# Class 10 Mathematics – Quarterly Examination

**Tamil Nadu State Syllabus | Perambalur District | English Medium**

Answer Key

## Part – I

**Answer all 14 questions – 14 × 1 = 14**

**1.** $f(x) = 2x^2, \quad g(x) = \frac{1}{3x}$.
$f \circ g(x) = 2\left(\frac{1}{3x}\right)^2 = \frac{2}{9x^2}$.
**Answer: (c)** $\frac{2}{9x^2}$

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**2.** $f(x) = \sqrt{1+x^2}$. Since

$$f(xy) = \sqrt{1+x^2y^2} \le \sqrt{(1+x^2)(1+y^2)} = f(x)f(y),$$

**Answer: (c)** $f(xy) \le f(x)f(y)$

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**3.** $7^4 = 2401 \equiv 1 \pmod{100}$.
**Answer: (a) 1**

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**4.** The printed question is

$$(1^3 + 2^3 + 3^3 + \cdots + 15^3) - (1 + 2 + 3 + \cdots + 15).$$

Using

$$1^3 + 2^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2,$$

we get

$$1^3 + \cdots + 15^3 = \left(\frac{15 \times 16}{2}\right)^2 = 120^2 = 14400.$$

Also,

$$1 + 2 + \cdots + 15 = \frac{15 \times 16}{2} = 120.$$

Therefore

$$14400 - 120 = \boxed{14280}.$$

**Answer: (c) 14280**

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**5.** $x^2 - 2x - 24 = (x-6)(x+4)$. For $x - 6$ to be a common factor of $x^2 - kx - 6$, $36 - 6k - 6 = 0$, giving $k = 5$
**Answer: (b) 5**

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**6.** $(2x-1)^2 = 9 \Rightarrow 2x - 1 = \pm 3$. Hence $x = 2, -1$.
**Answer: (c) −1, 2**

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**7.** In $\triangle LMN$, $\angle L = 60°, \angle M = 50°$, so $\angle N = 70°$. Since $\triangle LMN \sim \triangle PQR$, $N \leftrightarrow R$.
**Answer: (d) 70°**

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**8.** Since $(5,7), (3,p), (6,6)$ are collinear, their slopes are equal:

$$\frac{7-6}{5-6} = \frac{p-7}{3-5}.$$

Thus

$$-1 = \frac{p-7}{-2} \Rightarrow p - 7 = 2 \Rightarrow \boxed{p = 9}.$$

**Answer: (c) 9**

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**9.** At $(2,1)$, $x+y=3$ and $3x+y=7$.
**Answer: (b)** $x+y=3,\ 3x+y=7$

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**10.** If $a=\sin\theta+\cos\theta$ and $b=\sec\theta+\backslash\mathrm{cosec}\theta$,

$$b(a^2-1)=\left(\frac{1}{\cos\theta}+\frac{1}{\sin\theta}\right)(2\sin\theta\cos\theta)=2(\sin\theta+\cos\theta)=2a.$$

**Answer: (a)** $2a$

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**11.** If $n(A)=m,\ n(B)=n$, the number of relations from $A$ to $B$ is

$$2^{mn}.$$

**Answer: (d)** $2^{mn}$

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**12.** The paper visibly prints $44\equiv 8\pmod{12}$ and $113\equiv 85\pmod{12}$. The second congruence is a printing error because

$$113\equiv 5\pmod{12}.$$

Therefore,

$$44\times 113\equiv 8\times 5=40\equiv\boxed{4}\pmod{12}.$$

**Answer: (a) 4**

> **Question-paper issue:** $113\equiv 85\pmod{12}$ is false. It should be $113\equiv 5\pmod{12}$.



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**13.** The expression $\frac{x+10}{8x}$ is undefined when $8x=0$, i.e. $x=0$.
**Answer: (c) 0**

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**14.** The printed conditions are

$$x\sin^3\theta+y\cos^3\theta=\sin\theta\cos\theta$$

and

$$x \sin\theta = y\cos\theta.$$

From the second equation,

$$y = \frac{x\sin\theta}{\cos\theta}.$$

Substitute in the first:

$$x\sin^3\theta + x\sin\theta\cos^2\theta = \sin\theta\cos\theta.$$

Hence

$$x\sin\theta(\sin^2\theta + \cos^2\theta) = \sin\theta\cos\theta,$$

so $x = \cos\theta$. Then $y = \sin\theta$. Therefore

$$x^2 + y^2 = \cos^2\theta + \sin^2\theta = \boxed{1}.$$

**Answer: (a) 1**

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**15.** $B = \{(-2,3),(-2,4),(0,3),(0,4),(3,3),(3,4)\}$.
**Domain** $A = \{-2,0,3\}$; **Range** $B = \{3,4\}$.

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**16.** $A = \{-2,-1,0,1,2\}$, $f(x) = x^2 + x + 1$.

$$f(-2) = 3,\ f(-1) = 1,\ f(0) = 1,\ f(1) = 3,\ f(2) = 7.$$

Therefore the range is $\{1,3,7\}$. Since the function is stated to be onto,
$B = \{1,3,7\}$.
**Answer:** $B = \{1,3,7\}$.

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**17.** The paper asks to show $f\circ g = g\circ f$, where $f(x) = x - 6$ and $g(x) = x^2$.

$$(f\circ g)(x) = f(x^2) = x^2 - 6,$$

while

$$(g\circ f)(x) = g(x-6) = (x-6)^2 = x^2 - 12x + 36.$$

Therefore

$$\boxed{f \circ g \neq g \circ f}.$$

**Question-paper issue:** The requested equality is false for the functions printed in the paper. The question should be corrected before use.

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**18.** The printed condition is

$$a^b \times b^a = 800.$$

The positive-integer solutions are

$$(a, b) = (1, 800), (2, 5), (5, 2), (800, 1).$$

Thus the usual non-trivial pair is

$$\boxed{a = 2, \quad b = 5}$$

(or $a = 5, b = 2$ on interchanging the variables).

**Question-paper issue:** As printed, the question does not have a unique answer. An additional condition such as \(a1\) is needed for uniqueness.

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**19.** 100 hours = 4 days 4 hours. From 7 a.m., the time is **11 a.m.**

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**20.**

$$3 + \frac{1}{3} + \frac{1}{9} + \cdots = 3 + \frac{\frac{1}{3}}{1 - \frac{1}{3}} = 3 + \frac{1}{2} = \frac{7}{2}.$$

**Answer:** $\frac{7}{2}$

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**21.** For $15x^2 + 11x + 2 = 0$,

$$D = b^2 - 4ac = 121 - 120 = 1 > 0.$$

Hence the roots are real and distinct:

$$x = \frac{-11 \pm 1}{30} = -\frac{1}{3}, \ -\frac{2}{5}.$$

**Nature: two distinct real roots.**

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**22.** For $x^2 + 7x + 10 = 0$, $\alpha + \beta = -7,\ \alpha\beta = 10$.

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 49 - 20 = 29.$$

**Answer: 29**

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**23.**

$$\frac{x^2 + 6x + 8}{x^2 + x - 2} = \frac{(x+2)(x+4)}{(x+2)(x-1)}.$$

The excluded values come from the original denominator:

$$(x+2)(x-1) = 0 \Rightarrow x = -2, 1.$$

**Excluded values: $x = -2, 1$**

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**24.** Since $AQ \perp AB$ and $PB \perp AB$, and the two triangles share the vertically opposite angle at $O$, the right triangles are similar:

$$\frac{AQ}{PB} = \frac{AO}{BO} = \frac{10}{6}.$$

Thus

$$AQ = 9 \times \frac{10}{6} = 15 \text{ cm}.$$

**Answer: $AQ = 15$ cm**

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**25.**

$$m = \frac{2 - 1}{-3 - (-6)} = \frac{1}{3}.$$

**Slope $= \frac{1}{3}$**

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**26.** Through $(3, -4)$ with slope $-\frac{5}{7}$:

$$y+4=-\frac{5}{7}(x-3).$$

Therefore

$$\boxed{5x+7y+13=0}.$$

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**27.** Slopes are

$$m_1=-\frac{5}{23},\qquad m_2=\frac{23}{5}.$$

Thus $m_1m_2=-1$, so the lines are perpendicular.
**Verified: the lines are perpendicular.**

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**28.** The scanned paper clearly shows

$$\left[\frac{1+\sin\theta-\cos\theta}{1+\sin\theta-\cos\theta}\right]^2=\frac{1-\cos\theta}{1+\cos\theta}.$$

As printed, the left-hand side is simply $1$, so the identity is not generally true.

> **Question-paper issue:** The denominator appears to be exactly $1+\sin\theta-\cos\theta$, not $1+\sin\theta+\cos\theta$. Therefore the printed identity is mathematically incorrect.



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**29.** $f(x)=\frac{x}{2}-1$, $A=\{2,4,6,10,12\}$.

| $x$ | 2 | 4 | 6 | 10 | 12 |
|---|---|---|---|---|---|
| $f(x)$ | 0 | 1 | 2 | 4 | 5 |

Ordered pairs:

$$\boxed{\{(2,0),(4,1),(6,2),(10,4),(12,5)\}}.$$

Arrow diagram: $2\to0,\ 4\to1,\ 6\to2,\ 10\to4,\ 12\to5$.
For the graph, plot the five points $(2,0),(4,1),(6,2),(10,4),(12,5)$.

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**30.**

$$f(x) = \begin{cases} 2x + 7, & x < -2 \\ x^2 - 2, & -2 \leq x < 3 \\ 3x - 2, & x \geq 3 \end{cases}$$

(i) $f(4) = 3(4) - 2 = \boxed{10}$.
(ii) $f(-2) = (-2)^2 - 2 = \boxed{2}$.
(iii) $f(4) + 2f(1) = 10 + 2(1^2 - 2) = \boxed{8}$.
(iv) $\frac{f(-1)-3f(4)}{f(-3)} = \frac{-1-30}{1} = \boxed{-31}$.

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**31.** $f(x) = x - 1,\ g(x) = 3x + 1,\ h(x) = x^2$.

$$((f \circ g) \circ h)(x) = f(g(h(x))) = f(3x^2 + 1) = 3x^2.$$

Also,

$$(f \circ (g \circ h))(x) = f(g(x^2)) = f(3x^2 + 1) = 3x^2.$$

Hence

$$\boxed{(f \circ g) \circ h = f \circ (g \circ h)}.$$

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**32.**

$$504 - 396 = 108, \quad 396 = 3(108) + 72, \quad 108 = 1(72) + 36.$$

Thus $\gcd(396, 504) = 36$. Also $636 = 17(36) + 12$, so

$$\boxed{\gcd(396, 504, 636) = 12}.$$

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**33.** Let the three consecutive A.P. terms be $9 - d, 9, 9 + d$, since their sum is 27.

$$9(81 - d^2) = 288 \Rightarrow 81 - d^2 = 32 \Rightarrow d^2 = 49.$$

Taking $d = 7$, the terms are

$$\boxed{2, 9, 16}.$$

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**34.**

$$9^3 + 10^3 + \cdots + 21^3 = \left(\frac{21(22)}{2}\right)^2 - \left(\frac{8(9)}{2}\right)^2 = 231^2 - 36^2 = \boxed{52065}.$$

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**35.** By Euclidean algorithm for polynomials,

$$\gcd(6x^3 - 30x^2 + 60x - 48,\ 3x^3 - 12x^2 + 21x - 18) = \boxed{3x - 6}.$$

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**36.**

$$\frac{1}{x^2 - 5x + 6} + \frac{1}{x^2 - 3x + 2} - \frac{1}{x^2 - 8x + 15}$$
$$= \frac{1}{(x-2)(x-3)} + \frac{1}{(x-1)(x-2)} - \frac{1}{(x-3)(x-5)}$$
$$= \boxed{\frac{x-9}{(x-5)(x-3)(x-1)}}.$$

Restrictions: $x \neq 1, 2, 3, 5$.

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**37. Angle Bisector Theorem – Statement and Proof**

**Statement:** The internal angle bisector of an angle of a triangle divides the opposite side internally in the ratio of the other two sides.

In $\triangle ABC$, let $AD$ bisect $\angle A$, meeting $BC$ at $D$. We have to prove

$$\frac{BD}{DC} = \frac{AB}{AC}.$$

Draw $CE \parallel AD$, meeting the extension of $BA$ at $E$. Since $AD \parallel CE$, $\angle BAD = \angle AEC$ and $\angle CAD = \angle ACE$. Since $AD$ bisects $\angle A$, $\angle AEC = \angle ACE$, so $AE = AC$. By the basic proportionality theorem in $\triangle BCE$,

$$\frac{BD}{DC} = \frac{BA}{AE} = \frac{AB}{AC}.$$

Hence proved.

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**38.** Area of quadrilateral with vertices $(8,6),(5,11),(-5,12),(-4,3)$. Using the shoelace formula,

$$A=\frac{1}{2}\left|\sum x_iy_{i+1}-\sum y_ix_{i+1}\right|=\boxed{79\text{ square units}}.$$

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**39.** A line with equal positive x- and y-intercepts has equation

$$\frac{x}{a}+\frac{y}{a}=1\Rightarrow x+y=a.$$

Since it passes through $(-3,8)$,

$$-3+8=a=5.$$

Therefore

$$\boxed{x+y=5}.$$

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**40.** Solve

$$4x+5y=13,\qquad x-8y+9=0.$$

From the second, $x=8y-9$. Hence

$$32y-36+5y=13\Rightarrow 37y=49,$$

so $y=\frac{49}{37}$, $x=\frac{59}{37}$. A line parallel to the y-axis through the intersection is

$$\boxed{x=\frac{59}{37}}.$$

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**41.** The paper states

$$\frac{\cos\alpha}{\cos\beta}=m,\qquad \frac{\sin\alpha}{\sin\beta}=n$$

and asks to prove

$$(m^2+n^2)\cos^2\beta=n^2.$$

From the stated ratios,

$$\cos\alpha = m\cos\beta, \qquad \sin\alpha = n\sin\beta.$$

Therefore,

$$m^2\cos^2\beta + n^2\sin^2\beta = 1.$$

Using $\sin^2\beta = 1 - \cos^2\beta$,

$$\boxed{(m^2 - n^2)\cos^2\beta = 1 - n^2}.$$

Hence the requested identity does not follow from the printed hypotheses.

> **Question-paper issue:** Q.41 is mathematically inconsistent as printed.

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**42.**

$$289x^4 - 612x^3 + 970x^2 - 684x + 361$$

$$= (17x^2 - 18x + 19)^2.$$

Therefore

$$\boxed{\sqrt{289x^4 - 612x^3 + 970x^2 - 684x + 361} = 17x^2 - 18x + 19}$$

(for the usual textbook square-root interpretation; strictly, the principal square root is the absolute value of this polynomial).



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## Part – IV

**Answer both questions – 2 × 8 = 16**

**43(a). Construct a triangle similar to $\triangle PQR$ with scale factor $7/4$.**

1. Draw the given triangle $PQR$.
2. From $P$, draw a ray $PX$ making an acute angle with $PQ$.

3. Mark seven equal points $P_1, P_2, \ldots, P_7$ on $PX$.
4. Join $P_4Q$.
5. Through $P_7$, draw a line parallel to $P_4Q$, meeting the extension of $PQ$ at $Q'$.
6. Through $Q'$, draw a line parallel to $QR$, meeting the extension of $PR$ at $R'$.

The resulting $\triangle PQ'R'$ is similar to $\triangle PQR$ with scale factor $\frac{7}{4}$.

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**43(b). Construct $\triangle PQR$ with $QR = 6.5$ cm, $\angle P = 60°$, and altitude from $P$ to $QR = 4.5$ cm.**

1. Draw $QR = 6.5$ cm.
2. Construct a line parallel to $QR$ at a perpendicular distance of $4.5$ cm from $QR$. The vertex $P$ lies on this parallel.
3. Construct the locus of points from which $QR$ subtends $60°$ (equivalently, construct the corresponding circle/arc using the $60°$ inscribed-angle condition).
4. Take the intersection $P$ of this locus with the $4.5$ cm parallel.
5. Join $PQ$ and $PR$.

Thus the required triangle is obtained.



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**44(a). Distance-time graph for a bus travelling at 50 km/h.**

Since speed is constant,

$$d = 50t.$$

Useful plotting points are:

$$(0, 0), (1, 50), (2, 100), (3, 150), (4, 200), (5, 250), (6, 300).$$

The graph is a straight line through the origin.

i) Constant speed/variation = $\boxed{50 \text{ km/h}}$.
ii) In 90 minutes $= 1.5$ h:

$$d = 50(1.5) = \boxed{75 \text{ km}}.$$

iii) For 300 km:

$$t = \frac{300}{50} = \boxed{6\text{ h}}.$$

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**44(b). Graph of $xy = 24,\ x, y > 0$.**

$$y = \frac{24}{x}.$$

Some points for the graph are:

$$(1, 24), (2, 12), (3, 8), (4, 6), (6, 4), (8, 3), (12, 2), (24, 1).$$

Plot these points and draw the smooth first-quadrant branch of the rectangular hyperbola.

i) When $x = 3$,

$$y = \frac{24}{3} = \boxed{8}.$$

ii) When $y = 6$,

$$x = \frac{24}{6} = \boxed{4}.$$

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