HIGHER SECONDARY FIRST TERMINAL EXAMINATION - 2025

	Part - III	Max. Score	: 60
HSE II	MATHEMATICS	Time	: 2 Hrs
		Cool – Off Time	: 15 Mts

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Answer any 6 questions from 1 to 8. Each carries 3 scores.

1.(i) A function is said to be ……………..….. , if it is both one – one and onto. **[1]**

(ii) Prove that the Greatest Integer Function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = [x]$

is neither one-one nor onto. **[2]**

2. If $X + Y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$, find X and Y **[3]**

3. Prove that the determinant $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ is independent of θ **[3]**

4. Check whether the relation R in $\mathbb{R}$ defined as $R = \{(a, b): a \le b^3\}$ is

reflexive, symmetric or transitive **[3]**

5. (i) $\sin(\tan^{-1} x), |x| < 1$ is equal to

(A) $\frac{x}{\sqrt{1-x^2}}$ (B) $\frac{1}{\sqrt{1-x^2}}$ (C) $\frac{1}{\sqrt{1+x^2}}$ (D) $\frac{x}{\sqrt{1+x^2}}$ **[1]**

(ii) Write the function in the simplest form : $\tan^{-1} \left(\frac{1}{\sqrt{x^2-1}}\right), |x| > 1$ **[2]**

6. If $y = 3 \cos (\log x) + 4 \sin (\log x)$, show that $x^2 y_2 + x y_1 + y = 0$ **[3]**

7. Prove $3 \cos^{-1} x = \cos^{-1}(4x^3 - 3x), x \in \left[\frac{1}{2}, 1\right]$ **[3]**

8. If $y = \cos^{-1} \left(\frac{1-x^2}{1+x^2}\right), -1 < x < 1$, find $\frac{dy}{dx}$ **[3]**

Answer any 6 questions from 9 to 16. Each carries 4 scores.

9. (i) If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$, then

(A) $1 + \alpha^2 + \beta\gamma = 0$ (B) $1 - \alpha^2 + \beta\gamma = 0$

(C) $1 - \alpha^2 - \beta\gamma = 0$ (D) $1 + \alpha^2 - \beta\gamma = 0$ **[1]**

(ii) . If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find k so that $A^2 = kA - 2I$ **[3]**

10. Let $f:\{1,3,4\} \rightarrow \{1,2,5\}$ and $g:\{1,2,5\} \rightarrow \{1,3\}$ given by

$f = \{(1,2),(3,5),(4,1)\}$ and $g = \{(1,3),(2,3),(5,1)\}$. Write down gof   [4]

11. (i) Differentiate w.r.t. x, $\log_7 (\log x)$ [2]

(ii) If $2x + 3y = \sin x$, find $\frac{dy}{dx}$ [2]

12. (i) Find the values of $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ [2]

(ii) $\sin^{-1}(2x\sqrt{1-x^2}) = 2\sin^{-1} x$, $-\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$ [2]

13. Given $3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$ find the values of x, y, z and w. [4]

14. (i) If $A = \begin{bmatrix} 1 & -2 \\ -5 & 3 \end{bmatrix}$ then show that $|3A| = 9|A|$ [2]

(ii) Using determinants, find equation of line joining $(1, 2)$ and $(3, 6)$ [2]

15. (i) Find the derivative of $\sin(\sqrt{x})$, using chain rule   [2]

(ii) Find the values of k so that the function $f(x) = \begin{cases} kx^2, if\ x \le 2 \\ 3,\ if\ x > 2 \end{cases}$ is continuous [2]

16. (i) $\sin^{-1}\left(\sin\frac{5\pi}{6}\right) = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ [1]

(ii) Prove that $\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$ [3]

Answer any 3 questions from 17 to 20. Each carries 6 scores.

17.(i) If $y = \sin^{-1} x$, show that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$ [3]

(ii) Find $\frac{dy}{dx}$ if $x^y = y^x$ [3]

18.(i) Find the number of all one-one functions from the set $\{1,2,3,\ldots,n\}$ to itself.   [1]

(ii) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = 4x + 3$.

Find the function $g : \mathbb{R} \rightarrow \mathbb{R}$ such that $g = f^{-1}$. Also find $f^{-1}(x)$ [5]

19. Solve the following system of equations by matrix method.

$5x - y + 4z = 5$; $2x + 3y + 5z = 2$; $5x - 2y + 6z = -1$ [6]

20.(i) Construct a 2 X 2 matrix $A = [a_{ij}]$ where $a_{ij} = \frac{1}{2}|-3i + j|$ [2]

(ii) Express A as the sum of a symmetric and skew symmetric matrices   [4]

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