Sl. No. **8951**   **B–JGT–K–NBB**

MATHEMATICS

Paper—II

| Time Allowed : Three Hours | Maximum Marks : 200 |

INSTRUCTIONS

**Candidates should attempt Question Nos. 1 and 5
which are compulsory, and any THREE of the
remaining questions, selecting at least
ONE question from each Section.**

All questions carry equal marks.

**The number of marks carried by each part of a
question is indicated against each.**

Answers must be written in ENGLISH only.

**Assume suitable data, if considered necessary,
and indicate the same clearly.**

**Symbols and notations have their usual meanings,
unless indicated otherwise.**

Section—A

1. Answer any *four* parts from the following :

   (a) Let

   $$G = \left\{ \begin{bmatrix} a & a \\ a & a \end{bmatrix} \middle| a \in \mathbb{R},\ a \neq 0 \right\}$$

   Show that G is a group under matrix multiplication.   10

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(b) Let F be a field of order 32. Show that the only subfields of F are F itself and $\{0, 1\}$.   10

(c) If $f : \mathbb{R} \to \mathbb{R}$ is such that

$$f(x+y) = f(x)f(y)$$

for all x, y in $\mathbb{R}$ and $f(x) \neq 0$ for any x in $\mathbb{R}$, show that $f'(x) = f(x)$ for all x in $\mathbb{R}$ given that $f'(0) = f(0)$ and the function is differentiable for all x in $\mathbb{R}$.   10

(d) Determine the analytic function $f(z) = u + iv$ if $v = e^x(x \sin y + y \cos y)$.   10

(e) A captain of a cricket team has to allot four middle-order batting positions to four batsmen. The average number of runs scored by each batsman at these positions are as follows. Assign each batsman his batting position for *maximum* performance :   10

Batting position \ Batsman	IV	V	VI	VII
A	40	25	20	35
B	36	30	24	40
C	38	30	18	40
D	40	23	15	33

2. (a) A rectangular box open at the top is to have a surface area of 12 square units. Find the dimensions of the box so that the volume is maximum.   13

(b) Prove or disprove that $(\mathbb{R}, +)$ and $(\mathbb{R}^+, \cdot)$ are isomorphic groups where $\mathbb{R}^+$ denotes the set of all positive real numbers. 13

(c) Using the method of contour integration, evaluate

$$\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)^2(x^2+2x+2)}$$ 14

3. *(a)* Show that zero and unity are only idempotents of Z_n if $n = p^r$, where p is a prime. 13

(b) Evaluate

$$\iint_R (x-y+1)\,dx\,dy$$

where R is the region inside the unit square in which $x+y \geq \frac{1}{2}$. 13

(c) Solve the following linear programming problem by the simplex method : 14

Maximize $Z = 3x_1 + 4x_2 + x_3$

subject to

$$x_1 + 2x_2 + 7x_3 \leq 8$$
$$x_1 + x_2 - 2x_3 \leq 6$$
$$x_1, x_2, x_3 \geq 0$$

4. *(a)* Let R be a Euclidean domain with Euclidean valuation d. Let n be an integer such that $d(1) + n \geq 0$. Show that the function $d_n : R - \{0\} \rightarrow S$, where S is the set of all negative integers defined by $d_n(a) = d(a) + n$ for all $a \in R - \{0\}$ is a Euclidean valuation.   13

(b) Obtain Laurent's series expansion of the function

$$f(z) = \frac{1}{(z+1)(z+3)}$$

in the region $0 < |z+1| < 2$.   13

(c) *ABC* Electricals manufactures and sells two models of lamps, L_1 and L_2, the profit per unit being Rs 50 and Rs 30, respectively. The process involves two workers W_1 and W_2, who are available for 40 hours and 30 hours per week, respectively. W_1 assembles each unit of L_1 in 30 minutes and that of L_2 in 40 minutes. W_2 paints each unit of L_1 in 30 minutes and that of L_2 in 15 minutes. Assuming that all lamps made can be sold, determine the weekly production figures that maximize the profit.   14

Section—B

5. Answer any *four* parts from the following :

(a) Find the general solution of

$$x(y^2 + z)p + y(x^2 + z)q = z(x^2 - y^2)$$ 10

(b) Solve $x \log_{10} x = 1 \cdot 2$ by regula falsi method.   10

(c) Convert the following :   10

   (i) $(736 \cdot 4)_8$ to decimal number

   (ii) $(41 \cdot 6875)_{10}$ to binary number

   (iii) $(101101)_2$ to decimal number

   (iv) $(AF63)_{16}$ to decimal number

   (v) $(10111101111)_2$ to hexadecimal number

(d) Show that the sum of the moments of inertia of an elliptic area about any two tangents at right angles is always the same.   10

(e) A two-dimensional flow field is given by $\psi = xy$. Show that—

   (i) the flow is irrotational;

   (ii) ψ and ϕ satisfy Laplace equation.

   Symbols ψ and ϕ convey the usual meaning.   10

6. *(a)* Using Lagrange interpolation, obtain an approximate value of $\sin(0\cdot15)$ and a bound on the truncation error for the given data :   12

$$\sin(0\cdot1) = 0\cdot09983,\ \sin(0\cdot2) = 0\cdot19867$$

(b) Draw a flow chart for finding the roots of the quadratic equation $ax^2 + bx + c = 0$.   12

(c) Solve

$$\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}$$

given the conditions

(i) $u(0,\ t) = u(\pi,\ t) = 0,\ \ t > 0$

(ii) $u(x,\ 0) = \sin 2x,\ \ 0 < x < \pi$ 16

7. *(a)* Find the general solution of

$$(D - D' - 1)(D - D' - 2)z = e^{2x-y} + \sin(3x + 2y)$$

13

(b) Show that $\phi = (x - t)(y - t)$ represents the velocity potential of an incompressible two-dimensional fluid. Further show that the streamlines at time t are the curves

$$(x - t)^2 - (y - t)^2 = \text{constant}$$

13

(c) Find the interpolating polynomial for $(0, 2)$, $(1, 3)$, $(2, 12)$ and $(5, 147)$.   14

8. (a) A mass m_1, hanging at the end of a string, draws a mass m_2 along the surface of a smooth table. If the mass on the table be doubled, the tension of the string is increased by one-half. Show that $m_1 : m_2 = 2 : 1$.   13

(b) Solve the initial value problem

$$\frac{dy}{dx} = \frac{y-x}{y+x}, \quad y(0) = 1$$

for $x = 0 \cdot 1$ by Euler's method.   13

(c) Show that the vorticity vector $\vec{\Omega}$ of an incompressible viscous fluid moving under no external forces satisfies the differential equation

$$\frac{D\vec{\Omega}}{Dt} = (\vec{\Omega} \cdot \nabla)\vec{q} + v\nabla^2 \vec{\Omega}$$

where v is the kinematic viscosity.   14

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