HIGHER SECONDARY FIRST TERMINAL EXAMINATION - 2025

	Part - III	Max. Score	: 60
HSE II	MATHEMATICS	Time	: 2 Hrs
		Cool – Off Time	: 15 Mts

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<u>Answer any 6 questions from 1 to 8. Each carries 3 scores</u>.

1. Let f : X → Y be a function. Define a relation R in X given by R = {(a, b): f(a) = f(b)}
   Examine whether R is an equivalence relation or not. **[3]**

2. Write the function in the simplest form $\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$, $x \neq 0$ **[3]**

3. Find the value of x, y, and z from the equation: $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$ **[3]**

4. (i) A square matrix A is said to be singular if $|A|$ is **[1]**

   (A) zero (B) non zero (C) infinity (D) one

   (ii) Find values of x if $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$ **[2]**

5. Find the relationship between a and b so that the function f defined by

   $f(x) = \begin{cases} ax+1, if\ x \leq 3 \\ bx+3\ , if\ x > 3 \end{cases}$ is continuous at $x = 3$

6. (i) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined as f(x) = 2x. Choose the correct answer. **[1]**

   (A) f is one-one onto (B) f is many-one onto

   (C) f is one-one but not onto (D) f is neither one-one nor onto

   (ii) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \sin x$ and $g(x) = x^2$

   Show that fog $\neq$ gof **[2]**

7. Prove $3\sin^{-1} x = \sin^{-1}(3x - 4x^3)$, $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$ **[3]**

8. Differentiate $(\log x)^{\sin x}$, x > 0 w.r.t. x. **[3]**

<u>Answer any 6 questions from 9 to 16. Each carries 4 scores</u>.

9. Show that the matrix A = $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ satisfies the equation $A^2 - 4A + I = O$

   Using this equation, find A^{-1} **[4]**

10. (i) A function f is said to be ………………… if it is both one – one and onto **[1]**

(ii) Let A and B be sets. Show that f: A × B → B × A such that (a, b) = (b, a) is bijective function **[3]**

11. (i) $\sin^{-1}\left(\sin\frac{3\pi}{4}\right) =$ ……………………….. **[1]**

(ii) Prove that $\sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{77}{36}$ **[3]**

12. (i) Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ **[2]**

(ii) $\sin^{-1}\left(2x\sqrt{1-x^2}\right) = 2\cos^{-1}x\ , \frac{1}{\sqrt{2}} \leq x \leq 1$ **[2]**

13. If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, show that $F(x)F(y) = F(x+y)$ **[4]**

14. (i) If $A = \begin{bmatrix} 1 & -2 \\ -5 & 3 \end{bmatrix}$ then show that | 2A | = 4 | A | **[2]**

(ii) Using determinants, find equation of line joining (3, 1) and (9, 3) **[2]**

15. (i) Find all the points of discontinuity of f defined by $f(x) = |x| - |x+1|$ **[1]**

(ii) If x = a cos θ, y = a sin θ, where θ is the parameter , find $\frac{d^2y}{dx^2}$ **[3]**

16. (i) $\frac{d}{dx}(a^x) =$ ………………… **[1]**

(ii) If $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right)\ , -1 < x < 1$, find $\frac{dy}{dx}$ **[3]**

<u>Answer any 3 questions from 17 to 20. Each carries 6 scores</u>.

17. If $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ then verify that A adj A = |A| I. Also find A^{-1}

18.(i) Construct a 2 X 2 matrix A = $[a_{ij}]$ where $a_{ij} = 2i + j$ **[2]**

(ii) Express A as the sum of a symmetric and skew symmetric matrices **[4]**

19.(i) Find $\frac{dy}{dx}$ if $x^2y + xy^2 = 100$ **[2]**

(ii) If $y = (\tan^{-1}x)^2$, show that $(1+x^2)^2\, y_2 + 2x(1+x^2)y_1 = 2$ **[4]**

20.(i) Find the number of all one-one functions from the set $\{1, 2, 3, \ldots, n\}$ to itself. **[1]**

(ii) Let $f: \mathbb{R} \to \mathbb{R}$ be defined as f(x) = 5x + 3.

Find the function $g: \mathbb{R} \to \mathbb{R}$ such that g o f = f o g = 1_R. **[5]**

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