

HIGHER SECONDARY FIRST TERMINAL EXAMINATION - 2025

Second Year

Max. Score : 60

Time : 2 Hrs

Cool-off Time : 15 Mts

PART – III

MATHEMATICS (SCIENCE 60)

Answer any 6 questions from 1 to 8. Each carries 3 scores.

1. i) R is transitive only

ii) (d, d) and (d, c)

iii) 16

2. i) The expression $= -\frac{\pi}{4} + -\frac{\pi}{4} + \frac{\pi}{3} = -\frac{2\pi}{4} + \frac{\pi}{3} = -\frac{\pi}{2} + \frac{\pi}{3} = \frac{\pi}{6}$

3. i) 16

$$\text{ii) } \begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 2+y & 6+0 \\ 0+1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$$2+y=5 \Rightarrow y=5-2=3$$

$$2x+2=8 \Rightarrow 2x=6 \Rightarrow x=3$$

4. i) $2x-15=5 \Rightarrow 2x=5+15 \Rightarrow 2x=20 \Rightarrow x=10$

ii) Area= 4

$$\left| \frac{1}{2} \begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix} \right| = 4 \Rightarrow \frac{1}{2} \begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix} = \pm 4$$

$$\begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix} = \pm 8 \Rightarrow k[0-2] - 0 + 1[8-0] = \pm 8$$

$$\Rightarrow -2k + 8 = \pm 8$$

$$\Rightarrow -2k + 8 = 8 \text{ or } -2k + 8 = -8$$

$$\Rightarrow -2k = 8 - 8 \text{ or } -2k = -8 - 8$$

$$\Rightarrow -2k = 0 \text{ or } -2k = -16$$

$$\Rightarrow k = 0 \text{ or } k = 8$$

5. i) $|x| + 1$

ii) Sum of a modulus and polynomial function is continuous everywhere.

Hence, $|x| + 1$ is a continuous function.

iii) It is not differentiable at $x = 0$

6. i) D) $f: N \rightarrow N, f(x) = x^3$

ii) $f(x) = \cos x$

$$g(x) = 3x^2$$

$$f \circ g = f[g(x)] = f(3x^2) = \cos(3x^2)$$

$$g \circ f = g[f(x)] = g(\cos x) = 3(\cos x)^2 = 3 \cos^2 x$$

$$f \circ g \neq g \circ f$$

7 i) Put $x = \tan \theta$

$$\tan^{-1} x = \theta$$

$$\tan^{-1} \left(\frac{\sqrt{1+x^2}-1}{x} \right) = \tan^{-1} \left(\frac{\sqrt{1+\tan^2 \theta}-1}{\tan \theta} \right)$$

$$= \tan^{-1} \left(\frac{\sqrt{1+\tan^2 \theta}-1}{\tan \theta} \right)$$

$$= \tan^{-1} \left(\frac{\sqrt{\sec^2 \theta}-1}{\tan \theta} \right) = \tan^{-1} \left(\frac{\sec \theta - 1}{\tan \theta} \right) = \tan^{-1} \left[\frac{1 - \cos \theta}{\frac{\sin \theta}{\cos \theta}} \right]$$

$$= \tan^{-1} \left[\frac{1 - \cos \theta}{\sin \theta} \right] = \tan^{-1} \left(\frac{2 \sin^2 \frac{\theta}{2}}{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}} \right) = \tan^{-1} \tan \left(\frac{\theta}{2} \right) = \frac{\theta}{2} = \frac{1}{2} \tan^{-1} x$$

8. i) $y = \sqrt{\tan x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{\tan x}} \times \sec^2 x$$

ii) $x^2 + xy + y^2 = 100$

$$2x + x \frac{dy}{dx} + y + 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} (x + 2y) = -2x - y$$

$$\frac{dy}{dx} = \frac{-(2x + y)}{(x + 2y)}$$

Answer any 6 questions from 9 to 16. Each carries 4 scores.

9 i) $|a - a| = |0| = 0$, even $\forall (a, a) \in R$

$\therefore R$ is reflexive.

If $|a - b|$ is even, then $|b - a| = |-(a - b)| = |a - b|$ is also even.

i.e., if $(a, b) \in R$, then $(b, a) \in R$

$\therefore R$ is symmetric.

If $|a - b|$ is even and $|b - c|$ is even then $|a - b + b - c| = |a - c|$ is also even.

i.e., if $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$

$\therefore R$ is transitive.

Hence, R is an equivalence relation.

ii) Since 5 is an odd, the other odd numbers are 1 and 3.

$\therefore$ the set elements related to 5 is $\{1, 3, 5\}$

10. a) iii)

b) iv)

c) i)

d) ii)

11. i) Order of AB is 3×3

$$\text{ii) } AB = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix} \begin{bmatrix} 1 & 3 & 6 \end{bmatrix} = \begin{bmatrix} -2 & -6 & -12 \\ 4 & 12 & 24 \\ 5 & 15 & 30 \end{bmatrix}$$

$$(AB)' = \begin{bmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ -12 & 24 & 30 \end{bmatrix} \dots\dots\dots (1)$$

$$B'A' = \begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix} \begin{bmatrix} -2 & 4 & 5 \end{bmatrix} = \begin{bmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ -12 & 24 & 30 \end{bmatrix} \dots\dots\dots (2)$$

From (1) and (2) we have, $(AB)' = B'A'$

12. i) If a square matrix is singular, $\det = 0$

$$\begin{vmatrix} 2+x & 3 & 4 \\ 1 & -1 & 2 \\ x & 1 & 5 \end{vmatrix} = 0$$

$$(2+x)[-5-2] - 3[5-2x] + 4[1-(-x)] = 0$$

$$(2+x)(-7) - 3(5-2x) + 4(1+x) = 0$$

$$-14 - 7x - 15 + 6x + 4 + 4x = 0$$

$$3x - 25 = 0$$

$$3x = 25$$

$$\therefore x = \frac{25}{3}$$

$$\text{ii) } A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = 3 - -2 = 5$$

$$\text{adj } A = \begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}$$

$$\begin{aligned} A(\text{adj } A) &= \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 3 + 2 & 1 - 1 \\ 6 - 6 & 2 + 3 \end{bmatrix} \\ &= \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} \dots \dots \dots (1) \end{aligned}$$

$$|A|.I = 5 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} \dots \dots \dots (2)$$

From (1) and (2) we have

$$A(\text{adj } A) = |A|.I$$

$$13) \text{ i) } A = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$$

If A is symmetric, $A' = A$

$$\begin{bmatrix} 0 & 3 & 3a \\ 2b & 1 & 3 \\ -2 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$$

$$2b = 3 \Rightarrow b = \frac{3}{2}$$

$$3a = -2 \Rightarrow a = -\frac{2}{3}$$

$$\text{ii) } A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$\begin{aligned} A^2 &= A.A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 9 - 1 & 3 + 2 \\ -3 - 2 & -1 + 4 \end{bmatrix} \\ &= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} \end{aligned}$$

$$5A = 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$$

$$7I = 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$\text{LHS} = A^2 - 5A + 7I$$

$$\begin{aligned} &= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} \\ &= \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} \end{aligned}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

$$= \text{RHS}$$

Hence proved.

$$14) \text{ i) } y = \sin^{-1} x$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2} \cdot \frac{dy}{dx} = 1$$

Diff. w.r.t. x

$$\sqrt{1-x^2} \cdot \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \frac{-x}{\sqrt{1-x^2}} = 0$$

$$\text{Xing by } \sqrt{1-x^2}$$

$$(1-x^2) \cdot \frac{d^2y}{dx^2} - x \frac{dy}{dx} = 0$$

$$(1-x^2) \frac{d^2y}{dx^2} = x \frac{dy}{dx}$$

Hence proved.

$$\text{ii) } y = \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right)$$

$$= \cos^{-1} \left(\frac{1-\tan^2 \theta}{1+\tan^2 \theta} \right) \quad | \quad \text{Put } x = \tan \theta$$

$$= \cos^{-1} \cos 2\theta \quad | \quad \tan^{-1} x = \theta$$

$$= 2\theta$$

$$= 2 \tan^{-1} x$$

$$\therefore \frac{dy}{dx} = 2 \times \frac{1}{1+x^2} = \frac{2}{1+x^2}$$

$$15) \text{ i) } \sin^{-1} \left(\sin \frac{2\pi}{3} \right) = \sin^{-1} \sin \left(\pi - \frac{\pi}{3} \right)$$

$$= \sin^{-1} \sin \frac{\pi}{3}$$

$$= \frac{\pi}{3} \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$$

$$\text{ii) } \sin^{-1} \left(\frac{8}{17} \right) = \tan^{-1} \left(\frac{8}{15} \right)$$

$$\sin^{-1} \left(\frac{3}{5} \right) = \tan^{-1} \left(\frac{3}{4} \right)$$

$$\text{LHS} = \tan^{-1} \left(\frac{8}{15} \right) + \tan^{-1} \left(\frac{3}{4} \right)$$

$$\begin{aligned}
&= \tan^{-1} \left[\frac{\frac{8}{15} + \frac{3}{4}}{1 - \frac{8}{5} \times \frac{3}{4}} \right] \\
&= \tan^{-1} \left[\frac{32 + 45}{60 - 24} \right] \\
&= \tan^{-1} \left(\frac{77}{36} \right) = \text{RHS}
\end{aligned}$$

16) i) $f(x) = \cos(x^2)$

Let $h(x) = \cos x$

And $g(x) = x^2$

$\therefore f(x) = (h \circ g)(x) = h[g(x)]$

$= h(x^2)$

$= \cos(x^2)$

The composite function of two continuous functions is also continuous everywhere.

ii) Since f is continuous at $x = 5$

$$\lim_{x \rightarrow 5^-} f(x) = \lim_{x \rightarrow 5^+} f(x)$$

$$\lim_{x \rightarrow 5^-} (kx + 1) = \lim_{x \rightarrow 5^+} (3x - 5)$$

$$5k + 1 = 3(5) - 5$$

$$5k + 1 = 3(5) - 5$$

$$5k = 10 - 1$$

$$5k = 9$$

$$5k = 9$$

$$\therefore k = \frac{9}{5}$$

17. i) $A) = 0$

ii) The two functions $f(x) = (x - 1)^2 + 1$ can be made bijective by redefining its domain as

$[1, \infty)$ and its codomain as $[1, \infty)$. Thus f can be written as $f: [1, \infty) \rightarrow [1, \infty)$ by

$$f(x) = (x - 1)^2 + 1.$$

18. i) $a_{ij} = 2i - j$

$$a_{11} = 2(1) - 1 = 1$$

$$a_{12} = 2(1) - 2 = 0$$

$$a_{13} = 2(1) - 3 = -1$$

$$a_{21} = 2(2) - 1 = 3$$

$$a_{22} = 2(2) - 2 = 2$$

$$a_{23} = 2(2) - 3 = 1$$

$$a_{31} = 2(3) - 1 = 5$$

$$a_{32} = 2(3) - 2 = 4$$

$$a_{33} = 2(3) - 3 = 3$$

$$\therefore A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & 1 \\ 5 & 4 & 3 \end{bmatrix}$$

$$\begin{aligned} \text{ii) } A + A' &= \begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & 1 \\ 5 & 4 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 3 & 5 \\ 0 & 2 & 4 \\ -1 & 1 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \end{aligned}$$

$$\text{Let } P = \frac{1}{2}(A + A') = \frac{1}{2} \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}$$

$$P' = \frac{1}{2} \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}$$

= P, is symmetric

$$\text{Let } Q = \frac{1}{2}(A - A')$$

$$= \frac{1}{2} \begin{bmatrix} 1-1 & 0-3 & -1-5 \\ 3-0 & 2-2 & 1-4 \\ 5+1 & 4-1 & 3-3 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 0 & -3 & -6 \\ 3 & 0 & -3 \\ 6 & 3 & 0 \end{bmatrix}$$

$$Q' = \frac{1}{2} \begin{bmatrix} 0 & 3 & 6 \\ -3 & 0 & 3 \\ -6 & -3 & 0 \end{bmatrix}$$

$$= -\frac{1}{2} \begin{bmatrix} 0 & -3 & -6 \\ 3 & 0 & -3 \\ 6 & 3 & 0 \end{bmatrix} = -Q, \text{ skew-symmetric.}$$

$$\text{Now, } P + Q = \frac{1}{2} \left(\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 0 & -3 & -6 \\ 3 & 0 & -3 \\ 6 & 3 & 0 \end{bmatrix} \right)$$

$$= \frac{1}{2} \left(\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 0 & -3 & -6 \\ 3 & 0 & -3 \\ 6 & 3 & 0 \end{bmatrix} \right)$$

$$= \frac{1}{2} \begin{bmatrix} 2 & 0 & -2 \\ 6 & 4 & 2 \\ 10 & 8 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & 1 \\ 5 & 4 & 3 \end{bmatrix} = A, \text{ a square matrix}$$

Hence, verified.

Answer any 3 questions from 17 to 20. Each carries 6 scores.

19. The system equations can be written in the form $AX = B$

$$\text{Where, } A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix} = 1(1+3) + 1(2+3) + 1(2-1) = 4+5+1 = 10 \neq 0$$

$$\begin{array}{lll} A_{11} = 4 & A_{21} = 2 & A_{31} = 2 \\ A_{12} = -5 & A_{22} = 0 & A_{32} = 5 \\ A_{13} = 1 & A_{23} = -2 & A_{33} = 3 \end{array}$$

$$\text{adj}A = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \text{adj} A = \frac{1}{10} \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$\begin{aligned} \therefore X = A^{-1}B &= \frac{1}{10} \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} \\ &= \frac{1}{10} \begin{bmatrix} 16+0+4 \\ -20+0+10 \\ 4+0+6 \end{bmatrix} = \frac{1}{10} \begin{bmatrix} 20 \\ -10 \\ 10 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \end{aligned}$$

$$\Rightarrow x = 2, y = -1 \text{ and } z = 1$$

20. i) $a^x \log a$

ii) a) $x = a \cos \theta$

$$\frac{dx}{d\theta} = -a \sin \theta = -a 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} = -2a \sin \frac{\theta}{2} \cos \frac{\theta}{2}$$

$$y = a(\theta + \sin \theta)$$

$$\frac{dy}{d\theta} = a(1 + \cos \theta) = a \times 2 \cos^2 \frac{\theta}{2} = 2a \cos^2 \frac{\theta}{2}$$

$$\frac{dy}{dx} = \frac{dy}{d\theta} / \frac{dx}{d\theta}$$

$$= 2a \cos^2 \frac{\theta}{2} \times \frac{1}{-2a \sin \frac{\theta}{2} \cos \frac{\theta}{2}} = -\cot \frac{\theta}{2}$$

b) $y = x^x + x^{\sin x}$

$$\text{Let } u = x^x \text{ and } v = x^{\sin x}$$

$$\text{Then } y = u + v$$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \dots \dots \dots (1)$$

$$\text{Let } u = x^x$$

$$\log u = x \log x$$

Diff u w.r.t. x

$$\frac{1}{u} \frac{du}{dx} = x \cdot \frac{1}{x} + \log x \cdot 1$$

$$\frac{du}{dx} = u \left[x \cdot \frac{1}{x} + \log x \cdot 1 \right] = x^x (1 + \log x)$$

$$\text{Let } v = x^{\sin x}$$

$$\log v = \sin x \cdot \log x$$

Diff w.r.t. x

$$\frac{1}{v} \frac{dv}{dx} = \sin x \times \frac{1}{x} + \log x \cos x$$

$$\frac{dv}{dx} = v \left[\frac{\sin x}{x} + \cos x \log x \right] = x^{\sin x} \left[\frac{\sin x}{x} + \cos x \log x \right]$$

$$\frac{dy}{dx} = x^x (1 + \log x) + x^{\sin x} \left[\frac{\sin x}{x} + \cos x \log x \right]$$

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