II)

   Dividing (I) by (II) we get

$$\frac{m}{n} = \frac{a^x}{a^y} = a^{x-y}$$

Then by the definition of logarithm, we get

$$\log_a \frac{m}{n} = x - y = \log_a m - \log_a n$$

Similarly, $\log_a \frac{1}{n} = \log_a 1 - \log_a n = 0 - \log_a n = -\log_a n \; [\because \log_a 1 = 0]$

Illustration I: log ½ = log 1 – log 2 = –log 2

3. Logarithm of the number raised to the power is equal to the index of the power multiplied by the logarithm of the number to the same base i.e.

$\log_a m^n = n \log_a m$

Proof:

Let $\log_a m = x$ so that $a^x = m$

Raising the power n on both sides we get

$(a^x)^n = (m)^n$

$a^{xn} = m^n$ (by definition)

$\log_a m^n = nx$

i.e. $\log_a m^n = n \log_a m$

Illustrations II: 1(a) Find the logarithm of 1728 to the base $2\sqrt{3}$

Solution: We have $1728 = 2^6 \times 3^3 = 2^6 \times (\sqrt{3})^6 = (2\sqrt{3})^6$; and so, we may write

$\log_{2\sqrt{3}} 1728 = 6$

1(b) Solve $\frac{1}{2}\log_{10} 25 - 2\log_{10} 3 + \log_{10} 18$

Solution: The given expression

$$= \log_{10} 25^{\frac{1}{2}} - \log_{10} 3^2 + \log_{10} 18$$

$$= \log_{10} 5 - \log_{10} 9 + \log_{10} 18$$

$$= \log_{10} \frac{5 \times 18}{9} = \log_{10} 10 = 1$$

1.4.2 CHANGE OF BASE

If the logarithm of a number to any base is given, then the logarithm of the same number to any other base can be determined from the following relation

$$\log_a m = \log_b m \times \log_a b \Rightarrow \log_b m = \frac{\log_a m}{\log_a b}$$

Proof:

Let $\log_a m = x$, $\log_b m = y$ and $\log_a b = z$

Then by definition,

$a^x = m$, $b^y = m$ and $a^z = b$

Also $a^x = b^y = (a^z)^y = a^{yz}$

Therefore, $x = yz$

$\Rightarrow \log_a m = \log_b m \times \log_a b$

$$\log_b m = \frac{\log_a m}{\log_a b}$$

Putting m = a, we have

$\log_a a = \log_b a \times \log_a b$

$\Rightarrow \log_b a \times \log_a b = 1$, since $\log_a a = 1$.

Example 1: Change the base of $\log_5 31$ into the common logarithmic base.

Solution: Since $\log_a x = \frac{\log_b x}{\log_b a}$

$$\therefore \log_5 31 = \frac{\log_{10} 31}{\log_{10} 5}$$

Example 2: Prove that $\frac{\log_3 8}{\log_9 16 \log_4 10} = 3\log_{10} 2$

Solution: Change all the logarithms on L.H.S. to the base 10 by using the formula.

$\log_b x = \frac{\log_a x}{\log_a b}$, We may write

$$\log_3 8 = \frac{\log_{10} 8}{\log_{10} 3} = \frac{\log_{10} 2^3}{\log_{10} 3} = \frac{3\log_{10} 2}{\log_{10} 3}$$

$$\log_9 16 = \frac{\log_{10} 16}{\log_{10} 9} = \frac{\log_{10} 2^4}{\log_{10} 3^2} = \frac{4\log_{10} 2}{2\log_{10} 3}$$

$$\log_4 10 = \frac{\log_{10} 10}{\log_{10} 4} = \frac{1}{\log_{10} 2^2} = \frac{1}{2\log_{10} 2} \left[\log_{10} 10 = 1\right]$$

$$\therefore \text{L.H.S.} = \frac{3\log_{10} 2}{\log_{10} 3} \times \frac{2\log_{10} 3}{4\log_{10} 2} \times \frac{2\log_{10} 2}{1} \therefore \left[\log_{10} 10 = 1\right]$$

$= 3 \log_{10} 2 =$ R.H.S.

Logarithm Tables:

The logarithm of a number consists of two parts, the whole part or the integral part is called the **characteristic** and the decimal part is called the **mantissa** where the former can be known by mere inspection, the latter has to be obtained from the logarithm tables.

Characteristic:

The characteristic of the logarithm of any number greater than 1 is positive and is one less than the number of digits to the left of the decimal point in the given number. The characteristic of the logarithm of any number less than one (1) is negative and numerically one more than the number of zeros to the right of the decimal point. If there is no zero then obviously it will be –1. The following table will illustrate it.

Number		Characteristic
3 7	1	One less than the
4 6 2 3	3	number of digits to
6.21	0	the left of the decimal point

Number		Characteristic
.8	–1	One more than the
.07	–2	number of zeros on
.00507	–3	the right immediately
.000670	–4	after the decimal point.

Zero on positive characteristic when the number under consideration is greater than unity:

Since	$10^0 = 1$	,	log 1 = 0
	$10^1 = 10$	,	log 10 = 1
	$10^2 = 100$	,	log 100 = 2
	$10^3 = 1000$	,	log 1000 = 3

All numbers lying between 1 and 10 i.e. numbers with 1 digit in the integral part have their logarithms lying between 0 and 1. Therefore, their integral parts are zero only.

All numbers lying between 10 and 100 have two digits in their integral parts. Their logarithms lie between 1 and 2. Therefore, numbers with two digits have integral parts with 1 as characteristic.

In general, the logarithm of a number containing n digits only in its integral parts is (n – 1) + a decimal. For example, the characteristics of log 75, log 79326, log 1.76 are 1, 4 and 0 respectively.

Negative characteristics

$$\textit{Since } 10^{-1} = \frac{1}{10} = 0.1 \rightarrow \log 0.1 = -1$$

$$10^{-2} = \frac{1}{100} = 0.01 \rightarrow \log 0.01 = -2$$

All numbers lying between 1 and 0.1 have logarithms lying between 0 and –1, i.e. greater than –1 and less than 0. Since the decimal part is always written positive, the characteristic is –1.

All numbers lying between 0.1 and 0.01 have their logarithms lying between –1 and –2 as characteristic of their logarithms.

In general, the logarithm of a number having n zeros just after the decimal point is –

(n + 1) + a decimal.

Hence, we deduce that the characteristic of the logarithm of a number less than unity is one more than the number of zeros just after the decimal point and is negative.

Mantissa:

The mantissa is the fractional part of the logarithm of a given number

Number	Mantissa	Logarithm
Log 4594	= (……… 6625)	= 3.6625
Log 459.4	= (……… 6625)	= 2.6625
Log 45.94	= (……… 6625)	= 1.6625
Log 4.594	= (……… 6625)	= 0.6625
Log .4594	= (……… 6625)	= $\bar{1}.6625$

Thus with the same figures there will be difference in the characteristic only. It should be remembered, that the mantissa is always a positive quantity. The other way to indicate this is

Log .004594 = – 3 + .6625 = – 3.6625.

Negative mantissa must be converted into a positive mantissa before reference to a logarithm table. For example

$$-3.6872 = -4 + (1 - 3.6872) = \bar{4} + 0.3128 = \bar{4}.3128$$

It may be noted that $\bar{4}.3128$ is different from – 4.3128 as – 4.3128 is a negative number whereas, in $\bar{4}.3128$, 4 is negative while .3128 is positive.

Illustration I: Add $\bar{4}.74628$ and 3.42367

$-4 + .74628$

$\underline{3 + .42367}$

$-1 + 1.16995 = 1 - 0.16995$

Antilogarithms:

If x is the logarithm of a given number n with a given base then n is called the antilogarithm (antilog) of x to that base.

This can be expressed as follows:-

If $\log_a n = x$ then n = antilog x

For example, if log 61720 = 4.7904 then 61720 = antilog 4.7904

Number	Logarithm	Logarithm
206	2.3139	206.0
20.6	1.3139	20.60
2.06	0.3139	2.060
.206	–1.3139	.2060
.0206	–2.3139	.02060

Example 1: Find the value of log 5 if log 2 is equal to .3010

Solution : $\log 5 = \log \frac{10}{2} = \log 10 - \log 2$

$= 1 - .3010$

$= .6990$

Example 2: Find the number whose logarithm is 2.4678.

Solution: From the antilog table, for mantissa .467, the number = 2931

for mean difference 8, the number = 5

∴ for mantissa .4678, the number = 2936

The characteristic is 2, therefore, the number must have 3 digits in the integral part.

Hence, Antilog 2.4678 = 293.6

Example 3: Find the number whose logarithm is –2.4678.

Solution: $-2.4678 = -3 + 3 - 2.4678 = -3 + .5322 = \bar{3}.5322$

For mantissa .532, the number = 3404

For mean difference 2, the number = 2

$\therefore$ for mantissa .5322, the number = 3406

The characteristic is –3, therefore, the number is less than one and there must be two zeros just after the decimal point.

Thus, Antilog (–2.4678) = 0.003406

Properties of Logarithm

(I) $\log_a mn = \log_a m + \log_a n$

Ex. $\log (2 \times 3) = \log 2 + \log 3$

(II) $\log_a (m/n) = \log_a m - \log_a n$

Ex. $\log (3/2) = \log 3 - \log 2$

(III) $\log_a m^n = n \log_a m$

Ex. $\log 2^3 = 3 \log 2$

(IV) $\log_a a = 1, \quad a \neq 1$

Ex. $\log_{10} 10 = 1, \quad \log_2 2 = 1, \quad \log_3 3 = 1$ etc.

(V) $\log_a 1 = 0$

Ex. $\log_2 1 = 0, \quad \log_{10} 1 = 0$ etc.

(VI) $\log_b a \times \log_a b = 1$

Ex. $\log_3 2 \times \log_2 3 = 1$

(VII) $\log_b a \times \log_c b = \log_c a$

Ex. $\log_3 2 \times \log_5 3 = \log_5 2$

(VIII) $\log_b a = \log a / \log b$

Ex. $\log_3 2 = \log 2 / \log 3$

Note:

(A) If base is understood, base is taken as 10

(B) Thus log 10 = 1, log 1 = 0

(C) Logarithm using base 10 is called Common logarithm and logarithm using base e is called Natural logarithm {e = 2.33 (approx.) called exponential number}.

Relation between Indices and Logarithm

Let $x = \log_a m$ and $y = \log_a n$

$\therefore a^x = m$ and $a^y = n$

so $a^x . a^y = mn$

or $a^{x+y} = mn$

or $x+y = \log_a mn$

or $\log_a m + \log_a n = \log_a mn$ $[\because \log_a a = 1]$

or $\log_a mn = \log_a m + \log_a n$

Also, $(m/n) = a^x/a^y$

or $(m/n) = a^{x-y}$

or $\log_a (m/n) = (x-y)$

or $\log_a (m/n) = \log_a m - \log_a n$ $[\because \log_a a = 1]$

Again m^n = m.m.m. ———————— to n times

so $\log_a m^n = \log_a$(m.m.m ———————— to n times)

or $\log_a m^n = \log_a m + \log_a m + \log_a m +$ ———————— $+ \log_a m$

or $\log_a m^n = n \log_a m$

Now $a^0 = 1 \Rightarrow 0 = \log_a 1$

let $\log_b a = x$ and $\log_a b = y$

$\therefore$ a $= b^x$ and $b = a^y$

$\therefore$ so a $= (a^y)^x$

or $a^{xy} = a$

or $xy = 1$

or $\log_b a \times \log_a b = 1$

let $\log_b c = x$ & $\log_c b = y$

$\therefore$ $c = b^x$ & $b = c^y$

so $c = c^{xy}$ or $xy = 1$

$\log_b c \times \log_c b = 1$

Example 1: Find the logarithm of 64 to the base $2\sqrt{2}$

Solution: $\log_{2\sqrt{2}} 64 = \log_{2\sqrt{2}} 8^2 = 2\log_{2\sqrt{2}} 8 = 2\log_{2\sqrt{2}} (2\sqrt{2})^2 = 4\log_{2\sqrt{2}} 2\sqrt{2} = 4 \text{x} 1 = 4$

Example 2: If $\log_a bc = x$, $\log_b ca = y$, $\log_c ab = z$, prove that

$$\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1} = 1$$

Solution: $x+1 = \log_a bc + \log_a a = \log_a abc$

$y+1 = \log_b ca + \log_b b = \log_b abc$

$z+1 = \log_c ab + \log_c c = \log_c abc$

Therefore $\frac{1}{x+1}+\frac{1}{y+1}+\frac{1}{z+1}=\frac{1}{\log_a abc}+\frac{1}{\log_b abc}+\frac{1}{\log_c abc}$

$= \log_{abc}a + \log_{abc}b + \log_{abc}c$

$= \log_{abc}abc = 1$ (proved)

Example 3: If $a=\log_{24}12$, $b=\log_{36}24$, and $c=\log_{48}36$ then prove that $1+abc = 2bc$

Solution: $1+abc = 1+ \log_{24}12 \times \log_{36}24 \times \log_{48}36$

$= 1+ \log_{36}12 \times \log_{48}36$

$= 1 + \log_{48}12$

$= \log_{48}48 + \log_{48}12$

$= \log_{48}48\times12$

$= \log_{48} (2\times12)^2$

$= 2 \log_{48}24$

$= 2 \log_{36}24 \text{ x } \log_{48}36$

$= 2bc$

Exercise 1(D)

Choose the most appropriate option. (a) (b) (c) and (d)

1. log 6 + log 5 is expressed as
   (a) log 11 (b) log 30 (c) log 5/6 (d) none of these
2. $\log_2 8$ is equal to
   (a) 2 (b) 8 (c) 3 (d) none of these
3. log 32/4 is equal to
   (a) log 32/log 4 (b) log 32 – log 4 (c) 2^3 (d) none of these
4. log (1 × 2 × 3) is equal to
   (a) log 1 + log 2 + log 3 (b) log 3 (c) log 2 (d) none of these
5. The value of log 0.0001 to the base 0.1 is
   (a) –4 (b) 4 (c) ¼ (d) none of these
6. If 2 log x = 4 log 3, the *x* is equal to
   (a) 3 (b) 9 (c) 2 (d) none of these
7. $\log_{\sqrt{2}} 64$ is equal to
   (a) 12 (b) 6 (c) 1 (d) none of these
8. $\log_{2\sqrt{3}} 1728$ is equal to
   (a) $2\sqrt{3}$ (b) 2 (c) 6 (d) none of these

9. log (1/81) to the base 9 is equal to
(a) 2 (b) ½ (c) –2 (d) none of these

10. log 0.0625 to the base 2 is equal to
(a) 4 (b) 5 (c) 1 (d) none of these

11. Given log2 = 0.3010 and log3 = 0.4771 the value of log 6 is
(a) 0.9030 (b) 0.9542 (c) 0.7781 (d) none of these

12. The value of $\log_2 \log_2 \log_2 16$
(a) 0 (b) 2 (c) 1 (d) none of these

13. The value of log $\frac{1}{3}$ to the base 9 is
(a) – ½ (b) ½ (c) 1 (d) none of these

14. If log x + log y = log (x+y), y can be expressed as
(a) x–1 (b) x (c) x/x–1 (d) none of these

15. The value of $\log_2 [\log_2 \{\log_3 (\log_3 27^3)\}]$ is equal to
(a) 1 (b) 2 (c) 0 (d) none of these

16. If $\log_2 x + \log_4 x + \log_{16} x = 21/4$, these x is equal to
(a) 8 (b) 4 (c) 16 (d) none of these

17. Given that $\log_{10} 2 = x$ and $\log_{10} 3 = y$, the value of $\log_{10} 60$ is expressed as
(a) x – y + 1 (b) x + y + 1 (c) x – y – 1 (d) none of these

18. Given that $\log_{10} 2 = x$, $\log_{10} 3 = y$, then $\log_{10} 1.2$ is expressed in terms of x and y as
(a) x + 2y – 1 (b) x + y – 1 (c) 2x + y – 1 (d) none of these

19. Given that log x = m + n and log y = m – n, the value of log $10x/y^2$ is expressed in terms of m and n as
(a) 1 – m + 3n (b) m – 1 + 3n (c) m + 3n + 1 (d) none of these

20. The simplified value of $2 \log_{10} 5 + \log_{10} 8 - ½ \log_{10} 4$ is
(a) ½ (b) 4 (c) 2 (d) none of these

21. $\log [1 - \{1 - (1 - x^2)^{-1}\}^{-1}]^{-1/2}$ can be written as
(a) $\log x^2$ (b) $\log x$ (c) $\log 1/x$ (d) none of these

22. The simplified value of $\log \sqrt[4]{729\ 3\sqrt{9^{-1}.27^{-4/3}}}$ is
(a) log 3 (b) log 2 (c) log ½ (d) none of these

23. The value of $(\log_b a \times \log_c b \times \log_a c)^3$ is equal to
(a) 3 (b) 0 (c) 1 (d) none of these

24. The logarithm of 64 to the base 2√2 is
(a) 2 (b) √2 (c) ½ (d) none of these

25. The value of $\log_8 25$ given log 2 = 0.3010 is
(a) 1 (b) 2 (c) 1.5482 (d) none of these

ANSWERS

Exercise 1(A)

1. a	2. d	3. c	4. a	5. c	6. d	7. a	8. c
9. a	10. c	11. d	12. d	13. a	14. c	15. d	16. a
17. c	18. b	19. b	20. c	21. a	22. c	23. a	24. c
25. c							

Exercise 1(B)

1. a	2. b	3. c	4. d	5. a	6. c	7. a	8. c
9. c	10. b	11. c	12. d	13. a	14. d	15. d	16. a
17. a	18. b	19. d	20. a	21. c	22. d	23. c	24. a
25. b	26. b	27. c	28. b	29. a	30. b		

Exercise 1(C)

1. c	2. c	3. c	4. b	5. a	6. a	7. b	8. d
9. b	10. c	11. d	12. c	13. b	14. d	15. a	16. c
17. a	18. c	19. d	20. b	21. a	22. c	23. b	24. b
25. c	26. a	27. b	28. a	29. a	30 d		

Exercise 1(D)

1. b	2. c	3. b	4. a	5. b	6. b	7. a	8. c
9. c	10. d	11. c	12. c	13. a	14. c	15. c	16. a
17. b	18. c	19. a	20. c	21. b	22. a	23. c	24. d
25. c							

ADDITIONAL QUESTION BANK

1. The value of $\left(\frac{6^{-1}7^{2}}{6^{2}7^{-4}}\right)^{7/2} \times \left(\frac{6^{-2}7^{3}}{6^{3}7^{-5}}\right)^{-5/2}$ is

(A) 0 (B) 252 (C) 250 (D) 248

2. The value of $\frac{x^{2/7}}{z^{-1/2}} \times \frac{x^{2/5}}{z^{2/3}} \times \frac{x^{-9/7}}{z^{2/3}} \times \frac{z^{5/6}}{x^{-3/5}}$ is

(A) 1 (B) –1 (C) 0 (D) None

3. On simplification $\frac{2^{x+3} \times 3^{2x-y} \times 5^{x+y+3} \times 6^{y+1}}{6^{x+1} \times 10^{y+3} \times 15^{x}}$ reduces to

(A) –1 (B) 0 (C) 1 (D) 10

4. If $\frac{9^{y}.3^{2}.\left(3^{-y}\right)^{-1} - 27^{y}}{3^{3x}.2^{3}} = \frac{1}{27}$ then x–y is given by

(A) –1 (B) 1 (C) 0 (D) None

5. Show that $\left(x^{\frac{1}{a-b}}\right)^{\frac{1}{a-c}} \times \left(x^{\frac{1}{b-c}}\right)^{\frac{1}{b-a}} \times \left(x^{\frac{1}{c-a}}\right)^{\frac{1}{c-b}}$ is given by

(A) 1 (B) –1 (C) 3 (D) 0

6. Show that $\frac{16(32)^{x} - 2^{3x-2}.4^{x+1}}{15(2)^{x-1}(16)^{x}} - \frac{5(5)^{x-1}}{\sqrt{5^{2x}}}$ is given by

(A) 1 (B) –1 (C) 4 (D) 0

7. Show that $\left(\frac{x^{a}}{x^{b}}\right)^{a+b} \times \left(\frac{x^{b}}{x^{c}}\right)^{b+c} \times \left(\frac{x^{c}}{x^{a}}\right)^{c+a}$ is given by

(A) 0 (B) –1 (C) 3 (D) 1

8. Show that $\sqrt[(a+b)]{\frac{x^{a^2}}{x^{b^2}}} \times \sqrt[(b+c)]{\frac{x^{b^2}}{x^{c^2}}} \times \sqrt[(c+a)]{\frac{x^{c^2}}{x^{a^2}}}$ reduces to

(A) 1 (B) 0 (C) –1 (D) None

9. Show that $\left(x^{\frac{b+c}{c-a}}\right)^{\frac{1}{a-b}} \times \left(x^{\frac{c+a}{a-b}}\right)^{\frac{1}{b-c}} \times \left(x^{\frac{a+b}{b-c}}\right)^{\frac{1}{c-a}}$ reduces to

(A) 1 (B) 3 (C) –1 (D) None

10. Show that $\left(\frac{x^b}{x^c}\right)^a \times \left(\frac{x^c}{x^a}\right)^b \times \left(\frac{x^a}{x^b}\right)^c$ reduces to

(A) 1 (B) 3 (C) 0 (D) 2

11. Show that $\left(\frac{x^b}{x^c}\right)^{1/bc} \times \left(\frac{x^c}{x^a}\right)^{1/ca} \times \left(\frac{x^a}{x^b}\right)^{1/ab}$ reduces to

(A) –1 (B) 0 (C) 1 (D) None

12. Show that $\left(\frac{x^a}{x^b}\right)^{(a^2+ab+b^2)} \times \left(\frac{x^b}{x^c}\right)^{(b^2+bc+c^2)} \times \left(\frac{x^c}{x^a}\right)^{(c^2+ca+a^2)}$ is given by

(A) 1 (B) –1 (C) 0 (D) 3

13. Show that $\left(x^{\frac{1}{a-b}}\right)^{\frac{1}{a-c}} \times \left(x^{\frac{1}{b-c}}\right)^{\frac{1}{b-a}} \times \left(x^{\frac{1}{c-a}}\right)^{\frac{1}{c-b}}$ is given by

(A) 0 (B) 1 (C) –1 (D) None

14. Show that $\left(\frac{x^b}{x^c}\right)^{b+c-a} \times \left(\frac{x^c}{x^a}\right)^{c+a-b} \times \left(\frac{x^a}{x^b}\right)^{a+b-c}$ is given by

(A) 1 (B) 0 (C) –1 (D) None

15. Show that $\left(\frac{x^a}{x^{-b}}\right)^{a^2-ab+b^2} \times \left(\frac{x^b}{x^{-c}}\right)^{b^2-bc+c^2} \times \left(\frac{x^c}{x^{-a}}\right)^{c^2-ca+a^2}$ is reduces to

(A) 1 (B) $x^{-2(a^2+b^2+c^2)}$ (C) $x^{2(a^3+b^3+c^3)}$ (D) $x^{-2(a^3+b^3+c^3)}$

16. $x^{a^2b^{-1}c^{-1}}.x^{b^2c^{-1}a^{-1}}.x^{c^2a^{-1}b^{-1}} - x^3$ would reduce to zero if $a+b+c$ is given by

(A) 1 (B) –1 (C) 0 (D) None

17. The value of z is given by the following if $z^{z\sqrt{z}} = \left(z\sqrt{z}\right)^z$

(A) 2 (B) $\frac{3}{2}$ (C) $-\frac{3}{2}$ (D) $\frac{9}{4}$

18. $\frac{1}{x^b+x^{-c}+1} + \frac{1}{x^c+x^{-a}+1} + \frac{1}{x^a+x^{-b}+1}$ would reduce to one if $a+b+c$ is given by

(A) 1 (B) 0 (C) –1 (D) None

19. On simplification $\frac{1}{1+z^{a-b}+z^{a-c}}$ $\frac{1}{1+z^{b-c}+z^{b-a}}$ $\frac{1}{1+z^{c-a}+z^{c-b}}$ would reduces to

(A) $\frac{1}{z^{2(a+b+c)}}$ (B) $\frac{1}{z^{(a+b+c)}}$ (C) 1 (D) 0

20. If $(5.678)^x=(0.5678)^y=10^z$ then

(A) $\frac{1}{x}-\frac{1}{y}+\frac{1}{z}=1$ (B) $\frac{1}{x}-\frac{1}{y}-\frac{1}{z}=0$ (C) $\frac{1}{x}-\frac{1}{y}+\frac{1}{z}=-1$ (D) None

21. If $x=4^{1/3}+4^{-1/3}$ prove that $4x^3-12x$ is given by
(A) 12 (B) 13 (C) 15 (D) 17

22. If $x=5^{1/3}+5^{-1/3}$ prove that $5x^3-15x$ is given by
(A) 25 (B) 26 (C) 27 (D) 30

23. If $ax^{2/3}+bx^{1/3}+c=0$ then the value of $a^3x^2+b^3x+c^3$ is given by
(A) $3abcx$ (B) $-3abcx$ (C) $3abc$ (D) $-3abc$

24. If $a^p=b\ b^q=c\ c^r=a$ the value of pqr is given by
(A) 0 (B) 1 (C) –1 (D) None

25. If $a^p=b^q=c^r$ and $b^2=ac$ the value of $q(p+r)/pr$ is given by
(A) 1 (B) –1 (C) 2 (D) None

26. On simplification $\left[\frac{x^{\frac{a}{a-b}}}{x^{\frac{a}{a+b}}}\div\frac{x^{\frac{b}{b-a}}}{x^{\frac{b}{b+a}}}\right]^{a+b}$ reduces to

(A) 1 (B) –1 (C) 0 (D) None

27. On simplification $\left[\frac{x^{ab}}{x^{a^2+b^2}}\right]^{a+b}\times\left[\frac{x^{b^2+c^2}}{x^{bc}}\right]^{b+c}\times\left[\frac{x^{ca}}{x^{c^2+a^2}}\right]^{c+a}$ reduces to

(A) x^{-2a^3} (B) x^{2a^3} (C) $x^{-2(a^3+b^3+c^3)}$ (D) $x^{2(a^3+b^3+c^3)}$

28. On simplification $\left[\frac{x^{ab}}{x^{a^2+b^2}}\right]^{a+b}\times\left[\frac{x^{bc}}{x^{b^2+c^2}}\right]^{b+c}\times\left[\frac{x^{ca}}{x^{c^2+a^2}}\right]^{c+a}$ reduces to

(A) x^{-2a^3} (B) x^{2a^3} (C) $x^{-2(a^3+b^3+c^3)}$ (D) $x^{2(a^3+b^3+c^3)}$

29. On simplification $\left(\frac{m^x}{m^y}\right)^{x+y}\times\left(\frac{m^y}{m^z}\right)^{y+z}\div3\left(m^xm^z\right)^{x-z}\ m$ reduces to

(A) 3 (B) –3 (C) $-\frac{1}{3}$ (D) $\frac{1}{3}$

30. The value of $\frac{1}{1+a^{y-x}}+\frac{1}{1+a^{x-y}}$ is given by

(A) –1 (B) 0 (C) 1 (D) None

31. If $xyz=1$ then the value of $\frac{1}{1+x+y^{-1}}+\frac{1}{1+x+z^{-1}}+\frac{1}{1+z+x^{-1}}$ is

(A) 1 (B) 0 (C) 2 (D) None

32. If $2^a=3^b=(12)^c$ then $\frac{1}{c}-\frac{1}{b}-\frac{2}{a}$ reduces to

(A) 1 (B) 0 (C) 2 (D) None

33. If $2^a=3^b=6^{-c}$ then the value of $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$ reduce to

(A) 0 (B) 2 (C) 3 (D) 1

34. If $3^a=5^b=(75)^c$ then the value of $ab-c(2a+b)$ reduces to

(A) 1 (B) 0 (C) 3 (D) 5

35. If $2^a=3^b=(12)^c$ then the value of $ab-c(a+2b)$ reduces to

(A) 0 (B) 1 (C) 2 (D) 3

36. If $2^a=4^b=8^c$ and $abc=288$ then the value $\frac{1}{2a}+\frac{1}{4b}+\frac{1}{8c}$ is given by

(A) $\frac{1}{8}$ (B) $-\frac{1}{8}$ (C) $\frac{11}{96}$ (D) $-\frac{11}{96}$

37. If $a^p=b^q=c^r=d^s$ and ab = cd then the value of $\frac{1}{p}+\frac{1}{q}-\frac{1}{r}-\frac{1}{s}$ reduces to

(A) $1/a$ (B) $1/b$ (C) 0 (D) 1

38. If $a^b=b^a$ then the value of $\left(\frac{a}{b}\right)^{\frac{a}{b}}-a^{\frac{a}{b}-1}$ reduces to

(A) a (B) b (C) 0 (D) None

39. If $m=b^x, n=b^y$ and $(m^y n^x)=b^2$ the value of *xy* is given by

(A) –1 (B) 0 (C) 1 (D) None

40. If $a=xy^{m-1}$ $b=xy^{n-1}$ $c=xy^{p-1}$ then the value of $a^{n-p}\times b^{p-m}\times c^{m-n}$ reduces to
(A) 1 (B) –1 (C) 0 (D) None

41. If $a=x^{n+p}y^{m}$ $b=x^{p+m}y^{n}$ $c=x^{m+n}y^{p}$ then the value of $a^{n-p}\times b^{p-m}\times c^{m-n}$ reduces to
(A) 0 (B) 1 (C) –1 (D) None

42. If $a=\sqrt[3]{\sqrt{2}+1}-\sqrt[3]{\sqrt{2}-1}$ then the value of a^3+3a-2 is
(A) 3 (B) 0 (C) 2 (D) 1

43. If $a=x^{1/3}+x^{-1/3}$ then a^3-3a is
(A) $x+x^{-1}$ (B) $x-x^{-1}$ (C) $2x$ (D) 0

44. If $a=3^{1/4}+3^{-1/4}$ and $b=3^{1/4}-3^{-1/4}$ then the value of $3\left(a^2+b^2\right)^2$ is
(A) 67 (B) 65 (C) 64 (D) 62

45. If $x=\sqrt{3}+\frac{1}{\sqrt{3}}$ and $y=\sqrt{3}-\frac{1}{\sqrt{3}}$ then x^2-y^2 is
(A) 5 (B) $\sqrt{3}$ (C) $\frac{1}{\sqrt{3}}$ (D) 4

46. If $a=\frac{4\sqrt{6}}{\sqrt{2}+\sqrt{3}}$ then the value of $\frac{a+2\sqrt{2}}{a-2\sqrt{2}}+\frac{a+2\sqrt{3}}{a-2\sqrt{3}}$ is given by
(A) 1 (B) –1 (C) 2 (D) –2

47. If $P+\sqrt{3}Q+\sqrt{5}R+\sqrt{15}S=\frac{1}{1+\sqrt{3}+\sqrt{5}}$ then the value of P is
(A) 7/11 (B) 3/11 (C) -1/11 (D) -2/11

48. If $a=3+2\sqrt{2}$ then the value of $a^{1/2}+a^{-1/2}$ is
(A) $\sqrt{2}$ (B) $-\sqrt{2}$ (C) $2\sqrt{2}$ (D) $-2\sqrt{2}$

49. If $a=3+2\sqrt{2}$ then the value of $a^{1/2}-a^{-1/2}$ is
(A) $2\sqrt{2}$ (B) 2 (C) $2\sqrt{2}$ (D) $-2\sqrt{2}$

50. If $a=\frac{1}{2}\left(5-\sqrt{21}\right)$ then the value of $a^3+a^{-3}-5a^2-5a^{-2}+a+a^{-1}$ is
(A) 0 (B) 1 (C) 5 (D) –1

51. If $a=\sqrt{\frac{7+4\sqrt{3}}{7-4\sqrt{3}}}$ then the value of $\left[a\left(a-14\right)\right]^2$ is
(A) 14 (B) 7 (C) 2 (D) 1

52. If $a=3-\sqrt{5}$ then the value of $a^4-a^3-20a^2-16a+24$ is
(A) 10 (B) 14 (C) 0 (D) 15

53. If $a = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ then the value of $2a^4 - 21a^3 + 12a^2 - a + 1$ is

(A) 21 (B) 1 (C) 12 (D) None

54. The square root of $3+\sqrt{5}$ is

(A) $\sqrt{5/2} + \sqrt{1/2}$ (B) $-\left(\sqrt{5/2} + \sqrt{1/2}\right)$ (C) Both the above (D) None

55. If $x = \sqrt{2-\sqrt{2-\sqrt{2}}}$ …∝ the value of X is given by

(A) –2 (B) 1 (C) 2 (D) 0

56. If $a = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ $b = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ then the value of $a + b$ is

(A) 10 (B) 100 (C) 98 (D) 99

57. If $a = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ $b = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ then the value of $a^2 + b^2$ is

(A) 10 (B) 100 (C) 98 (D) 99

58. If $a = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ $b = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ then the value of $\frac{1}{a^2} + \frac{1}{b^2}$ is

(A) 10 (B) 100 (C) 98 (D) 99

59. The square root of $x + \sqrt{x^2-y^2}$ is given by

(A) $\frac{1}{2}\left[\sqrt{x+y} + \sqrt{x-y}\right]$ (B) $\frac{1}{2}\left[\sqrt{x+y} - \sqrt{x-y}\right]$ (C) $\left[\sqrt{x+y} + \sqrt{x-y}\right]$ (D) $\left[\sqrt{x+y} - \sqrt{x-y}\right]$

60. The square root of $11-\sqrt{120}$ is given by

(A) $\sqrt{6}+\sqrt{5}$ (B) $\sqrt{6}-\sqrt{5}$ (C) $2\sqrt{3}-3\sqrt{2}$ (D) $2\sqrt{3}+3\sqrt{2}$

61. log(1+2+3) is exactly equal to

(A) log 1 + log 2 + log 3 (B) log (1×2×3) (C) Both the above (D) None

62. The logarithm of 21952 to the base of $2\sqrt{7}$ and 19683 to the base of $3\sqrt{3}$ are

(A) Equal (B) Not equal (C) Have a difference of 2269 (D) None

63. The value of is $4\log\frac{8}{25} - 3\log\frac{16}{125} - \log 5$ is

(A) 0 (B) 1 (C) 2 (D) –1

64. $a^{\log b - \log c} \times b^{\log c - \log a} \times c^{\log a - \log b}$ has a value of

(A) 1 (B) 0 (C) –1 (D) None

65. $\frac{1}{\log_{ab}(abc)} + \frac{1}{\log_{bc}(abc)} + \frac{1}{\log_{ca}(abc)}$ is equal to

(A) 0 (B) 1 (C) 2 (D) –1

66. $\frac{1}{1+\log_a(bc)} + \frac{1}{1+\log_b(ca)} + \frac{1}{1+\log_c(ab)}$ is equal to

(A) 0 (B) 1 (C) 3 (D) –1

67. $\frac{1}{\log_{a/b}(x)} + \frac{1}{\log_{b/c}(x)} + \frac{1}{\log_{c/a}(x)}$ is equal to

(A) 0 (B) 1 (C) 3 (D) –1

68. $\log_b(a).\log_c(b).\log_a(c)$ is equal to

(A) 0 (B) 1 (C) –1 (D) None

69. $\log_b\left(a^{\frac{1}{2}}\right).\log_c(b^3).\log_a(c^{\frac{2}{3}})$ is equal to

(A) 0 (B) 1 (C) –1 (D) None

70. The value of is $a^{\log b/c}.b^{\log c/a}.c^{\log a/b}$

(A) 0 (B) 1 (C) –1 (D) None

71. The value of $(bc)^{\log b/c}.(ca)^{\log c/a}.(ab)^{\log a/b}$ is

(A) 0 (B) 1 (C) –1 (D) None

72. The value of $\log\frac{a^n}{b^n} + \log\frac{b^n}{c^n} + \log\frac{c^n}{a^n}$ is

(A) 0 (B) 1 (C) –1 (D) None

73. The value of $\log\frac{a^2}{bc} + \log\frac{b^2}{ca} + \log\frac{c^2}{ab}$ is

(A) 0 (B) 1 (C) –1 (D) None

74. $\log(a^9) + \log a = 10$ if the value of *a* is given by

(A) 0 (B) 10 (C) –1 (D) None

75. If $\frac{\log a}{y\text{-}z} = \frac{\log b}{z\text{-}x} = \frac{\log c}{x\text{-}y}$ the value of *abc* is

(A) 0 (B) 1 (C) –1 (D) None

76. If $\frac{\log a}{y-z} = \frac{\log b}{z-x} = \frac{\log c}{x-y}$ the value of $a^{y+z}.b^{z+x}.c^{x+y}$ is given by

(A) 0 (B) 1 (C) –1 (D) None

77. If $\log a = \frac{1}{2}\log b = \frac{1}{5}\log c$ the value of $a^4b^3c^{-2}$ is

(A) 0 (B) 1 (C) –1 (D) None

78. If $\frac{1}{2}\log a = \frac{1}{3}\log b = \frac{1}{5}\log c$ the value of $a^4 - bc$ is

(A) 0 (B) 1 (C) –1 (D) None

79. If $\frac{1}{4}\log_2 a = \frac{1}{6}\log_2 b = -\frac{1}{24}\log_2 c$ the value of a^3b^2c is

(A) 0 (B) 1 (C) –1 (D) None

80. The value of $\frac{1}{\log_a(ab)} + \frac{1}{\log_b(ab)}$ is

(A) 0 (B) 1 (C) –1 (D) None

81. If $\frac{1}{\log_a t} + \frac{1}{\log_b t} + \frac{1}{\log_c t} = \frac{1}{\log_z t}$ then the value if z is given by

(A) abc (B) $a + b + c$ (C) $a(b + c)$ (D) $(a + b)c$

82. If $l = 1+\log_a bc$, $m = 1 + \log_b ca$, , $n = 1 + \log_c ab$ then the value of $\frac{1}{l} + \frac{1}{m} + \frac{1}{n} - 1$ is

(A) 0 (B) 1 (C) –1 (D) 3

83. If $a = b^2 = c^3 = d^4$ then the value of $\log_a(abcd)$ is

(A) $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$ (B) $1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!}$ C) 1+2+3+4 (D) None

84. The sum of the series $\log_a b + \log_{a^2} b^2 + \log_{a^3} b^3 + ...\log_{a^n} b^n$ is given by

(A) $\log_a b^n$ (B) $\log_{a^n} b^n$ (C) $\log_{a^n} b^n$ (D) None

85. $\frac{1}{a^{\log_b a}}$ has a value of

(A) a (B) b (C) (a + b) (D) None

86. The value of the following expression $a^{\log_a b.\log_b c.\log_c d.\log_d t}$ is given by

(A) t (B) abcdt (C) (a + b +c + d + t) (D) None

87. For any three consecutive integers x y z the equation $\log(1+xz) - 2\log y = 0$ is

(A) True (B) False (C) Sometimes true

(D) cannot be determined in the cases of variables with cyclic order.

88. If $\log \frac{a+b}{3} = \frac{1}{2}(\log a + \log b)$ then the value of $\frac{a}{b}+\frac{b}{a}$ is

(A) 2 (B) 5 (C) 7 (D) 3

89. If $a^2 + b^2 = 7ab$ then the value of is $\log\frac{a+b}{3} - \frac{\log a}{2} - \frac{\log b}{2}$

(A) 0 (B) 1 (C) –1 (D) 7

90. If $a^3 + b^3 = 0$ then the value of $\log(a+b) - \frac{1}{2}(\log a + \log b + \log 3)$ is equal to

(A) 0 (B) 1 (C) –1 (D) 3

91. If $x = \log_a bc \quad y = \log_b ca \quad z = \log_c ab$ then the value of $xyz - x - y - z$ is

(A) 0 (B) 1 (C) –1 (D) 2

92. On solving the equation $\log t + \log(t-3) = 1$ we get the value of t as

(A) 5 (B) 2 (C) 3 (D) 0

93. On solving the equation $\log_3[\log_2(\log_3 t)] = 1$ we get the value of t as

(A) 8 (B) 18 (C) 81 (D) 6561

94. On solving the equation $\log_{1/2}[\log_t(\log_4 32)] = 2$ we get the value of t as

(A) $5/2$ (B) $25/4$ (C) $625/16$ (D) None

95. If $(4.8)^x = (0.48)^y = 1{,}000$ then the value of $\frac{1}{x}-\frac{1}{y}$ is

(A) 3 (B) –3 (C) $\frac{1}{3}$ (D) $-\frac{1}{3}$

96. If $x^{2a-3}y^{2a} = x^{6-a}y^{5a}$ then the value of $a\log(x/y)$ is

(A) $3\log x$ (B) $\log x$ (C) $6\log x$ (D) $5\log x$

97. If $x = \frac{e^n - e^{-n}}{e^n + e^{-n}}$ then the value of n is

(A) $\frac{1}{2}\log_e\frac{1+x}{1-x}$ (B) $\log_e\frac{1+x}{1-x}$ (C) $\log_e\frac{1-x}{1+x}$ (D) $\frac{1}{2}\log_e\frac{1-x}{1+x}$

ANSWERS

1)	B	18)	B	35)	A	52)	C	69)	B	86)	A
2)	A	19)	C	36)	C	53)	B	70)	B	87)	A
3)	C	20)	B	37)	C	54)	C	71)	B	88)	C
4)	B	21)	D	38)	C	55)	B	72)	A	89)	A
5)	A	22)	B	39)	C	56)	A	73)	A	90)	A
6)	A	23)	B	40)	A	57)	C	74)	B	91)	D
7)	D	24)	B	41)	B	58)	C	75)	B	92)	A
8)	A	25)	C	42)	B	59)	A	76)	B	93)	D
9)	A	26)	A	43)	A	60)	C	77)	B	94)	C
10)	A	27)	A	44)	C	61)	C	78)	A	95)	C
11)	C	28)	C	45)	D	62)	A	79)	B	96)	A
12)	A	29)	D	46)	C	63)	B	80)	B	97)	A
13)	B	30)	C	47)	A	64)	A	81)	A		
14)	A	31)	A	48)	C	65)	C	82)	A		
15)	C	32)	B	49)	B	66)	B	83)	A		
16)	C	33)	A	50)	A	67)	A	84)	A		
17)	D	34)	B	51)	D	68)	B	85)	B		