Model Question Paper 2

Half-Yearly Examination 2026-27

MATHEMATICS

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

1. This question paper has 5 sections: A, B, C, D and E.
2. Section A has 20 objective type questions (Multiple Choice Questions and Assertion-Reason questions) carrying 1 mark each.
3. Section B has 5 questions carrying 2 marks each. Answers to these should not exceed 40-50 words each.
4. Section C has 6 questions carrying 3 marks each. Answers to these should not exceed 60-80 words each.
5. Section D has 4 questions carrying 5 marks each. Answers to these should not exceed 120 words each.
6. Section E has 3 source/case-based questions carrying 4 marks each, with four sub-parts of 1 mark each.
7. There is no overall choice. However, internal choices have been provided in all four questions of Section D and in one sub-part of each case-based question of Section E. A student has to attempt only one of the choices in such questions.
8. Use of calculators is not permitted.
9. Wherever necessary, neat and labelled figures/graphs should be drawn.

SECTION A

Question numbers 1 to 20 carry 1 mark each. Choose the most appropriate answer.

1. Which of the following has a terminating decimal expansion? *[1]*

(a) 5/16
(b) 1/3
(c) 2/7
(d) 5/9

Answer: (a) — 16 = 2^4 (only prime factors 2 and/or 5), so 5/16 = 0.3125 terminates.

2. Simplify using laws of exponents: 3^4 × 3^-2. *[1]*

(a) 3^2 = 9
(b) 3^6 = 729
(c) 3^-8
(d) 3^8

Answer: (a) — 3^4 × 3^-2 = 3^(4-2) = 3^2 = 9.

3. Which of the following numbers is irrational? *[1]*

(a) 0.101001000100001...
(b) 9/4
(c) √16
(d) 22/7

Answer: (a) — 0.101001000100001... is non-terminating and non-repeating, so it cannot be written as p/q; the others are all rational (9/4, √16=4, 22/7).

4. On rationalising the denominator, 1/(√6 + √2) is equal to: *[1]*

(a) (√6 - √2)/4
(b) (√6 + √2)/4
(c) (√6 - √2)/8
(d) √6 - √2

Answer: (a) — Multiply by (√6-√2)/(√6-√2): denominator becomes 6-2=4, giving (√6-√2)/4.

5. *This question consists of two statements — Assertion (A) and Reason (R). Choose the correct option:* *[1]*

Assertion (A): $3 + \sqrt{5}$ is an irrational number.

Reason (R): The sum of a rational number and an irrational number is always irrational.

(a) Both A and R are true, and R is the correct explanation of A
(b) Both A and R are true, but R is NOT the correct explanation of A
(c) A is true, but R is false
(d) A is false, but R is true

Answer: (a) — Both true; R is the general rule that directly explains A.

6. Using a suitable identity, the value of $(98)^2$ is: *[1]*

(a) 9604
(b) 9608
(c) 9600
(d) 9404

Answer: (a) — $(100-2)^2 = 100^2 - 2\times100\times2 + 2^2 = 10000-400+4 = 9604$.

7. The expansion of $(x + 7)(x - 4)$ is: *[1]*

(a) $x^2 + 3x - 28$
(b) $x^2 - 3x - 28$
(c) $x^2 + 3x + 28$
(d) $x^2 - 11x - 28$

Answer: (a) — Using $(x+a)(x+b) = x^2+(a+b)x+ab$ with $a=7$, $b=-4$: $x^2 + 3x - 28$.

8. If $a - b = 6$ and $ab = 16$, the value of $a^2 + b^2$ is: *[1]*

(a) 68
(b) 36
(c) 52
(d) 16

Answer: (a) — $a^2+b^2 = (a-b)^2 + 2ab = 36 + 32 = 68$.

9. One of the factors of $4x^2 + 12x + 9$ is: *[1]*

(a) $(2x + 3)$
(b) $(2x - 3)$
(c) $(4x + 3)$
(d) $(2x + 9)$

Answer: (a) — $4x^2+12x+9 = (2x+3)^2$, so $(2x+3)$ is a (repeated) factor.

10. *This question consists of two statements — Assertion (A) and Reason (R). Choose the correct option: [1]*

Assertion (A): For all real numbers a and b, $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$.

Reason (R): Expanding $(a-b)^3$ gives $a^3 - 3a^2b + 3ab^2 - b^3$, and the middle two terms $-3a^2b + 3ab^2$ can be regrouped as $-3ab(a-b)$.

(a) Both A and R are true, and R is the correct explanation of A
(b) Both A and R are true, but R is NOT the correct explanation of A
(c) A is true, but R is false
(d) A is false, but R is true

Answer: (a) — Both true; R correctly derives the identity stated in A.

11. Which of the following is a linear polynomial in one variable? *[1]*

(a) $7 - 2x$
(b) $x^2 - 4$
(c) $2x^3 + 1$
(d) $1/x + 3$

Answer: (a) — $7-2x$ has degree 1, so it is a linear polynomial.

12. The zero of the linear polynomial $p(x) = 9 - 3x$ is: *[1]*

(a) 3
(b) -3
(c) 9
(d) 1/3
Answer: (a) — $9-3x=0 \Rightarrow x=3$.

13. A car rental company charges $C(d) = 15d + 100$ (in ₹) for a trip of d km. The fixed charge, applied even for a zero-distance booking, is: *[1]*

(a) ₹100
(b) ₹15
(c) ₹115
(d) ₹0
Answer: (a) — The constant term, ₹100, is the fixed charge independent of distance.

14. The value of k for which $x = -2$ is a zero of the polynomial $p(x) = 4x + k$ is: *[1]*

(a) 8
(b) -8
(c) 2
(d) 4
Answer: (a) — $p(-2) = -8 + k = 0 \Rightarrow k = 8$.

15. The point (4, -7) lies in which quadrant? *[1]*

(a) Quadrant IV
(b) Quadrant I
(c) Quadrant II
(d) Quadrant III
Answer: (a) — x positive, y negative ⇒ Quadrant IV.

16. The distance of the point (-5, 12) from the origin is: *[1]*

(a) 13 units
(b) 17 units
(c) 7 units
(d) 60 units
Answer: (a) — $\sqrt{(-5)^2+12^2} = \sqrt{25+144} = \sqrt{169} = 13$.

17. The coordinates of the midpoint of the segment joining A(-4, 6) and B(8, -2) are: *[1]*

(a) (2, 2)
(b) (-2, 2)
(c) (2, -2)
(d) (4, 4)
Answer: (a) — Midpoint = $((-4+8)/2, (6-2)/2) = (2, 2)$.

18. A cyclist rides in a straight line from P(2, 3) to Q(10, 9) on a coordinate grid (units in km). The distance covered is: *[1]*

(a) 10 km
(b) 14 km
(c) 100 km
(d) 8 km
Answer: (a) — $\sqrt{(10-2)^2+(9-3)^2} = \sqrt{64+36} = \sqrt{100} = 10$ km.

19. A triangular flag has sides 9 cm, 10 cm and 11 cm. Its semi-perimeter s is: *[1]*

(a) 15 cm

(b) 30 cm
(c) 9.5 cm
(d) 10 cm
Answer: (a) — $s = (9+10+11)/2 = 30/2 = 15$ cm.

20. Using Heron's formula, the area of a triangle with sides 9 m, 40 m and 41 m is: *[1]*

(a) 180 m^2
(b) 360 m^2
(c) 90 m^2
(d) 1476 m^2
Answer: (a) — $s=45$; Area$=\sqrt{45\times36\times5\times4}=\sqrt{32400}=180$ m^2 (also checks out as $\frac{1}{2}\times9\times40=180$, a right triangle).

SECTION B

Question numbers 21 to 25 carry 2 marks each.

21. Using the distance formula, find the distance between the points A(0, 5) and B(12, 0). *[2]*

Marking Scheme / Value Points:
- $AB = \sqrt{(0-12)^2 + (5-0)^2}$ — 1 mark
- $= \sqrt{144+25} = \sqrt{169} = 13$ units — 1 mark

22. Simplify $(\sqrt{7} - \sqrt{3})^2$ and state, with reason, whether the result is rational or irrational. *[2]*

Marking Scheme / Value Points:
- $(\sqrt{7}-\sqrt{3})^2 = 7 - 2\sqrt{21} + 3 = 10 - 2\sqrt{21}$ — 1 mark
- Since $\sqrt{21}$ is irrational, $10-2\sqrt{21}$ is irrational (a rational number minus an irrational number is always irrational) — 1 mark

23. Using a suitable identity, evaluate 95×105. *[2]*

Marking Scheme / Value Points:
- Write as $(100-5)(100+5)$ — 1 mark
- $= 100^2 - 5^2 = 10000 - 25 = 9975$ — 1 mark

24. The height of a candle remaining after burning for t minutes is given by the linear polynomial $H(t) = 20 - 2t$ (in cm). Find the zero of this polynomial and explain what it represents physically. *[2]*

Marking Scheme / Value Points:
- Set $H(t)=0$: $20-2t=0 \Rightarrow t = 10$ — 1 mark
- Interpretation: the candle burns out completely 10 minutes after it is lit — 1 mark

25. A triangular warning sign has sides 9 cm, 10 cm and 17 cm. Using Heron's formula, find its area. *[2]*

Marking Scheme / Value Points:
- $s = (9+10+17)/2 = 18$; set up Area $= \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{18\times9\times8\times1}$ — 1 mark
- $= \sqrt{1296} = 36$ cm^2 — 1 mark

SECTION C

Question numbers 26 to 31 carry 3 marks each.

26. Prove that $\sqrt{5}$ is an irrational number. *[3]*

Marking Scheme / Value Points:
- Assume, to the contrary, that $\sqrt{5}$ is rational; then $\sqrt{5} = p/q$ where p, q are coprime integers, $q \neq 0$ — 1 mark
- Squaring: $5q^2 = p^2 \Rightarrow 5$ divides $p^2 \Rightarrow 5$ divides p; let $p=5m$, so $5q^2 = 25m^2 \Rightarrow q^2 = 5m^2 \Rightarrow 5$ divides q — 1 mark
- This means both p and q are divisible by 5, contradicting that they are coprime; hence the assumption is wrong and $\sqrt{5}$ is irrational — 1 mark

27. If $x - y = 8$ and $xy = 65$, find the value of $x^2 + y^2$. *[3]*

Marking Scheme / Value Points:

- Write $(x-y)^2 = x^2+y^2 - 2xy$ — 1 mark
- Substitute: $8^2 = 64 = x^2+y^2 - 2(65) = x^2+y^2 - 130$ — 1 mark
- $x^2+y^2 = 64 + 130 = 194$ — 1 mark

28. Simplify using a suitable identity: $(5a+3b)^3 - (5a-3b)^3$. Express your answer in simplest form. *[3]*

Marking Scheme / Value Points:

- Use $(p+q)^3-(p-q)^3 = 2(3p^2q+q^3)$ with $p=5a$, $q=3b$ — 1 mark
- $= 2[3(5a)^2(3b) + (3b)^3] = 2[3\times25a^2\times3b + 27b^3] = 2[225a^2b + 27b^3]$ — 1 mark
- $= 450a^2b + 54b^3$ (final simplified answer) — 1 mark

29. If $p(x) = 3x + 9$ is a linear polynomial, find (a) its zero, (b) the value of $p(-2)$, and (c) state the degree of $p(x)$. *[3]*

Marking Scheme / Value Points:

- (a) Zero: $3x+9=0 \Rightarrow x = -3$ — 1 mark
- (b) $p(-2) = 3(-2)+9 = -6+9 = 3$ — 1 mark
- (c) Degree of $p(x)$ is 1 (it is a linear polynomial) — 1 mark

30. The vertices of a triangle are P(1,1), Q(1,5) and R(4,1). Using the distance formula, verify that triangle PQR is right-angled at P, and find the length of its hypotenuse. *[3]*

Marking Scheme / Value Points:

- $PQ = \sqrt{(1-1)^2+(5-1)^2} = 4$; $PR = \sqrt{(4-1)^2+(1-1)^2} = 3$ — 1 mark
- $QR = \sqrt{(4-1)^2+(1-5)^2} = \sqrt{9+16} = \sqrt{25} = 5$ — 1 mark
- $PQ^2 + PR^2 = 16+9 = 25 = QR^2 \Rightarrow$ triangle is right-angled at P; hypotenuse QR = 5 units — 1 mark

31. A quadrilateral field ABCD has AB=7 m, BC=24 m, CD=20 m, DA=15 m, with diagonal AC=25 m and angle ABC=90°. Find the total area of the field by splitting it into two triangles using the diagonal AC. *[3]*

Marking Scheme / Value Points:

- Triangle ABC is right-angled at B since $7^2+24^2=49+576=625=25^2$; Area(ABC) = $\frac{1}{2}\times7\times24 = 84$ m² — 1 mark
- For triangle ACD: $s=(25+20+15)/2=30$; Area = $\sqrt{30\times(30-25)\times(30-20)\times(30-15)} = \sqrt{30\times5\times10\times15} = \sqrt{22500} = 150$ m² — 1 mark
- Total area of field = 84 + 150 = 234 m² — 1 mark

SECTION D

Question numbers 32 to 35 carry 5 marks each.

32. (a) Simplify: $(\sqrt{11}+\sqrt{7})(\sqrt{11}-\sqrt{7})$. (b) Rationalise the denominator of $1/(4-\sqrt{3})$ and express it in the form $a+b\sqrt{3}$. (c) Using laws of exponents, simplify: $(3^{3/4})^4 \times 3^{-2}$. *[5]*

OR

Prove that $\sqrt{3}$ is irrational, and hence show that $4 - 2\sqrt{3}$ is also irrational.

Marking Scheme / Value Points:

- (a) $(\sqrt{11}+\sqrt{7})(\sqrt{11}-\sqrt{7}) = (\sqrt{11})^2-(\sqrt{7})^2 = 11-7 = 4$ — 1 mark
- (b) Multiply numerator and denominator by $(4+\sqrt{3})$: $[1\times(4+\sqrt{3})] / [(4-\sqrt{3})(4+\sqrt{3})] = (4+\sqrt{3})/(16-3) = (4+\sqrt{3})/13$ — 2 marks
- (c) $(3^{3/4})^4 = 3^3 = 27$; then $27 \times 3^{-2} = 27 \times 1/9 = 3$ — 2 marks

OR (value points):

- Assume $\sqrt{3}$ is rational: $\sqrt{3} = p/q$, where p, q are coprime integers, $q\neq0$ — 1 mark

• Then $p^2 = 3q^2 \Rightarrow$ 3 divides $p^2 \Rightarrow$ 3 divides p; let $p=3m \Rightarrow 9m^2 = 3q^2 \Rightarrow q^2 = 3m^2 \Rightarrow$ 3 divides q — this contradicts p, q being coprime, so $\sqrt{3}$ is irrational — 2 marks
• If $4-2\sqrt{3}$ were rational, then $2\sqrt{3} = 4-(4-2\sqrt{3})$ would be a difference of two rationals, hence rational, making $\sqrt{3} = (2\sqrt{3})/2$ rational — a contradiction — 1 mark
• Hence $4 - 2\sqrt{3}$ is irrational — 1 mark

33. (a) If $x + 1/x = 9$, find the values of $x^2 + 1/x^2$ and $x^3 + 1/x^3$. (b) Factorise $64a^3 + 27b^3$ using a standard identity. *[5]*

OR

Factorise completely: (a) $a^3 - 27b^3$, using a standard identity. (b) $x^2 - y^2 + 8y - 16$ (Hint: group the last three terms and use the difference-of-squares identity).

Marking Scheme / Value Points:

• (a) $x^2 + 1/x^2 = (x+1/x)^2 - 2 = 9^2 - 2 = 81-2 = 79$ — 1.5 marks
• $x^3 + 1/x^3 = (x+1/x)^3 - 3(x+1/x) = 729 - 27 = 702$ — 1.5 marks
• (b) $64a^3+27b^3 = (4a)^3+(3b)^3 = (4a+3b)(16a^2-12ab+9b^2)$ — 2 marks

OR (value points):

• (a) $a^3-27b^3 = a^3-(3b)^3 = (a-3b)(a^2+3ab+9b^2)$ — 2 marks
• (b) Group: $x^2 - (y^2-8y+16) = x^2 - (y-4)^2$ — 1.5 marks
• Apply $a^2-b^2 = (a-b)(a+b)$: $= [x-(y-4)][x+(y-4)] = (x-y+4)(x+y-4)$ — 1.5 marks

34. A bakery's estimated daily profit (in ₹) after selling x cakes is modelled by the linear polynomial $P(x) = 45x - 900$. (a) Find the zero of P(x) and state what it represents. (b) Find P(30) and interpret the sign of your answer. (c) How many cakes must be sold for a profit of ₹2700? (d) Is the profit increasing or decreasing as x increases? Justify. *[5]*

OR

A parking garage charges a linear polynomial $F(h) = 20 + 15h$ (in ₹) for h hours of parking. (a) Find the fixed base charge. (b) Find F(5). (c) If the total charge was ₹185, find the number of hours parked. (d) Find the zero of F(h), and comment on whether it has a meaningful real-life interpretation here.

Marking Scheme / Value Points:

• (a) Zero: $45x-900=0 \Rightarrow x=20$; this is the break-even number of cakes, where profit is exactly zero — 1.5 marks
• (b) $P(30) = 45(30)-900 = 1350-900 =$ ₹450; the positive value means the bakery makes a profit of ₹450 when 30 cakes are sold — 1 mark
• (c) $45x-900 = 2700 \Rightarrow 45x = 3600 \Rightarrow x = 80$ cakes — 1.5 marks
• (d) Increasing, since the coefficient of x is positive (+45), so profit rises steadily as more cakes are sold — 1 mark

OR (value points):

• (a) Fixed base charge = ₹20 (the constant term of F(h)) — 1 mark
• (b) $F(5) = 20 + 15\times5 = 20+75 =$ ₹95 — 1.5 marks
• (c) $185 = 20 + 15h \Rightarrow 15h = 165 \Rightarrow h = 11$ hours — 1.5 marks
• (d) Zero: $20+15h=0 \Rightarrow h = -4/3$; since parking hours cannot be negative, this zero has no meaningful real-life interpretation ($F(h) > 0$ for all valid $h \geq 0$) — 1 mark

35. A triangular park has sides 11 m, 60 m and 61 m. (a) Find its area using Heron's formula. (b) Find the perimeter of the park, and the area of a 1 m wide flower border to be planted just inside the 61 m side. (c) If fencing costs ₹40 per metre, find the total cost of fencing the park. *[5]*

OR

The floor of a rectangular hall is 20 m long and 16 m wide. A triangular stage with sides 9 m, 12 m and 15 m is to be built in one corner. (a) Find the area of the rectangular floor. (b) Find the area of the triangular stage using Heron's formula. (c) Find the area of the floor that remains after building the stage.

Marking Scheme / Value Points:

• (a) $s = (11+60+61)/2 = 66$; Area $= \sqrt{(66\times55\times6\times5)} = \sqrt{108900} = 330$ m² — 2 marks
• (b) Perimeter $= 11+60+61 = 132$ m; flower border area $= 61\times1 = 61$ m² — 2 marks

- (c) Cost of fencing = $132 \times ₹40 = ₹5280$ — 1 mark

OR (value points):

- (a) Area of rectangular floor = $20 \times 16 = 320\ m^2$ — 1.5 marks
- (b) $s = (9+12+15)/2 = 18$; Area = $\sqrt{(18 \times 9 \times 6 \times 3)} = \sqrt{2916} = 54\ m^2$ — 2 marks
- (c) Remaining area = $320 - 54 = 266\ m^2$ — 1.5 marks

SECTION E

Question numbers 36 to 38 are source/case-based questions carrying 4 marks each, with four sub-parts of 1 mark each.

Case Study 1 — Question 36 [4 Marks]

A theme park's map is laid out on a coordinate grid (in hundreds of metres), with the Main Gate taken as the origin O(0,0). On this map, the Ferris Wheel is at F(6,8), the Food Court is at C(-5,12), and the Exit Gate is at E(6,-8).

(i) In which quadrant does the Food Court C(-5,12) lie? *[1]*

Answer: Quadrant II (x is negative, y is positive).

(ii) Write the coordinates of the mirror image of the Exit Gate E(6,-8) in the x-axis. *[1]*

Answer: (6, 8) — reflecting in the x-axis keeps x the same and reverses the sign of y.

(iii) Find the distance between the Ferris Wheel F(6,8) and the Exit Gate E(6,-8). *[1]*

Answer: $FE = \sqrt{((6-6)^2 + (8-(-8))^2)} = \sqrt{(0+256)} = 16$ (hundred metres).

(iv) Find the coordinates of the midpoint of the segment joining the Main Gate O(0,0) and the Food Court C(-5,12). *[1]*

OR Find the distance of the Ferris Wheel F(6,8) from the Main Gate O(0,0).

Answer: Midpoint = $((0-5)/2, (0+12)/2) = (-2.5, 6)$.

OR Answer: $OF = \sqrt{(6^2+8^2)} = \sqrt{(36+64)} = \sqrt{100} = 10$ (hundred metres).

Case Study 2 — Question 37 [4 Marks]

In a class activity titled 'Numbers Around Us', students measured the diagonal of a square tile of side 1 m and found it to be $\sqrt{2}$ m — a length that cannot be written as a simple fraction p/q. They also noted that 3.14 is commonly used as a rational approximation for the irrational number π, and looked at the decimal expansion of 5/6.

(i) Is $\sqrt{2}$ a rational or an irrational number? Give a reason. *[1]*

Answer: Irrational; it cannot be expressed as p/q (p, q integers, $q \neq 0$) and its decimal expansion is non-terminating and non-repeating.

(ii) Is 3.14, used to approximate π, itself a rational or an irrational number? *[1]*

Answer: 3.14 is rational, since it is a terminating decimal that can be written as 314/100; it is only an approximation to the actual (irrational) value of π.

(iii) What type of decimal expansion does 5/6 have? *[1]*

Answer: 5/6 = 0.8333... — a non-terminating, repeating (recurring) decimal.

(iv) Briefly outline why $\sqrt{2}$ cannot be written in the form p/q (p, q integers, $q \neq 0$) — the key idea of the proof that $\sqrt{2}$ is irrational. *[1]*

OR Find one rational number that lies between 5/6 and 2/9.

Answer: Assuming $\sqrt{2} = p/q$ in lowest terms leads, after squaring and simplifying, to both p and q being even — contradicting that they have no common factor; hence no such p/q exists and $\sqrt{2}$ is irrational.

OR Answer: Using a common denominator of 18: 5/6=15/18 and 2/9=4/18; any fraction between them works, e.g. 10/18 = 5/9.

Case Study 3 — Question 38 [4 Marks]

A municipal corporation is developing a small triangular traffic island with sides 21 m, 20 m and 13 m. The cost of laying topsoil and planting a garden is proportional to the island's area, while the cost of building a concrete curb around it is proportional to its perimeter.

(i) Find the perimeter of the triangular traffic island. *[1]*

Answer: Perimeter = 21+20+13 = 54 m.

(ii) Find the semi-perimeter s of the triangle, to be used in Heron's formula. *[1]*

Answer: $s = 54/2 = 27$ m.

(iii) Using Heron's formula, find the area of the traffic island. *[1]*

Answer: Area = $\sqrt{(27\times(27-21)\times(27-20)\times(27-13))} = \sqrt{(27\times6\times7\times14)} = \sqrt{15876} = 126$ m^2.

(iv) If topsoil and planting costs ₹15 per square metre, find the total cost for the island. *[1]*

OR If curbing costs ₹60 per metre, find the total cost of curbing the island.

Answer: Cost = 126 × ₹15 = ₹1890.

OR Answer: Cost = 54 × ₹60 = ₹3240.