TVL12BM

Tirunelveli District
Common Half Yearly Examination - 2024

Standard 12
BUSINESS MATHEMATICS AND STATISTICS

Time: 3.00 Hours | Marks: 90

I. Choose the best answer: **20×1=20**

1) If $T = \begin{matrix} A \\ B \end{matrix}\begin{bmatrix} 0.7 & 0.3 \\ 0.6 & x \end{bmatrix}$ is a transition probability matrix, then the value of x is
   a) 0.2 b) 0.3 c) 0.4 d) 0.7
2) If $|A| \neq 0$, then A is
   a) non-singluar matrix b) Singular matrix
   c) zero matrix d) none of these
3) $\int \frac{1}{x^3} dx$ is
   a) $\frac{-3}{x^2} + c$ b) $\frac{-1}{2x^2} + c$ c) $\frac{-1}{3x^2} + c$ d) $\frac{-2}{x^2} + c$
4) $\int_0^\infty x^4 e^{-x} dx$ is
   a) 12 b) 4 c) 4! d) 64
5) When $x_0 = 2$ and $P_0 = 12$ the producer's surplus for the supply function $P_s = 2x^2 + 4$ is
   a) $\frac{31}{5}$ units b) $\frac{32}{2}$ units c) $\frac{32}{3}$ units d) $\frac{30}{7}$ units
6) Area bounded by $y = |x|$ between the limits 0 and 2 is
   a) 1 sq.units b) 3 sq.units c) 2 sq.units d) 4 sq.units
7) The differential equation of $y = mx + c$ is (m and c are arbitrary constants)
   a) $\frac{d^2y}{dx^2} = 0$ b) $y = x\frac{dy}{dx} + c$ c) $xdy + ydx = 0$ d) $ydx - xdy = 0$
8) The particular integral of the differential equation $f(D)\, y = e^{ax}$ where $f(D) = (D-a)^2$
   a) $\frac{x^2}{2}e^{ax}$ b) xe^{ax} c) $\frac{x}{2}e^{ax}$ d) x^2e^{ax}
9) $E \equiv$
   a) $1 + \Delta$ b) $1 - \Delta$ c) $1 + \nabla$ d) $1 - \nabla$
10) $\nabla \equiv$
   a) 1+E b) 1−E c) $1-E^{-1}$ d) $1+E^{-1}$
11) Given E(x)=5 and E(Y)=−2, then E(x − y) is
   a) 3 b) 5 c) 7 d) −2
12) The probability density function p(x) cannot exceed
   a) zero b) one c) mean d) infinity
13) In a parametric distribution the mean is equal to variance is
   a) binomial b) normal c) poisson d) all the above
14) Using the standard normal table, the sum of the probabilities to the right of z = 2.18 and to the left of z=−1.75 is
   a) 0.4854 b) 0.4599 c) 0.0146 d) 0.0547
15) A ..... may be finite or infinite according as the number of observations or items in it is finite or infinite
   a) Population b) census c) parameter d) none of these
16) An estimator is said to be ..... if it contains all the inforamtion in the data about the parameter it estimates
   a) efficient b) sufficient c) unbiased d) consistent
17) Laspeyre's index = 110, Paache's index = 108, then Fisher's Ideal index is equal to
   a) 110 b) 108 c) 100 d) 109
18) A typical control charts consists of
   a) CL, UCL b) CL, LCL c) CL, LCL, UCL d) UCL, LCL

19) In a non-degenerate solution number of allocations is
a) Equal to m+n−1 b) Equal to m+n+1
c) Not equal to m+n−1 d) Not equal to m+n+1

20) In an assignment problem involving four workers and three jobs, total number of assignments possible are
a) 4 b) 3 c) 7 d) 12

II. Answer any seven of the following. Q.No. 30 is compulsory. 7×2=14

21) Find the rank of the matrix $\begin{bmatrix} 2 & -1 & 1 \\ 3 & 1 & -5 \\ 1 & 1 & 1 \end{bmatrix}$

22) Evaluate $\int_0^\infty e^{-2x} x^5 \, dx$

23) Calculate consumer's surplus if the demand function $P=122-5x-2x^2$ and $x=6$

24) Find the differential equation of the family of all straight lines passing through the origin

25) Find $\Delta \log x$

26) In a Poisson distribution the first probability term is 0.2725. Find the next probability term

27) A sample of 100 students is chosen from a large group of students. The average height of these students is 162cm and standard deviation (S.D.) is 8cm. Obtain the standard error for the average height of large group of students of 160cm?

28) Fit a trend line by the method of semi-averages for the given data

Year	2000	2001	2002	2003	2004	2005	2006
production	105	115	120	100	110	125	135

29) State the uses of Index Number

30) The mean of Binomials distribution is 20 and standard deviation is 4. Find the parameter of the distribution

III. Answer any seven of the following. Q.No. 40 is compulsory. 7×3=21

31) Solve the equations $2x+3y=7$, $3x+5y=9$ by Cramer's Rule

32) Evaluate $\int x e^x \, dx$

33) Find the area bounded by $y=x$ between the lines $x = -1$ and $x = 2$ with x-axis

34) Use Lagrange's formula and estimate from the following data the number of workers getting income not exceeding Rs.26 per month

Income not exceeding(₹)	15	25	30	35
No. of workers	36	40	45	48

35) Two unbiased dice are thrown simultaneously and sum of the upturned faces considered as random variable. Construct a probability mass function.

36) Determine the binomial distribution for which the mean is 4 and variance 3. Also find P(x=15)

37) Assuming one in 80 births is a case of twins, calculate the probability of 2 or more sets twins on a day when 30 births occur

38) A sample of 100 items, draw from a universe with mean value 4 and SD 3, has a mean value 3.5. Is the difference in the mean significant at 0.05 level of significance?

39) The following figures relates to the profits of a commerical for 8 years.

Year	1986	1987	1988	1989	1990	1991	1992	1993
Profit	15420	15470	15520	21020	26500	31950	35600	34900

Find the trend of profits by the method of three yearly moving averages.

40) If $y=x^3-x^2+x-1$, calculate the value of y for $x=0, 1, 2, 3, 4, 5$ and form the forward difference table.

IV. Answer all the questions. **7×5=35**

41) a) Two types of soaps A and B are in the market. Their present market shares are 15% for A and 85% for B. Of those who bought A the previous year, 65% continue to buy it again while 85% switch over to B. Of those who bought B the previous year, 35% buy it again and 45% switch over to A. Find their market shares after one year and when is the equilibrium reached?

(OR)

b) Compute (i) Laspeyre's (ii) Paasche's (iii) Fisher's Index numbers for the 2010 from the following data

Commodity	Price		Quantity	
	2000	2010	2000	2010
A	12	14	18	16
B	15	16	20	15
C	14	15	24	20
D	12	12	29	23

42) a) Evaluate $\int_2^5 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{7-x}}dx$

(OR)

b) Given $y_3=2$, $y_4=-6$, $y_5=8$, $y_6=9$ and $y_7=17$, Calcuate $\Delta^4 y_3$

43) a) The elasticity of demand with respect to price P for a commodity is $\eta_d = \frac{P+2P^2}{100-P-P^2}$. Find demand function where price is ₹5 and the demand is 70.

(OR)

b) Calcuate the seasonal indices from the following data using the average from the following data using the average method.

	I Quarter	II Quarter	III Quarter	IV Quarter
2008	72	68	62	76
2009	78	74	78	72
2010	74	70	72	76
2011	76	74	74	72
2012	72	72	76	68

44) a) Suppose that the quantity demanded $Q_d = 29 - 2P - 5\frac{dp}{dt} + \frac{d^2P}{dt^2}$ and quantity supplied $Q_s = 5+4P$ where P is the price. Find the equilibrium price for market clearance.

(OR)

b) Forty percent of business travellers carry a laptop. In a sample of 15 business travellers.

i) What is the probability that 3 will have a laptop?

ii) What is the probability that 12 of the travellers will not have a laptop?

iii) What is the probability that atleast three of the travellers have a laptop?

45) a) The population of a certain town is as follows

Year	1941	1951	1961	1971	1981	1991
Population in lakhs. y	20	24	29	36	46	51

b) Using appropriate interpolation formula estimate the population during the period 1946

(OR)

a) Determine how much quantity should be stepped from factory to various destinations for the following transportation problem using least cost method.

24] Soln:

$y = mx + c$ — ①

$\frac{dy}{dx} = m$ [Diff w.r.to x]

Sub value of m in ①

$y = x\frac{dy}{dx}$

25] Soln:

$\Delta \log x = \log(x+h) - \log x$

$\Delta f(x) = f(x+h) - f(x)$

$= \log \frac{(x+h)}{x}$

$= \log\left(1 + \frac{h}{x}\right)$

26] Soln:

$P_0 = 0.2725$

$\frac{e^{-\lambda}\lambda^0}{0!} = 0.2725$

$e^{-\lambda} = 0.2725$

$\lambda = 1.3$

$P(x=1) = \frac{e^{-1.3}(1.3)^1}{1!}$

$= 0.2725 \times 1.3$

$= 0.3543$

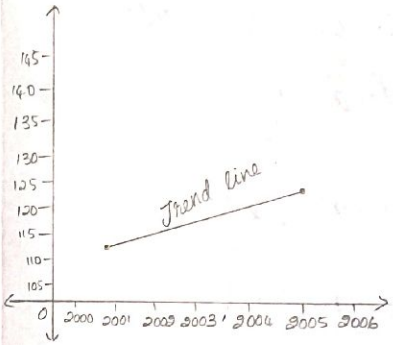
27] Soln:

$SE = \frac{\sigma}{\sqrt{n}} = \frac{8}{\sqrt{100}} = \frac{8}{10}$

$SE = 0.8$

28] Soln:

YEAR	PRODUCTION	AVERAGES
2000	105	$\Rightarrow \frac{105+115+120}{3} \Rightarrow \frac{340}{3}$
2001	115	
2002	120	$\Rightarrow 113.3$
2003	100	(Left out)
2004	110	$\Rightarrow \frac{110+125+135}{3} \Rightarrow \frac{370}{3}$
2005	125	
2006	135	$\Rightarrow 123.3$



29] Soln:

(i) To regulate inflation and deflation in an economy.

(ii) To study trends.

30] Soln:

$np = 20$, $\sqrt{npq} = 4 \Rightarrow npq = 16$

$\frac{npq}{np} = \frac{16}{20} \Rightarrow q = \frac{4}{5}$, $p = \frac{1}{5}$

$np \Rightarrow n\left(\frac{1}{5}\right) = 20 \Rightarrow \boxed{n = 100}$

31] Soln:

$$\Delta = \begin{vmatrix} 2 & 3 \\ 3 & 5 \end{vmatrix} = 10 - 9 = 1 \neq 0$$

$$\Delta x = \begin{vmatrix} 7 & 3 \\ 9 & 5 \end{vmatrix} = 35 - 27 = 8$$

$$\Delta y = \begin{vmatrix} 2 & 7 \\ 3 & 9 \end{vmatrix} = 18 - 21 = -3$$

$$x = \frac{\Delta x}{\Delta} = \frac{8}{1} = 8$$

$$y = \frac{\Delta y}{\Delta} = \frac{-3}{1} = -3$$

32] Soln:

$$\int x e^x dx$$

$$\int uv = uv - \int v\,du$$

$u = x \quad dv = e^x$

$du = dx \quad v = e^x$

$$\Rightarrow x e^x - \int e^x dx$$

$$\Rightarrow x e^x - e^x + c$$

$$\Rightarrow e^x (x - 1) + c$$

33] Soln:

$$\text{Area} = -\int y\,dx + \int y\,dx$$

$$= -\int_{-1}^{0} x\,dx + \int_{0}^{2} x\,dx$$

$$= -\left[\frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_{0}^{2}$$

$$= -\left[0 - \tfrac{1}{2}\right] + 2$$

$$= \tfrac{1}{2} + 2 = \tfrac{5}{2} \text{ sq. units.}$$

34] Soln:

$x_0 = 15,\ x_1 = 25,\ x_2 = 30,\ x_3 = 35$

$y_0 = 36,\ y_1 = 40,\ y_2 = 45,\ y_3 = 48$

$$y = f(x) = \frac{(x - x_1)(x - x_2)(x - x_3)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)}(y_0) +$$

$$\frac{(x - x_0)(x - x_2)(x - x_3)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)}(y_1) +$$

$$\frac{(x - x_0)(x - x_1)(x - x_3)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)}(y_2) +$$

$$\frac{(x - x_0)(x - x_1)(x - x_2)}{(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)}(y_3)$$

$$f(26) = \frac{(26 - 25)(26 - 30)(26 - 35)}{(15 - 25)(15 - 30)(15 - 35)}(36) +$$

$$\frac{(26 - 15)(26 - 30)(26 - 35)}{(25 - 15)(25 - 30)(25 - 35)}(40) +$$

$$\frac{(26 - 15)(26 - 25)(26 - 35)}{(30 - 15)(30 - 25)(30 - 35)}(45) +$$

$$\frac{(26 - 15)(26 - 25)(26 - 30)}{(35 - 15)(35 - 25)(35 - 30)}(48)$$

$$= 41 \text{ persons.}$$

35] Soln:

x	2	3	4	5	6	7	8	9	10	11	12
$P(x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

36] Soln:

$q = 4/3,\ p = 1/4,\ n = 16$

$$P(x = 15) = {}^{16}C_{15} \left(\frac{1}{4}\right)^{15} \left(\frac{4}{3}\right)^{16-15}$$

$$= 16 \left(\frac{1}{4}\right)^{15} \left(\frac{4}{3}\right)$$

$$= \frac{3}{(4)^3}$$

37] Soln:

$np = 30 \times 0.16 = 0.375$

$P(X \geq 2) = 1 - P(X < 2)$

$= 1 - [P(x=0) + P(x=1)]$

$= 1 - [0.6873 + 1.375]$

$= 0.055$

38] Soln:

$|z| = 1.667$

$n = 100, \ \mu = 3.5, \ \sigma = 3$

$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{3.5 - 4}{3/\sqrt{100}}$

$= \frac{-0.5}{3/10} = \frac{-0.5}{0.3} = -1.667$

$|z| = |-1.667| = 1.667$

39] Soln:

YEAR	PROFIT	AVERAGE
1986	15420	-
1987	15470	15470
1988	15520	17336.67
1989	21020	21013.33
1990	26500	26490
1991	31930	31350
1992	35600	34150
1993	34900	-

40]

x	y_0	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$
0	-1				
		1			
1	0		4		
		5		6	
2	5		10		0
		15		6	
3	20		16		0
		31		6	
4	51		22		
		53			
5	104				

41] a)

Soln:

$$T = \begin{matrix} \\ A \\ B \end{matrix} \begin{matrix} \begin{matrix} A & \quad B \end{matrix} \\ \begin{pmatrix} 0.65 & 0.35 \\ 0.45 & 0.55 \end{pmatrix} \end{matrix}$$

After one year,

$$\Rightarrow (0.15 \quad 0.85) \begin{pmatrix} 0.65 & 0.35 \\ 0.45 & 0.55 \end{pmatrix}$$

$$\Rightarrow (0.48 \quad 0.52)$$

At equilibrium,

$$(A \ B) \begin{pmatrix} 0.65 & 0.35 \\ 0.45 & 0.55 \end{pmatrix} = (A \ B)$$

$0.65A + 0.45B = A$

$0.65A + 0.45(1 - A) = A$

$0.65A + 0.45 - 0.45A = A$

$A = 0.5625$

$B = 0.4375$

$A = 56.25\%$

$B = 43.75\%$

42) b) Soln:

$\Sigma P_0 q_0 = 1200$, $\Sigma P_0 q_1 = 973$

$\Sigma P_1 q_0 = 1280$, $\Sigma P_1 q_1 = 1040$

$$P_0^L = \frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times 100$$

$$= \frac{1280}{1200} \times 100 = 106.6$$

$$P_0^P = \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1} \times 100$$

$$= \frac{1040}{973} \times 100 = 106.8$$

$$P_0^F = \sqrt{P_0^L \times P_0^P} \times 100$$

$$= \sqrt{106.6 \times 106.8} \times 100$$

$$= 106.7$$

42) a) Soln:

$$I = \int_2^5 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{7-x}} dx \quad -①$$

$$I = \int_2^5 \frac{\sqrt{7-x}}{\sqrt{7-x} + \sqrt{x}} dx \quad -②$$

① + ② ⇒

$$2I = \int_2^5 \frac{\sqrt{x} + \sqrt{7-x}}{\sqrt{x} + \sqrt{7-x}} dx$$

$$2I = \int_2^5 dx$$

$$2I = [x]_2^5$$

$$I = 3/2$$

43) b) Soln:

$$\Delta^4 y_3 = (E-1)^4 y_3$$

$$= E^4 y_3 - 4E^3 y_3 + 6E^2 y_3 - 4E y_3 + y_3$$

$$= y_7 - 4y_6 + 6y_5 - 4y_4 + y_3$$

$$= 17 - 36 + 48 + 24 + 2$$

$$= 55$$

43) a)

Soln:

$$\eta_d = \frac{2P + 2P^2}{100 - P - P^2}$$

$$\frac{+P}{x} \cdot \frac{dx}{dP} = \int \frac{P(2P+1)}{P^2 + P - 100} dP$$

$$\int \frac{dx}{x} = \int \frac{2P+1}{P^2+P-100} dP$$

$$\log x = \log(P^2 + P - 100) + \log k$$

$$x = (P^2 + P - 100)k$$

$$R = Px = P(100 - P - P^2)$$

43) c)

	I-Qu	II-Qu	III-Qu	IV-Qu
Total	372	358	362	364
Average	74.4	71.6	72.4	72.8

$$G.A = \frac{74.4 + 71.6 + 72.4 + 72.8}{4}$$

$$= 72.8$$

$$SI \cdot IQ = \frac{\text{Av of QI}}{GA} \times 100$$

$$\Rightarrow \frac{74.4}{72.8} \times 100 = 102.19$$

$\Rightarrow \frac{71.6}{72.8} \times 100 = 98.35$

$\Rightarrow \frac{72.4}{72.8} \times 100 = 99.45$

$\Rightarrow \frac{72.8}{72.8} \times 100 = 100$

44]

a) Soln:

$$29 - 2P - 5\frac{dP}{dt} + \frac{d^2P}{dt^2} = 5 + 4P$$

$$\frac{d^2P}{dt^2} - 5\frac{dP}{dt} - 6P = -24$$

$$(D^2 - 5D - 6)P = -24$$

AE $m^2 - 5m - 6 = 0$

$(m-6)(m+1) = 0$

$m = -1, 6$

$CF = Ae^{6x} + Be^{-x}$

$PI = \frac{-24e^{0x}}{D^2 - 5m - 6} = \frac{-24}{-6} = 4$

$P = CF + PI$

$= Ae^{6x} + Be^{-x} + 4$

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b) Soln:

$P = 0.4$, $q = 0.6$, $n = 15$

(i) $P(X=3) = {}^{15}C_3 (0.4)^3 (0.6)^{12}$

$= \frac{15 \times 14 \times 13}{3 \times 2 \times 1} (0.064)(0.002)$

$= 0.0634$

(ii) $P(X=3) = 0.0634$

(iii) $P(X \geq 3) = 1 - P(X < 3)$

$= 0.9728$

45] a)

Soln:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1941	20					
		4				
1951	24		1			
		5		1		
1961	29		2		0	
		7		1		-9
1971	36		3		-9	
		10		-8		
1981	46		-5			
		5				
1991	51					

$$y = y_0 + \frac{n}{1!}\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \dots$$

$$y = 20 + 0.5(4) + \frac{0.5(0.5-1)}{2!}(1) + \frac{(0.5)(0.5-1)(0.5-2)}{3!}(1) + \frac{0.5(0.5-1)(0.5-2)(0.5-4)}{5!}(-9)$$

$= 21.69$

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b)

	C	H	K	P	
T	6	8 (5)	8	5 (25)	30
B	5 (35)	11 (5)	9	7	40
M	8	9 (18)	9 (32)	13	50
	35	28	32	25	

(i) $T \to P$, $B \to C$, $B \to H$, $M \to H$, $M \to K$, $T \to H$

(ii) $\Rightarrow (5 \times 8) + (25 \times 5) + (35 \times 5) + (5 \times 11) + (18 \times 9) + (32 \times 7)$

$\Rightarrow 2781$

46)

a)

x	1	2	3	4	5	6
P(x)	1/6	1/6	1/6	1/6	1/6	1/6

$E(X) = \sum x_i P_i$

$= \frac{1}{6}(1+2+3+4+5+6)$

$= 7/2$

$E(X^2) = \frac{1}{6}(1^2+2^2+3^2+4^2+5^2+6^2)$

$= \frac{1}{6}(1+4+9+16+25+36)$

$= \frac{91}{6}$

$V(X) = \frac{91}{6} - \frac{49}{4} = \frac{35}{12}$

b)

Soln:

$$(A/B) = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 2 & 5 & 7 & 52 \\ 2 & -1 & & 0 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & 3 & 5 & 34 \\ 0 & 0 & -4 & -20 \end{array}\right)$$

$-4z = -20 \Rightarrow z = 5$

$3y + 5z = 34 \Rightarrow y = 3$

$x + y + z = 9 \Rightarrow x = 1$

$\{1, 3, 5\}$

47) a)

Soln:

$\mu = 68, \quad \sigma = 3$

i) $P(X > 72) = P\left(Z > \frac{72-68}{3}\right)$

$= P(Z < -1.3)$

$= 0.5 - 0.4082$

$= 0.0918$

$\Rightarrow 0.0918 \times 500 \Rightarrow 46$

ii) $P(65 < X < 71) =$

$P\left(\frac{65-68}{3} < Z < \frac{71-68}{3}\right)$

$P(-1 < Z < 1) = 0.6826$

$\Rightarrow 0.6826 \times 100 \approx 341.$

iii) $P(X < 64) = P\left(Z < \frac{64-68}{3}\right)$

$= P(Z < -1.3)$

$= 0.5 - 0.4082$

$= 0.0918$

$\Rightarrow 500 \times 0.0918 = 45.9 \approx 46$

b)

$n = 900, \bar{x} = 3.4, s = 2.61,$

$\mu = 3.25, \sigma = 2.61$

$H_0 : \mu = 3.25$

$H_1 : \mu \neq 3.25$

i) $Z = \frac{3.4 - 3.25}{2.61/\sqrt{900}} = \frac{0.15}{0.087} = 1.724$

ii) $(x_1 - z_{\alpha/2}\ SE \leq \mu \leq \bar{x} + z_{\alpha/2}$

$3.4 - (1.96)(0.087) \leq \mu \leq 3.4 + (1.96)(0.087)$

$3.197 \leq \mu \leq 3.603$