HALF-YEARLY EXAMINATION (2025 -26)

CLASS X — MATHEMATICS (Standard)

Chapters Covered: 1 to 8 (Real Numbers, Polynomials, Pair of Linear Equations in Two Variables, Quadratic Equations, Arithmetic Progressions, Triangles, Coordinate Geometry, Introduction to Trigonometry)

Maximum Marks: 80 | **Time Allowed:** 3 Hours

General Instructions:

1. This question paper contains **38 questions** divided into **5 Sections**: A, B, C, D, and E.
2. **Section A** comprises 20 Multiple Choice Questions (MCQs) (Q1–Q20) of 1 mark each.
3. **Section B** comprises 5 Very Short Answer (VSA) type questions (Q21–Q25) of 2 marks each
4. **Section C** comprises 6 Short Answer (SA) type questions (Q26–Q31) of 3 marks each.
5. **Section D** comprises 4 Long Answer (LA) type questions (Q32–Q35) of 5 marks each.
6. **Section E** comprises 3 Case-Based integrated units (Q36–Q38) of 4 marks each with sub-parts.
7. All Questions are compulsory. However, internal choices are provided in 2 questions of 2 marks, 2 questions of 3 marks, and 2 questions of 5 marks. An internal choice is provided in the 2-mark sub-parts of Section E.
8. Use of calculators is not permitted.

SECTION A (Multiple Choice Questions — 1 Mark Each)

(Q1 to Q20 contain 18 MCQs and 2 Assertion-Reasoning questions)

Q1.

The HCF and LCM of two numbers are 9 and 360 respectively. If one number is 45, the other number is:

(a) 72

(b) 40

(c) 80

(d) 90

Q2.

If one zero of the quadratic polynomial $p(x) = x^2 - 6x + k$ is 3, then the value of k is:

(a) 9

(b) -9

(c) -3

(d) 3

Q3.

The pair of equations $x + 2y + 5 = 0$ and $-3x - 6y + 1 = 0$ has:

(a) A unique solution

(b) Exactly two solutions

(c) Infinitely many solutions

(d) No solution

Q4.

If the quadratic equation $kx^2 - 5x + k = 0$ has real and equal roots, then the value(s) of k is/are:

(a) $\pm\frac{5}{2}$

(b) $\frac{5}{2}$ only

(c) $\pm\frac{2}{5}$

(d) 5

Q5.

The 11th term of the A.P. $-3, -\frac{1}{2}, 2, \ldots$ is:

(a) 28

(b) 22

(c) -38

(d) $-48\frac{1}{2}$

Q6.

The distance of the point $P(-6,8)$ from the origin is:

(a) 8 units

(b) $2\sqrt{7}$ units

(c) 10 units

(d) 6 units

Q7.

In $\triangle ABC$, $DE \parallel BC$ such that $\frac{AD}{DB} = \frac{3}{5}$. If $AC = 5.6$ cm, then the length of AE is:

(a) 4.2 cm

(b) 3.15 cm

(c) 2.1 cm

(d) 5.2 cm

Q8.

If $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$, then the value of $\tan\theta$ is:

(a) $\sqrt{2} - 1$

(b) $\sqrt{2} + 1$

(c) 1

(d) $\sqrt{2}$

Q9.

If the vertices of a parallelogram $ABCD$ taken in order are $A(3, a), B(5,6), C(5,2)$, and $D(3,3)$, then the value of a is:

(a) 4

(b) 5

(c) 2

(d) 3

Q10.

The sum of the first 20 terms of an A.P. whose n^{th} term is given by $an = 3 + 2n$ is:

(a) 440

(b) 460

(c) 420

(d) 480

Q11.

If α and β are the zeroes of the polynomial $f(x) = x^2 - p(x+1) - c$, then $(\alpha+1)(\beta+1)$ is equal to:

(a) $c-1$

(b) $1-c$

(c) c

(d) $1+c$

Q12.

The ratio in which the point $P(-4,6)$ divides the line segment joining the points $A(-6,10)$ and $B(3,-8)$ is:

(a) $2:7$

(b) $7:2$

(c) $2:5$

(d) $5:2$

Q13.

If $\triangle ABC \sim \triangle PQR$ with $\frac{\text{Area}(\triangle ABC)}{\text{Area}(\triangle PQR)} = \frac{9}{16}$, and $BC = 4.5$ cm, then the length of QR is:

(a) 6 cm

(b) 5 cm

(c) 4 cm

(d) 6.5 cm

Q14.

If $\sec\theta + \tan\theta = x$, then the value of $\sin\theta$ is:

(a) $\frac{x^2+1}{x^2-1}$

(b) $\frac{x^2-1}{x^2+1}$

(c) $\frac{2x}{x^2+1}$

(d) $\frac{x^2-1}{2x}$

Q15.

The coordinate of the point which divides the line segment joining $(4,-3)$ and $(8,5)$ in the ratio $3:1$ internally is:

(a) $(7,3)$

(b) $(3,7)$

(c) $(5,4)$

(d) $(4,5)$

Q16.

The number of quadratic polynomials having -3 and 5 as their zeroes is:

(a) only 1

(b) exactly 2

(c) infinitely many

(d) cannot be determined

Q17.

If $\cos A + \cos^2 A = 1$, then the value of $\sin^2 A + \sin^4 A$ is:

(a) -1

(b) 1

(c) 0

(d) 2

Q18.

The common difference of the A.P. for which $a_{30} - a_{20} = 50$ is:

(a) 5

(b) 10

(c) -5

(d) 50

Direction for Q19 & Q20: In question numbers 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option:

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Assertion (A) is false, but Reason (R) is true.

Q19.

- **Assertion (A):** The HCF of two numbers is 5 and their product is 150, then their LCM is 30.
- **Reason (R):** For any two positive integers a and b, HCF$(a, b) \times$ LCM$(a, b) = a \times b$.

Q20.

- **Assertion (A):** The point $(-1,6)$ divides the line segment joining the points $(-3,10)$ and $(6,-8)$ in the ratio $2:7$internally.
- **Reason (R):** The coordinates of the point $P(x, y)$ which divides the line segment joining $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ in the ratio $m_1 : m_2$ are given by the section formula.

SECTION B (Very Short Answer Type — 2 Marks Each)

(Q21 to Q25)

Q21.

Prove that $\sqrt{5}$ is an irrational number.

Q22.

For what value of k will the pair of linear equations have infinitely many solutions?

$$kx + 3y = k - 3$$
$$12x + ky = k$$

Q23.

In $\triangle ABC$, $DE \parallel BC$ such that $AD = x$, $DB = x - 2$, $AE = x + 2$, and $EC = x - 1$. Find the value of x.

(OR)

Diagonals AC and BD of a trapezium $ABCD$ with $AB \parallel DC$ intersect each other at the point O. Show that $\frac{AO}{OC} = \frac{BO}{OD}$.

Q24.

Evaluate:

$$\frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$$

Q25.

Find the coordinates of the point which divides the join of $(-1,7)$ and $(4,-3)$ in the ratio $2:3$.

(OR)

Find the relation between x and y such that the point (x, y) is equidistant from the points $(7,1)$ and $(3,5)$.

SECTION C (Short Answer Type — 3 Marks Each)

(Q26 to Q31)

Q26.

Find the zeroes of the quadratic polynomial $6x^2 - 3 - 7x$ and verify the relationship between the zeroes and the coefficients.

Q27.

Solve the following pair of linear equations by the substitution method:

$$7x - 15y = 2$$
$$x + 2y = 3$$

Q28.

Find the roots of the quadratic equation $2x^2 - 7x + 3 = 0$ by using the quadratic formula.

(OR)

A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the original speed of the train.

Q29.

An A.P. consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.

Q30.

Prove the following identity:

$$\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$$

(OR)

If $\sin(\alpha+\beta) = 1$ and $\cos(\alpha-\beta) = \frac{\sqrt{3}}{2}$, where $0^\circ \leq \alpha, \beta \leq 90^\circ$ and $\alpha \geq \beta$, find the values of α and β.

Q31.

Prove that the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle. (Thales Theorem application).

SECTION D (Long Answer Type — 5 Marks Each)

(Q32 to Q35)

Q32.

Find the values of a and b for which the following system of linear equations has an infinite number of solutions:

$$(2a-1)x + 3y = 5$$
$$3x + (b-1)y = 2$$

Q33.

State and prove Basic Proportionality Theorem (Thales Theorem).

Q34.

A motor boat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

(OR)

Sum of the areas of two squares is $468\ \text{m}^2$. If the difference of their perimeters is 24 m, find the sides of the two squares.

Q35.

If $\sec\theta + \tan\theta = p$, prove that $\sin\theta = \frac{p^2-1}{p^2+1}$.

(OR)

Prove that:

$$\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta\csc\theta$$

SECTION E (Case-Based Integrated Units — 4 Marks Each)

(Q36 to Q38 with sub-parts)

Q36. Case Study 1: Real Numbers in Architecture (Low to Medium)

To enhance the safety of an ancient monument, architects plan to install warning lights at regular intervals along a perimeter wall. Three indicator lights flash at intervals of 48 seconds, 72 seconds, and 108 seconds respectively.

- **(i)** Find the prime factorization of 108. *(1 Mark)*
- **(ii)** Find the HCF of 48, 72, and 108 using the prime factorization method. *(1 Mark)*
- **(iii)** If all three lights flash together at 8:00 AM, after what time interval will they next flash together again? *(2 Marks)*

  (OR)

  Check whether 6^n can end with the digit 0 for any natural number n. *(2 Marks)*

Q37. Case Study 2: Tracking Paths in Coordinate Geometry (Medium to High)

A school has organized a sports day event. To run a coordinate race, lime powder is used to mark a grid on the ground. Four friends Rama, Sneha, Tina, and Geeta are sitting at positions $A(3,4)$, $B(6,7)$, $C(9,4)$, and $D(6,1)$ respectively as shown in a coordinate layout.

- **(i)** Using the distance formula, find the length of side AB. *(1 Mark)*
- **(ii)** Find the coordinates of the midpoint of diagonal AC. *(1 Mark)*
- **(iii)** Show that the quadrilateral $ABCD$ formed by joining these students is a square. *(2 Marks)*

  (OR)

  Find the area of $\Delta\, ABC$ using its coordinate vertices. *(2 Marks)*

Q38. Case Study 3: Applications of Trigonometry / Height & Track

A rescue helicopter hovers at a specific height above a straight highway. Two milestone markers on opposite sides of the helicopter are observed at specific depression angles.

- **(i)** If a pole casts a shadow of length $20\sqrt{3}$ meters when the angle of elevation of the sun is 60°, find the height of the pole. *(1 Mark)*
- **(ii)** Draw a neat labeled schematic line diagram representing an observer looking up at an object at an angle of elevation θ. *(1 Mark)*

- **(iii)** If the angle of elevation of a cloud from a point h meters above a lake is θ and the angle of depression of its reflection in the lake is ϕ, prove that the height of the cloud is given by $\frac{h(\tan\phi+\tan\theta)}{\tan\phi-\tan\theta}$. *(2 Marks)*