CHAPTER – 9

BASIC CONCEPTS OF DIFFERENTIAL AND INTEGRAL CALCULUS

LEARNING OBJECTIVES

After studying this chapter, you will be able to:

- Understand the basics of differentiation and integration;
- Know how to compute derivative of a function by the first principle, derivative of a: function by the application of formulae and higher order differentiation;
- Appreciate the various techniques of integration; and
- Understand the concept of definite integrals of functions and its properties.

INTRODUCTION TO DIFFERENTIAL AND INTEGRAL CALCULUS (EXCLUDING TRIGONOMETRIC FUNCTIONS)

(A) DIFFERENTIAL CALCULUS

9.A.1 INTRODUCTION

Differentiation is one of the most important fundamental operations in calculus. Its theory primarily depends on the idea of limit and continuity of function.

To express the rate of change in any function we introduce concept of derivative which involves a very small change in the dependent variable with reference to a very small change in independent variable.

Thus differentiation is the process of finding the derivative of a continuous function. It is defined as the limiting value of the ratio of the change (increment) in the function corresponding to a small change (increment) in the independent variable (argument) as the later tends to zero.

9.A.2 DERIVATIVE OR DIFFERENTIAL COEFFICIENT

Let y = f(x) be a function. If h (or Δx) be the small increment in x and the corresponding increment in y or f(x) be $\Delta y = f(x+h) - f(x)$ then the derivative of f(x) is defined

as $\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ i.e.

$$= \lim_{\Delta x\to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

This is denoted as $f'(x)$ or dy/ dx or $\frac{d}{dx}f(x)$. The derivative of f(x) is also known as differential coefficient of f(x) with respect to x. This process of differentiation is called the first principle (or definition or abinitio).

Note: In the light of above discussion a function f (x) is said to differentiable at x = c if

$\lim_{h\to c} \frac{f(x)-f(c)}{x-c}$ exist which is called the differential coefficient of f(x) at x = c and is denoted by $f'(c)$ or $\left[\frac{dy}{dx}\right]_{x=c}$.

We will now study this with an example.

Consider the function $f(x) = x^2$.

By definition

$$\frac{d}{dx}f(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - x^2}{\Delta x} = \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 - x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} (2x + \Delta x) = 2x + 0 = 2x$$

Thus, derivative of f(x) exists for all values of x and equals 2x at any point x.

Examples of Differentiations from the 1st principle

i) $f(x) = c$, c being a constant.

Since c is constant we may write $f(x+h) = c$.

So $f(x+h) - f(x) = 0$

Hence $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{0}{h} = 0$

So $\frac{d(c)}{dx} = 0$

ii) Let $f(x) = x^n$; then $f(x+h) = (x+h)^n$

let $x+h = t$ or $h = t - x$ and as $h \to 0$, $t \to x$

Now $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \to 0} \frac{(x+h)^n - x^n}{h}$$

$$= \lim_{t \to x} (t^n - x^n) / (t - x) = nx^{n-1}$$

Hence $\frac{d}{dx}(x^n) = nx^{n-1}$

iii) $f(x) = e^x \therefore f(x+h) = e^{x+h}$

So $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \to 0} \frac{e^{x+h} - e^x}{h} = \lim_{h \to 0} \frac{e^x(e^h - 1)}{h}$$

$= e^x \lim_{h\to 0} \frac{e^h - 1}{h} \qquad = e^x.1$

Hence $\frac{d}{dx}(e^x) = e^x$

iv) Let $f(x) = a^x$ then $f(x+h) = a^{x+h}$

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{a^{x+h}-a^x}{h} = \lim_{h\to 0} \left[\frac{a^x(a^h-1)}{h}\right]$

$= a^x \lim_{h\to 0} \frac{a^h-1}{h}$

$= a^x \log_e a$

Thus $\frac{d}{dx}(a^x) = a^x \log_e a$

v) Let $f(x) = \sqrt{x}$. Then $f(x+h) = \sqrt{x+h}$

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h}$

$= \lim_{h\to 0} \frac{(\sqrt{x+h}-\sqrt{x})(\sqrt{x+h}+\sqrt{x})}{h(\sqrt{x+h}+\sqrt{x})}$

$= \lim_{h\to 0} \frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x}}$

$= \lim_{h\to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$

Thus $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

vi) $f(x) = \log x \therefore f(x+h) = \log(x+h)$

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} \frac{\log(x+h)-\log x}{h}$

$$= \lim_{h\to 0} \frac{\log\left(\frac{x+h}{x}\right)}{h}$$

$$= \lim_{h\to 0} \frac{1}{h}\left\{\log\left(1+\frac{h}{x}\right)\right\}$$

Let $\frac{h}{x} = t$ i.e. $h=tx$ and as $h \to 0$, $t \to 0$

$\therefore f'(x) = \lim_{t\to 0} \frac{1}{tx}\log(1+t) = \frac{1}{x} \lim_{t\to 0} \frac{1}{t}\log(1+t) = \frac{1}{x}\times 1 = \frac{1}{x}$, since $\lim_{t\to 0} \frac{\log(1+t)}{t} = 1$

Thus $\frac{d}{dx}(\log x) = \frac{1}{x}$

9.A.3 SOME STANDARD RESULTS (FORMULAS)

(1) $\frac{d}{dx}(x^n) = nx^{n-1}$ (2) $\frac{d}{dx}(e^x) = e^x$ (3) $\frac{d}{dx}(a^x) = a^x \log_e a$

(4) $\frac{d}{dx}(\text{constant}) = 0$ (5) $\frac{d}{dx}(e^{ax}) = ae^{ax}$ (5) $\frac{d}{dx}(\log x) = \frac{1}{x}$

Note: $\frac{d}{dx}\{c\, f(x)\} = cf'(x)$ c being constant.

In brief we may write below the above functions and their derivatives:

Table: Few functions and their derivatives

Function	Derivative of the function
$f(x)$	$f'(x)$
x^n	$n x^{n-1}$
e^{ax}	ae^{ax}
$\log x$	$1/x$
a^x	$a^x \log_e a$
c (a constant)	0

We also tabulate the basic laws of differentiation.

Table: Basic Laws for Differentiation

Function	Derivative of the function
(i) h(x) = c.f(x) where c is any real constant (Scalar multiple of a function)	$\frac{d}{dx}\{h(x)\} = c.\frac{d}{dx}\{f(x)\}$
(ii) h(x) = f(x) ± g(x) (Sum/Difference of function)	$\frac{d}{dx}\{h(x)\} = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}\{g(x)\}$
(iii) h(x) = f(x). g(x) (Product of functions)	$\frac{d}{dx}\{h(x)\} = f(x)\frac{d}{dx}\{g(x)\}+g(x)\frac{d}{dx}\{f(x)\}$
(iv) $h(x) = \frac{f(x)}{g(x)}$ (quotient of function)	$\frac{d}{dx}\{h(x)\} = \frac{g(x)\frac{d}{dx}\{f(x)\}-f(x)\frac{d}{dx}\{g(x)\}}{\{g(x)\}^2}$
(v) h(x) = f{g(x)}	$\frac{d}{dx}\{h(x)\} = \frac{d}{dz}f(z).\frac{dz}{dx}$, where z = g(x)

It should be noted here even though in (ii) (iii) (iv) and (v) we have considered two functions f and g, it can be extended to more than two functions by taking two by two.

Example: Differentiate each of the following functions with respect to x:

(a) $3x^2 + 5x - 2$ (b) $a^x + x^a + a^a$ (c) $\frac{1}{3}x^3-5x^2+6x-2\log x+3$

(d) $e^x \log x$ (e) $2^x x^5$ (f) $\frac{x^2}{e^x}$

(g) $e^x / \log x$ (h) $2^x. \log x$ (i) $\frac{2x}{3x^3+7}$

Solution: (a) Let $y = f(x) = 3x^2 + 5x - 2$

$$\frac{dy}{dx} = 3\frac{d}{dx}(x)^2 + 5\frac{d}{dx}(x) - \frac{d}{dx}(2)$$

$= 3 \times 2x + 5.1 - 0 = 6x + 5$

(b) Let $h(x) = a^x + x^a + a^a$

$$\frac{d}{dx}\{h(x)\} = \frac{d}{dx}(a^x + x^a + a^a) = \frac{d}{dx}(a^x) + \frac{d}{dx}(x^a) + \frac{d}{dx}(a^a),\ a^a \text{ is a constant}$$

$= a^x \log a + ax^{a-1} + 0 = a^x \log a + ax^{a-1}.$

(c) Let $f(x) = \frac{1}{3}x^3 - 5x^2 + 6x - 2\log x + 3$ $\therefore$ $\frac{d}{dx}\{f(x)\} = \frac{d}{dx}\left(\frac{1}{3}x^3 - 5x^2 + 6x - 2\log x + 3\right)$

$= \frac{1}{3}.3x^2 - 5.2x + 6.1 - 2.\frac{1}{x} + 0 \quad = x^2 - 10x + 6 - \frac{2}{x}.$

(d) Let $y = e^x \log x$

$\frac{dy}{dx} = e^x \cdot \frac{d}{dx}(\log x) + \log x \cdot \frac{d}{dx}(e^x)$ (Product rule)

$= \frac{e^x}{x} + e^x \log x = \frac{e^x}{x}(1 + x \log x)$

So $\frac{dy}{dx} = \frac{e^x}{x}(1 + x \log x)$

(e) $y = 2^x x^5$

$\frac{dy}{dx} = x^5 \frac{d}{dx}(2^x) + 2^x \frac{d}{dx}(x^5)$ (Product Rule)

$= x^5 2^x \log_e 2 + 5.\ 2^x x^4$

(f) let $y = \frac{x^2}{e^x}$

$\frac{dy}{dx} = \frac{e^x \frac{8}{dx}(x^2) - x^2 \frac{d}{dx}(e^x)}{(e^x)^2}$ (Quotient Rule)

$= \frac{2xe^x - x^2 e^x}{(e^x)^2} = \frac{x(2-x)}{e^x}$

(g) Let $y = e^x / \log x$

so $\frac{dy}{dx} = \frac{(\log x)\frac{d}{dx}(e^x) - e^x \frac{d}{dx}(\log x)}{(\log x)^2}$ (Quotient Rule)

$= \frac{e^x \log x - e^x/x}{(\log x)^2} = \frac{e^x\ x \log x - e^x}{x(\log x)^2}$

So $\frac{dy}{dx} = \frac{e^x(x \log x - 1)}{x(\log x)^2}$

(h) Let $h(x) = 2^x . \log x$

The given function h(x) is appearing here as product of two functions

$f(x) = 2^x$ and $g(x) = \log x$.

$$= \frac{d}{dx}\{h(x)\} = \frac{d}{dx}(2^x . \log x) = 2^x \frac{d}{dx}(\log x) + \log x \frac{d}{dx}(2^x).$$

$$2^x \times \frac{1}{x} + \log x.(2^x \log 2) = \frac{2^x}{x} + 2^x \log 2 \log x$$

(i) Let $h(x) = \frac{2x}{3x^3+7}$ [Given function appears as the quotient of two functions]

$f(x) = 2x$ and $g(x) = 3x^3 + 7$

$$\frac{d}{dx}\{h(x)\} = \frac{(3x^3+7)\frac{d}{dx}(2x) - 2x\frac{d}{dx}(3x^3+7)}{(3x^3+7)^2} = \frac{(3x^3+7).2-2x.(9x^2+0)}{(3x^3+7)^2}$$

$$= \frac{2\left\{(3x^3+7)-9x^3\right\}}{(3x^3+7)^2} = \frac{2(7-6x^3)}{(3x^3+7)^2}.$$

9.A.4 DERIVATIVE OF A FUNCTION OF FUNCTION

If $y = f[h(x)]$ then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = f'(u) \times h'(x)$ where $u = h(x)$

Example: Differentiate $\log(1 + x^2)$ wrt. x

Solution: Let $y = \log(1 + x^2) = \log t$ when $t = 1 + x^2$

$$\frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx} = \frac{1}{t} \times (0+2x) = \frac{2x}{t} = \frac{2x}{(1+x^2)}$$

This is an example of derivative of function of a function and the rule is called Chain Rule.

9.A.5 IMPLICIT FUNCTIONS

A function in the form $f(x, y) = 0$. For example $x^2y^2 + 3xy + y = 0$ where y cannot be directly defined as a function of x is called an implicit function of x.

In case of implicit functions if y be a differentiable function of x, no attempt is required to express y as an explicit function of x for finding out $\frac{dy}{dx}$. In such case differentiation of both sides with respect of x and substitution of $\frac{dy}{dx} = y_1$ gives the result. Thereafter y_1 may be obtained by solving the resulting equation.

Example: Find $\frac{dy}{dx}$ for $x^2y^2 + 3xy + y = 0$

Solution: $x^2y^2 + 3xy + y = 0$

Differentiating with respect to x we see

$$x^2 \frac{d}{dx}(y^2) + y^2 \frac{d}{dx}(x^2) + 3x \frac{d(y)}{dx} y + 3y \frac{d}{dx}(x) + \frac{dy}{dx} = 0$$

or $\quad 2yx^2 \frac{dy}{dx} + 2xy^2 + 3x \frac{dy}{dx} + 3y \frac{d(x)}{dx} + \frac{dy}{dx} = 0, \frac{d}{dx}(x) = 1, \frac{d(y^2)}{dx} = 2y\frac{dy}{dx}$ (chain rule)

or $\quad (2yx^2 + 3x + 1) \frac{dy}{dx} + 2xy^2 + 3y = 0$

or $\quad \frac{dy}{dx} = -\frac{(2xy^2 + 3y)}{(2x^2y + 3x + 1)}$

This is the procedure for differentiation of Implicit Function.

9.A.6 PARAMETRIC EQUATION

When both the variables x and y are expressed in terms of a parameter (a third variable), the involved equations are called parametric equations.

For the parametric equations $x = f(t)$ and $y = h(t)$ the differential coefficient $\frac{dy}{dx}$ is obtained by using $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} . = \frac{dy}{dt} \cdot \frac{dt}{dx}$

Example: Find $\frac{dy}{dx}$ if $x = at^3$, $y = a / t^3$

Solution : $\frac{dx}{dt} = 3at^2; \quad \frac{dy}{dt} = -3a/t^4$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{-3a}{t^4} \times \frac{1}{3at^2} = \frac{-1}{t^6}$$

This is the procedure for differentiation of parametric functions.

9.A.7 LOGARITHMIC DIFFERENTIATION

The process of finding out derivative by taking logarithm in the first instance is called logarithmic differentiation. The procedure is convenient to adopt when the function to be differentiated involves a function in its power or when the function is the product of number of functions.

Example: Differentiate x^x w.r.t. x

Solution: let $y = x^x$

Taking logarithm,

log y = x log x

Differentiating with respect to x,

$$\frac{1}{y}\frac{dy}{dx} = \log x + \frac{x}{x} = 1 + \log x$$

or $\frac{dy}{dx} = y\,(1 + \log x) = x^x\,(1 + \log x)$

This procedure is called logarithmic differentiation.

9.A.8 SOME MORE EXAMPLES

(1) If $y = \sqrt{\frac{1-x}{1+x}}$ show that $(1 - x^2)\frac{dy}{dx} + y = 0.$

Solution: Taking logarithm, we may write $\log y = \frac{1}{2}\{\log(1 - x) - \log(1 + x)\}$

Differentiating throughout we have

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}\{\log(1 - x) - \log(1 + x)\} = \frac{1}{2}\left(\frac{-1}{1-x} - \frac{1}{1+x}\right) = -\frac{1}{1-x^2}$$

By cross–multiplication $(1 - x^2)\frac{dy}{dx} = -y$

Transposing $(1 - x^2)\frac{dy}{dx} + y = 0.$

(2) Differentiate the following w.r.t. x:

(a) $\log\left(x + \sqrt{x^2 + a^2}\right)$

(b) $\log\left(\sqrt{x-a} + \sqrt{x-b}\right)$.

Solution: (a) $y = \log\left(x + \sqrt{x^2 + a^2}\right)$

$$\frac{dy}{dx} = \frac{1}{(x+\sqrt{x^2+a^2})}\left(1 + \frac{1}{2\sqrt{x^2+a^2}}(2x)\right)$$

$$= \frac{1}{(x+\sqrt{x^2+a^2})} + \frac{x}{(x+\sqrt{x^2+a^2})\sqrt{x^2+a^2}}$$

$$= \frac{(x+\sqrt{x^2+a^2})}{(x+\sqrt{x^2+a^2})\ \sqrt{x^2+a^2}} = \frac{1}{\sqrt{x^2+a^2}}$$

(b) Let $y = \log(\sqrt{x-a}+\sqrt{x-b})$

or $$\frac{dy}{dx} = \frac{1}{\sqrt{x-a}+\sqrt{x-b}}\left(\frac{1}{2\sqrt{x-a}}+\frac{1}{2\sqrt{x-b}}\right) = \frac{(\sqrt{x-b}+\sqrt{x-a})}{(\sqrt{x-a}+\sqrt{x-b})\ 2\sqrt{x-a}\sqrt{x-b}}.$$

$$= \frac{1}{2\sqrt{x-a}\sqrt{x-b}}$$

(3) If $x^m y^n = (x+y)^{m+n}$ prove that $\frac{dy}{dx} = \frac{y}{x}$

Solution : $x^m y^n = (x+y)^{m+n}$

Taking log on both sides

$\log x^m y^n = (m+n) \log (x + y)$
or $m \log x + n \log y = (m+n) \log (x+y)$

so $$\frac{m}{x}+\frac{n}{y}\frac{dy}{dx} = \frac{(m+n)}{(x+y)}\left(1+\frac{dy}{dx}\right)$$

or $$\left(\frac{n}{y}-\frac{m+n}{x+y}\right)\frac{dy}{dx} = \frac{m+n}{(x+y} = \frac{m}{x}$$

or $$\frac{(nx+ny-my-ny)}{y(x+y)}\frac{dy}{dx} = \frac{mx+nx-mx-my}{x(x+y)}$$

or $$\frac{(nx-my)}{y}\frac{dy}{dx} = \frac{nx-my}{x}$$

or $\frac{dy}{dx} = \frac{y}{x}$ Proved.

(4) If $x^y = e^{x-y}$ Prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$

Solution : $x^y = e^{x-y}$

So $y \log x = (x - y) \log e$

or $y \log x = (x - y)$ ....(a)

Differentiating w.r.t. x we get

$$\frac{y}{x} + \log x \frac{dy}{dx} = 1 - \frac{dy}{dx}$$

or $(1 + \log x)\frac{dy}{dx} = 1 - \frac{y}{x}$

or $\frac{dy}{dx} = \frac{(x-y)}{x(1+\log x)}$, substituting x–y = log x, from (a) we have

or $\frac{dy}{dx} = \frac{y(\log x)}{x(1+\log x)}$ ……………….. (b)

From (a) $y(1 + \log x) = x$

or $\frac{y}{x} = \frac{1}{(1+\log x)}$

From (b) $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$

9.A.9 BASIC IDEA ABOUT HIGHER ORDER DIFFERENTIATION

Let $y = f(x) = x^4 + 5x^3 + 2x^2 + 9$

$\frac{dy}{dx} = \frac{d}{dx}f(x) = 4x^3 + 15x^2 + 4x = f'(x)$

Since f(x) is a function of x it can be differentiated again.

Thus $\frac{d}{dx}\left(\frac{dy}{dx}\right) = f''(x) = \frac{d}{dx}(4x^3 + 15x^2 + 4x) = 12x^2 + 30x + 4$

$\frac{d}{dx}\left(\frac{dy}{dx}\right)$ is written as $\frac{d^2y}{dx^2}$ (read as d square y by dx square) and is called the second derivative of y with respect to x while $\frac{dy}{dx}$ is called the first derivative. Again the second derivative here being a function of x can be differentiated again and $\frac{d}{dx}\left(\frac{dy}{dx}\right)$

$= f'''(x) = 24x + 30.$

Example: If $y = ae^{mx} + be^{-mx}$ prove that = m^2y.

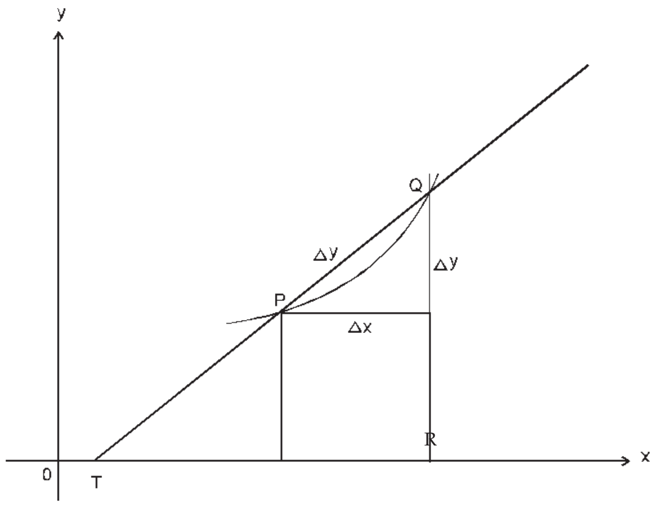
Solution: $\frac{dy}{dx} = \frac{d}{dx}(ae^{mx} + be^{-mx}) = ame^{mx} - bme^{-mx}$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(ame^{mx} - bme^{-mx})$

$= am^2e^{mx} + bm^2e^{-mx} = m^2(ae^{mx} + be^{-mx}) = m^2y.$

9.A.10 GEOMETRIC INTERPRETATION OF THE DERIVATIVE

Let f(x) represent the curve in the Fig. We take two adjacent pair's P and Q on the curve



Let f(x) represent the curve in the fig. We take two adjacent points P and Q on the curve whose coordinates are (x y) and (x + Δx, y+Δy) respectively. The slope of the chord TPQ is given by Δy/Δx when $\Delta x \to 0$, $Q \to P$. TPQ becomes the tangent at P and $\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$

The derivative of f(x) at a point x represents the slope (or sometime called the gradient of the curve) of the tangent to the curve y = f(x) at the point x. If $\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$ exists for a particular point say x =a and f(a) is finite we say the function is differentiable at x = a and continuous at that point.

Example : Find the gradient of the curve $y = 3x^2 - 5x + 4$ at the point (1, 2).

Solution : $y = 3x^2 - 5x + 4 \quad \therefore \frac{dy}{dx} = 6x - 5$

so $[dy/dx]_{x=1\ y=2} = 6.1 - 5 = 6 - 5 = 1$

Thus the gradient of the curve at (1, 2) is 1.

Exercise 9 (A)

Choose the most appropriate option (a) (b) (c) or (d)

1. The gradient of the curve $y = 2x^3 - 3x^2 - 12x + 8$ at $x = 0$ is

   a) –12 b) 12 c) 0 d) none of these

2. The gradient of the curve $y = 2x^3 - 5x^2 - 3x$ at $x = 0$ is

   a) 3 b) –3 c) 1/3 d) none of these

3. The derivative of $y = \sqrt{x+1}$ is

   a) $1 / \sqrt{x+1}$ b) $-1 / \sqrt{x+1}$ c) $1 / 2\sqrt{x+1}$ d) none of these

4. If $f(x) = e^{ax^2+bx+c}$ the $f'(x)$ is

   a) e^{ax^2+bx+c} b) $e^{ax^2+bx+c}(2ax+b)$ c) $2ax+b$ d) none of these

5. If $f(x) = \dfrac{x^2+1}{x^2-1}$ then $f'(x)$ is

   a) $-4x / (x^2-1)^2$ b) $4x / (x^2-1)^2$ c) $x / (x^2-1)^2$ d) none of these

6. If $y = x(x-1)(x-2)$ then $\dfrac{dy}{dx}$ is

   a) $3x^2 - 6x + 2$ b) $-6x + 2$ c) $3x^2 + 2$ d) none of these

7. The gradient of the curve $y - xy + 2px + 3qy = 0$ at the point (3, 2) is $\dfrac{-2}{3}$. The values of p and q are

   a) (1/2, 1/2) b) (2, 2) c) (–1/2, –1/2) d) (1/2, 1/6)

8. The curve $y^2 = ux^3 + v$ passes through the point P(2, 3) and $\dfrac{dy}{dx} = 4$ at P. The values of u and v are

   a) (u = 2, v = 7) b) (u = 2, v =– 7) c) (u = –2, v =– 7) d) (0, -1)

9. The gradient of the curve $y + px + qy = 0$ at (1, 1) is $^1/_2$. The values of p and q are

   a) (–1, 1) b) (2, –1) c) (1, 2) d) (0, -1)

10. If $xy = 1$ then $y^2 + dy/dx$ is equal to

    a) 1 b) 0 c) –1 d) none of these

11. The derivative of the function $\sqrt{x+\sqrt{x}}$ is

    a) $\dfrac{1}{2\sqrt{x+\sqrt{x}}}$ b) $1+\dfrac{1}{2\sqrt{x}}$ c) $\dfrac{1}{2\sqrt{x+\sqrt{x}}}\left(1+\dfrac{1}{2\sqrt{x}}\right)$ d) none of these

12. Given $e^{-xy} - 4xy = 0$, $\frac{dy}{dx}$ can be proved to be

a) $-y/x$ b) y/x c) x/y d) none of these

13. If $\frac{x^2}{a^2} - \frac{y^2}{a^2} = 1$, $\frac{dy}{dx}$ can be expressed as

a) $\frac{x}{a}$ b) $\frac{x}{\sqrt{x^2-a^2}}$ c) $\frac{1}{\sqrt{\frac{x^2}{a^2}-1}}$ d) none of these

14. If $\log(x/y) = x + y$, $\frac{dy}{dx}$ may be found to be

a) $\frac{y(1-x)}{x(1+y)}$ b) $\frac{y}{x}$ c) $\frac{1-x}{1+y}$ d) none of these

15. If $f(x, y) = x^3 + y^3 - 3axy = 0$, $\frac{dy}{dx}$ can be found out as

a) $\frac{ay-x^2}{y^2+ax}$ b) $\frac{ay-x^2}{y^2-ax}$ c) $\frac{ay+x^2}{y^2+ax}$ d) none of these

16. Given $x = at^2$, $y = 2at$; $\frac{dy}{dx}$ is calculated as

a) t b) $-1/t$ c) $1/t$ d) none of these

17. Given $x = 2t + 5$, $y = t^2 - 2$; $\frac{dy}{dx}$ is calculated as

a) t b) $-1/t$ c) $1/t$ d) none of these

18. If $y = \frac{1}{\sqrt{x}}$ then $\frac{dy}{dx}$ is equal to

a) $\frac{1}{2x\sqrt{x}}$ b) $\frac{-1}{x\sqrt{x}}$ c) $-\frac{1}{2x\sqrt{x}}$ d) none of these

19. If $x = 3t^2 - 1$, $y = t^3 - t$, then $\frac{dy}{dx}$ is equal to

a) $\frac{3t^2-1}{6t}$ b) $3t^2-1$ c) $\frac{3t\ -1}{6t}$ d) none of these

20. The slope of the tangent to the curve y = $\sqrt{4-x^2}$ at the point, where the ordinate and the abscissa are equal, is

a) –1 b) 1 c) 0 d) none of these

21. The slope of the tangent to the curve $y = x^2 - x$ at the point, where the line y = 2 cuts the curve in the Ist quadrant, is

a) 2 b) 3 c) –3 d) none of these

22. For the curve $x^2 + y^2 + 2gx + 2hy = 0$, the value of $\frac{dy}{dx}$ at (0, 0) is

a) -g/h b) g/h c) h/g d) none of these

23. If $y = \frac{e^{3x}-e^{2x}}{e^{3x}+e^{2x}}$, then $\frac{dy}{dx}$ is equal to

a) $2e^{5x}$ b) $1/(e^{5x} + e^{2x})^2$ c) $e^{5x}/(e^{5x} + e^{2x})$ d) none of these

24. If $x^y \cdot y^x = M$, where M is constant then $\frac{dy}{dx}$ is equal to

a) $\frac{-y}{x}$ b) $\frac{-y(y+x\log y)}{x(y\log x+x)}$ c) $\frac{y+x\log y}{y\log x+x}$ d) none of these

25. Given $x = t + t^{-1}$ and $y = t - t^{-1}$ the value of $\frac{dy}{dx}$ at t = 2 is

a) 3/5 b) –3/5 c) 5/3 d) none of these

26. If $x^3 - 2x^2 y^2 + 5x + y - 5 = 0$ then $\frac{dy}{dx}$ at x = 1, y = 1 is equal to

a) 4/3 b) – 4/3 c) 3/4 d) none of these

27. The derivative of $x^2 \log x$ is

a) 1+2log x b) x(1 + 2 log x) c) 2 log x d) none of these

28. The derivative of $\frac{3-5x}{3+5x}$ is

a) $30/(3+5x)^2$ b) $1/(3+5x)^2$ c) $-30/(3+5x)^2$ d) none of these

29. Let $y = \sqrt{2x} + 3^{2x}$ then $\frac{dy}{dx}$ is equal to

a) $(1/\sqrt{2x}) + 2.3^{2x} \log_e 3$ b) $1/\sqrt{2x}$

c) $2.3^{2x} \log_e 3$ d) none of these

30. The derivative of $\log\left[e^x\left\{\frac{x-2}{x+2}\right\}^{3/4}\right]$ is

a) $\frac{x^2+1}{x^2+4}$ b) $\frac{x^2-1}{x^2-4}$ c) $\frac{1}{x^2-4}$ d) none of these

31. The derivative of e^{3x^2-6x+2} is

a) $30(1-5x)^5$ b) $(1-5x)^5$ c) $6(x-1)e^{3x^2-6x+2}$ d) none of these

32. If $y=\frac{e^x+1}{e^x-1}$ then $\frac{dy}{dx}$ is equal to

a) $\frac{-2e^x}{(e^x-1)^2}$ b) $\frac{-2e^x}{(e^x-1)^2}$ c) $\frac{-2}{(e^x-1)^2}$ d) none of these

33. If $f(x)=\left\{\frac{(a+x)}{(1+x)}\right\}^{a+1+2x}$ the value of $f'(0)$ is

a) a^{a+1} b) $a^{a+1}\left[\frac{1-a^2}{a}+2\log a\right]$ c) $2\log a$ d) none of these

34. If $x = at^2$ $y = 2at$ then $\left[\frac{dy}{dx}\right]_{t=2}$ is equal to

a) 1/2 b) –2 c) –1/2 d) none of these

35. Let $f(x)=\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2$ then $f'(2)$ is equal to

a) 3/4 b) 1/2 c) 0 d) none of these

36. If $f(x) = x^2 - 6x+8$ then $f'(5) - f'(8)$ is equal to

a) $f'(2)$ b) $3f'(2)$ c) $2f'(2)$ d) none of these

37. If $y=\left(x+\sqrt{x^2+m^2}\right)^n$ then dy/dx is equal to

a) ny b) $ny/\sqrt{x^2+m^2}$ c) $-ny/\sqrt{x^2+m^2}$ d) none of these

38. If $y = +\sqrt{x/m} + \sqrt{m/x}$ then $2xy\ dy/dx - x/m + m/x$ is equal to

a) 0 b) 1 c) –1 d) none of these

39. If $y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots\ldots + \frac{x^n}{n} + \ldots\ldots\ldots\ldots$ then $\frac{dy}{dx} - y$ is proved to be

a) 1 b) –1 c) 0 d) none of these

40. If $f(x) = x^k$ and $f'(1) = 10$ the value of k is

a) 10 b) –10 c) 1/10 d) none of these

41. If $y = \sqrt{x^2+m^2}$ then $y\,y_1$ (where $y_1 = dy/dx$) is equal to

a) –x b) x c) 1/x d) none of these

42. If $y = e^x + e^{-x}$ then $\frac{dy}{dx} - \sqrt{y^2-4}$ is equal to

a) 1 b) –1 c) 0 d) none of these

43. The derivative of $(x^2–1)/x$ is

a) $1 + 1/x^2$ b) $1 – 1/x^2$ c) $1/x^2$ d) none of these

44. The differential coefficients of $(x^2 +1)/x$ is

a) $1 + 1/x^2$ b) $1 – 1/x^2$ c) $1/x^2$ d) none of these

45. If $y = e^{\sqrt{2x}}$ then $\frac{dy}{dx}$ is equal to ____________.

a) $\frac{e^{\sqrt{2x}}}{\sqrt{2x}}$ b) $e^{\sqrt{2x}}$ c) $\frac{e^{\sqrt{2x}}}{\sqrt{2x}}$ d) none of these

46. If $y = \sqrt{x}^{\sqrt{x}^{\cdots\infty}}$ then $\frac{dy}{dx}$ is equal to ____________.

a) $\frac{y^2}{2-y\log x}$ b) $\frac{y^2}{x(2-y\log x)}$ c) $\frac{y^2}{\log x}$ d) none of these

47. If $x = (1 – t^2)/(1 + t^2)$ $y = 2t/(1 + t^2)$ then dy/dx at t =1 is ____________.

a) 1/2 b) 1 c) 0 d) none of these

48. $f(x) = x^2/e^x$ then $f'(1)$ is equal to ____________.

a) – 1/e b) 1/e c) e d) none of these

49. If $y = (x + \sqrt{x^2-1})^m$ then $(x^2– 1)(dy/dx)^2 – m^2y^2$ is proved to be

a) –1 b) 1 c) 0 d) none of these

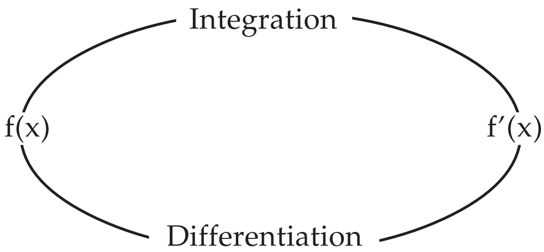
50. If $f(x) = \frac{4-2x}{2+3x+3x^2}$ then the values of x for which $f'(x) = 0$ is

a) $2\left(1 \pm \sqrt{\frac{5}{3}}\right)$ b) $(1 \pm \sqrt{3})$ c) 2 d) none of these

(B) INTEGRAL CALCULUS

9.B.1 INTEGRATION

Integration is the reverse process of differentiation.



we know

$$\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right)=\frac{(n+1)x^{n}}{(n+1)}$$

or $$\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right)=x^{n} \qquad \text{.....................(1)}$$

Itnegration is the inverse operation of differentiation and is denoted by the symbol $\int$.

Hence, from equation (1), it follows that

$$\int x^{n}\,dx=\frac{x^{n+1}}{n+1}$$

i.e. Integral of x^n with respect to variable x is equal to $\frac{x^{n+1}}{n+1}$

Thus if we differentiate $\frac{\left(x^{n+1}\right)}{n+1}$ we can get back x^n

Again if we differentiate $\frac{\left(x^{n+1}\right)}{n+1}$ + c and c being a constant, we get back the same x^n .

ii.e. $$\frac{d}{dx}\left[\frac{x^{n+1}}{n+1}+c\right]=x^{n}$$

Hence $\int x^{n}\,dx = \frac{\left(x^{n+1}\right)}{n+1}$ + c and this c is called the constant of integration.

Integral calculus was primarily invented to determine the area bounded by the curves dividing the entire area into infinite number of infinitesimal small areas and taking the sum of all these small areas.

9.B.2 BASIC FORMULAS

i) $\int x^{n}\,dx=\frac{x^{n+1}}{n+1}$ + c, $n\neq-1$ (If n=-1, $\frac{x^{n+1}}{n+1}=\frac{1}{0}$ which is not defined)

ii) $\int dx$ = x, since $\int$ 1dx = $\int x^{\circ}dx=\frac{x1}{1}$ = x

iii) $e^x\,dx = e^x + c$, since $\frac{d}{dx}e^x = e^x$

iv) $\int e^{ax}\,dx = \frac{e^{ax}}{a} + c$, since $\frac{d}{dx}\left(\frac{e^{ax}}{a}\right) = e^{ax}$

v) $\int \frac{dx}{x} = \log x + c$, since $\frac{d}{dx}\log x = \frac{1}{x}$

vi) $\int a^x\,dx = a^x / \log_e a + c$, since $\frac{d}{dn}\left(\frac{a^n}{\log^a_e}\right) = a^x$

Note: *In the answer for all integral sums we add +c (constant of integration) since the differentiation of constant is always zero.*

Elementary Rules:

$\int c\,f(x)\,dx = c\int f(x)\,dx$ where c is constant.

$\int \{f(x)\,dx \pm g(x)\}\,dx = \int f(x)dx \pm \int g(x)dx$

Examples : Find (a) $\int \sqrt{x}\,dx$, (b) $\int \frac{1}{\sqrt{x}}\,dx$, (c) $\int e^{-3x}dx$ (d) $\int 3^x dx$ (e) $\int x\sqrt{x}\ dx$.

Solution: (a) $\int \sqrt{x}\,dx = x^{1/2+1} / (1/2+1) = \frac{x^{3/2}}{3/2} = \frac{2x^{3/2}}{3} + c$

(b) $\int \frac{1}{\sqrt{x}}dx = \int x^{-\frac{1}{2}}dx = \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c = 2\sqrt{x} + c$ where c is arbitrary constant.

(c) $\int e^{-3x}\ dx = \frac{e^{-3x}}{-3} + c = -\frac{1}{3}e^{-3x} + c$

(d) $\int 3^x\ dx = \frac{3^x}{\log_e 3} + c.$

(e) $\int x\sqrt{x}\ dx. = x^{\frac{3}{2}}dx = \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1}dx = \frac{2}{5}x^{5/2} + c.$

Examples : Evaluate the following integral:

i) $\int (x + 1/x)^2\,dx = \int x^2\,dx + 2\int dx + \int dx / x^2$

$$= \frac{x^3}{3} + 2x + \frac{x^{-2+1}}{-2+1}$$

$$= \frac{x^3}{3} + 2x - \frac{1}{x} + c$$

ii) $\int \sqrt{x}\ (x^3+2x-3)\ dx = \int x^{7/2}\ dx + 2\int x^{3/2}\ dx - 3\int x^{1/2}\ dx$

$$= \frac{x^{7/2+1}}{7/2+1} + \frac{2\,x^{3/2+1}}{3/2+1} - \frac{3\,x^{1/2+1}}{1/2+1}$$

$$= \frac{2x^{9/2}}{9} + \frac{4x^{5/2}}{5} - 2\,x^{3/2} + c$$

iii) $\int e^{3x} + e^{-3x}\ dx \quad = \int e^{2x}\ dx \quad + \quad \int e^{-4x}\ dx$

$$= \frac{e^{2x}}{2} + \frac{e^{-4x}}{-4} = \frac{e^{2x}}{2} - \frac{1}{4e^{4x}} + c$$

iv) $\int \frac{x^2}{x+1} dx = \int \frac{x^2-1+1}{x+1} dx$

$$= \int \frac{(x^2-1)}{x+1} dx + \int \frac{dx}{x+1}$$

$$= \int (x-1)dx + \log(x+1) = \frac{x^2}{2} - x + \log(x+1) + c$$

v) $\int \frac{x^3 + 5x^2 - 3}{(x+2)} dx$

By simple division $\int \frac{x^3 + 5x^2 - 3}{(x+2)} dx$

$$= \int \left\{ x^2 + 3x - 6 + \frac{9}{(x+2)} \right\} dx$$

$$= \frac{x^3}{3} + \frac{3x^2}{2} - 6x + 9\log(x+2) + c$$

9.B.3 METHOD OF SUBSTITUTION (CHANGE OF VARIABLE)

It is sometime possible by a change of independent variable to transform a function into another which can be readily integrated.

We can show the following rules.

To put z = f (x) and also adjust dz = f ′(x) dx

Example: $\int F\{h(x)\}\ h'(x)\,dx$, take e^z = h(x) and to adjust dz = h′ (x) dx

then integrate $\int F(z)\ dz$ using normal rule.

Example: $\int (2x+3)^7\ dx$

We put (2x + 3) = t ⇒ so 2 dx = dt or dx = dt / 2

Therefore

This method is known as Method of Substitution

Example: $\int \frac{x^3}{\left(x^2+1\right)^3} dx$ We put $(x^2 + 1) = t$

so 2x dx = dt or x dx = dt / 2

$$= \int \frac{x^2.x}{t^3} dx$$

$$= \frac{1}{2}\int \frac{t-1}{t^3} dt$$

$$= \frac{1}{2}\int \frac{dt}{t^2} - \frac{1}{2}\int \frac{dt}{t^3}$$

$$= \frac{1}{2}\times\frac{t^{-2+1}}{(-2+1)} - \frac{1}{2}\times\frac{t^{-3+1}}{(-3+1)}$$

$$= -\frac{1}{2}\ \frac{1}{t} + \frac{1}{4}\ \frac{1}{t^2}$$

$$= \frac{1}{4}\frac{1}{t^2} - \frac{1}{2}\frac{1}{t}$$

$$= \frac{1}{4}.\frac{1}{(x^2+1)} - \frac{1}{2}.\frac{1}{(x^2+1)} + c$$

IMPORTANT STANDARD FORMULAE

(a) $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\log\frac{x-a}{x+a}$ + c

b) $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\log\frac{a+x}{a-x}$ + c

c) $\int \frac{dx}{\sqrt{x^2+a^2}} = \log\left|x+\sqrt{x^2+a^2}\right|$ + c

d) $\int \frac{dx}{\sqrt{x^2 - a^2}} = \log\left(x + \sqrt{x^2 - a^2}\right) + c$

e) $\int e^x\{f(x) + f'(x)\}dx = e^x f(x) + c$

f) $\int \sqrt{x^2 + a^2}\,dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log\left(x + \sqrt{x^2 + a^2}\right) + c$

g) $\int \sqrt{x^2 - a^2}\,dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log(x + \sqrt{x^2 - a^2}) + c$

h) $\int \frac{f'(x)}{f(x)}\,dx = \log f(x) + c$

Examples :(a) $\int \frac{e^x}{e^{2x} - 4}dx = \int \frac{dz}{z^2 - 2^2}$ where $z = e^x$ $dz = e^x dx$

$$= \frac{1}{4}\log\left(\frac{e^x - 2}{e^x + 2}\right) + c$$

(b) $\int \frac{1}{x + \sqrt{x^2 - 1}}dx = \int \frac{x - \sqrt{x^2 - 1}}{(x + \sqrt{x^2 - 1})\,(x - \sqrt{x^2 - 1})}dx = \int (x - \sqrt{x^2 - 1})\,dx$

$$= \frac{x^2}{2} - \frac{x}{2}\sqrt{x^2 - 1} + \frac{1}{2}\log(x + \sqrt{x^2 - 1}) + c$$

(c) $\int e^x(x^3 + 3x^2)dx = \int e^x\{f(x) + f'(x)\}dx, \text{where} f(x) = x^3$

[by (e) above)] $= e^x x^3 + c$

9.B.4 INTEGRATION BY PARTS

$$\int uv\,dx = u\int v\,dx - \int [\frac{d(u)}{dx}\int v\,dx]\,dx$$

where u and v are two different functions of x

Evaluate:

i) $\int xe^x\,dx$

Integrating by parts we see

$$\int xe^x\,dx = x\int e^x\,dx - \int\left\{\frac{d}{dx}(x)\int e^x\,dx\right\}dx$$

$$= xe^x - \int 1.e^x dx = xe^x - e^x + c$$

ii) $\int x \log x \, dx$

Integrating by parts,

$$= \log x \int x \, dx - \int \left\{ \frac{d}{dx}(\log x) \int x dx \right\} dx$$

$$= \frac{x^2}{2} \log x - \int \left[\frac{1}{x} . \frac{x^2}{2} \right] dx$$

$$= \frac{x^2}{2} \log x - \frac{1}{2} \int x dx$$

$$= \frac{x^2}{2} \log x - \frac{x^2}{4} + c$$

iii) $\int x^2 e^{ax} \, dx$

$$= x^2 \int e^{ax} \, dx - \int \left\{ \frac{d}{dx}(x^2) \int e^{ax} \, dx \right\} dx$$

$$= \frac{x^2}{a} e^{ax} - \int 2x . \frac{e^{ax}}{a} \, dx$$

$$= \frac{x^2}{a} e^{ax} - \frac{2}{a} \int x . e^{ax} \, dx$$

$$= \frac{x^2}{a} e^{ax} - \frac{2}{a} \times \int e^{ax} \, dx - \int \left[\frac{d}{dx}(x) \int e^{ax} \, dx \right] dx$$

$$= \frac{x^2 e^{ax}}{a} - \frac{2}{a} \left[\frac{x e^{ax}}{a} - \int 1 . \frac{e^{ax}}{a} dx \right]$$

$$= \frac{x^2 e^{ax}}{a} - \frac{2x e^{ax}}{a^2} + \frac{2}{a^3} e^{ax} + c$$

9.B.5 METHOD OF PARTIAL FRACTION

Type I :

Example : $\int \frac{(3x+2) \, dx}{(x-2)(x-3)}$

Solution : let $\frac{(3x+2)}{(x-2)(x-3)}$

$$= \frac{A}{(x-2)} + \frac{B}{(x-3)}$$

[Here degree of the numerator must be lower than that of the denominator; the denominator contains non–repeated linear factor]

so 3x +2 = A (x – 3) + B (x – 2)

We put x = 2 and get

3.2 +2 = A (2–3) + B (2–2) => A = –8

we put x = 3 and get

3.3 +2 = A (3–3) + B (3–2) => B= 11

$$\int \frac{(3x+2)dx}{(x-2)^2\,(x-3)} \quad -8\int \frac{dx}{x-2}+11\int \frac{dx}{x-3}$$

$$=-\log(x–2)+11\log(x\text{-}3)+c$$

Type II:

Example : $\int \frac{(3x+2)\,dx}{(x-2)^2\,(x-3)}$

Solution : let $\frac{(3x + 2)}{(x\text{-}2)^2\ (x\text{-}3)}=\frac{A}{(x–2)}+\frac{B}{(x-2)^2}+\frac{C}{(x-3)}$

or 3x +2 = A (x –2) (x–3) + B (x–3) +C $(x–2)^2$

Comparing coefficients of x^2, x and the constant terms of both sides, we find

A+C = 0 …………(i)

–5A +B – 4C = 3 ……(ii)

6A –3B +4C = 2 …….(iii)

By (ii) + (iii) A –2B = 5 ..…….(iv)

(i) – (iv) 2B + C = –5 …….(v)

From (iv) A = 5+2B

From (v) C = –5–2B

From (ii) –5 (5+2B) + B –4 (–5 –2B) = 3

or –25 – 10B + B + 20 + 8B = 3

or –B –5 = 3

or B = –8, A = 5 –16 = –11, from (iv) C = –A = 11

TTherefore $\int \frac{(3x+2)\,dx}{(x\text{ - }2)^2(x\text{ - }3)}$

$$= -11\int \frac{dx}{(x-2)}-8\int \frac{dx}{(x-2)^2}+11\int \frac{dx}{(x-3)}$$

$= -11 \log (x-2) + \frac{8}{(x-2)} + 11 \log (x-3)$

$= 11 \log \frac{(x-3)}{(x-2)} + \frac{8}{(x-2)} + c$ Ans.

Type III:

Example: $\int \frac{(3x^2 - 2x + 5)}{(x-1)^2 (x^2 + 5)} dx$

Solution: Let $\frac{3x^2 - 2x + 5}{(x-1)^2 (x^2 + 5)} = \frac{A}{x-1} + \frac{Bx + C}{(x^2 + 5)}$

so $3x^2 - 2x + 5 = A(x^2 + 5) + (Bx + C)(x-1)$

Equating the coefficients of x^2, x and the constant terms from both sides we get

$A + B = 3$ ....(i)

$C - B = -2$ ....(ii)

$5A - C = 5$ ....(iii)

by (i) + (ii) $A + C = 1$ .... (iv)

by (iii) + (iv) $6A = 6$ .... (v)

or $A = 1$

therefore $B = 3-1 = 2$ and $C = 0$

Thus $\int \frac{(3x^2 - 2x + 5)}{(x-1)^2 (x^2 + 5)} dx$

$= \int \frac{dx}{x-1} + \int \frac{2x}{x^2 + 5} dx$

$= \log (x-1) + \log (x^2 + 5)$

$= \log (x^2 + 5)(x-1) + c$

Example: $\int \frac{dx}{x(x^3 + 1)}$

Solution : $\int \frac{dx}{x(x^3 + 1)}$

$= \int \frac{x^2 dx}{x^3 (x^3 + 1)}$ we put $x^3 = z$, $3x^2\, dx = dz$

$$= \frac{1}{3}\int \frac{dz}{z(z+1)}$$

$$= \frac{1}{3}\int \left(\frac{1}{z} - \frac{1}{z+1}\right)dz$$

$$= \frac{1}{3}\left[\log z - \log(z+1)\right]$$

$$= \frac{1}{3}\log\left(\frac{x^3}{x^3+1}\right) + c$$

Example : Find the equation of the curve where slope at (x, y) is 9x and which passes through the origin.

Solution : $\frac{dy}{dx} = 9x$

$\therefore \int dy =$ or $y = 9x^2/2 + c$

Since it passes through the origin, c = 0; thus required eqn . is $9x^2 = 2y$.

9.B.6 DEFINITE INTEGRATION

Suppose F(x) dx = f(x)

As x changes from a to b the value of the integral changes from f(a) to f(b). This is as

$$\int_a^b F(x)\,dx = f(b) - f(a)$$

'b' is called the upper limit and 'a' the lower limit of integration. We shall first deal with indefinite integral and then take up definite integral.

Example : $\int_0^2 x^5\,dx$

Solution : $\int_0^2 x^5\,dx = \frac{x^6}{6}$

$$\int_0^2 x^5\,dx = \left(\frac{x^6}{6}\right)_0^2$$

$= \frac{1}{6}(2^6 - 0) \qquad = 64/6 = 32/3$

Note: In definite integration the constant C should not be added

Example: $\int_1^2 (x^2 - 5x + 2)dx$

Solution: $\int_1^2 (x^2 - 5x + 2)dx = \frac{x^3}{3} - \frac{5x^2}{2} + 2x$. Now, $\int_1^2 (x^2 - 5x + 2)dx = \left[\frac{x^3}{3} - \frac{5x^2}{2} + 2x\right]_1^2$

$$= \left[\frac{2^3}{3} - \frac{5x2^2}{2} + 2x2\right] - \left[\frac{1}{3} - \frac{5}{2} + 2\right] = -19/6$$

9.B.7 IMPORTANT PROPERTIES

Important Properties of Definite Integral

(I) $\int_a^b f(x)dx = \int_a^b f(t)\,dt$

(II) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

(III) $\int_a^b f(x)\,dx = \int_b^c f(x)\,dx + \int_c^b f(x)\,dx, a < c < b$

(IV) $\int_0^a f(x)\,dx = \int_o^a f(a-x)\,dx$

(V) When f(x) =f (a+x) = $\int_0^{na} f(x)\,dx = n\int_0^a f(x)\,dx$

(VI) $\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$ if $f(-x) = f(x)$

= 0 if $f(-x) = -f(x)$

Example : $\int_0^2 \frac{x^2 dx}{x^2 + (2-x)^2}$

Solution : Let I = $\int_0^2 \frac{x^2 dx}{x^2 + (2-x)^2}$

$= \int_0^2 \frac{(2-x)^2\ dx}{(2-x)^2\ + x^2}$ [by prop IV]

$$\therefore \quad 21 = \int_0^2 \frac{x^2 dx}{x^2 + (2-x)^2} + \int_0^2 \frac{(2-x)^2}{(2-x)^2 + x^2} \, dx$$

$$\int_0^2 \frac{x^2 + (2-x)^2}{x^2 + (2-x)^2} dx$$

$$= \int_0^2 dx \quad = \quad [x]_0^2 = 2\text{–}0 = 2$$

or $\quad I = 2/2 = 1$

Example : Evaluate $\displaystyle\int_{-2}^{2} \frac{x^4 \, dx}{a^{10} - x^{10}} \quad (a > 2)$

Solution : $\displaystyle\frac{x^4 \, dx}{a^{10} - x^{10}} = \frac{x^4 \, dx}{(a^5)^2 - (x^5)^2}$

let $x^5 = t$ so that $5x^4 \, dx = dt$

Now $\displaystyle\int \frac{x^4 dx}{(a^5)^2 - (x^5)^2}$

$$= \frac{1}{5} \int \frac{5x^4 dx}{(a^5)^2 - (x^5)^2}$$

$$= \frac{1}{5} \int \frac{dt}{(a^5)^2 - t^2}$$

$$= \frac{1}{10a^5} \log \frac{a^5 + x^5}{a^5 - x^5} \qquad \text{(by standard formula b)}$$

Therefore, $\displaystyle\int_{-2}^{2} \frac{x^4 dx}{a^{10} - x^{10}}$

$$= 2\int_0^2 \frac{x^4 dx}{a^{10} - x^{10}} \qquad \text{(by prop. VI)}$$

$$= 2 \times \frac{1}{10a^5} \log \left[\frac{a^5 + x^5}{a^5 - x^5} \right]_0^2$$

$$= \frac{1}{5a^5} \log \frac{a^5 + 32}{a^5 - 32} \qquad \text{Ans.}$$

EXERCISE 9 (B) [K = constant]

Choose the most appropriate option (a) (b) (c) or (d)

1. Evaluate $=\int 5x\,dx$:

   (a) $5 / 3x^3 + k$ (b) $\frac{5x^3}{3}+k$ (c) $5x^3$ (d) none of these

2. Integration of $3 - 2x - x^4$ will become

   (a) $- x^2 - x^5 / 5$ (b) $3x-x^2-\frac{x^5}{5}+k$ (c) $3x-x^2+\frac{x^5}{5}+k$ (d) none of these

3. Given $f(x) = 4x^3 + 3x^2 - 2x + 5$ and $\int f(x)\,dx$ is

   (a) $x^4 + x^3 - x^2 + 5x$ (b) $x^4 + x^3 - x^2 + 5x + k$

   (c) $12x^2 + 6x - 2x^2$ (d) none of these

4. Evaluate $\int (x^2 - 1)\,dx$

   (a) $x^5/5 - 2/3\, x^3 + x + k$ (b) $\frac{x^3}{3}-x+k$

   (c) 2x (d) none of these

5. $\int (1-3x)(1+x)\,dx$ is equal to

   (a) $x - x^2 - x^3$ (b) $x^3 - x^2 + x$ (c) $x - x^2 - x^3 + k$ (d) none of these

6. $\int \left[\sqrt{x}-1/\sqrt{x}\right]$ dx is equal to

   (a) $\frac{2}{3}x^{3/2} - 2\,x^{½}$ +k (b) $\frac{2}{3}\sqrt{x}-2\sqrt{x}+k$ (c) $\frac{1}{2\sqrt{x}}+\frac{1}{2x\sqrt{x}}+k$ (d) none of these

7. The integral of $px^3 + qx^2 + rk + w/x$ is equal to

   (a) $px^2 + qx + r + k$ (b) $px^3/3 + qx^2/2 + rx$

   (c) $3px + 2q - w/x^2$ (d) none of these

8. Use method of substitution to integrate the function $f(x) = (4x + 5)^6$ and the answer is

   (a) $1/28\,(4x + 5)^7 + k$ (b) $(4x + 5)^7/7 + k$ (c) $(4x + 5)^7/7$ (d) none of these

9. Use method of substitution to evaluate $\int x(x^2+4)^5$ dx and the answer is

   (a) $(x^2 + 4)^6 + k$ (b) $1/12\,(x^2 + 4)^6 + k$

   (c) $(x^2 + 4)^6/ + k$ (d) none of these

10. Integrate $(x + a)^n$ and the result will be

    (a) $\frac{(x+a)^{n+1}}{n+1}+k$ (b) $\frac{(x+a)^{n+1}}{n+1}$

    (c) $(x + a)^{n+1}$ (d) none of these

11. $\int 8x^2/(x^3+2)^3\, dx$ is equal to

(a) $-4/3(x^3 + 2)^2 + k$ (b) $-\dfrac{4}{3(x^3+2)^2} + k$

(c) $\dfrac{4}{3(x^3+2)^2} + k$ (d) none of these

12. Using method of partial fraction the integration of f(x) when $f(x) = \dfrac{1}{x^2 - a^2}$ and the answer is

(a) $\log x - \dfrac{a}{x+a} + k$ (b) $\log (x - a) - \log (x + a) + k$

(c) $\dfrac{1}{2a}\log\left(\dfrac{x-a}{x+a}\right) + k$ (d) none of these

13. Use integration by parts to evaluate $\int x^2 e^{3x} dx$

(a) $x^2\, e^{3x}/3 - 2x\, e^{3x}/9 + 2/27\, e^{3x} + k$ (b) $x^2\, e^{3x} - 2x\, e^{3x} + 2e^{3x} + k$

(c) $e^{3x}/3 - x\, e^{3x}/9 + 2e^{3x} + k$ (d) none of these

14. $\int$ logx dx is equal to

(a) x logx + k (b) x logx – x^2 + k (c) x logx + k (d) none of these

15. $\int$ xe^x dx is

(a) $(x - 1)e^x + k$ (b) $(x - 1)\, e^x$ (c) $x\, e^x + k$ (d) none of these

16. $\int (\log x)^2$ dx and the result is

(a) $x\, (\log x)^2 - 2\, x \log x + 2x + k$ (b) $x\, (\log x)^2 - 2x + k$

(c) $2x \log x - 2x + k$ (d) none of these

17. Using method of partial fraction to evaluate $\int (x + 5)\, dx/(x + 1)\, (x + 2)^2$ we get

(a) $4 \log (x + 1) - 4 \log (x + 2) + 3/x + 2 + k$

(b) $4 \log (x + 2) - 3/x + 2) + k$

(c) $4 \log (x + 1) - 4 \log (x + 2)$

(d) none of these

18. Evaluate $\int_0^1 (2x^2 - x^3)$ dx and the value is

(a) 4/3 + k (b) 5/12 (c) – 4/3 (d) none of these

19. Evaluate $\int_2^4 (3x-2)^2$ dx and the value is

(a) 104 (b) 100 (c) 10 (d) none of these.

20. Evaluate $\int_0^1 xe^x dx$ dx and the value is

(a) –1 (b) 10 (c) 10/9 (d) +1

21. $\int x^x$ (1 + logx) dx is equal to

(a) x^x logx + k (b) e^{x2} + k (c) $\frac{x^2}{2}$ + k (d) x^x + c

22. If $f(x) = \sqrt{1+x^2}$ then $\int f(x)dx$ is

(a) $\frac{2}{3}$ x (1 + x^2) $^{3/2}$ + k (b) $\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\log(x+\sqrt{x^2+1})$ +k

(c) $\frac{2}{3}$ x (1 + x^2) $^{3/2}$ + k (d) none of these

23. $\int d(x^2+1)/\sqrt{x^2+2}$ is equal to

(a) $\frac{x}{\sqrt{2}}\left(\sqrt{x^2+2}\right)+k$ (b) $\sqrt{x^2+2}$ + k (c) 1/(x^2 + 2) $^{3/2}$ + k (d) none of these

24. $\int (e^x+e^{-x})^2(e^x-e^{-x})$ dx is

(a) $\frac{1}{3}(e^x+e^{x.})^3+k$ (b) $\frac{1}{2}(e^x-e^{-x})^2+k$

(c) e^x+k (d) none of these

25. $\int_0^a [f(x)+f(-x)]\,dx$ is equal to

(a) $\int_0^a 2\ f(x)\,dx$ (b) $\int_{-a}^a f(x)\,dx$ (c) 0 (d) $\int_{-a}^a -f(-x)\,dx$

26. $\int xe^x/(x+1)^2$ dx is equal to

(a) e^x/(x + 1) + k (b) e^x/x + k (c) e^x + k (d) none of these

27. $\int (x^4+3/x)$ dx is equal to

(a) x^5/5 + 3 log |x| (b) 1/5 x^5 + 3 log |x| + k

(c) 1/5 x^5 + k (d) none of these

28. Evaluate the integral $\int (1-x)^3 / x \ dx$ and the answer is equal to

(a) $\log |x| - 3x + 3/2x^2 + k$ (b) $\log x - 2 + 3x^2 + k$

(c) $\log x + 3x^2 + k$ (d) none of these

29. The equation of the curve in the form y = f(x) if the curve passes through the point (1, 0) and $f'(x) = 2x - 1$ is

(a) $y = x^2 - x$ (b) $x = y^2 - y$ (c) $y = x^2$ (d) none of these

30. Evaluate $\int_1^4 (2x+5)\ dx$ and the value is

(a) 3 (b) 10 (c) 30 (d) none of these

31. $\int_1^2 \frac{2x}{1+x^2} dx$ is equal to

(a) $\log_e (5/2)$ (b) $\log_e 5 - \log_e 2 + k$

(c) $\log_e (2/5)$ (d) none of these

32. $\int_0^4 \sqrt{3x+4}\ dx$ is equal to

(a) 9/112 (b) 112/9 (c) 11/9 (d) none of these

33. $\int_0^2 \frac{x+2}{x+1} dx$ is

(a) $2 + \log_e 2$ (b) $2 + \log_e 3$ (c) $\log_e 3$ (d) none of these

34. Evaluate $\int_1^{e^2} \frac{dx}{x(1+\log x)^2}$ and the value is

(a) 3/2 (b) 1/3 (c) 26/3 (d) 1/2 ($\log_e 5$)

35. $\int_0^4 \frac{(x+1)(x+4)}{\sqrt{x}} dx$ is equal is

(a) $51\frac{1}{5}$ (b) 48/5 (c) 48 (d) $55\frac{7}{15}$

36. The equation of the curve which passes through the point (1, 3) and has the slope 4x – 3 at any point (x, y) is

(a) $y = 2x^3 - 3x + 4$ (b) $y = 2x^2 - 3x + 4$

(c) $x = 2y^2 - 3y + 4$ (d) none of these

37. The value of $\int_2^3 f(5-x)\,dx - \int_2^3 f(x)\,dx$ is

(a) 1 (b) 0 (c) –1 (d) none of these

38. $\int (x-1)e^x / x^2 dx$ is equal to

(a) $e^x/x + k$ (b) $e^{-x}/x + k$ (c) $-e^x/x + k$ (d) none of these

39. $\int \frac{e^x(x\log x+1)}{x}dx$ is equal to

(a) $e^x \log x + k$ (b) $e^x + k$ (c) $\log x + k$ (d) none of these

40. $\int \log x^2\, dx$ is equal to

(a) $x(\log x - 1) + k$ (b) $2x(\log x - 1) + k$

(c) $2(\log x - 1) + k$ (d) none of these

41. $\int_1^2 x \log x\, dx$ is equal to

(a) 2 log 2 (b) – 3/4 (c) 2 log 2 – ¾ (d) none of these

42. Evaluate $\int_1^2 \frac{x^2-1}{x^2} e^{x+\frac{1}{x}} dx$. The value is

(a) $e^2(\sqrt{e}-1)$ (b) $e^2[\sqrt{e}-1]+k$ (c) $e^2\sqrt{e}$ (d) none of these

43. $\int_0^2 3x^2\, dx$ is

(a) 7 (b) –8 (c) 8 (d) none of these

44. Evaluate $\int \frac{(2-x)e^x}{(1-x)^2}dx$ and the value is

(a) $\frac{e^x}{1-x} + k$ (b) $e^x + k$ (c) $\frac{1}{1-x}+k$ (d) none of these

45. Using integration by parts $\int x^3 \log x dx$

(a) $x^4/16 + k$ (b) $x^4/16\,(4\log x - 1) + k$

(c) $4\log x - 1 + k$ (d) none of these

46. $\int \log(\log x)/x dx$ is

(a) $\log(\log x - 1) + k$ (b) $\log x - 1 + k$

(c) $[\log(\log x - 1)]\log x + k$ (d) none of these

47. $\int (\log x)^2 \, x \, dx$ is equal to

(a) $\frac{x^2}{2}\left[(\log x)^2 - \log x + \frac{1}{2}\right] + k$

(b) $(\log x)^2 - \log x + \frac{1}{2} + k$

(c) $\frac{x^2}{2}\left[(\log x)^2 + \frac{1}{2}\right] + k$

(d) none of these

48. Evaluate $\int \left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right) dx$ and the value is

(a) $\log_e |e^x + e^{-x}|$

(b) $\log^e |e^x + e^{-x}| + k$

(c) $\log_e |e^x - e^{-x}| + k$

(d) none of these

49. By the method of partial fraction $\int \frac{3x}{(x^2 - x - 2)} dx$ is

(a) $2 \log_e |x - 2| + \log_e |x + 1| + k$

(b) $2 \log_e |x - 2| - \log_e |x + 1| + k$

(c) $\log_e |x - 2| + \log_e |x + 1| + k$

(d) none of these

50. If $f'(x) = x - 1$, the equation of a curve $y = f(x)$ passing through the point (1, 0) is given by

(a) $y = x^2 - 2x + 1$

(b) $y = x^2/2 - x + 1$

(c) $y = x^2/2 - x + 1/2$

(d) none of these

ANSWERS

Exercise 9(A)

1. a	2. b	3. c	4. b	5. a	6. a	7. d	8. b
9. d	10. b	11. c	12. a	13. b	14. a	15. b	16. c
17. a	18. c	19. a	20. a	21. b	22. a	23. d	24. b
25. c	26. a	27. b	28. c	29. a	30. b	31. c	32. a
33. b	34. a	35. a	36. b	37. b	38. a	39. c	40. a
41. b	42. c	43. a	44. b	45. a	46. b	47. c	48. d
49. c	50. a						

Exercise 9(B)

1. b	2. b	3. b	4. a	5. c	6. a	7. d	8. a
9. b	10. a	11. b	12. c	13. a	14. d	15. a	16. d
17. a	18. b	19. a	20. d	21. d	22. b	23. a	24. a
25. b	26. a	27. b	28. d	29. a	30. c	31. a	32. b
33. b	34. d	35. d	36. b	37. b	38. a	39. a	40. b
41. c	42. a	43. c	44. a	45. b	46. c	47. a	48. b
49. a	50. c						

ADDITIONAL QUESTION BANK

(A) Differential Calculus

1. If $y=x^3$ then dy/dx is

(A) $x^4/4$ (B) $-x^4/4$ (C) $3x^2$ (D) $-3x^2$

2. If $y=x^{2/3}$ then dy/dx is

(A) $(2/3)x^{-1/3}$ (B) $(3/5)x^{5/3}$ (C) $(-3/5)x^{5/3}$ (D) None

3. If $y=x^{-8}$ then dy/dx is

(A) $-8x^{-9}$ (B) $8x^{-9}$ (C) $-8x^9$ (D) $8x^9$

4. If $y=5x^2$ then dy/dx is

(A) $10x$ (B) $5x$ (C) $2x$ (D) None

5. If $y = 2x^2 + x$ then dy/dx is

(A) $4x +1$ (B) $2(x-1)$ (C) $x + 1$ (D) $x – 1$

6. If $y=4x^3-7x^4$ then dy/dx is

(A) $2x(-14x^2+6x)$ (B) $2x(14x^2-6x)$ (C) $2x(14x^2+6x)$ (D) None

7. If $y=(4/3)x^3-(6/7)x^7+4x^{-3}$ then dy/dx is

(A) $4x^2-6x^6-12x^{-4}$ (B) $4x^2+6x^6-12x^{-4}$ (C) $4x^2+6x^6+12x^{-4}$ (D) None

8. If $y=9x^4-7x^3+8x^2-8x^{-1}+10x^{-3}$ then dy/dx is

(A) $36x^3-21x^2+16x+8x^{-2}-30x^{-4}$ (B) $36x^3-21x^2+16x-8x^{-2}+30x^{-4}$

(C) $36x^3+21x^2+16x+8x^{-2}+30x^{-4}$ (D) None

9. If $y=[(1-x)/x]^2$ then dy/dx is

(A) $2(x^{-3}+x^{-2})$ (B) $2(-x^{-3}+x^{-2})$ (C) $2(x^{-3}-x^{-2})$ (D) None

10. If $y=(3x^2+1)(x^3+2x)$ then dy/dx is

(A) $15x^4+21x^2+2$ (B) $15x^3+21x^2+2$ (C) $15x^3+21x+2$ (D) None

11. If $y=(3x^2+5)(2x^3+x+7)$ then dy/dx is

(A) $30x^4+39x^2+42x+5$ (B) $30x^4+39x^3+42x^2+5$

(C) $30x^4+39x^3+42x^2+5x$ (D) None

12. If $y=2x^{3/2}(x^{1/2}+2)(x^{1/2}-1)$ then dy/dx is

(A) $4x+5x(x-6)^{1/2}x^{1/2}$ (B) $4x+5x(x-3)^{1/2}x^{1/2}$

(C) $4x+5x(x-2)^{1/2}x^{1/2}$ (D) None

13. If $y=(x^2-1)/(x^2+1)$ then dy/dx is

(A) $4x(x^2+1)^{-2}$ (B) $4x(x^2+1)^2$ (C) $4x(x^2-1)^{-2}$ (D) None

14. If $y=(x+1)(2x-1)/(x-3)$ then dy/dx is

(A) $2(x^2-6x-1)/(x-3)^2$ (B) $2(x^2+6x-1)/(x-3)^2$

(C) $2(x^2+6x+1)/(x-3)^2$ (D) None

15. If $y=(x^{1/2}+2)/x^{1/2}$ then dy/dx is

(A) $-x^{-3/2}$ (B) $x^{-3/2}$ (C) $x^{3/2}$ (D) None

16. If $y=(3x^2-7)^{1/2}$ then dy/dx is

(A) $3x(3x^2-7)^{-1/2}$ (B) $6x(3x^2-7)^{-1/2}$ (C) $3x(3x^2-7)^{1/2}$ (D) None

17. If $y=(3x^3-5x^2+8)^3$ then dy/dx is

(A) $3(3x^3-5x^2+8)^2\ (9x^2-10x)$ (B) $3(3x^3-5x^2+8)^2\ (9x^2+10x)$

(C) $3(3x^3-5x^2+8)^2\ (10x^2-9x)$ (D) None

18. If $y=(6x^5-7x^3+9)^{-1/3}$ then dy/dx is

(A) $(-1/3)(6x^5-7x^3+9)^{-4/3}(30x^4-21x^2)$ (B) $(1/3)(6x^5-7x^3+9)^{-4/3}(30x^4-21x^2)$

(C) $(-1/3)(6x^5-7x^3+9)^{4/3}(30x^4-21x^2)$ (D) None

19. If $y=[(x^2+a^2)^{1/2}+(x^2+b^2)^{1/2}]^{-1}$ then dy/dx is

(A) $x(a^2-b^2)^{-1}[(x^2+a^2)^{1/2}-(x^2+b^2)^{1/2}]$

(B) $(a^2-b^2)^{-1}[(x^2+a^2)^{1/2}-(x^2+b^2)^{1/2}]$

(C) $x(a^2-b^2)^{-1}[(x^2+a^2)^{1/2}+(x^2+b^2)^{1/2}]$

(D) $(a^2-b^2)^{-1}[(x^2+a^2)^{1/2}+(x^2+b^2)^{1/2}]$

20. If $y=\log 5x$ then dy/dx is

(A) x^{-1} (B) x (C) $5x^{-1}$ (D) $5x$

21. If $y=x^{-1/2}$ then dy/dx is

(A) $(-1/2)x^{-3/2}$ (B) $(1/2)x^{-3/2}$ (C) $(1/2)x^{3/2}$ (D) None

22. If $y=-3x^{-7/3}$ then dy/dx is

(A) $7x^{-10/3}$ (B) $-7x^{-10/3}$ (C) $(-7/3)x^{-10/3}$ (D) None

23. If $y=7x^4+3x^3-9x+5$ then dy/dx is

(A) $28x^3+9(x+1)(x-1)$ (B) $28x^3+9(x+1)^2$

(C) $28x^3+9(x-1)^2$ (D) None

24. If $y=x+4x^{-1}-2x^{-7}$ then dy/dx is

(A) $1-4x^{-2}+14x^{-8}$ (B) $1+4x^{-2}-14x^{-8}$ (C) $1+4x^{-2}+14x^{-8}$ (D) None

25. If $y=(x-x^{-1})^2$ then dy/dx is

(A) $2x-2x^{-3}$ (B) $2x+2x^{-3}$ (C) $2x+2x^3$ (D) $2x-2x^3$

26. If $y=(x^{1/3}-x^{-1/3})^3$ then dy/dx is

(A) $1-x^{-2}+x^{-2/3}-x^{-4/3}$ (B) $1+x^{-2}+x^{-2/3}-x^{-4/3}$

(C) $1+x^{-2}+x^{-2/3}+x^{-4/3}$ (D) None

27. If $y=(x+a)(x+b)(x+c)$ then dy/dx is

(A) $3x^2+2ax+2bx+2cx+ab+bc+ca$ (B) $2x^2+3ax+3bx+3cx+ab+bc+ca$

(C) $3x^2+2ax+2bx+2cx+2ab+2bc+2ca$ (D) None

28. If $y=(3x^2+5x)(7x+4)^{-1}$ then dy/dx is

(A) $(21x^2+24x+20)(7x+4)^{-2}$ (B) $(21x^2+20x+24)(7x+4)^{-2}$

(C) $(21x^2+24x+4)(7x+4)^{-2}$ (D) None

29. If $y=(2x+1)(3x+1)(4x+1)^{-1}$ then dy/dx is

(A) $(24x^2+12x+1)(4x+1)^{-2}$ (B) $(24x^2+12x+3)(4x+1)^{-2}$

(C) $(24x^2+12x+5)(4x+1)^{-2}$ (D) None

30. If $y=(5x^4-6x^2-7x+8)/(5x-6)$ then dy/dx is

(A) $(75x^4-120x^3-30x^2+72x+2)(5x-6)^{-2}$

(B) $(75x^4-120x^3+30x^2-72x+2)(5x-6)^{-2}$

(C) $(75x^4-120x^3-30x^2+72x-2)(5x-6)^{-2}$ (D) None

31. If $y=(ax^2+bx+c)^{1/2}$ then dy/dx is

(A) $(1/2)(2ax+b)(ax^2+bx+c)^{-1/2}$

(B) $(-1/2)(2ax+b)(ax^2+bx+c)^{-1/2}$

(C) $(1/2)(ax+2b)(ax^2+bx+c)^{-1/2}$ (D) None

32. If $y=(2x^4+3x^3-5x+6)^{-1/3}$ then dy/dx is

(A) $(-1/3)(2x^4+3x^3-5x+6)^{-4/3}(8x^3+9x^2-5)$

(B) $(1/3)(2x^4+3x^3-5x+6)^{-4/3}(8x^3+9x^2-5)$

(C) $(1/3)(2x^4+3x^3-5x+6)^{4/3}(8x^3+9x^2-5)$ (D) None

33. If $y=\log[(x-1)^{1/2}-(x+1)^{1/2}]$ then dy/dx is

(A) $(1/2)(x^2-1)^{-1/2}$ (B) $(-1/2)(x^2-1)^{-1/2}$ (C) $(1/2)(x^2-1)^{1/2}$ (D) None

34. If $y=\log\sqrt{x+\sqrt{x^2+a^2}}$ then dy/dx is

(A) $(1/2)(x^2+a^2)^{-1/2}$ (B) $(-1/2)(x^2+a^2)^{-1/2}$

(C) $(1/2)(x^2+a^2)^{1/2}$ (D) None

35. If $x=3at/(1+t^3)$, $y=3at^2/(1+t^3)$, then dy/dx is

(A) $(2t-t^4)/(1-2t^3)$ (B) $(2t-t^4)/(1+2t^3)$ (C) $(2t+t^4)/(1+2t^3)$ (D) None

36. If $y=\log[e^{3x}(5x-3)^{1/3}(4x+2)^{-1/3}]$ then dy/dx is

(A) $3+(1/3)[5/(5x-3)-4/(4x+2)]$ (B) $3-(1/3)[5/(5x-3)-4/(4x+2)]$

(C) $3+(1/3)[5/(5x-3)+4/(4x+2)]$ (D) None

37. If $y=x^{x^x}$ then the value of dy/dx is

(A) $x^{x^x}[x^{x-1}+\log x.x^x(1+\log x)]$ (B) $x^{x^x}[x^{x-1}+\log x.(1+\log x)]$

(C) $x^{x^x}[x^{x-1}+\log x.x^x(1-\log x)]$ (D) $x^{x^x}[x^{x-1}+\log x.(1-\log x)]$

38. If $x^y=e^{x-y}$ then dy/dx is

(A) $\log x/(1-\log x)^2$ (B) $\log x/(1+\log x)^2$ (C) $\log x/(1-\log x)$ (D) $\log x/(1+\log x)$

39. If $y=(x+a)(x+b)(x+c)(x+d)/(x-a)(x-b)(x-c)(x-d)$ then the value of dy/dx is

(A) $(x+a)^{-1}+(x+b)^{-1}+(x+c)^{-1}+(x+d)^{-1}-(x-a)^{-1}-(x-b)^{-1}-(x-c)^{-1}-(x-d)^{-1}$

(B) $(x+a)^{-1}-(x+b)^{-1}+(x+c)^{-1}-(x+d)^{-1}+(x-a)^{-1}-(x-b)^{-1}+(x-c)^{-1}-(x-d)^{-1}$

(C) $(x-a)^{-1}+(x-b)^{-1}+(x-c)^{-1}+(x-d)^{-1}-(x+a)^{-1}-(x+b)^{-1}-(x+c)^{-1}-(x+d)^{-1}$

(D) None

40. If $y=x(x^2-4a^2)^{1/2}(x^2-a^2)$ then dy/dx is

(A) $(x^4-2a^2x^2+4a^4)(x^2-a^2)^{-3/2}(x^2-4a^2)^{-1/2}$

(B) $(x^4+2a^2x^2-4a^4)(x^2-a^2)^{-3/2}(x^2-4a^2)^{-1/2}$

(C) $(x^4+2a^2x^2+4a^4)(x^2-a^2)^{-3/2}(x^2-4a^2)^{-1/2}$ (D) None

41. If $y=(2-x)(3-x)^{1/2}(1+x)^{-1/2}$ then the value of $[dy/dx]/y$ is

(A) $(x-2)^{-1}+(1/2)(x-3)^{-1}-(1/2)(1+x)^{-1}$

(B) $(x-2)^{-1}+(x-3)^{-1}-(1+x)^{-1}$

(C) $(x-2)^{-1}-(1/2)(x-3)^{-1}+(1/2)(1+x)^{-1}$ (D) None

42. If $y=\log[e^x\{x-2)/(x+3)\}^{3/4}$ then dy/dx is

(A) $1+(3/4)(x-2)^{-1}-(3/4)(x+3)^{-1}$ (B) $1-(3/4)(x-2)^{-1}+(3/4)(x+3)^{-1}$

(C) $1+(3/4)(x-2)^{-1}+(3/4)(x+3)^{-1}$ (D) None

43. If $y=e^{5/x}(2x^2-1)^{1/2}$ then the value of $[dy/dx]/y$ is

(A) $(2x^3-10x^2+5)x^{-2}(2x^2-1)^{-1/2}$ (B) $(2x^3-5x^2+10)x^{-2}(2x^2-1)^{-1/2}$

(C) $(2x^3+10x^2-5)x^{-2}(2x^2-1)^{-1/2}$ (D) None

44. If $y=x^2e^{5x}(3x+1)^{-1/2}(2x-1)^{-1/3}$ then the value of $[dy/dx]/y$ is

(A) $5+2x^{-1}-(3/2)(3x+1)^{-1}-(2/3)(2x-1)^{-1}$

(B) $5+2x^{-1}-(2/3)(3x+1)^{-1}-(3/2)(2x-1)^{-1}$

(C) $5+2x^{-1}-(2/3)(3x+1)^{-1}+(3/2)(2x-1)^{-1}$ (D) None

45. If $y=x^{1/2}(5-2x)^{2/3}(4-3x)^{-3/4}(7-4x)^{-4/5}$ then the value of $[dy/dx]/y$ is

(A) $(1/2)x^{-1}-(4/3)(5-2x)^{-1}+(9/4)(4-3x)^{-1}+(16/5)(7-4x)^{-1}$

(B) $(1/2)x^{-1}-(3/4)(5-2x)^{-1}+(9/4)(4-3x)^{-1}+(16/5)(7-4x)^{-1}$

(C) $(1/2)x^{-1}+(4/3)(5-2x)^{-1}+(9/4)(4-3x)^{-1}+(16/5)(7-4x)^{-1}$

(D) None

46. If $y=x^x$ then the value of $[dy/dx]/y$ is

(A) $\log x+1$ (B) $\log x-1$ (C) $\log(x+1)$ (D) None

47. If $y=(1+x)^{2x}$ then the value of $[dy/dx]/y$ is

(A) $2[x(x+1)^{-1}+\log(x+1)]$ (B) $x(x+1)^{-1}+\log(x+1)$

(C) $2[x(x+1)^{-1}-\log(x+1)]$ (D) None

48. If $y=x^{1/x}$ then the value of $[dy/dx]/y$ is

(A) $x^{-2}(1-\log x)$ (B) $x^{2}(1-\log x)$ (C) $x^{-2}(1+\log x)$ (D) None

49. If $y=(x^{x})^{x}$ then dy/dx is

(A) $x^{x^2+1}(1+2\log x)$ (B) $x^{x^2+1}(1+\log x)$ (C) $x^{x^2+1}(1-\log x)$ (D) None

50. If $y=x^{\log x}$ then dy/dx is

(A) $2x^{\log x-1}.\log x$ (B) $x^{\log x-1}.\log x$ (C) $2x^{\log x+1}.\log x$ (D) None

51. If $y=x^{\log(\log x)}$ then the value of $[dy/dx]/y$ is given by

(A) $x^{-1}[1+\log(\log x)]$ (B) $x^{-1}[1-\log(\log x)]$

(C) $x[1+\log(\log x)]$ (D) $x[1-\log(\log x)]$

52. If $y=x^{a}+a^{x}+x^{x}+a^{a}$ a being a constant then dy/dx is

(A) $ax^{a-1}+a^{x}\log a+x^{x}(\log x+1)$ (B) $ax^{a-1}+a^{x}\log a+x^{x}(\log x-1)$

(C) $ax^{a-1}+a^{x}\log a-x^{x}(\log x+1)$ (D) None

53. If $x(1+y)^{1/2}+y(1+x)^{1/2}=0$ then dy/dx is

(A) $-(1+x^{2})^{-1}$ (B) $(1+x^{2})^{-1}$ (C) $-(1+x^{2})^{-2}$ (D) $(1+x^{2})^{-2}$

54. If $x^{2}-y^{2}+3x-5y=0$ then dy/dx is

(A) $(2x+3)(2y+5)^{-1}$ (B) $(2x+3)(2y-5)^{-1}$ (C) $(2x-3)(2y-5)^{-1}$ (D) None

55. If $x^{3}-xy^{2}+3y^{2}+2=0$ then dy/dx is

(A) $(y^{2}-3x^{2})/[2y(3-x)]$ (B) $(y^{2}-3x^{2})/[2y(x-3)]$

(C) $(y^{2}-3x^{2})/[2y(3+x)]$ (D) None

56. If $ax^{2}+2hxy+by^{2}+2gx+2fy+c=0$ then dy/dx is

(A) $-(ax+hy+g)/(hx+by+f)$ (B) $(ax+hy+g)/(hx+by+f)$

(C) $(ax-hy+g)/(hx-by+f)$ (D) None

57. If $y=x^{x^{x\ldots\ldots\infty}}$ then dy/dx is

(A) $y^{2}/[x(1-y\log x)]$ (B) $y^{2}/(1-y\log x)$

(C) $y^{2}/[x(1+y\log x)]$ (D) $y^{2}/(1+y\log x)]$

58. The slope of the tangent at the point (2 -2) to the curve $x^2+xy+y^2-4=0$ is given by

(A) 0 (B) 1 (C) –1 (D) None

59. If $x^2+y^2-2x=0$ then dy/dx is

(A) $(1-x)/y$ (B) $(1+x)/y$ (C) $(x-1)/y$ (D) None

60. If $x^2+3xy+y^2-4=0$ then dy/dx is

(A) $-(2x+3y)/(3x+2y)$ (B) $(2x+3y)/(3x+2y)$

(C) $-(3x+2y)/(2x+3y)$ (D) $(3x+2y)/(2x+3y)$

61. If $x^3+5x^2y+xy-5=0$ then dy/dx is

(A) $-(3x^2+10xy+y)/[x(5x+1)]$ (B) $(3x^2+10xy+y)/[x(5x+1)]$

(C) $-(3x^2+10xy+y)/[x(5x-1)]$ (D) None

62. If $(x+y)^{m+n}-x^m y^n=0$ then dy/dx is

(A) y/x (B) $-y/x$ (C) x/y (D) $-x/y$

63. Find the fourth derivative of $\log[(3x+4)^{1/2}]$

(A) $-243(3x+4)^{-4}$ (B) $243(3x+4)^{-4}$ (C) $-243(4x+3)^{-4}$ (D) None

64. If $y=[x+(1+x^2)^{1/2}]^m$ then the value of the expression $(1+x^2)d^2y/dx^2+xdy/dx-m^2y$ is

(A) 0 (B) 1 (C) –1 (D) None

65. If $y=x^m e^{nx}$ then d^2y/dx^2 is

(A) $m(m-1)x^{m-2}e^{nx}+2mnx^{m-1}e^{nx}+n^2x^me^{nx}$

(B) $m(1-m)x^{m-2}e^{nx}+2mnx^{m-1}e^{nx}+n^2x^me^{nx}$

(C) $m(m+1)x^{m-2}e^{nx}+2mnx^{m-1}e^{nx}+n^2x^me^{nx}$ (D) None

66. If $y=(\log x)/x$ then d^2y/dx^2 is

(A) $(2\log x-3)/x^3$ (B) $(3\log x-2)/x^3$ (C) $(2\log x+3)/x^3$ (D) None

67. If $y=ae^{mx}+be^{-mx}$ then d^2y/dx^2 is

(A) m^2y (B) my (C) $-m^2y$ (D) $-my$

68. If $y=ae^{2x}+bxe^{2x}$ where a and b are constants the value of the expression

$d^2y/dx^2-4dy/dx+4y$ is __________.

(A) 0 (B) 1 (C) –1 (D) None

69. If $y=a[x+(x^2-1)^{1/2}]^n+b[x-(x^2-1)^{1/2}]^n$ the value of the expression $(x^2-y)d^2y/dx^2+xdy/dx-n^2y$ is __________.

(A) 0 (B) 1 (C) –1 (D) None

70. If $y=(x+1)^{1/2}-(x-1)^{1/2}$ the value of the expression $(x^2-1)d^2y/dx^2+xdy/dx-y/4$ is given by

(A) 0 (B) 1 (C) –1 (D) None

71. If $y=\log[x+(1+x^2)^{1/2}]$ the value of the expression $(x^2+1)d^2y/dx^2+xdy/dx$ is ___.

(A) 0 (B) 1 (C) –1 (D) None

72. If $x=at^2$ and $y=2at$ then d^2y/dx^2 is

(A) $1/(2at^3)$ (B) $-1/(2at^3)$ (C) $2at^3$ (D) None

73. If $x=(1-t)/(1+t)$ and $t=(2t)/(1+t)$ then d^2y/dx^2 is

(A) 0 (B) 1 (C) –1 (D) None

(B) Integral Calculus

1. Integrate w.r.t x, $5x^2$

(A) $(5/3)x^3$ (B) $(3/5)x^3$ (C) $5x$ (D) $10x$

2. Integrate w.r.t x, $(3-2x-x^4)$

(A) $3x-x^2-x^5/5$ (B) $3x+x^2-x^5/5$ (C) $3x+x^2+x^5/5$ (D) None

3. Integrate w.r.t x, $(4x^3+3x^2-2x+5)$

(A) $x^4+x^3-x^2+5x$ (B) $x^4-x^3+x^2-5x$ (C) $x^4+x^3-x^2+5$ (D) None

4. Integrate w.r.t x, $(x^2-1)^2$

(A) $x^5/5-(2/3)x^3+x$ (B) $x^5/5+(2/3)x^3+x$

(C) $x^5/5+(3/2)x^3+x$ (D) None

5. Integrate w.r.t x, $(x^{1/2}-x/2+2x^{-1/2})$

(A) $(2/3)x^{3/2}-(1/4)x^2+4x^{1/2}+k$ (B) $(3/2)x^{3/2}-(1/4)x^2+4x^{1/2}$

(C) $(2/3)x^{3/2}+(1/4)x^2+4x^{1/2}$ (D) None

6. Integrate w.r.t x, $(1-3x)(1+x)$

(A) $x-x^2-x^3+k$ (B) $x-x^2+x^3+k$ (C) $x+x^2+x^3+k$ (D) None

7. Integrate w.r.t x, $(x^4+1)/x^2$

(A) $x^3/3-1/x+k$ (B) $1/x-x^3/3$ (C) $x^3/3+1/x$ (D) None

8. Integrate w.r.t x, $(3x^{-1}+4x^2-3x+8)$

(A) $3\log x-(4/3)x^3+(3/2)x^2-8x$ (B) $3\log x+(4/3)x^3-(3/2)x^2+8x$

(C) $3\log x+(4/3)x^3+(3/2)x^2+8x$ (D) None

9. Integrate w.r.t x, $(x-1/x)^3$

(A) $x^4/4-(3/2)x^2+3\log x+x^{-2}/2$ (B) $x^4/4+(3/2)x^2+3\log x+x^{-2}/2$

(C) $x^4/4-(2/3)x^2+3\log x+x^{-2}/2$ (D) None

10. Integrate w.r.t x, $(x^2-3x+x^{1/3}+7)x^{-1/2}$

(A) $(2/5)x^{5/2}-2x^{3/2}+(6/5)x^{5/6}-14x^{1/2}$ (B) $(5/2)x^{5/2}-2x^{3/2}+(5/6)x^{5/6}+14x^{1/2}$

(C) $(2/5)x^{5/2}+2x^{3/2}+(6/5)x^{5/6}+14x^{1/2}$ (D) None

11. Integrate w.r.t x, $(ax^2+bx^{-3}+cx^{-7})x^2$

(A) $(1/4)ax^4+b\log x-(1/4)cx^{-4}$ (B) $4ax^4+b\log x-4cx^{-4}$

(C) $(1/4)ax^4+b\log x+(1/4)cx^{-4}$ (D) None

12. Integrate w.r.t x, $x^{6/5}$

(A) $(5/11)x^{11/5}$ (B) $(11/5)x^{11/5}$ (C) $(1/5)x^{1/5}$ (D) None

13. Integrate w.r.t x, $x^{4/3}$

(A) $(3/7)x^{7/3}$ (B) $(7/3)x^{7/3}$ (C) $(1/3)x^{1/3}$ (D) None

14. Integrate w.r.t x, $x^{-1/2}$

(A) $2x^{1/2}$ (B) $(1/2)x^{1/2}$ (C) $-(3/2)x^{-3/2}$ (D) None

15. Integrate w.r.t x, $(x^{1/2}-x^{-1/2})$

(A) $(2/3)x^{3/2}-2x^{1/2}$ (B) $(3/2)x^{3/2}-(1/2)x^{1/2}$

(C) $-(1/2)x^{-1/2}-(3/2)x^{-3/2}$ (D) None

16. Integrate w.r.t x, $(7x^2-3x+8-x^{-1/2}+x^{-1}+x^{-2})$

(A) $(7/3)x^3-(3/2)x^2+8x-2x^{1/2}+\log x-x^{-1}$

(B) $(3/7)x^3-(2/3)x^2+8x-(1/2)x^{1/2}+\log x+x^{-1}$

(C) $(7/3)x^3+(3/2)x^2+8x+2x^{1/2}+\log x+x^{-1}$ (D) None

17. Integrate w.r.t x, $x^{-1}[ax^3+bx^2+cx+d]$

(A) $(1/3)ax^3+(1/2)bx^2+cx+d\log x$

(B) $3ax^3+2bx^2+cx+d\log x$

(C) $2ax+b-dx^{-2}$ (D) None

18. Integrate w.r.t x, $x^{-3}[4x^6+3x^5+2x^4+x^3+x^2+1]$

(A) $x^4+x^3+x^2+x+\log x-(1/2)x^{-2}$

(B) $x^4+x^3+x^2+x+\log x+(1/2)x^{-2}$

(C) $x^4+x^3+x^2+x+\log x+2x^{-2}$ (D) None

19. Integrate w.r.t x, $[2^x+(1/2)e^{-x}+4x^{-1}-x^{-1/3}]$

(A) $2^x/\log 2-(1/2)e^{-x}+4\log x-(3/2)x^{2/3}$

(B) $2^x/\log 2+(1/2)e^{-x}+4\log x+(3/2)x^{2/3}$

(C) $2^x/\log 2-2e^{-x}+4\log x-(2/3)x^{2/3}$ (D) None

20. Integrate w.r.t x, $(4x+5)^6$

(A) $(1/28)(4x+5)^7+k$ (B) $(1/7)(4x+5)^7+k$ (C) $7(4x+5)^7+k$ (D) None

21. Integrate w.r.t x, $x(x^2+4)^5$

(A) $(1/12)(x^2+4)^6+k$ (B) $(1/6)(x^2+4)^6+k$ (C) $6(x^2+4)^6+k$ (D) None

22. Integrate w.r.t x, $(x+a)^n$

(A) $(x+a)^{n+1}/(n+1)+k$ (B) $(x+a)^n/n+k$ (C) $(x+a)^{n-1}/(n-1)+k$ (D) None

23. Integrate w.r.t x, $(x^3+2)^2 3x^2$

(A) $(1/3)(x^3+2)^3+k$ (B) $3(x^3+2)^3+k$ (C) $3x^2(x^3+2)^3+k$ (D) $9x^2(x^3+2)^3+k$

24. Integrate w.r.t x, $(x^3+2)^{1/2}x^2$

(A) $(2/9)(x^3+2)^{3/2}+k$ (B) $(2/3)(x^3+2)^{3/2}+k$ (C) $(9/2)(x^3+2)^{3/2}+k$ (D) None

25. Integrate w.r.t x, $(x^3+2)^{-3}8x^2$

(A) $-(4/3)(x^3+2)^{-2}+k$ (B) $(4/3)(x^3+2)^{-2}+k$ (C) $(2/3)(x^3+2)^{-2}+k$ (D) None

26. Integrate w.r.t x, $(x^3+2)^{-1/4}x^2$

(A) $(4/9)(x^3+2)^{3/4}+k$ (B) $(9/4)(x^3+2)^{3/4}+k$ (C) $(3/4)(x^3+2)^{3/4}+k$ (D) None

27. Integrate w.r.t x, $(x^2+1)^{-n}3x$

(A) $(3/2)(x^2+1)^{1-n}/(1-n)+k$ (B) $(3/2)(x^2+1)^{n-1}/(1-n)$

(C) $(2/3)(x^2+1)^{1-n}/(1-n)+k$ (D) None

28. Integrate w.r.t x, $(x^2+1)^{-3}x^3$

(A) $-(1/4)(2x^2+1)/(x^2+1)^2+k$ (B) $(1/4)(2x^2+1)/(x^2+1)^2+k$

(C) $-(1/4)(2x^2+1)/(x^2+1)+k$ (D) $(1/4)(2x^2+1)/(x^2+1)+k$

29. Integrate w.r.t x, $1/[x\log x\log(\log x)]$

(A) $\log[\log(\log x)]+k$ (B) $\log(\log x)+k$ (C) $\log x+k$ (D) x^{-1}

30. Integrate w.r.t x, $1/[x(\log x)^2]$

(A) $-1/\log x+k$ (B) $1/\log x+k$ (C) $\log x$ (D) None

31. Integrate w.r.t x, $x(x^2+3)^{-2}$

(A) $-(1/2)\,(x^2+3)^{-1}+k$ (B) $(1/2)\,(x^2+3)^{-1}+k$ (C) $2(x^2+3)^{-1}+k$ (D) None

32. Integrate w.r.t x, $(3x+7)(2x^2+3x-2)^{-1}$

(A) $(3/4)\log(2x^2+3x-2)+(19/20)\log[(2x-1)/\{2(x+2)\}]+k$

(B) $(3/4)\log(2x^2+3x-2)+\log[(2x-1)/\{2(x+2)\}]+k$

(C) $(3/4)\log(2x^2+3x-2)+(19/20)\log[2(2x-1)(x+2)]+k$ (D) None

33. Integrate w.r.t x, $1/(2x^2-x-1)$

(A) $(1/3)\log[2(x-1)/(2x+1)]+c$ (B) $-(1/3)\log[2(x-1)/(2x+1)]+c$

(C) $(1/3)\log[2(1-x)/(2x+1)]$ (D) None

34. Integrate w.r.t x, $(x+1)(3+2x-x^2)^{-1}+c$

(A) $-(1/2)\log(3+2x-x^2)+(1/2)\log[(x+1)/(x-3)]+c$

(B) $(1/2)\log(3+2x-x^2)+(1/2)\log[(x+1)/(x-3)]+c$

(C) $-(1/2)\log(3+2x-x^2)+(1/2)\log[(x-3)/(x+1)]+c$ (D) None

35. Integrate w.r.t x, $(5x^2+8x+4)^{-1/2}$

(A) $(1/\sqrt{5})\log[\{\sqrt{5}x+4/\sqrt{5}+(5x^2+8x+4)^{1/2}\}]+c$

(B) $\sqrt{5}\log[\{\sqrt{5}x+4/\sqrt{5}+(5x^2+8x+4)^{1/2}\}]+c$

(C) $(1/\sqrt{5})\log[\{\sqrt{5}x+4/\sqrt{5}+(5x^2+8x+4)^{-1/2}\}]+c$ (D) None

36. Integrate w.r.t x, $(x+1)(5x^2+8x-4)^{-1/2}$

(A) $(1/5)(5x^2+8x-4)^{1/2}+[1/(5\sqrt{5})]\log[5\{x+4/5+(x^2+8x/5-4/5)^{1/2}(1/6)\}]+c$

(B) $(1/5)(5x^2+8x-4)^{1/2}+[1/(5\sqrt{5})]\log[5\{x+4/5+(x^2+8x/5-4/5)^{-1/2}(1/6)\}]+c$

(C) $(1/5)(5x^2+8x-4)^{1/2}+[1/(5\sqrt{5})]\log[5\{x+4/5+(x^2+8x/5-4/5)^{1/2}\}]+c$

(D) None

37. Integrate w.r.t x, $(x^2-1)(x^4-x^2+1)^{-1}$

(A) $[1/(2\sqrt{3})]\log[(x^2-\sqrt{3}x+1)/(x^2+\sqrt{3}x+1)]+c$

(B) $[1/(2\sqrt{3})]\log[(x^2+\sqrt{3}x+1)/(x^2-\sqrt{3}x+1)]+c$

(C) $[3/(2\sqrt{3})]\log[(x^2-\sqrt{3}x+1)/(x^2+\sqrt{3}x+1)]+c$

(D) None

38. Integrate w.r.t x, x^2e^{3x}

(A) $(1/3)(x^2e^{3x})-(2/9)(xe^{3x})+(2/27)e^{3x}+c$

(B) $(1/3)(x^2e^{3x})+(2/9)(xe^{3x})+(2/27)e^{3x}+c$

(C) $(1/3)(x^2e^{3x})-(1/9)(xe^{3x})+(1/27)e^{3x}+c$ (D) None

39. Integrate w.r.t x, $\log x$

(A) $x(\log x-1)+c$ (B) $x(\log x+1)+c$ (C) $\log x-1+c$ (D) $\log x+1+c$

40. Integrate w.r.t x, $x^n\log x$

(A) $x^{n+1}(n+1)^{-1}[\log x-(n+1)^{-1}]+c$ (B) $x^{n-1}(n-1)^{-1}[\log x-(n-1)^{-1}]+c$

(C) $x^{n+1}(n+1)^{-1}[\log x+(n+1)^{-1}]+c$ (D) None

41. Integrate w.r.t x, $xe^x(x+1)^{-2}$

(A) $e^x(x+1)^{-1}+c$ (B) $e^x(x+1)^{-2}$ (C) $xe^x(x+1)^{-1}+c$ (D) None

42. Integrate w.r.t x, xe^x

(A) $e^x(x-1)+k$ (B) $e^x(x+1)$ (C) $xe^x(x-1)+k$ (D) None

43. Integrate w.r.t x, x^2e^x

(A) $e^x(x^2-2x+2)+k$ (B) $e^x(x^2+2x+2)$ (C) $e^x(x+2)^2+k$ (D) None

44. Integrate w.r.t x, $x\log x$

(A) $(1/4)x^2\log(x^2/e)+k$ (B) $(1/2)x^2\log(x^2/e)+k$

(C) $(1/4)x^2\log(x/e)+k$ (D) None

45. Integrate w.r.t x, $(\log x)^2$

(A) $x(\log x)^2-2x\log x+2x+k$ (B) $x(\log x)^2+2x\log x+2x+k$

(C) $x(\log x)^2-2\log x+2x+k$ (D) $x(\log x)^2+2\log x+2x+k$

46. Integrate w.r.t x, $e^x(1+x)(2+x)^{-2}$

(A) $e^x(2+x)^{-1}+k$ (B) $-e^x(2+x)^{-1}+k$ (C) $(1/2)e^x(2+x)^{-1}+k$ (D) None

47. Integrate w.r.t x, $e^x(1+x\log x)x^{-1}$

(A) $e^x\log x+k$ (B) $-e^x\log x+k$ (C) $e^x x^{-1}+k$ (D) None

48. Integrate w.r.t x, $x(x-1)^{-1}(2x+1)^{-1}$

(A) $(1/3)[\log(x-1)+(1/2)\log(2x+1)]+k$ (B) $(1/3)[\log(x-1)+\log(2x+1)]+k$

(C) $(1/3)[\log(x-1)-(1/2)\log(2x+1)]+k$ (D) None

49. Integrate w.r.t x, $(x-x^3)^{-1}$

(A) $(1/2)\log[x^2/(1-x^2)]+k$ (B) $(1/2)\log[x^2/(1-x)^2]+k$

(C) $(1/2)\log[x^2/(1+x)^2]+k$ (D) None

50. Integrate w.r.t x, $x^3[(x-a)(x-b)(x-c)]^{-1}$ given that

$1/A=(a-b)(a-c)/a^3$, $1/B=(b-a)(b-c)/b^3$, $1/C=(c-a)(c-b)/c^3$

(A) $x+A\log(x-a)+B\log(x-b)+C\log(x-c)+k$

(B) $A\log(x-a)+B\log(x-b)+C\log(x-c)+k$

(C) $1+A\log(x-a)+B\log(x-b)+C\log(x-c)+k$ (D) None

51. Integrate w.r.t x, $(25-x^2)^{-1}$ from lower limit 3 to upper limit 4 of x

(A) $(3/4)\log(1/5)+k$ (B) $(1/5)\log(3/4)$ (C) $(1/5)\log(4/3)+k$ (D) $(3/4)\log 5+k$

52. Integrate w.r.t x, $(2x+3)^{1/2}$ from lower limit 3 to upper limit 11 of x

(A) 33 (B) 100/3 (C) 98/3 (D) None

ANSWERS

(A) Differential Calculus

1)	C	2)	A	3)	A	4)	A	5)	A	6)	A
7)	A	8)	A	9)	B	10)	A	11)	A	12)	A
13)	A	14)	A	15)	A	16)	A	17)	A	18)	A
19)	A	20)	A	21)	A	22)	A	23)	A	24)	A
25)	A	26)	A	27)	A	28)	A	29)	A	30)	A
31)	A	32)	A	33)	A	34)	A	35)	A	36)	A
37)	A	38)	B	39)	A	40)	A	41)	A	42)	A
43)	A	44)	A	45)	A	46)	A	47)	A	48)	A
49)	A	50)	A	51)	A	52)	A	53)	A	54)	A
55)	A	56)	A	57)	A	58)	B	59)	A	60)	A
61)	A	62)	A	63)	A	64)	A	65)	A	66)	A
67)	A	68)	A	69)	A	70)	A	71)	A	72)	A
73)	A										

(B) Integral Calculus

1)	A	2)	A	3)	A	4)	A	5)	A	6)	A
7)	A	8)	B	9)	A	10)	A	11)	A	12)	A
13)	A	14)	A	15)	A	16)	A	17)	A	18)	A
19)	A	20)	A	21)	A	22)	A	23)	A	24)	A
25)	A	26)	A	27)	A	28)	A	29)	A	30)	A
31)	A	32)	A	33)	A	34)	A	35)	A	36)	A
37)	A	38)	A	39)	A	40)	A	41)	A	42)	A
43)	A	44)	A	45)	A	46)	A	47)	A	48)	A
49)	A	50)	A	51)	B	52)	C				