FIRST YEAR HIGHER SECONDARY SECOND TERMINAL EXAMINATION

DECEMBER 2023

1. i) A

   ii) $\{-1,0,1\}, \{-1,0\}, \{-1,1\}, \{0,1\}, \{-1\}, \{0\}, \{1\}, \phi$

2. i) b) $|x| + 2$

   ii) Domain – R

   Range – $[2, \infty)$

3. $LHS = \frac{sin3x+sinx}{cos3x+cosx} = \frac{2\sin\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)}{2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)} = \frac{\sin\left(\frac{3x+x}{2}\right)}{\cos\left(\frac{3x+x}{2}\right)} = \frac{sin2x}{c0s2x} = tan2x = RHS$

4. i) $2n + 1$

   ii) $\left(x^2 - \frac{1}{x}\right)^4 = (x^2)^4 - {}^4C_1\,(x^2)^3\left(\frac{1}{x}\right) + {}^4C_2\,(x^2)^2\left(\frac{1}{x}\right)^2 - {}^4C_3\,(x^2)^1\left(\frac{1}{x}\right)^3 + \left(\frac{1}{x}\right)^4$

   $= x^8 - 4 \times x^6 \times \frac{1}{x} + 6 \times x^4 \times \frac{1}{x^2} - 4 \times x^2 \times \frac{1}{x^3} + \frac{1}{x^4}$

   $= x^8 - 4x^5 + 6x^2 - \frac{4}{x} + \frac{1}{x^4}$

5. i) No. of selections $= {}^5C_3 = 10$

   ii) No. of selections $= {}^2C_1 \times {}^3C_2 + {}^2C_2 \times {}^3C_1 = 2 \times 3 + 1 \times 3 = 9$

6. i) Slope $= \frac{6-5}{-3-2} = \frac{1}{-5} = -\frac{1}{5}$

   ii) Slope of the required line $= \frac{-1}{\left(-\frac{1}{5}\right)} = 5$

   Equation of the required line is $y - y_1 = m(x - x_1)$

   $y - (-3) = 5(x - 2)$

   $y + 3 = 5x - 10$

   $5x - 10 - y - 3 = 0$

   $5x - y - 13 = 0$

7. i) Equation is $x^2 + y^2 = r^2$

   ii) $2g = 8 \Rightarrow g = 4,\ 2f = -10 \Rightarrow f = -5$

   $\therefore centre\ is\ (-g, -f) = (-4, 5)$

   Radius $= \sqrt{g^2 + f^2 - c} = \sqrt{16 + 25 - -8} = \sqrt{49} = 7$ units.

8. i) $a_n = ar^{n-1}$

   ii) $a = -3$

   $a_4 = a_2^2 \Rightarrow ar^3 = (ar)^2 \Rightarrow ar^3 = a^2r^2$

   $\Rightarrow ar^2 \times r = a \times ar^2 \Rightarrow r = a = -3$

   $\therefore a_7 = ar^6 = (-3)(-3)^6 = (-3)^7 = -2187$

9. i) When $x = 1, y = 1^2 + 1 = 2 \in A$

When $x = 2, y = 2^2 + 1 = 5 \in A$

When $x = 3, y = 3^2 + 1 = 10 \in A$

When $x = 4, y = 4^2 + 1 = 17 \notin A$

$\therefore R = \{(1,2), (2,5), (3,10)]$

ii) Domain $= \{1, 2, 3\}$

Range $= \{2, 5, 10\}$

Codomain = set A

10. i) $\bar{z} = 1 - 3i$

ii) $|z| = \sqrt{1 + 9} = \sqrt{10}$

iii) $\frac{1}{z} = \frac{1}{1-3i} = \frac{1}{1-3i} \times \frac{1+3i}{1+3i} = \frac{1+3i}{1-9i^2} = \frac{1+3i}{1-9(-1)} = \frac{1+3i}{10} = \frac{1}{10} - i\,\frac{3}{10}$

11. $Let\ S_n = 3 + 33 + 333 + \cdots\ to\ n\ terms$

$= 3(1 + 11 + 111 + \ldots$ to n terms)

$= \frac{3}{9}\ (9 + 99 + 999 + \ldots$ to n terms)

$= \frac{1}{3}\ [(10 - 1) + (100 - 1) + (1000 - 1) + \ldots$ to n terms]

$= \frac{1}{3}\ [(10 + 100 + 1000 + \cdots to\ n\ terms + -1 \times n)]$

$= \frac{1}{3}\left[\frac{10}{9}(10^n - 1) - n\right]$

12. i) $y = 3x^2$

$\frac{dy}{dx} = 3(2x) = 6x$

ii) $y = sinxcosx$

$\frac{dy}{dx} = sinx(-sinx) + cosx(sinx) = \cos^2 x - \sin^2 x = cos2x$

13. $2(x + 3) - 10 \leq 3(x - 2)$

$2x + 6 - 10 \leq 3x - 6$

$2x - 4 \leq 3x - 6$

$2x - 3x \leq -6 + 4$

$-x \leq -2 \Longrightarrow x \geq 2$

Solution $= [2, \infty)$

Graph:

14. i) Equation is $\frac{x}{a}+\frac{y}{a}=1$ .... (1)

It passes through (2,3) we have

$\frac{2}{a}+\frac{3}{a}=1 \Longrightarrow \frac{5}{a}=1 \Longrightarrow a=5$

(1) becomes, $\frac{x}{5}+\frac{y}{5}=1 \Longrightarrow x+y=5$ is the required equation.

ii) Equation of the line is $x+y-5=0$

distance of the line from $(-3,2)=\left|\frac{(-3)+(2)-5}{\sqrt{1^2+1^2}}\right|=\left|\frac{-6}{\sqrt{2}}\right|=\frac{6}{\sqrt{2}}=\frac{6\sqrt{2}}{2}=3\sqrt{2}$ units.

15. i) $\sin(\pi-x)=sinx$

ii) $sin15=\sin(45-30)=sin45cos30-cos45sin30$

$=\frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}}\times\frac{1}{2}=\frac{\sqrt{3}}{2\sqrt{2}}-\frac{1}{2\sqrt{2}}=\frac{\sqrt{3}-1}{2\sqrt{2}}$

16. i) $(-3,2,-1)$ $-$ 6th octant.

ii) $AB=\sqrt{(-1-0)^2+(6-7)^2+(6-10)^2}=\sqrt{1+1+16}=\sqrt{18}$

$BC=\sqrt{(-4+1)^2+(9-6)^2+(6-6)^2}=\sqrt{9+9+0}=\sqrt{18}$

$AC=\sqrt{(-4-0)^2+(9-7)^2+(6-10)^2}=\sqrt{16+4+16}=\sqrt{36}$

Using Pythagoras' theorem,

$AB^2+BC^2=\left(\sqrt{18}\right)^2+\left(\sqrt{18}\right)^2=18+18=36=AC^2$

$\therefore$ ΔABC is a right triangle.

17. i) $\mathrm{A}=\{1,2,3,4,5,6\}$

$\mathrm{B}=\{2,3,5,7\}$

ii) $\mathrm{A}\cup\mathrm{B}=\{1,2,3,4,5,6,7\}$

$\mathrm{A}\cap\mathrm{B}=\{2,3,5\}$

iii) $(\mathrm{A}\cap\mathrm{B})'=\{1,4,6,7,8,9,10\}$

$A'=\{7,8,9,10\}$

$B'=\{1,4,6,8,9,10\}$

RHS $=A'\cup B'$

$=\{1,4,6,7,8,9,10\}$

$\therefore LHS=RHS$

18. i) The word TRIGONOMETRY has:

T	2
R	2
I	1
G	1
N	1
O	2
M	1
E	1
Y	1
Total	12

$\therefore$ Total no. of arrangements $= \frac{12!}{2!\times2!\times2!} = 59875200$

ii) a) The word that starts with `T` then T is fixed. The remaining letters can be permuted.

No. of permutations $= \frac{11!}{2!\times2!} = 9979200$

b) If all vowels occure together, we have

IOOE	1
T	2
R	2
G	1
N	1
M	1
Y	1
Total	9

And, the word IOOE has 4 letters and has $I-1, O-2, E-1$.

$\therefore$ No of permutation$= \frac{9!}{2!\times2!} \times \frac{4!}{2!} = 1088640$

19. i) $\lim\limits_{n\to2} f(x) = \lim\limits_{n\to2}\left(\frac{x^4-16}{x^3-8}\right)$

$$= \lim_{n\to2}\left(\frac{x^4-2^4}{x^3-2^3}\right)$$

$$= \lim_{n\to2}\left[\frac{\frac{x^4-2^4}{x-2}}{\frac{x^3-2^3}{x-2}}\right]$$

$$= \frac{4\times2^3}{3\times2^2} = \frac{8}{3}$$

ii) $f(x) = \begin{cases} x^2, & x \le 2 \\ 2x, & x > 2 \end{cases}$

LHL $= \lim_{n\to 2}(x)^2 = 2^2 = 4$

RHL $= \lim_{n\to 2}\ (2x) = 2 \times 2 = 4$

Now $f(2) = \lim_{n\to 2} f(x) = \lim_{n\to 2}(x^2) = 2^2 = 4$

$\therefore$ LHL = RHL = f(x)

Hence, f(x) exists at $x = 2$.

20. $9x^2 + 16y^2 = 144$

Dividing by 144

$$\frac{9x^2}{144} + \frac{16y^2}{144} = \frac{144}{144}$$

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

$$\frac{x^2}{4^2} + \frac{y^2}{3^2} = 1$$

$a = 4, b = 3$

$c = \sqrt{4^2 - 3^2} = \sqrt{16 - 9} = \sqrt{7}$

1. Verticles : $(\pm a, 0) = (\pm 4, 0)$

2. Foci : $(\pm c, 0) = (\pm\sqrt{7}, 0)$

3. Length of major axis $= 2a = 2 \times 4 = 8$

4. Length of minor axis $= 2b = 2 \times 3 = 6$

5. Length of L.R $= \frac{2b^2}{a} = \frac{2\times 3^2}{4} = \frac{9}{2}$

6. $e = \frac{c}{a} = \frac{\sqrt{7}}{4}$

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