2011
PART 01 — ENGINEERING MATHEMATICS
(Common to all candidates)
(Answer ALL questions)

1. **If the rank of a matrix $\begin{pmatrix} 1 & 1 & -1 & 0 \\ 4 & 4 & -3 & 1 \\ b & 2 & 2 & 2 \\ 9 & 9 & b & 3 \end{pmatrix}$ is 3, then value of b is**

1) 3 2) 1 3) –6 4) 4

2. **If the rank of non-square matrix A and rank of the augmented matrix of system of linear equations are equal, then the system**

1) is inconsistent 2) has no solution
3) is consistent 4) does not have solution

3. **If the system $-2x+y+z=a$, $x-2y+z=b$, $x+y-2z=c$, where a, b, c are constants, is consistent, then it has infinite solutions only when**

1) $a+b+c=0$ 2) $a-b+c=0$
3) $a+b-c=0$ 4) $a+b+c\neq 0$

4. **If $A=\begin{pmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{pmatrix}$, then the algebraic and geometric multiplicity are respectively**

1) 2, 2 2) 1, 2 3) 1, 1 4) 2, 1

5. **The signature of quadratic form $2xy+2yz+2zx$ is**

1) 3 2) –1 3) 2 4) 1

6. **If $u=\log\left(\frac{x^2}{y}\right)$, then xu_x+yu_y is equal to**

1) 2u 2) u 3) 0 4) 1

7. **If $u=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, then $x^2u_{xx}+2xyu_{xy}+y^2u_{yy}$ equals**

1) 0 2) sin u cos 3u
3) sin 3u cos u 4) 2sin u cos 3u

8. **If $u=xyz$, $v=x^2+y^2+z^2$, $w=x+y+z$, then $\frac{\partial(x,y,z)}{\partial(u,v,w)}$ is equal to**

1) $-2(x-y)(y-z)(z-x)$ 2) $(x-y)(y-z)(z-x)$
3) $\frac{1}{2(x-y)(y-z)(z-x)}$ 4) xyz

9. **The particular integral of $(D^2+D)y=x^2+2x+4$ is**

1) x^2+4 2) $\frac{x^3}{3}+2x$
3) $\frac{x^3+12x}{3}$ 4) $\frac{1}{3}(x^3+4x)$

10. **In the equation $x'(t)+2y(t)=-\sin t$, $y'(t)-2x(t)=\cos t$, given $x(0)=0$ and $y(0)=1$, if $x=\cos 2t-\sin 2t-\cos t$, then y is equal to**

1) cos 2t–sin 2t+sin t 2) cos 2t+sin 2t–sin t
3) sin 2t–cos 2t–sin t 4) cos 2t+sin 2t+sin t

11. **If minimum value of $f(x)=x^2+2bx+2c^2$ is greater than maximum value of $g(x)=-x^2-2cx+b^2$, then for x is real,**

1) $0<c<\sqrt{2b}$ 2) no real value of a
3) $|c|>\sqrt{2}\,|b|$ 4) $\sqrt{2}\,|c|>b$

12. **Form the partial differential equation by eliminating the arbitrary constants a and b from $z=a\log\left\{\frac{b(y-1)}{1-x}\right\}$ as**

1) $xp = yq$ 2) $p+q=xp+yq$
3) $yp = xq$ 4) $p+q = z$

13. **The particular integral of $(2D^2-3DD'+D'^2)z=e^{x+2y}$ is**

1) $\frac{1}{2}e^{x+2y}$ 2) $-\frac{x}{2}e^{x+2y}$ 3) xe^{x+2y} 4) $\frac{x^2}{2}e^{x+2y}$

14. **If $f=\tan^{-1}\left(\frac{y}{x}\right)$, then div (grad f) is equal to**

1) –1 2) 1 3) 2 4) 0

15. **If $\vec{F}=ax\vec{i}+by\vec{j}+cz\vec{k}$, then $\iint_S \vec{F}.d\vec{S}$, where S is the surface of a unit sphere, is**

1) $\frac{4\pi}{3(a+b+c)}$ 2) $\frac{4}{3}\pi(a+b+c)^2$
3) $\frac{4\pi}{3}(a+b+c)$ 4) 0

16. The value of $\int_C [(y-\sin x)dx+\cos y dy]$, where C is the plane triangle enclosed by the lines y=0, $y=\frac{\pi}{2}$ and $y=\frac{2}{\pi}x$, is

1) $\frac{8}{\pi}$ 2) $-\frac{1}{4\pi}(\pi^2+8)$ 3) $\frac{1}{8\pi}(\pi^2+4)$ 4) π^2+2

17. If f(z)=u+iv is analytic, then its first derivative equals

1) $\frac{\partial u}{\partial x}-i\frac{\partial v}{\partial x}$ 2) $\frac{\partial u}{\partial x}+i\frac{\partial v}{\partial y}$ 3) $\frac{\partial v}{\partial y}-i\frac{\partial u}{\partial x}$ 4) $\frac{\partial u}{\partial x}-i\frac{\partial u}{\partial y}$

18. The value of $\int_C \frac{3z+4}{2z+1}dz$, where C is the circle $|z|=1$, is

1) πi 2) $3\pi i$ 3) $2\pi i$ 4) $\frac{\pi}{3}$

19. The value of $\int_C \frac{4z^2+z+5}{z-4}dz$, where C is the ellipse $\left(\frac{3x}{2}\right)^2+y^2=3^2$, is

1) 3 2) 0 3) $\frac{2}{3}$ 4) –1

20. The pole of $\frac{1}{\cos z-\sin z}$ is

1) $\frac{\pi}{2}$ 2) $\frac{\pi}{3}$ 3) π 4) $\frac{\pi}{4}$

21. The value of $\int_0^\infty \frac{1}{t}\left(e^{-t}\sin^2 t\right)dt$ is

1) $\frac{1}{5}\log 2$ 2) $\frac{1}{4}\log 5$ 3) $\log 3$ 4) 0

22. The solution of $(D^2+9)y=\cos 2t$, $y(0)=1$ and $y(\pi/2)=1$ is given by

1) $y=\frac{1}{5}(\cos 3t+4\sin 3t+4\cos 2t)$

2) $y=\frac{1}{5}(2\cos 2t+\sin 3t+\cos 3t)$

3) $y=\frac{1}{5}(\cos 2t+4\sin 3t+4\cos 3t)$

4) $y=\frac{-1}{5}(\cos 2t-4\sin 3t+4\cos 3t)$

23. The Fourier sine transform of $e^{\frac{-ax}{x}}$ is

1) $\tan^{-1}(s/a)$ 2) $\tan^{-1}(s/2a)$

3) $\tanh^{-1}(s/a)$ 4) $\frac{1}{2}\tan^{-(s/a)}$

24. If $Z(u_n)=\frac{2z^2+3z+4}{(z-3)^3}$, $|z|>3$, then the value of u_3 is equal to

1) 21 2) 193 3) 46 4) 139

25. As soon as a new value of a variable is found, it is used immediately in the equations, such method is known as

1) Gauss-Jordan method
2) Gauss-Jacobi's method
3) Gauss Elimination method
4) Gauss-Seidal method

26. The value of x for the data (0, 1), (1, 3), (2, 9), (3, x) and (4, 81) is

1) 31 2) 18 3) 27 4) 36

27. If y(0)=2, y(1)=4, y(2)=8 and y(4)=32, then y(3) is equal to

1) 12 2) 16.5 3) 18 4) 20

28. The joint probability density function of a random variable (x, y) is given by $f(x, y)=kxye^{-(x^2+y^2)}$, where x, y>0. Then the value of k is

1) 1 2) 3 3) 4 4) 2

29. The two lines of regression are perpendicular to each other if the co-efficient of correlation equals

1) 0 2) 1 3) –1 4) ±1

30. Let the random variable X have the probability density function

$$f(x)=\begin{cases} xe^{-x} & \text{for } x>0 \\ 0 & \text{otherwise}\end{cases}$$

Then the moment generating function is

1) $\frac{1}{1-2t}$ 2) $\frac{1}{1-t}$ 3) $\frac{1}{1+t}$ 4) $\frac{2}{2-t}$

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1 3	2 3	3 1	4 1	5 2	6 4	7 4	8 3	9 3	10 2
11 3	12 2	13 2	14 4	15 3	16 .... *	17 4	18 3	19 2	20 4
21 2	22 .... *	23 1	24 3	25 4	26 3	27 2	28 3	29 1	30 .... *

PART 01 — ENGINEERING MATHEMATICS

DETAILED SOLUTIONS

1. **(3)**

$$\begin{bmatrix} 1 & 1 & -1 & 0 \\ 4 & 4 & -3 & 1 \\ b & 2 & 2 & 2 \\ 9 & 9 & b & 3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 4 & 0 & 1 & 1 \\ b & 2-b & 2+b & 2 \\ 9 & 0 & 9+b & 3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 4 & 1 & 1 & 0 \\ b & 2+b & 2 & 2-b \\ 9 & 9+b & 3 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 4 & 1 & 0 & 0 \\ b & 2+b & -b & 2-b \\ 9 & 9+b & -6-b & 0 \end{bmatrix}$$

Since the rank is 3 any determinant of order 4=0

$$\therefore 1\times \begin{vmatrix} 1 & 0 & 0 \\ 2+b & -b & 2-b \\ 9+b & -6-b & 0 \end{vmatrix} = 0$$

$\Rightarrow 1(2-b)(6+b) = 0$

$\therefore b = -6$ (or) $b = 2$

2. **(3)**

If $\rho(A) = \rho(A, B)$ then the given system is consistent.

3. **(1)**

$$[A, B] = \begin{vmatrix} -2 & 1 & 1 & a \\ 1 & -2 & 1 & b \\ 1 & 1 & -2 & c \end{vmatrix}$$

$$\sim \begin{vmatrix} 1 & -2 & 1 & b \\ -2 & 1 & 1 & a \\ 1 & 1 & -2 & c \end{vmatrix} R_1 \leftrightarrow R_2$$

$$\sim \begin{vmatrix} 1 & -2 & 1 & b \\ 0 & -3 & 3 & a+2b \\ 0 & 3 & -3 & c-b \end{vmatrix}$$

$$\sim \begin{vmatrix} 1 & -2 & 1 & b \\ 0 & -3 & 3 & a+2b \\ 0 & 0 & 0 & a+b+c \end{vmatrix}$$

Since the system has infinite solutions implies rank is less than 3.

$\therefore a+b+c = 0$

4. **(1)**

Algebraic multiplicity = 2

Geometric multiplicity = 2

5. **(2)**

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$|A| = -1[0-1]+1[1-0]$$
$$= 1+1 = 2$$
$$D_1 = |a_{11}| = 0$$
$$D_2 = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = 0-1 = -1$$
$$D_3 = |A| = 2$$

Difference between positive square terms and non positive square terms

$= 1-2 = -1$

$\therefore$ Signature = –1

6. **(4)**

$$u = \log\left(\frac{x^2}{y}\right)$$

$$ux = \frac{\partial u}{\partial x} = \frac{1}{\left(\frac{x^2}{y}\right)}.\frac{2x}{y}$$

$$= \frac{y}{x^2}.\frac{2x}{y} = \frac{2}{x}$$

$$u_y = \frac{\partial u}{\partial y} = \frac{1}{\left(\frac{x^2}{y}\right)} \cdot \left(\frac{-x^2}{y^2}\right)$$

$$= \frac{y}{x^2}\left(\frac{-x^2}{y^2} = \frac{-1}{y}\right)$$

$$\therefore xu_x + yu_y = x\left(\frac{2}{x}\right) + y\left(\frac{-1}{y}\right)$$

$$= 2-1 = 1$$

7. (4)

$$\text{Let } f(u) = z = \tan u$$

$$= \frac{x^3+y^3}{x-y}$$

Clearly z is a homogeneous function of degree 2.

$$g(u) = \frac{nf(u)}{f'(u)}$$

$$= \frac{2 \times \tan u}{\sec^2 u}$$

$$= 2 \times \frac{\sin u}{\cos u} \times \cos^2 u$$

$$= 2 \sin u \cos u$$

$$= \sin 2u$$

Formula :

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$$

$$= g(u)[g'(u)-1]$$

$$= \sin 2u(2\cos 2u-1)$$

$$= 2\sin 2u \cos 2u - \sin 2u$$

$$= \sin 4u - \sin 2u$$

$$= 2\sin u \cos 3u$$

8. (3)

$$u = xyz$$

$$\therefore \frac{\partial u}{\partial x} = yz ; \frac{\partial u}{\partial y} = xz ; \frac{\partial u}{\partial z} = xy$$

$$v = x^2+y^2+z^2$$

$$\frac{\partial v}{\partial x} = 2x ; \frac{\partial v}{\partial y} = 2y ; \frac{\partial v}{\partial z} = 2z$$

$$w = x+y+z$$

$$\frac{\partial w}{\partial x} = 1 ; \frac{\partial w}{\partial y} = 1 ; \frac{\partial w}{\partial z} = 1$$

Now

$$\frac{\partial(u, v, w)}{\partial(x, y, z)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix}$$

$$= \begin{vmatrix} yz & xz & xy \\ 2x & 2y & 2z \\ 1 & 1 & 1 \end{vmatrix} = 2\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ yz & xz & xy \end{vmatrix}$$

$$= 2(x-y)(y-z)(z-x) \left[\because \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a) \right]$$

Now

$$\frac{\partial(x, y, z)}{\partial(u, v, w)} = \frac{1}{\frac{\partial(u, v, w)}{\partial(x, y, z)}}$$

$$= \frac{1}{2(x-y)(y-z)(z-x)}$$

9. (3)

Auxillary equation is

$$m^2+m = 0$$

$$m(m+1) = 0$$

$$m = 0, \; m = -1$$

$$C.F = Ae^0+Be^{-x} = A+Be^{-x}$$

$$\text{If P.I.} = \frac{x^3+12x}{3}, \text{ then}$$

$$\text{Solution} = y = A+Be^{-x}+\frac{x^3}{3}+4x$$

$$\text{then } \frac{dy}{dx} = -Be^{-x}+x^2+4$$

$$\frac{d^2y}{dx^2} = Be^{-x}+2x$$

$$\therefore (D^2+D)y = \frac{d^2y}{dx^2}+\frac{dy}{dx} = x^2+2x+4$$

$\therefore$ Correct option is (3)

10. (2)

$$x'(t)+2y(t) = -\sin t \quad \ldots (1)$$
$$y'(t)-2x(t) = \cos t \quad \ldots (2)$$
$$x = \cos 2t - \sin 2t - \cos t$$
$$x'(t) = -2\sin 2t - 2\cos 2t + \sin t$$

$\therefore (1) \Rightarrow$

$-2\sin 2t - 2\cos 2t + \sin t + 2y(t) = -\sin t$

$$\therefore 2y(t) = 2\sin 2t + 2\cos 2t - 2\sin t$$
$$\therefore y(t) = \sin 2t + \cos 2t - \sin t$$

11. (3)

$$f(x) = x^2+2bx+2c^2$$
$$f'(x) = 2x+2b$$
$$f''(x) = 2$$
$$f'(x) = 0 \Rightarrow 2x+2b=0$$
$$\Rightarrow x = -b$$
$$f''(-b) = 2>0$$

$\therefore x = -b$ gives minimum

Minimum value $= (-b)^2+2b(-b)+2c^2$
$= b^2-2b^2+2c^2$
$= -b^2+2c^2$

$$g(x) = -x^2-2cx+b^2$$
$$g'(x) = -2x-2c$$
$$g''(x) = -2$$
$$g'(x) = 0 \Rightarrow -2x-2c = 0$$
$$\therefore x = -c$$

Now $g''(-c) = -2<0$

$\therefore x = -c$ gives maximum

Maximum value $= -(-c)^2-2c(-c)+b^2$
$= -c^2+2c^2+b^2$
$= c^2+b^2$

Minimum value of $f(x)>$ Maximum value of $g(x)$

$\Rightarrow -b^2+2c^2>c^2+b^2$

$\Rightarrow c^2>2b^2$

$\therefore |c|>\sqrt{2}\,|b|$

12. (2)

$$z = a\log\left(\frac{b(y-1)}{1-x}\right)$$

$$p = \frac{\partial z}{\partial x} = \frac{a}{\frac{b(y-1)}{1-x}}\times\frac{-b(y-1)}{(1-x)^2}\times -1$$

$$= \frac{a(1-x)}{b(y-1)}\times\frac{b(y-1)}{(1-x)^2} = \frac{a}{(1-x)}$$

$$\Rightarrow (1-x)p = a \quad \ldots (1)$$

$$q = \frac{\partial z}{\partial y} = \frac{a}{\frac{b(y-1)}{1-x}}\times\frac{b}{(1-x)}$$

$$= \frac{a(1-x)}{b(y-1)}\times\frac{b}{(1-x)} = \frac{a}{y-1}$$

$$\therefore (y-1)q = a \quad \ldots (2)$$

From (1) and (2)

$$(1-x)p = (y-1)q$$
$$\Rightarrow p-xp = yq-q$$
$$\therefore p+q = xp+yq$$

13. (2)

$$PI = \frac{e^{x+2y}}{2D^2-3DD'+D'^2}$$

$$= \frac{xe^{x+2y}}{4D-3D'} = \frac{xe^{x+2y}}{4(1)-3(2)}$$

$$= \frac{-xe^{x+2y}}{2}$$

14. (4)

Formula :

$$\text{div (grad f)} = \nabla.\nabla f$$
$$= \nabla^2 f$$
$$= \frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}$$

Now $f = \tan^{-1}\left(\frac{y}{x}\right)$

$$\frac{\partial f}{\partial x} = \frac{1}{1+\left(\frac{y}{x}\right)^2}\times\frac{-y}{x^2}$$

$$= \frac{x^2}{x^2+y^2}\times\frac{-y}{x^2} = \frac{-y}{\left(x^2+y^2\right)}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\left(x^2+y^2\right)0+y.2x}{\left(x^2+y^2\right)^2}$$

$$= \frac{2xy}{\left(x^2+y^2\right)^2}$$

$$\frac{\partial f}{\partial y} = \frac{1}{1+\left(\frac{y}{x}\right)^2}.\frac{1}{x}$$

$$= \frac{x^2}{x^2+y^2}.\frac{1}{x} = \frac{x}{x^2+y^2}$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{(x^2+y^2).0 - x(2y)}{(x^2+y^2)^2}$$

$$= \frac{-2xy}{(x^2+y^2)^2}$$

$$\therefore \text{div (grad f)} = \nabla^2 f$$

$$= \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$= \frac{2xy}{(x^2+y^2)^2} - \frac{2xy}{(x^2+y^2)^2}$$

$$= 0$$

15. (3)

$$\nabla.\vec{F} = \left(\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}\right).\left(ax\,\vec{i} + by\vec{j} + cz\vec{k}\right)$$

$$= a+b+c$$

By Gauss divergence theorem

$$\iint_S \vec{F}.\hat{n}ds = \iiint_V \nabla.\vec{F}dv$$

$$= \iiint_V (a+b+c)dv$$

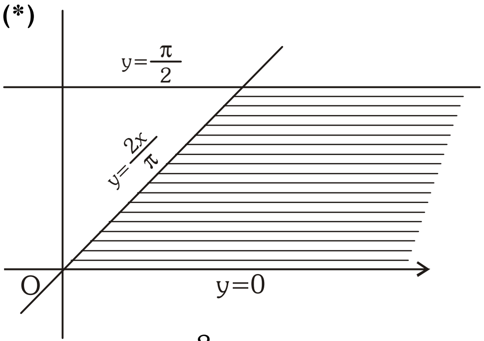
$$= (a+b+c)\iiint_V dv$$

$= (a+b+c)$ volume of the unit sphere

$$= (a+b+c)\times\frac{4\pi}{3}(1)^3$$

$$= \frac{4\pi(a+b+c)}{3}$$

16. (*)



$y=0,\ y=\dfrac{\pi}{2},\ y=\dfrac{2x}{\pi}$

will not form a triangle

$\therefore$ The data given in the problem are not correct.

17. (4)

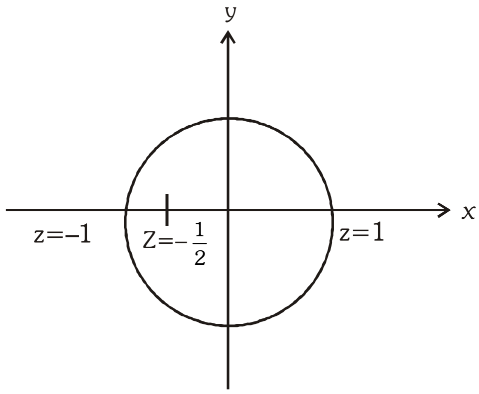
$$f(z) = u+iv$$

$$\Rightarrow f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$$

$$= \frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y}$$

$$\left[\because \text{By CR equations } \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}\right]$$

18. (3)



$$\text{Let } f(z) = \frac{3z+4}{2z+1} = \frac{3z+4}{2\left(z+\frac{1}{2}\right)}$$

$\therefore z = -\dfrac{1}{2}$ is a simple pole

$$\text{Residue at } z = -\frac{1}{2}$$

$$= \lim_{z\to -\frac{1}{2}}\left(z-\left(-\frac{1}{2}\right)\right)\frac{3z+4}{2\left(z+\frac{1}{2}\right)}$$

$$= \lim_{z\to -\frac{1}{2}}\frac{3z+4}{2}$$

$$= \frac{3\left(-\frac{1}{2}\right)+4}{2} = \frac{-3+8}{4} = \frac{5}{4}$$

$$\text{Now } \int_C \frac{3z+4}{2z+1}dz$$

$$= \int_C f(z)dz$$

$= 2\pi i$ (sum of residues of poles within C)

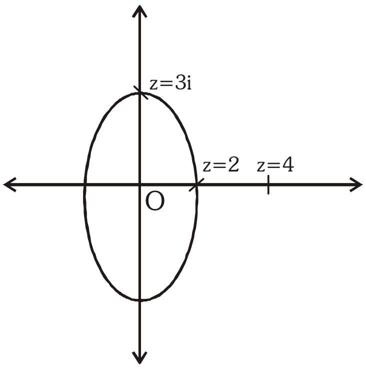
[By Cauchy's Residue Theorem]

$$= 2\pi i\times\frac{5}{4} = \frac{5\pi i}{2}$$

19. **(2)**

$$\left(\frac{3x}{2}\right)^2 + y^2 = 3^2$$

$$\Rightarrow \frac{x^2}{2^2}+\frac{y^2}{3^2} = 1$$



z = 4 lies outside of the ellipse

$$\therefore f(z) = \frac{4z^2+z+5}{z-4} \text{ is analytic inside C}$$

$\therefore$ By Cauchy's theorem $\int_C f(z)dz = 0$

$$\Rightarrow \int_C \frac{4z^2+z+5}{z-4}dz = 0$$

20. **(4)**

To final pole of $\frac{1}{\cos z - \sin z}$ is put cos z–sin z =0

$$\Rightarrow \cos z = \sin z$$
$$\Rightarrow \tan z = 1$$
$$\therefore z = \frac{\pi}{4}$$
$$\therefore \text{Pole is } z = \frac{\pi}{4}$$

21. **(2)**

$$L(\sin^2 t) = L\left(\frac{1-\cos 2t}{2}\right)$$
$$= \frac{1}{2}[L(1)–L(\cos 2t)]$$
$$= \frac{1}{2}\left[\frac{1}{S}-\frac{S}{S^2+4}\right]$$

$$L\left(\frac{\sin^2 t}{t}\right) = \frac{1}{2}\int_S^\infty \left[\frac{1}{S}-\frac{S}{S^2+4}\right]ds$$

$\Big[\because$ If $\frac{f(t)}{t}$ has a limit as $t\to 0$ and $L(f(t))=F(s)$, then

$$L\left(\frac{f(t)}{t}\right) = \int_S^\infty F(s)ds\Big]$$

$$= \left[\log s - \frac{1}{2}\log\left(s^2+4\right)\right]_S^\infty$$
$$= \frac{1}{4}\left[\log\frac{s^2}{s^2+4}\right]_s^\infty$$
$$= \frac{1}{4}\left[0-\log\frac{s^2}{s^2+4}\right]$$
$$= \frac{-1}{4}\log\frac{s^2}{s^2+4}$$

$$L\left(\frac{\sin t}{t}\right) = \frac{-1}{4}\log\frac{s^2}{s^2+4}$$

$$\therefore \int_0^\infty e^{-st}\left(\frac{\sin t}{t}\right) = \frac{-1}{4}\log\frac{s^2}{s^2+4}$$

Put s=1

$$\Rightarrow \int_0^\infty e^{-t}\left(\frac{\sin t}{t}\right) = \frac{-1}{4}\log\frac{1}{5}$$
$$= \frac{1}{4}\log 5$$

22. **(*)**

Auxillary equation is given by

$$m^2+9 = 0$$
$$m^2 = 9$$
$$m = \pm 3i$$
$$C.F = A\cos 3t + B\sin 3t$$
$$P.I. = \frac{1}{D^2+9}\cos 2t$$
$$= \frac{1}{-4+9}\cos 2t = \frac{\cos 2t}{5}$$
$$y(t) = A\cos 3t + B\sin 3t + \frac{\cos 2t}{5}$$

$y(0) = 1 \Rightarrow$

$$1 = A + \frac{1}{5} \Rightarrow A = 1 - \frac{1}{5} = \frac{4}{5}$$

$y\left(\frac{\pi}{2}\right) = 1 \Rightarrow$

$$1 = -B - \frac{1}{5} \Rightarrow B + \frac{1}{5} = -1$$

$$B = -1\frac{-1}{5} = \frac{-6}{5}$$

$$\therefore y(t) = \frac{4}{5}\cos 3t \frac{-6\sin 3t}{5} + \frac{\cos 2t}{5}$$

$$= \frac{1}{5}(4\cos 3t - 6\sin 3t + \cos 2t)$$

23. **(1)**

Fourier sine transform of $e^{\frac{-ax}{x}}$ is $\tan^{-1}\left(\frac{s}{a}\right)$

25. **(4)**

Required method – Gauss siedal method.

26. **(3)**

$$(0, 1) = (0, 3^{o})$$
$$(1, 3) = (1, 3^{1})$$
$$(2, 9) = (2, 3^{2})$$
$$(4, 81) = (4, 3^{4})$$
$$\therefore (3, x) = (3, 3^{3})$$
$$\therefore x = 3^{3} = 27$$

27. **(2)**

$$y(0) = 2 = 2^{1}$$
$$y(1) = 4 = 2^{2}$$
$$y(2) = 8 = 2^{3}$$
$$y(4) = 32 = 2^{5}$$
$$\therefore y(x) = 2^{x+1}$$

Now $y(3) = 2^{3+1} = 2^{4}$

$= 16$

≈ 16.5

If we use any interpolation method we get the value near to 16.5

28. **(3)**

$$\int_0^\infty\int_0^\infty kxye^{-\left(x^2+y^2\right)}dx\,dy = 0$$

i.e., $k\int_0^\infty ye^{-y^2}dy\int_0^\infty xe^{-x^2}dx = 1$

i.e., $\frac{k}{4} = 1$

$\therefore k = 4$

29. **(1)**

If the two regression lines are perpendicular to each other, then the coefficient of correlation is equal to 0.

30. **(*)**

Moment generating function is

$$\int_0^\infty e^{tx}\left(xe^{-x}\right)dx = \int_0^\infty xe^{x(t-1)}dx$$

$$= \left[x\frac{e^{x}(t-1)}{(t-1)}\right]_0^\infty - \frac{1}{(t-1)}\int_0^\infty e^{x(t-1)}dx$$

$$= 0 - \frac{1}{t-1}\left(\frac{e^{x(t-1)}}{(t-1)}\right)_0^\infty$$

$$= \frac{1}{(t-1)^2}$$