

HIGHER SECONDARY FIRST TERMINAL EXAMINATION

Max. Score : 60

September : 2024

Part III

Time : 2 Hrs

Second Year

MATHEMATICS (SCIENCE)

Cool-off Time : 15 Mts.

Answer any 6 questions from 1 to 8. Each carries 3 scores

1. Show that the relation R in the set Z of integers given by

$R = \{(x, y) : |x - y| \text{ is a multiple of } 2\}$ is an equivalence relation [3]

2. Find all points of discontinuity of the function, $f(x) = \begin{cases} x^3 - 3, & \text{if } x \leq 2 \\ x^2 + 1 & \text{if } x > 2 \end{cases}$ [3]

3. (a) Give an example of a relation on set $A = \{1, 2, 3, 4\}$ which is reflexive, symmetric but not transitive [1]

(b) Show that the function $f(x) = 8x + 2$ is one – one and onto [2]

4. Consider the matrix $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$

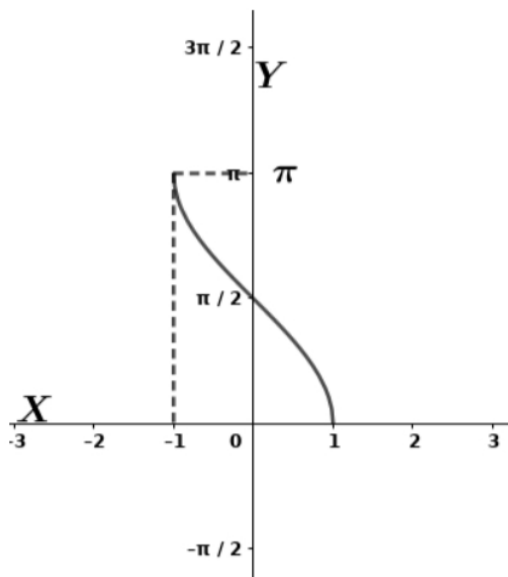
(a) Find A^2 [2]

(b) Find k so that $A^2 = kA - 2I$ [1]

5. (a) If $\begin{vmatrix} x & 3 \\ 1 & x \end{vmatrix} = \begin{vmatrix} 6 & 0 \\ 2 & 1 \end{vmatrix}$ the value of x is [1]

(b) Find the area of a triangle whose vertices are (1, 0), (6, 0) and (4, 3) [2]

6. The graph of a function is given below :



(a) Write the domain of the function [1]

(b) Write the range of the function [1]

(c) Identify the function [1]

7. (a) What is the value of $\sin^{-1}(\sin 160^\circ)$
- (i) 160° (ii) 70° (iii) -20° (iv) 20° [1]

(b) Show that $\sin^{-1}(2x\sqrt{1-x^2}) = 2\sin^{-1}x$ [2]

8. Find $\frac{dy}{dx}$, where $x^2 + y^2 + xy = 100$ [3]

Answer any 6 questions from 9 to 16. Each carries 4 scores

9. (a) Consider $A = \{1, 2, 3\}$. The number of one - one functions from A to itself is

(i) 2 (ii) 3 (iii) 6 (iv) 8 [1]

(b) Find fog and gof for the real functions $f(x) = 8x^3$ and $g(x) = x^{\frac{1}{3}}$ [3]

10. (a) If A is a singular matrix then $|A| = \dots\dots\dots$ [1]

(b) Given that $\begin{bmatrix} 2+x & 3 & 4 \\ 1 & -1 & 2 \\ x & 1 & 5 \end{bmatrix}$ is a singular matrix. Find the value of x. [3]

11. Find 'a' and 'b' such that the function defined by $f(x) = \begin{cases} 10, & x \leq 3 \\ ax + b, & 3 < x < 4 \\ 20, & x \geq 4 \end{cases}$ is a continuous function. [4]

12. (a) Construct a 2×2 matrix $A = [a_{ij}]$, where $a_{ij} = 2i - j$ [2]

(b) Express A as the sum of symmetric and skew symmetric matrices [2]

13. (a) Write the principal value branch of $\sin^{-1}(x)$ [1]

(b) Find the value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$ [3]

14. (a) If $x = \sin^{-1}\left(\frac{3}{5}\right)$, then which of the following is true ?

(i) $x = \cos^{-1}\left(\frac{5}{3}\right)$ (ii) $x = \tan^{-1}\left(\frac{3}{4}\right)$

(iii) $x = \operatorname{cosec}^{-1}\left(\frac{5}{4}\right)$ (iv) $x = \cot^{-1}\left(\frac{3}{4}\right)$ [1]

(b) Write following in simplest form $\tan^{-1}\left[\sqrt{\frac{1-\cos x}{1+\cos x}}\right]$, $0 < x < \pi$ [3]

15. (a) Find the derivative of $\cos(2x + 3)$ [2]

(b) Find $\frac{dy}{dx}$ if $x + 2y = e^x$ [2]

16. Find $\frac{dy}{dx}$ if $y = x^x + x^{\sin x}$ [4]

Answer any 3 questions from 17 to 20. Each carries 6 scores

17. (a) Find $\frac{dy}{dx}$ if $x = 2at$, $y = at^2$, where 't' is the parameter [3]

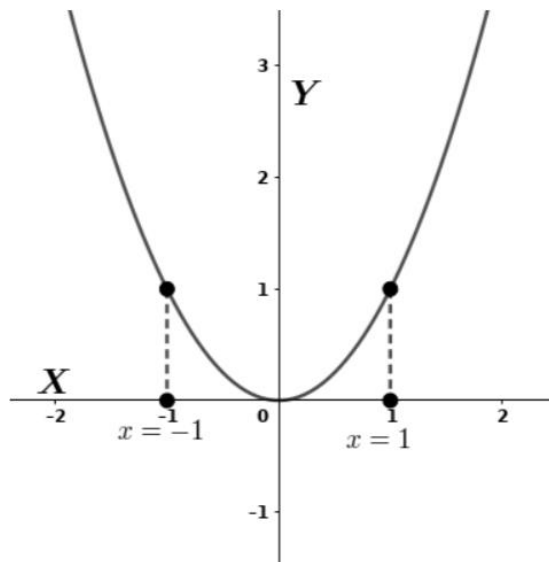
(b) If $y = \tan^{-1}(x)$, prove that $(1 + x^2)y'' + 2xy' = 0$ [3]

18. Consider the set $A = \{1, 2, 3, 4, 5\}$, $B = \{1, 4, 9, 16, 25\}$ and a function $f: A \rightarrow B$ defined by $f(1) = 1$, $f(2) = 4$, $f(3) = 9$, $f(4) = 16$ and $f(5) = 25$

(a) Show that f is bijective [3]

(b) Find f^{-1} [1]

(c) Using the following function $f: \mathbb{R} \rightarrow \mathbb{R}$, state whether the function is one – one or onto. Give reason [2]



19. Consider $A = \begin{bmatrix} 2 & -6 \\ 1 & 2 \end{bmatrix}$ and given $A - 3B = \begin{bmatrix} 5 & -3 \\ -2 & -1 \end{bmatrix}$

(a) Find matrix B [2]

(b) Find A' and B' [1]

(c) Show that $(AB)' = B'.A'$ [3]

20. (a) Show that the system of equations $x - y + 2z = 7$

$$3x + 4y - 5z = -5$$

$$2x - y + 3z = 12 \quad \text{is consistent} \quad [1]$$

(b) Solve the above system using matrix method. [5]

=====L=====