HIGHER SECONDARY MODEL EXAMINATION, FEB 2025

Max. Score : 60

Part III — Time : 2 Hrs

Second Year — MATHEMATICS (SCIENCE) — Cool- off Time : 15 Mts.

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ANSWER KEY

1. Domain = {1, 2, 3, 4}, Range = {1, 2, 3, 4}

   R is reflexive and transitive, but not symmetric. It is not symmetric. ∴ It is not an equivalence relation

2. $\frac{\pi}{4}$; Put x = sin θ ; ∴ θ = $\sin^{-1}$ x ;

   $\sin^{-1}(2x\sqrt{1-x^2}) = \sin^{-1}(2\sin\theta\sqrt{1-sin^2\theta}) = \sin^{-1}(2sin\theta cos\theta)$

   $= \sin^{-1}(sin2\theta) = 2\theta = 2\sin^{-1}x$

3. 7 x 1 ; A = $\begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}$

4. $2^x log2$;

   cosx – siny $\frac{dy}{dx}$ = x $\frac{dy}{dx} + y$; $\frac{dy}{dx} = \frac{\cos x - y}{\sin y + x}$

5. f(x) = $2x^2 - 8x + 5$; $f'(x) = 4x - 8$; $f'(x) = 0$ => $4x - 8 = 0$ => x = 2

   f(1) = –1 ; f(2) = – 3 ; f(5) = 15

   Absolute maximum value is 15 at x = 5

   Absolute minimum value is –3 at x = 2

6. $I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}$ dx -----→(1) $I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}$ dx -----→(2)

   Adding (1) and (2) $2I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}$ dx + $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}$ dx

   2I = $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}$ dx = $\int_0^{\frac{\pi}{2}} 1\, dx = [x]_0^{\frac{\pi}{2}} = \frac{\pi}{2}$ ∴ $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}$ dx = $\frac{\pi}{4}$

7. $\vec{a} + \vec{b} = 3\,\hat{\imath} + 5\,\hat{\jmath} + 3\,\hat{k}$

   $\vec{a}.\vec{b} = (2\,\hat{\imath} + 3\,\hat{\jmath} + 2\,\hat{k}).(\hat{\imath} + 2\,\hat{\jmath} + \hat{k}) = 2 + 6 + 2 = 10$

   $|\vec{b}| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6}$

   Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}.\vec{b}}{|\vec{b}|} = \frac{10}{\sqrt{6}}$

8. A = {(1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4), (4, 1), (4, 2), (4, 3), (4, 5), (4, 6)}

   B = {(1, 4), (2, 3), (3, 2), (4, 1)}

   A∩B = {(1, 4), (4, 1)}

$P(A) = \frac{11}{36}$: $P(B) = \frac{4}{36}$; $P(A \cap B) = \frac{2}{36}$; $P(A/B) = \frac{p(A\cap B)}{P(B)} = \frac{\frac{2}{36}}{\frac{4}{36}} = \frac{1}{2}$

9. $f(x) = f(y) => x^3 = y^3 = > x = y$; $\therefore f$ is injective.

$f(a_1, b_1) = f(a_2, b_2) \; => \; (b_1, a_1) = (b_2, a_2)$

$=> \; (a_1, b_1) = (a_2, b_2) \; \therefore f$ is one – one

For every (b, a) $\epsilon\ BXA$, there exists (a, b) $\epsilon\ AXB$, such that f(a, b) = (b, a). $\therefore f$ is onto

Or codomain = range of f

10. $A + A^T = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$

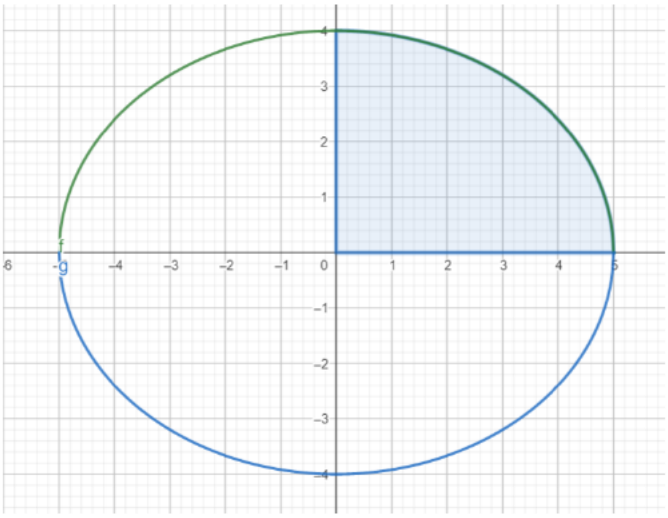
$A - A^T = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix}$

$\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix}$

11. Increasing function

$A = \pi r^2$; $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$; $\frac{dA}{dt}$ (r = 3.2) = 2π. (3.2). (0.05) = $0.32\ \pi\ cm^2/sec$

12. Area = $\int_a^b y dx$



Area of the shaded region = $\int_0^5 y\, dx = \int_0^5 \frac{4}{5}\sqrt{25 - x^2} dx = \frac{4}{5}\int_0^5 \sqrt{25 - x^2} dx$

$= \frac{4}{5} . \left[\frac{x}{2} . \sqrt{25 - x^2} + \frac{25}{2} . \sin^{-1}\left(\frac{x}{5}\right)\right]_0^5 = \frac{4}{5}\left[\frac{25}{2} . \frac{\pi}{2}\right] = 5\pi$

$\therefore$ Area of the ellipse = 4. 5 π = 20 π Sq. Unit

$\therefore$ Area of the circle $x^2 + y^2 = 25$ is 25 π Sq. Unit

13. $|\vec{a}| = |\vec{b}| = 1$; $|\vec{a} + \vec{b}| = 1$

$(\vec{a} + \vec{b})^2 = 1$

$(\vec{a}+\vec{b}).(\vec{a}+\vec{b}) = 1$

$\vec{a}.\vec{a} + \vec{a}.\vec{b} + \vec{b}.\vec{a} + \vec{b}.\vec{b} = 1$

$|\vec{a}|^2 + 2\vec{a}.\vec{b} + |\vec{b}|^2 = 1$

$|\vec{a}|^2 + 2|\vec{a}||\vec{b}|cos\theta + |\vec{b}|^2 = 1$

$1 + 2. 1. 1.\cos\theta + 1 = 1$

$\text{Cos}\ \theta = \frac{-1}{2}, \theta = \frac{2\pi}{3}$

14. $\vec{r} = \vec{a}_1 + \lambda\, \vec{b}_1$; $\vec{r} = \vec{a}_2 + \mu\, \vec{b}_2$

$\vec{a}_1 = \hat{\imath} + 2\,\hat{\jmath} + \hat{k}$; $\vec{a}_2 = 2\,\hat{\imath} - \hat{\jmath} - \hat{k}$; $\vec{b}_1 = \hat{\imath} - \hat{\jmath} - \hat{k}$; $\vec{b}_2 = 2\,\hat{\imath} + \hat{\jmath} + 2\,\hat{k}$

$\vec{a}_2 - \vec{a}_1 = \hat{\imath} - 3\,\hat{\jmath} - 2\,\hat{k}$; $\vec{b}_1 \text{ x } \vec{b}_2 = -3\hat{\imath} + 3\,\hat{k}$; $|\vec{b}_1 \text{ x } \vec{b}_2| = \sqrt{18}$; $(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \text{ x } \vec{b}_2) = -9$

Shortest Distance = $\left|\frac{(\vec{a}_2-\vec{a}_1).(\vec{b}_1 \text{ x } \vec{b}_2)}{|\vec{b}_1 \text{ x } \vec{b}_2|}\right| = \frac{9}{\sqrt{18}} = \frac{3}{\sqrt{2}}$

15. E_1 : Choosing bag I ; E_2 : Choosing bag II ; A : Drawing a black ball

$\text{P}(E_1) = \frac{1}{2}$; $\text{P}(E_2) = \frac{1}{2}$; $\text{P}(A/E_1) = \frac{4}{7}$; $\text{P}(A/E_2) = \frac{6}{11}$

$\text{P}(E_1/A) = \frac{P(E_1).\text{P}(A/E_1)}{P(E_1).\text{P}(A/E_1)+\text{P}(E_2).\text{P}(A/E_2)} = \frac{\frac{1}{2}.\frac{4}{7}}{\frac{1}{2}.\frac{4}{7}+\frac{1}{2}.\frac{6}{11}} = \frac{22}{43}$

16. Order 3 ; Degree 2

$\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$; $\frac{dy}{1+y^2} = \frac{dx}{1+x^2}$;

On integration, $\int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2} + \text{C}$; $\tan^{-1} y = \tan^{-1} x + C$

17. AX = B ; Where, A = $\begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$, B = $\begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$

$|A|$ = - 17 ; Adj A = $\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$; x = 1, y = 2, z = 3

18. Every polynomial function is continuous, every modulus function is continuous and difference of two continuous function is continuous. Hence x - |x| is continuous at x = 0 in ℝ

$\frac{dx}{dt} = 2at$; $\frac{dy}{dt} = 2a$; $\frac{dy}{dx} = \frac{1}{t}$; $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right).\left(\frac{dt}{dx}\right) = \frac{-1}{t^2}.\frac{1}{2at} = \frac{-1}{2at^3}$

$y = x^{sinx}$; $\log y = \sin x \log x$; $\frac{1}{y}.\frac{dy}{dx} = \frac{\sin x}{x} + \log x.\cos x$;

$\frac{dy}{dx} = y\left[\frac{\sin x}{x} + \log x.\cos x\right]$; $\frac{dy}{dx} = x^{sinx}.\left[\frac{\sin x}{x} + \log x.\cos x\right]$

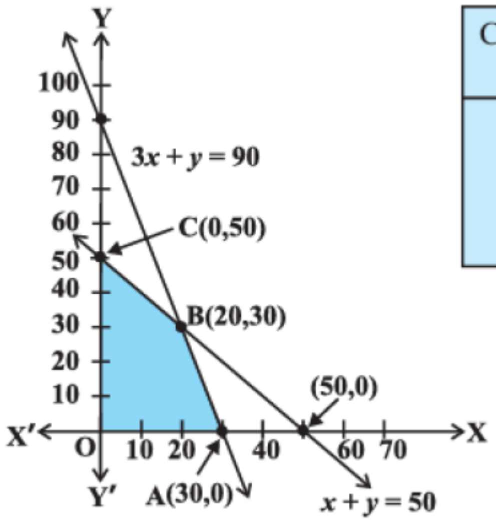
19. $\int \frac{1}{1+\frac{x^2}{4}} dx = \int \frac{1}{\frac{4+x^2}{4}} \text{dx} = 4\int \frac{1}{x^2+2^2} dx = 4.\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C = 2.\tan^{-1}\left(\frac{x}{2}\right) + C$

$\frac{1}{(x-1).(x-2)} = \frac{A}{(x-1)} + \frac{B}{(x-2)}$ => A = -1, B = 1

$\int \frac{1}{(x-1)(x-2)}dx = \int \frac{-1}{(x-1)}dx + \int \frac{1}{(x-2)}dx = -\log(x-1) + \log(x-2) + C = \log\frac{(x-2)}{(x-1)} + C$

$\int_0^{\frac{\pi}{2}} x\cos x dx = [x\sin x]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin x dx = [x\sin x]_0^{\frac{\pi}{2}} + [\cos x]_0^{\frac{\pi}{2}} = \frac{\pi}{2} - 1$

20.



Corner Point	Corresponding value of Z	
(0, 0)	0	
(30, 0)	**120** ←	Maximum
(20, 30)	110	
(0, 50)	50	

Fig 12.2

Hence, maximum value of Z is 120 at the point (30, 0).

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