Model Question Paper 1

Half-Yearly Examination 2026-27

MATHEMATICS

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

1. This question paper has 5 sections: A, B, C, D and E.
2. Section A has 20 objective type questions (Multiple Choice Questions and Assertion-Reason questions) carrying 1 mark each.
3. Section B has 5 questions carrying 2 marks each. Answers to these should not exceed 40-50 words each.
4. Section C has 6 questions carrying 3 marks each. Answers to these should not exceed 60-80 words each.
5. Section D has 4 questions carrying 5 marks each. Answers to these should not exceed 120 words each.
6. Section E has 3 source/case-based questions carrying 4 marks each, with four sub-parts of 1 mark each.
7. There is no overall choice. However, internal choices have been provided in all four questions of Section D and in one sub-part of each case-based question of Section E. A student has to attempt only one of the choices in such questions.
8. Use of calculators is not permitted.
9. Wherever necessary, neat and labelled figures/graphs should be drawn.

SECTION A

Question numbers 1 to 20 carry 1 mark each. Choose the most appropriate answer.

1. While measuring the length of a table with three different tools, Aisha recorded three readings: 4.0 cm, 2.6666... cm and $\sqrt{5}$ cm. Which of these readings represents an irrational number? *[1]*

(a) 4.0 cm
(b) 2.6666... cm
(c) $\sqrt{5}$ cm
(d) 22/7 cm
Answer: (c) — $\sqrt{5}$ — its decimal expansion is non-terminating and non-repeating, and it cannot be written as p/q.

2. Simplify using laws of exponents: $2^5 \times 2^{-3}$. *[1]*

(a) $2^2 = 4$
(b) $2^8 = 256$
(c) $2^{-2} = 1/4$
(d) 2^{15}
Answer: (a) — $2^5 \times 2^{-3} = 2^{(5-3)} = 2^2 = 4$.

3. A packaging engineer computes a length as 13/125 metres. The decimal expansion of 13/125 is: *[1]*

(a) 0.104, a terminating decimal
(b) 0.104104..., a non-terminating repeating decimal
(c) $0.\overline{1}04$, a repeating decimal
(d) A non-terminating, non-repeating decimal
Answer: (a) — $125 = 5^3$ (only prime factor 5), so the expansion terminates: 13/125 = 0.104.

4. On rationalising the denominator, $1/(\sqrt{7} - \sqrt{5})$ is equal to: *[1]*

(a) $(\sqrt{7} + \sqrt{5})/2$
(b) $(\sqrt{7} - \sqrt{5})/2$
(c) $(\sqrt{7} + \sqrt{5})/12$
(d) $\sqrt{7} + \sqrt{5}$
Answer: (a) — Multiply by $(\sqrt{7}+\sqrt{5})/(\sqrt{7}+\sqrt{5})$: denominator becomes 7-5=2, giving $(\sqrt{7}+\sqrt{5})/2$.

5. *This question consists of two statements — Assertion (A) and Reason (R). Choose the correct option:* *[1]*

Assertion (A): $\sqrt{2}$ cannot be expressed in the form p/q, where p and q are integers and $q \neq 0$; hence $\sqrt{2}$ is irrational.
Reason (R): If $\sqrt{2}$ were rational, say $\sqrt{2} = p/q$ in lowest terms, then $p^2 = 2q^2$ would force p to be even, which in turn would force q to also be even — contradicting that p/q is already in lowest terms.

(a) Both A and R are true, and R is the correct explanation of A
(b) Both A and R are true, but R is NOT the correct explanation of A
(c) A is true, but R is false
(d) A is false, but R is true

Answer: (a) — Both true; R is exactly the standard proof-by-contradiction argument that establishes A.

6. Using a suitable identity, the value of $(102)^2$ is: *[1]*

(a) 10404
(b) 10402
(c) 10204
(d) 10408

Answer: (a) — $(100+2)^2 = 100^2 + 2\times100\times2 + 2^2 = 10000+400+4 = 10404$.

7. The expansion of $(x - 3)(x - 5)$ is: *[1]*

(a) $x^2 - 8x + 15$
(b) $x^2 + 8x + 15$
(c) $x^2 - 2x - 15$
(d) $x^2 - 8x - 15$

Answer: (a) — Using $(x+a)(x+b) = x^2+(a+b)x+ab$ with $a=-3$, $b=-5$: $x^2 -8x +15$.

8. If $a + b = 12$ and $ab = 32$, the value of $a^2 + b^2$ is: *[1]*

(a) 80
(b) 112
(c) 176
(d) 144

Answer: (a) — $a^2+b^2 = (a+b)^2 - 2ab = 144 - 64 = 80$.

9. One of the factors of $9x^2 - 24x + 16$ is: *[1]*

(a) $(3x - 4)$
(b) $(3x + 4)$
(c) $(9x - 4)$
(d) $(3x - 16)$

Answer: (a) — $9x^2-24x+16 = (3x-4)^2$, so $(3x-4)$ is a (repeated) factor.

10. *This question consists of two statements — Assertion (A) and Reason (R). Choose the correct option: [1]*

Assertion (A): For all real numbers a and b, $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$.
Reason (R): Expanding $(a+b)^3$ gives $a^3 + 3a^2b + 3ab^2 + b^3$, and the middle two terms $3a^2b + 3ab^2$ can be regrouped as $3ab(a+b)$.

(a) Both A and R are true, and R is the correct explanation of A
(b) Both A and R are true, but R is NOT the correct explanation of A
(c) A is true, but R is false
(d) A is false, but R is true

Answer: (a) — Both true; R correctly derives the identity stated in A.

11. Which of the following is a linear polynomial in one variable? *[1]*

(a) $3x + 5$
(b) $x^2 + 2x + 1$
(c) $2x^3 - 1$
(d) $1/x + 3$

Answer: (a) — $3x+5$ has degree 1, so it is a linear polynomial.

12. The zero of the linear polynomial $p(x) = 5x - 20$ is: *[1]*

(a) 4
(b) -4
(c) 5
(d) 20
Answer: (a) — $5x-20=0 \Rightarrow x=4$.

13. The fare for a taxi ride of x km is modelled by the linear polynomial $C(x) = 25x + 50$ (in rupees). The fixed base fare, charged even for zero distance, is: *[1]*

(a) ₹50
(b) ₹25
(c) ₹75
(d) ₹0
Answer: (a) — The constant term, ₹50, is the base fare independent of distance.

14. The value of k for which $x = 2$ is a zero of the polynomial $p(x) = 3x - 2k$ is: *[1]*

(a) 3
(b) 2
(c) 1
(d) 6
Answer: (a) — $p(2) = 6 - 2k = 0 \Rightarrow k = 3$.

15. The point (-3, 5) lies in which quadrant? *[1]*

(a) Quadrant II
(b) Quadrant I
(c) Quadrant III
(d) Quadrant IV
Answer: (a) — x negative, y positive ⇒ Quadrant II.

16. The distance of the point (6, 8) from the origin is: *[1]*

(a) 10 units
(b) 14 units
(c) 48 units
(d) 100 units
Answer: (a) — $\sqrt{(6^2+8^2)} = \sqrt{(36+64)} = \sqrt{100} = 10$.

17. The coordinates of the midpoint of the segment joining A(2, -3) and B(-6, 7) are: *[1]*

(a) (-2, 2)
(b) (2, -2)
(c) (-2, -2)
(d) (4, -2)
Answer: (a) — Midpoint = ((2-6)/2, (-3+7)/2) = (-2, 2).

18. A delivery drone flies in a straight line from P(1, 1) to Q(4, 5) on a coordinate grid (units in km). The distance it travels is: *[1]*

(a) 5 km
(b) 7 km
(c) 25 km
(d) 4 km
Answer: (a) — $\sqrt{((4-1)^2+(5-1)^2)} = \sqrt{(9+16)} = \sqrt{25} = 5$ km.

19. A triangular park has sides 12 m, 16 m and 20 m. Its semi-perimeter s is: *[1]*

(a) 24 m

(b) 48 m
(c) 20 m
(d) 16 m
Answer: (a) — s = (12+16+20)/2 = 48/2 = 24 m.

20. Using Heron's formula, the area of a triangle with sides 5 cm, 12 cm and 13 cm is: *[1]*
(a) 30 cm^2
(b) 60 cm^2
(c) 15 cm^2
(d) 45 cm^2
Answer: (a) — s=15; Area=$\sqrt{15\times10\times3\times2}$=$\sqrt{900}$=30 cm^2 (also checks out as ½×5×12=30, a right triangle).

SECTION B

Question numbers 21 to 25 carry 2 marks each.

21. Using the distance formula, find the distance between the points A(3, 0) and B(0, 4). *[2]*
Marking Scheme / Value Points:
- AB = $\sqrt{(3-0)^2 + (0-4)^2}$ — 1 mark
- = $\sqrt{9+16}$ = $\sqrt{25}$ = 5 units — 1 mark

22. Simplify $(\sqrt{3} + \sqrt{2})^2$ and state, with reason, whether the result is rational or irrational. *[2]*
Marking Scheme / Value Points:
- $(\sqrt{3}+\sqrt{2})^2 = 3 + 2\sqrt{6} + 2 = 5 + 2\sqrt{6}$ — 1 mark
- Since $\sqrt{6}$ is irrational, $5+2\sqrt{6}$ is irrational (a rational number plus an irrational number is always irrational) — 1 mark

23. Using a suitable identity, evaluate 96 × 104. *[2]*
Marking Scheme / Value Points:
- Write as (100-4)(100+4) — 1 mark
- = $100^2 - 4^2$ = 10000 - 16 = 9984 — 1 mark

24. The volume of water remaining in a draining tank after t minutes is given by the linear polynomial V(t) = 500 - 20t (litres). Find the zero of this polynomial and explain what it represents physically. *[2]*
Marking Scheme / Value Points:
- Set V(t)=0: 500-20t=0 ⇒ t = 25 — 1 mark
- Interpretation: the tank becomes completely empty 25 minutes after draining begins — 1 mark

25. A triangular field has sides 13 m, 14 m and 15 m. Using Heron's formula, find its area. *[2]*
Marking Scheme / Value Points:
- s = (13+14+15)/2 = 21; set up Area = $\sqrt{s(s-a)(s-b)(s-c)}$ = $\sqrt{21\times8\times7\times6}$ — 1 mark
- = $\sqrt{7056}$ = 84 m^2 — 1 mark

SECTION C

Question numbers 26 to 31 carry 3 marks each.

26. Prove that $\sqrt{3}$ is an irrational number. *[3]*
Marking Scheme / Value Points:
- Assume, to the contrary, that $\sqrt{3}$ is rational; then $\sqrt{3}$ = p/q where p, q are coprime integers, q≠0 — 1 mark
- Squaring: $3q^2 = p^2$ ⇒ 3 divides p^2 ⇒ 3 divides p; let p=3m, so $3q^2 = 9m^2$ ⇒ $q^2 = 3m^2$ ⇒ 3 divides q — 1 mark
- This means both p and q are divisible by 3, contradicting that they are coprime; hence the assumption is wrong and $\sqrt{3}$ is irrational — 1 mark

27. Using the identity $(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$, find the value of $x^2+y^2+z^2$ if $x+y+z=9$ and $xy+yz+zx=23$. *[3]*

Marking Scheme / Value Points:

- Write $(x+y+z)^2 = x^2+y^2+z^2 + 2(xy+yz+zx)$ — 1 mark
- Substitute: $9^2 = 81 = x^2+y^2+z^2 + 2(23) = x^2+y^2+z^2 + 46$ — 1 mark
- $x^2+y^2+z^2 = 81 - 46 = 35$ — 1 mark

28. Simplify using a suitable identity: $(3x+2y)^3 - (3x-2y)^3$. Express your answer in simplest form. *[3]*

Marking Scheme / Value Points:

- Use $(a+b)^3-(a-b)^3 = 2(3a^2b+b^3)$ with $a=3x$, $b=2y$ — 1 mark
- $= 2[3(3x)^2(2y) + (2y)^3] = 2[3\times9x^2\times2y + 8y^3] = 2[54x^2y + 8y^3]$ — 1 mark
- $= 108x^2y + 16y^3$ (final simplified answer) — 1 mark

29. If $p(x) = 2x - 7$ is a linear polynomial, find (a) its zero, (b) the value of $p(5)$, and (c) state the degree of $p(x)$. *[3]*

Marking Scheme / Value Points:

- (a) Zero: $2x-7=0 \Rightarrow x = 7/2 = 3.5$ — 1 mark
- (b) $p(5) = 2(5)-7 = 10-7 = 3$ — 1 mark
- (c) Degree of $p(x)$ is 1 (it is a linear polynomial) — 1 mark

30. The vertices of a triangle are A(0,0), B(4,0) and C(4,3). Using the distance formula, verify that triangle ABC is right-angled at B, and find the length of its hypotenuse. *[3]*

Marking Scheme / Value Points:

- $AB = \sqrt{(4-0)^2+(0-0)^2} = 4$; $BC = \sqrt{(4-4)^2+(3-0)^2} = 3$ — 1 mark
- $AC = \sqrt{(4-0)^2+(3-0)^2} = \sqrt{16+9} = \sqrt{25} = 5$ — 1 mark
- $AB^2 + BC^2 = 16+9 = 25 = AC^2 \Rightarrow$ triangle is right-angled at B; hypotenuse AC = 5 units — 1 mark

31. A quadrilateral field ABCD has AB=9 m, BC=40 m, CD=28 m, DA=15 m, with diagonal AC=41 m and angle ABC=90°. Find the total area of the field by splitting it into two triangles using the diagonal AC. *[3]*

Marking Scheme / Value Points:

- Triangle ABC is right-angled at B since $9^2+40^2=81+1600=1681=41^2$; Area(ABC) $= \frac{1}{2}\times9\times40 = 180$ m² — 1 mark
- For triangle ACD: $s=(41+28+15)/2=42$; Area $= \sqrt{42\times(42-41)\times(42-28)\times(42-15)} = \sqrt{42\times1\times14\times27} = \sqrt{15876} = 126$ m² — 1 mark
- Total area of field = 180 + 126 = 306 m² — 1 mark

SECTION D

Question numbers 32 to 35 carry 5 marks each.

32. (a) Simplify: $(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})$. (b) Rationalise the denominator of $1/(3+\sqrt{2})$ and express it in the form $a+b\sqrt{2}$. (c) Using laws of exponents, simplify: $(2^{(2/3)})^3 \times 2^{-1}$. *[5]*

OR

Prove that $\sqrt{2}$ is irrational, and hence show that $5 - 3\sqrt{2}$ is also irrational.

Marking Scheme / Value Points:

- (a) $(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2}) = (\sqrt{5})^2-(\sqrt{2})^2 = 5-2 = 3$ — 1 mark
- (b) Multiply numerator and denominator by $(3-\sqrt{2})$: $[1\times(3-\sqrt{2})] / [(3+\sqrt{2})(3-\sqrt{2})] = (3-\sqrt{2})/(9-2) = (3-\sqrt{2})/7$ — 2 marks
- (c) $(2^{(2/3)})^3 = 2^2 = 4$; then $4 \times 2^{-1} = 4 \times 1/2 = 2$ — 2 marks

OR (value points):

- Assume $\sqrt{2}$ is rational: $\sqrt{2} = p/q$, where p, q are coprime integers, $q\neq0$ — 1 mark
- Then $p^2 = 2q^2 \Rightarrow$ p is even; let $p=2m \Rightarrow 4m^2 = 2q^2 \Rightarrow q^2 = 2m^2 \Rightarrow$ q is also even — this contradicts p, q being coprime, so $\sqrt{2}$ is irrational — 2 marks

• If $5-3\sqrt{2}$ were rational, then $3\sqrt{2} = 5-(5-3\sqrt{2})$ would be a difference of two rationals, hence rational, making $\sqrt{2} = (3\sqrt{2})/3$ rational — a contradiction — 1 mark
• Hence $5 - 3\sqrt{2}$ is irrational — 1 mark

33. (a) If $x + 1/x = 7$, find the values of $x^2 + 1/x^2$ and $x^3 + 1/x^3$. (b) Factorise $27x^3 + 64y^3$ using a standard identity. *[5]*

OR

Factorise completely: (a) $a^3 - 8b^3$, using a standard identity. (b) $x^2 - y^2 + 6y - 9$ (Hint: group the last three terms and use the difference-of-squares identity).

Marking Scheme / Value Points:

• (a) $x^2 + 1/x^2 = (x+1/x)^2 - 2 = 7^2 - 2 = 49-2 = 47$ — 1.5 marks
• $x^3 + 1/x^3 = (x+1/x)^3 - 3(x+1/x) = 343 - 21 = 322$ — 1.5 marks
• (b) $27x^3+64y^3 = (3x)^3+(4y)^3 = (3x+4y)(9x^2-12xy+16y^2)$ — 2 marks

OR (value points):

• (a) $a^3-8b^3 = a^3-(2b)^3 = (a-2b)(a^2+2ab+4b^2)$ — 2 marks
• (b) Group: $x^2 - (y^2-6y+9) = x^2 - (y-3)^2$ — 1.5 marks
• Apply $a^2-b^2 = (a-b)(a+b)$: $= [x-(y-3)][x+(y-3)] = (x-y+3)(x+y-3)$ — 1.5 marks

34. A mobile-phone company's monthly bill (in ₹) for x minutes of calls used is modelled by the linear polynomial $B(x) = 199 + 0.50x$. (a) Find the fixed monthly rental. (b) Find the bill for 240 minutes of usage. (c) In a month when the bill was ₹349, find the number of minutes used. (d) Find the zero of $B(x)$, and state whether this zero has a meaningful real-life interpretation here. *[5]*

OR

A company's profit (in thousands of rupees) after x months of launching a new product is modelled by the linear polynomial $P(x) = 15x - 45$. (a) Find the zero of $P(x)$ and explain what it represents. (b) Find $P(2)$ and interpret the sign of your answer. (c) After how many months will the profit reach ₹3,00,000? (d) Is this an increasing or decreasing situation? Justify.

Marking Scheme / Value Points:

• (a) Fixed rental = ₹199 (the constant term of $B(x)$) — 1 mark
• (b) $B(240) = 199 + 0.50\times240 = 199+120 =$ ₹319 — 1.5 marks
• (c) $349 = 199 + 0.50x \Rightarrow 0.50x = 150 \Rightarrow x = 300$ minutes — 1.5 marks
• (d) Zero: $199+0.50x = 0 \Rightarrow x = -398$; since minutes used cannot be negative, this zero has no meaningful real-life interpretation ($B(x) > 0$ for all valid $x \geq 0$) — 1 mark

OR (value points):

• (a) Zero: $15x-45=0 \Rightarrow x=3$; this is the break-even month, when profit is exactly zero — 1.5 marks
• (b) $P(2) = 15(2)-45 = 30-45 = -15$; the negative value means the company is in a loss of ₹15,000 in month 2 — 1 mark
• (c) $15x-45 = 300 \Rightarrow 15x = 345 \Rightarrow x = 23$ months — 1.5 marks
• (d) Increasing, since the coefficient of x is positive (+15), so profit rises steadily as x increases — 1 mark

35. A triangular garden has sides 30 m, 26 m and 28 m. (a) Find its area using Heron's formula. (b) Find the perimeter of the garden, and the area of a 1 m wide flower border to be planted just inside the 28 m side. (c) If fencing costs ₹50 per metre, find the total cost of fencing the garden. *[5]*

OR

The floor of a rectangular hall is 24 m long and 18 m wide. A triangular stage with sides 10 m, 17 m and 21 m is to be built in one corner. (a) Find the area of the rectangular floor. (b) Find the area of the triangular stage using Heron's formula. (c) Find the area of the floor that remains after building the stage.

Marking Scheme / Value Points:

• (a) $s = (30+26+28)/2 = 42$; Area $= \sqrt{(42\times12\times16\times14)} = \sqrt{112896} = 336\ m^2$ — 2 marks
• (b) Perimeter $= 30+26+28 = 84$ m; flower border area $= 28\times1 = 28\ m^2$ — 2 marks
• (c) Cost of fencing = 84 × ₹50 = ₹4200 — 1 mark

OR (value points):

- (a) Area of rectangular floor = $24\times18 = 432$ m² — 1.5 marks
- (b) $s = (10+17+21)/2 = 24$; Area = $\sqrt{24\times14\times7\times3} = \sqrt{7056} = 84$ m² — 2 marks
- (c) Remaining area = $432 - 84 = 348$ m² — 1.5 marks

SECTION E

Question numbers 36 to 38 are source/case-based questions carrying 4 marks each, with four sub-parts of 1 mark each.

Case Study 1 — Question 36 [4 Marks]

A city's roads are laid out on a rectangular grid, and the town-planning office marks every important location with coordinates (in km), taking the Clock Tower as the origin O(0,0). On this map, the Public Library is at L(4,3), the Hospital is at H(-3,4), and the Railway Station is at R(4,-3).

(i) In which quadrant does the Hospital H(-3,4) lie? *[1]*

Answer: Quadrant II (x is negative, y is positive).

(ii) Write the coordinates of the mirror image of the Railway Station R(4,-3) in the x-axis. *[1]*

Answer: (4, 3) — reflecting in the x-axis keeps x the same and reverses the sign of y.

(iii) Find the distance between the Public Library L(4,3) and the Railway Station R(4,-3). *[1]*

Answer: LR = $\sqrt{(4-4)^2 + (3-(-3))^2} = \sqrt{0+36} = 6$ km.

(iv) Find the coordinates of the midpoint of the segment joining the Clock Tower O(0,0) and the Hospital H(-3,4). *[1]*

OR Find the distance of the Public Library L(4,3) from the Clock Tower O(0,0).

Answer: Midpoint = $((0-3)/2, (0+4)/2) = (-1.5, 2)$.

OR Answer: OL = $\sqrt{4^2+3^2} = \sqrt{16+9} = \sqrt{25} = 5$ km.

Case Study 2 — Question 37 [4 Marks]

A farmer owns a triangular plot of land with side lengths 15 m, 17 m and 8 m, as recorded in the local land registry. He wants to find its area to estimate the fertilizer needed (fertilizer requirement is proportional to area) and the wire needed to fence its boundary.

(i) Find the perimeter of the triangular plot. *[1]*

Answer: Perimeter = 8+15+17 = 40 m.

(ii) Find the semi-perimeter s of the triangle, to be used in Heron's formula. *[1]*

Answer: s = 40/2 = 20 m.

(iii) Using Heron's formula, find the area of the triangular plot. *[1]*

Answer: Area = $\sqrt{20\times(20-8)\times(20-15)\times(20-17)} = \sqrt{20\times12\times5\times3} = \sqrt{3600} = 60$ m².

(iv) If fertilizer costs ₹8 per square metre, find the total cost of fertilizing the plot. *[1]*

OR If wire fencing costs ₹25 per metre, find the total cost of fencing the plot.

Answer: Cost = 60 × ₹8 = ₹480.

OR Answer: Cost = 40 × ₹25 = ₹1000.

Case Study 3 — Question 38 [4 Marks]

During a school project titled 'Numbers Around Us', students measured the diagonal of a square tile of side 1 m and found it to be √2 m — a length that cannot be written as a simple fraction p/q. They also discussed the value 22/7, commonly used by engineers to approximate π, and looked at the decimal expansions of everyday fractions such as 1/3 and 7/8.

(i) Is √2 a rational or an irrational number? Give a reason. *[1]*

Answer: Irrational; it cannot be expressed as p/q (p, q integers, q≠0) and its decimal expansion is non-terminating and non-repeating.

(ii) Is 22/7, used to approximate π, itself a rational or an irrational number? *[1]*

Answer: 22/7 is rational, since it is of the form p/q with $q \neq 0$; it is only an approximation to the actual (irrational) value of π.

(iii) What type of decimal expansion does 1/3 have? *[1]*

Answer: 1/3 = 0.333... — a non-terminating, repeating (recurring) decimal.

(iv) Briefly outline why $\sqrt{2}$ cannot be written in the form p/q (p, q integers, $q \neq 0$) — the key idea of the proof that $\sqrt{2}$ is irrational. *[1]*

OR Find one rational number that lies between 1/3 and 7/8.

Answer: Assuming $\sqrt{2}$ = p/q in lowest terms leads, after squaring and simplifying, to both p and q being even — contradicting that they have no common factor; hence no such p/q exists and $\sqrt{2}$ is irrational.

OR Answer: Using a common denominator of 24: 1/3=8/24 and 7/8=21/24; any fraction between them works, e.g. 5/12 (=10/24).