COMMON HALF YEARLY EXAMINATION - 2024	Reg. No.						

XII - MATHEMATICS

Time Allowed : 3-00 Hrs. *Maximum Marks: 90*

Part - I

I. **Choose the correct answer:** $20 \times 1 = 20$

1. If $A\begin{bmatrix}1 & -2\\1 & 4\end{bmatrix}=\begin{bmatrix}6 & 0\\0 & 6\end{bmatrix}$, then A =

a) $\begin{bmatrix}1 & -2\\1 & 4\end{bmatrix}$ b) $\begin{bmatrix}1 & 2\\-1 & 4\end{bmatrix}$ c) $\begin{bmatrix}4 & 2\\-1 & 1\end{bmatrix}$ d) $\begin{bmatrix}4 & -1\\2 & 1\end{bmatrix}$

2. If $A=\begin{bmatrix}\cos\theta & \sin\theta\\-\sin\theta & \cos\theta\end{bmatrix}$ and $A(\text{adj } A)=\begin{bmatrix}k & 0\\0 & k\end{bmatrix}$, then k =

a) 0 b) $\sin\theta$ c) $\cos\theta$ d) 1

3. The area of the triangle formed by the complex numbers z, iz and $z+iz$ in the Argand's diagram is

a) $\frac{1}{2}|z|^2$ b) $|z|^2$ c) $\frac{3}{2}|z|^2$ d) $2|z|^2$

4. If $\omega \neq 1$ is a cubic root of unity and $(1+\omega)^7 = A + B\omega$, then (A, B) equals

a) (1,0) b) (–1 1) c) (0,1) d) (1,1)

5. The number of positive zeros of the polynomial $\sum_{r=0}^{n} nCr(-1)^r x^r$ is

a) 0 b) n c) < n d) r

6. If $\cot^{-1}(\sqrt{\sin\alpha})+\tan^{-1}(\sqrt{\sin\alpha})=u$, then cos 2u is equal to

a) $\tan^2\alpha$ b) 0 c) –1 d) $\tan 2\alpha$

7. If the normal of the parabola $y^2 = 4x$ drawn at the end points of its latus rectum are tangents to the circle $(x-3)^2+(y+2)^2=r^2$, then the value of r^2 is

a) 2 b) 3 c) 1 d) 4

8. If the direction cosines of a line are $\frac{1}{c}, \frac{1}{c}, \frac{1}{c}$ then

a) $c = \pm 3$ b) $c \pm \sqrt{3}$ c) $c > 0$ d) $0 < c < 1$

9. The number given by the Rolle's theorem for the function $x^3 - 3x^2$, $x \in [0, 3]$ is

a) 1 b) $\sqrt{2}$ c) $\frac{3}{2}$ d) 2

10. If $f(x, y) = e^{xy}$, then $\frac{\partial^2 f}{\partial x \partial y}$ is equal to

a) $xy\, e^{xy}$ b) $(1 + xy)\, e^{xy}$ c) $(1 + y)\, e^{xy}$ d) $(1 + x)\, e^{xy}$

11. Linear approximation for $g(x) = \cos x$ at $x = \frac{\pi}{2}$ is

a) $x+\frac{\pi}{2}$ b) $-x+\frac{\pi}{2}$ c) $x-\frac{\pi}{2}$ d) $-x-\frac{\pi}{2}$

12. The value of $\int_0^1 x(1-x)^{99}\,dx$ is

a) $\frac{1}{11000}$ b) $\frac{1}{10100}$ c) $\frac{1}{10010}$ d) $\frac{1}{10001}$

13. The solution of $\frac{dy}{dx}+p(x)y=0$ is

a) $y=ce^{\int pdx}$ b) $y=ce^{-\int pdx}$ c) $x=ce^{-\int pdy}$ d) $x=ce^{\int pdy}$

14. If $P(X=0)=1-P(X=1)$. If $E[X]=3\,Var(X)$, then $P(X=0)$ is

a) $\frac{2}{3}$ b) $\frac{2}{5}$ c) $\frac{1}{5}$ d) $\frac{1}{3}$

15. The operation * defined by $a*b=\frac{ab}{7}$ is not a binary operation on

a) Q^+ b) Z c) R d) C

16. $\sin(\sin^{-1}x)=x$ if

a) $|x|\le 1$ b) $|x|\ge 1$ c) $|x|<1$ d) $|x|\le \frac{\pi}{2}$

17. The non parametric form of a vector equation passing through two points whose position vectors are $\vec{a}$ and $\vec{b}$ and parallel to $\vec{u}$ is

a) $\left[\vec{r}-\vec{u},\vec{b}-\vec{a},\vec{u}\right]=0$ b) $\left[\vec{r}-\vec{a},\vec{u}-\vec{a},\vec{u}\right]=0$

c) $\left[\vec{r}-\vec{u},\vec{a}-\vec{b},\vec{u}\right]=0$ d) $\left[\vec{r}-\vec{a},\vec{b}-\vec{a},\vec{u}\right]=0$

18. If f(x) is continuous on [a, b] then f has both absolute maximum and absolute minimum in [a, b]. This statement is

a) Extreme value theorem b) Intermediate value theorem

c) Lagrange mean value theorem d) Taylors theorem

19. If $X \sim B(n, p)$ then

a) $\mu=np, \sigma^2=np(1-p)$ b) $\mu=nq, \sigma^2=np(1-p)$

c) $\mu=nq, \sigma^2=np(1-q)$ d) $\mu=np, \sigma^2=nq(1-p)$

20. $y=mx+c$ is a tangent to the parabola $y^2=4ax$ then

a) $c=\frac{a}{m}$ b) $c=\frac{m}{a}$ c) $c^2=a^2m^2+b^2$ d) $m=c$

Part - II

II. Answer any 7 questions. (Q.No.30 is compulsory) **$7\times2=14$**

21. If $\operatorname{adj}A=\begin{bmatrix}-1&2&2\\1&1&2\\2&2&1\end{bmatrix}$, find A^{-1}

22. Show that the following equations represent a circle, and find its center and radius.
$|z-2-i|=3$

23. Find a polynomial equation of minimum degree with rational coefficients, having $2i+3$ as a root.

24. Find the general equation of the circle whose diameter is the line segment joining the points (–4,–2) and (1,1).

25. Find the angle between the line $\vec{r} = \left(2\hat{i} - \hat{j} + \hat{k}\right) + t\left(\hat{i} + 2\hat{j} - 2\hat{k}\right)$ and the plane $\vec{r}.\left(6\hat{i} + 3\hat{j} + 2\hat{k}\right) = 8$

26. Evaluate the following: $\int_0^{\pi/2} \sin^{10} x \, dx$

27. Find the differential equation for the family of all straight lines passing through the origin.

28. Suppose X is the number of tails occurred when three fair coins are tossed once simultaneously. Find the values of the random variable X and number of points in its inverse images.

29. Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ be any two boolean matrices of the same type. Find $A \vee B$ and $A \wedge B$.

30. Evaluate $\lim_{x \to \infty} \left(\frac{e^x}{x^m}\right)$, $m \in N$

Part - III

III. Answer any 7 questions. (Q.No.40 is compulsory) **$7 \times 3 = 21$**

31. Find the rank of the matrix $\begin{bmatrix} 2 & -2 & 4 & 3 \\ -3 & 4 & -2 & -1 \\ 6 & 2 & -1 & 7 \end{bmatrix}$ by reducing it to an echelon form.

32. The complex numbers u, v and w are related by $\frac{1}{u} = \frac{1}{v} + \frac{1}{w}$. If $v = 3 - 4i$ and $w = 4 + 3i$, find u in rectangular form.

33. Solve the equation $x^3 - 5x^2 - 4x + 20 = 0$

34. Find the domain of $\cos^{-1}\left(\frac{2 + \sin x}{3}\right)$

35. The maximum and minimum distances of the Earth from the Sun respectively are 152×10^6 km and 94.5×10^6 km. The Sun is at one focus of the elliptical orbit. Find the distance from the Sun to the other focus.

36. A variable plane moves in such a way that the sum of the reciprocals of its intercepts on the coordinate axes is a constant. Show that the plane passes through a fixed point.

37. Write the Maclaurin series expansion of the following function: $\tan^{-1}(x)$; $-1 \le x \le 1$

38. Suppose the amount of milk sold daily at a milk booth is distributed with a minimum of 200 liters and a maximum of 600 liters with probability density function

$f(x) = \begin{cases} k, & 200 \le x \le 600 \\ 0, & \text{Otherwise} \end{cases}$. Find (i) the value of k (ii) the distribution function

(iii) the probability that daily sales will fall between 300 liters and 500 liters?

39. Define an operation* on Q as follows $a * b = \left(\frac{a+b}{3}\right)$; a, b $\in$ Q. Examine the closure, commutative, and associative properties satisfied by * on Q.

40. Evaluate $\int_2^4 \frac{\sqrt{x}}{\sqrt{6-x} + \sqrt{x}} dx$

Part - IV

IV. Answer all the questions. **$7 \times 5 = 35$**

41 a) Solve the following systems of linear equations by Cramer's rule:

$$\frac{3}{x}-\frac{4}{y}-\frac{2}{z}-1=0,\ \frac{1}{x}+\frac{2}{y}+\frac{1}{z}-2=0\ \ \frac{2}{x}-\frac{5}{y}-\frac{4}{z}+1=0$$

(OR)

b) Find the non–parametric form of vector equation, and Cartesian equation of the plane passing through the point (2,3,6) and parallel to the straight lines

$$\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-3}{1} \text{ and } \frac{x+3}{2}=\frac{y-3}{-5}=\frac{z+1}{-3}$$

42. a) If $z=x+iy$ and $\arg\left(\frac{z-i}{z+2}\right)=\frac{\pi}{4}$, show that $x^2+y^2+3x-3y+2=0$ ***(OR)***

b) The rate of increase in the number of bacteria in a certain bacteria culture is proportional to the number present. Given that the number triples in 5 hours, find how many bacteria will be present after 10 hours?

43. a) If $2+i$ and $3-\sqrt{2}$ are roots of the equation $x^6-13x^5+62x^4-126x^3+65x^2+127x-140=0$ find all roots. ***(OR)***

b) A conical water tank with vertex down of 12 meters height has a radius of 5 meters at the top. If water flows into the tank at a rate 10 cubic m/min, how fast is the depth of the water increases when the water is 8 meters deep?

44. a) Prove that $\tan^{-1}x+\tan^{-1}\frac{2x}{1-x^2}=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$, $|x|<\frac{1}{\sqrt{3}}$ ***(OR)***

b) If $u=\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}\tan u$

45. a) A tunnel through a mountain for a four–lane highway is to have a elliptical opening. The total width of the highway (not the opening) is to be 16m, and the height at the edge of the road must be sufficient for a truck 4m high to clear if the highest point of the opening is to be 5m approximately. How wide must the opening be?

(OR)

b) A random variable X has the following probability mass function:

X	1	2	3	4	5
f(x)	k^2	$2k^2$	$3k^2$	$2k$	$3k$

Find (i) the value of k (ii) $P(2 \le X < 5)$ (iii) $P(3 < X)$

46. a) By vector method, prove that $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

(OR)

b) Find the area of the region bounded between the parabolas $y^2=x$ and $x^2=y$

47. a) A rectangular page is to contain 24 cm^2 of print. The margins at the top and bottom of the page are 1.5 cm and the margins at other sides of the page is 1 cm. What should be the dimensions of the page so that the area of the paper used is minimum?

(OR)

b) Prove that $p\rightarrow(\neg q\vee r)\equiv\neg q\vee(\neg q\vee r)$ using truth table.