ANSWER KEY

1/14

SECOND YEAR HIGHER SECONDARY EXAMINATION ....MARCH... 2024

PART-~~I/II~~/III

SUBJECT: ......MATHEMATICS [COMMERCE]

CODE NO: **SY 555** VERSION:...3...

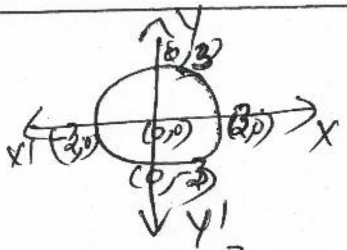
..80.. SCORES ..$2\frac{1}{2}$.. HOURS

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
1	(a)	Range of f = Y	1	
	(b)	$f(x_1)=f(x_2) \Rightarrow 3-4x_1=3-4x_2 \Rightarrow x_1=x_2$ f is one-one	1	
		For every $y=3-4x \in R$ there exist an element $x=\frac{3-y}{4}$ in R such that $f(x)=f\left(\frac{3-y}{4}\right)=3-4\left(\frac{3-y}{4}\right)=y$ ∴ f is onto	1	3
		Since f is one-one and onto, f is bijective		
2	(a)	AB is of order $\underline{3\times2}$	1	
	(b)	$A=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$	1	3
		$=\begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$	1	
3	(a)	$\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} = \cos^2\theta+\sin^2\theta=1$	1	
	(b)	Area $=\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$	1	
		$=\frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix}$	$\frac{1}{2}$	3
		$=\frac{1}{2}\times -3(1-6)=\frac{15}{2}$ sq. units	$\frac{1}{2}$	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
4	(a)	$\lim_{x\to 2}\frac{x^3-8}{x-2} = \lim_{x\to 2}\frac{x^3-2^3}{x-2} = 3\times 2^{3-1} = 3\times 4 = 12$	1	
	(b)	$x^2+6y = e^x$		
		$2x+6\frac{dy}{dx} = e^x$	1	3
		$6\frac{dy}{dx} = e^x-2x$	$\frac{1}{2}$	
		$\frac{dy}{dx} = \frac{e^x-2x}{6}$	1/2	
5	a)	$\int e^x \sec x(1+\tan x)\,dx = e^x\sec x + C$	1	
	b)	Put $\tan^{-1}x = t$		
		$\frac{1}{1+x^2}dx = dt$	$\frac{1}{2}$	
		$\int \frac{e^{\tan^{-1}(x)}}{1+x^2}dx = \int e^t\,dt = e^t+C$	1	3
		$= e^{\tan^{-1}(x)}+C$	$\frac{1}{2}$	
6	a)	$\vec{a} = \hat{i}+\hat{j}+\hat{k}$, $\vec{b} = -\hat{j}+\hat{k}$		
		$\vec{a}+\vec{b} = \hat{i}+2\hat{k}$	1	
	b)	Unit Vector along $\vec{a}+\vec{b} = \frac{\vec{a}+\vec{b}}{\lvert\vec{a}+\vec{b}\rvert}$	1	3
		$= \frac{\hat{i}+2\hat{k}}{\sqrt{5}}$	1	
7	a)	$l^2+m^2+n^2 = 1$	1	
	b)	$a=2,\ b=-1,\ c=-2$	1/2	
		$l = \frac{2}{\sqrt{2^2+(-1)^2+(-2)^2}} = 2/3$	$\frac{1}{2}$	
		$m = \frac{-1}{\sqrt{2^2+(-1)^2+(-2)^2}} = -1/3$	1/2	3
		$n = \frac{-2}{\sqrt{2^2+(-1)^2+(-2)^2}} = -2/3$ (Formula	1/2	
			1)	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		Answer any 8 questions from 8 to 17. Each carries 4 scores (8×4=32)		
8	(a)	$gof(x) = g(f(x)) = g(8x^3) = (8x^3)^{1/3} = 2x$	1	
	(b)	R is reflexive, as 2 divides $(a-a)$, $a \in Z$	1	
		$(a,b) \in R \Rightarrow 2$ divides $(a-b)$ $\Rightarrow 2$ divides $b-a$ $\Rightarrow (b,a) \in R$ R is symmetric	1	
		$(a,b) \in R$ & $(b,c) \in R \Rightarrow a-b$ and $b-c$ are divisible by 2 $\Rightarrow a-c = (a-b)+(b-c)$ is even $\Rightarrow 2$ divides $a-c$ $\Rightarrow (a,c) \in R$ ∴ R is Transitive. Hence R is an equivalence relation	1	4
9	(a)	$\cos^{-1}(\sqrt{3}/2) = \pi/6$	1	
	(b)	$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$	1	
		$\tan^{-1}(1/3) - \tan^{-1}(1/5) = \tan^{-1}\left(\frac{1/3 - 1/5}{1 - 1/3 \times 1/5}\right)$	1	4
		$\tan^{-1}(1/3) - \tan^{-1}(1/5) = \tan^{-1}\left(\frac{2}{16}\right)$	1/2	
		$= \tan^{-1}(1/8)$	1/2	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
10	(a)	$A' = -A$	1	
	b)	$A = \begin{bmatrix} 1 & 4 & -1 \\ 2 & 5 & 4 \\ -1 & -6 & 3 \end{bmatrix}$		
		$A' = \begin{bmatrix} 1 & 2 & -1 \\ 4 & 5 & -6 \\ -1 & 4 & 3 \end{bmatrix}$		
		$A + A' = \begin{bmatrix} 2 & 6 & -2 \\ 6 & 10 & -2 \\ -2 & -2 & 6 \end{bmatrix}$	$\frac{1}{2}$	
		$A - A' = \begin{bmatrix} 0 & 2 & 0 \\ -2 & 0 & 10 \\ 0 & -10 & 0 \end{bmatrix}$	$\frac{1}{2}$	
		$P = \frac{1}{2}(A + A') = \frac{1}{2}\begin{bmatrix} 2 & 6 & -2 \\ 6 & 10 & -2 \\ -2 & -2 & 6 \end{bmatrix}$		
		$P' = P$ $\therefore$ P is symmetric	$\frac{1}{2}$	4
		$Q = \frac{1}{2}(A - A') = \frac{1}{2}\begin{bmatrix} 0 & 2 & 0 \\ -2 & 0 & 10 \\ 0 & -10 & 0 \end{bmatrix}$		
		$Q' = -Q$ $\therefore$ Q is skew-symmetric	$\frac{1}{2}$	
		$P + Q = \frac{1}{2}\begin{bmatrix} 2 & 6 & -2 \\ 6 & 10 & -2 \\ -2 & -2 & 6 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & 2 & 0 \\ -2 & 0 & 10 \\ 0 & -10 & 0 \end{bmatrix}$		
		$= \frac{1}{2}\begin{bmatrix} 2 & 8 & -2 \\ 4 & 10 & 8 \\ -2 & -12 & 6 \end{bmatrix}$		
		$= \begin{bmatrix} 1 & 4 & -1 \\ 2 & 5 & 4 \\ -1 & -6 & 3 \end{bmatrix}$	1	
		$= A$		

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
11	a)	$\int_0^a f(x)\,dx = (B)\int_0^a f(a-x)\,dx$	1	
	b)	Let $I = \int_0^{\pi/2} \frac{\cos^5 x}{\sin^5 x+\cos^5 x}\,dx \rightarrow (1)$		
		$= \int_0^{\pi/2} \frac{\cos^5(\pi/2 - x)}{\sin^5(\pi/2-x)+\cos^5(\pi/2-x)}\,dx$	$\frac{1}{2}$	
		$I = \int_0^{\pi/2} \frac{\sin^5 x}{\cos^5 x+\sin^5 x}\,dx \rightarrow (2)$	$\frac{1}{2}$	
		$(1)+(2) \Rightarrow 2I = \int_0^{\pi/2} \frac{\sin^5 x+\cos^5 x}{\sin^5 x+\cos^5 x}\,dx$	1	4
		$= \int_0^{\pi/2} dx$		
		$2I = \left[x\right]_0^{\pi/2}$		
		$2I = \pi/2 - 0$	1	
		$I = \pi/4$		
12	a)		1	
	b)	Area $= 4\int_0^3 \sqrt{9-x^2}\,dx$	1	
		$= 4\int_0^3 \sqrt{3^2-x^2}\,dx$		4
		$= 4\left[\frac{x}{2}\sqrt{3^2-x^2} + \frac{3^2}{2}\sin^{-1}\left(\frac{x}{3}\right)\right]_0^3$	1	
		$= 4\left[\left(\frac{3}{2}\sqrt{3^2-3^2}-0\right)+\frac{9}{2}\left(\sin^{-1}\left(\frac{3}{3}\right)-\sin 0\right)\right]$	$\frac{1}{2}$	
		$= 4 \times \frac{9}{2} \times \frac{\pi}{2}$		
		$= 9\pi$	$\frac{1}{2}$	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
13	a)	$I \cdot F = e^{\int P\,dx} = e^{\int \frac{1}{x}dx} = e^{\log x} = \underline{x}$	1	
	b)	$P = \frac{1}{x}, \quad Q = x^2, \quad I \cdot F = x$	$\frac{1}{2}$	
		Soln is		
		$y \times I \cdot F = \int (Q \cdot IF)\,dx + C$	1	4
		$y \times x = \int x^2 \times x\,dx + C$	1	
		$= \int x^3\,dx + C$		
		$xy = \dfrac{x^4}{4} + C$	$\frac{1}{2}$	
14	a)	$\vec{a} \cdot \vec{b} = 1 \times 3 + -7 \times -2 + 7 \times 2 = \underline{31}$	1	
	b)	$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{vmatrix}$	1	
		$= +19\hat{j} + 19\hat{k}$	$\frac{1}{2}$	
		Area $= \lvert \bar{a} \times \bar{b} \rvert$	1	4
		$= \sqrt{19^2 + 19^2}$		
		$= 19\sqrt{2}$	$\frac{1}{2}$	
		Area of Parallelogram $= 19\sqrt{2}$ sq. units		

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
15	(a)	(C) $k^3\lvert A\rvert$	1	
	(b)	$R_1 \to R_1+R_2+R_3$ $\begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix} = \begin{vmatrix} 3y+k & 3y+k & 3y+k \\ y & y+k & y \\ y & y & y+k \end{vmatrix}$	1	
		$= (3y+k)\begin{vmatrix} 1 & 1 & 1 \\ y & y+k & y \\ y & y & y+k \end{vmatrix}$	1	4
		$C_2 \to C_2 - C_1,\ C_3 \to C_3 - C_1$ $= (3y+k)\begin{vmatrix} 1 & 0 & 0 \\ y & k & 0 \\ y & 0 & k \end{vmatrix}$	$\frac{1}{2}$	
		$= (3y+k)\, 1\, k^2$ $= k^2(3y+k)$	$\frac{1}{2}$	
16		$\vec{a_1} = \hat{i}+\hat{j} \qquad \vec{a_2} = 2\hat{i}+\hat{j}-\hat{k}$ $\vec{b_1} = 2\hat{i}-\hat{j}+\hat{k} \qquad \vec{b_2} = 3\hat{i}-5\hat{j}+2\hat{k}$	1	
		$\vec{a_2}-\vec{a_1} = \hat{i}-\hat{k}$	$\frac{1}{2}$	
		$\vec{b_1}\times\vec{b_2} = \begin{vmatrix} i & j & k \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix}$ $= i(-2+5) - j(4-3) + k(-10+3)$ $= 3\hat{i}-\hat{j}-7\hat{k}$	$\frac{1}{2}$	4
		$(\vec{a_2}-\vec{a_1})\cdot(\vec{b_1}\times\vec{b_2}) = 1\times 3 + 0 + -1\times -7$ $= 3+7 = 10$	$\frac{1}{2}$	
		$S\cdot D = \left\lvert \dfrac{(\vec{a_2}-\vec{a_1})\cdot(\vec{b_1}\times\vec{b_2})}{\lvert \vec{b_1}\times\vec{b_2}\rvert} \right\rvert$	1	
		$= \dfrac{10}{\sqrt{59}}$ Units	$\frac{1}{2}$	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
17	a)	(C) not defined	1	
	b)	$P(A/B) = 2/5$		
		$\frac{P(A\cap B)}{P(B)} = 2/5$	$\frac{1}{2}$	
		$P(A\cap B) = 2/5 \, P(B) = 2/5 \times 5/13 = 2/13$		
		$2P(A) = 5/13$	$\frac{1}{2}$	4
		$P(A) = 5/26$		
		$P(A\cup B) = P(A)+P(B)-P(A\cap B)$	1	
		$= 5/26 + 5/13 - 2/13$	$\frac{1}{2}$	
		$= 11/26$	$\frac{1}{2}$	
		Answer any 5 questions from 18 to 24. Each carries 6 scores $(5\times6=30)$		
18		$AX = B$	$\frac{1}{2}$	
		$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & -1 \\ 2 & -1 & 1 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $B = \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}$	$\frac{1}{2}$	
		$\|A\| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -2 & -1 \\ 2 & -1 & 1 \end{vmatrix} = 1(-2-1)-1(1+2)+1(-1+4)$	1	
		$= -3 \neq 0$		
		$\therefore A$ is non-singular. A^{-1} exists		
		Cofactor matrix $= \begin{bmatrix} -3 & -3 & 3 \\ -2 & -1 & 3 \\ 1 & 2 & -3 \end{bmatrix}$	1	6
		$\text{adj}A = \begin{bmatrix} -3 & -2 & 1 \\ -3 & -1 & 2 \\ 3 & 3 & -3 \end{bmatrix}$		
		$A^{-1} = \frac{\text{adj}A}{\|A\|} = -1/3 \begin{bmatrix} -3 & -2 & 1 \\ -3 & -1 & 2 \\ 3 & 3 & -3 \end{bmatrix}$	1	
		$X = A^{-1}B = 1/3 \begin{bmatrix} 3 & 2 & -1 \\ 3 & 1 & -2 \\ -3 & -3 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix} = 1/3 \begin{bmatrix} 3 \\ -3 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$	2	
		$x = 1, \; y = -1, \; z = 2$		

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
19	a)	$A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$		
		$A = IA$	½	
		$\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$	½	
		$R_2 \to R_2 - 2R_1$ $\begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$	½	
		$R_2 \to R_2/5$ $\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2/5 & 1/5 \end{bmatrix} A$	½	
		$R_1 \to R_1 + R_2$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3/5 & 1/5 \\ -2/5 & 1/5 \end{bmatrix} A$	½	
		$I = BA$ where $B = \begin{bmatrix} 3/5 & 1/5 \\ -2/5 & 1/5 \end{bmatrix} = A^{-1}$	½	
		Remark: Any other method give 1		
	b)	$A^2 = A \cdot A$ $= \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ $= \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$	1	6
		$kA - 2I = k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $= \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$	1	
		$A^2 = kA - 2I$ $\begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$	½	
		$-2k = -2$ $\Rightarrow k = 1$	½	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
20	(a)	$x = \sin t, \quad y = \cos t$		
		$\frac{dx}{dt} = \cos t \quad \frac{dy}{dt} = -\sin t$	1	
		$\frac{dy}{dx} = \frac{dy/dt}{\frac{dx}{dt}}$	1	
		$= \frac{-\sin t}{\cos t}$	$\frac{1}{2}$	
		$= -\tan t$	$\frac{1}{2}$	
	b)	$y = x^2$ in $[-2, 2]$		
		$y = f(x) = x^2$ is continuous in $[-2,2]$	$\frac{1}{2}$	6
		f is differentiable in $(-2,2)$	$\frac{1}{2}$	
		$f(-2) = f(2) = 4$	$\frac{1}{2}$	
		$f'(x) = 2x$	$\frac{1}{2}$	
		$f'(c) = 0 \Rightarrow 2c = 0$	$\frac{1}{2}$	
		$\Rightarrow c = 0$ in $(-2,2)$	$\frac{1}{2}$	
		Hence Rolle's theorem verified		

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
21	(a)	Let $y=\sqrt{x}$ Let $x=36$ Let $\Delta x=0.6$	$\frac{1}{2}$	
		$\Delta y=\sqrt{x+\Delta x}-\sqrt{x}=\sqrt{36.6}-\sqrt{36}=\sqrt{36.6}-6$	1	
		$\sqrt{36.6}=6+\Delta y$		
		$dy=\left(\frac{dy}{dx}\right)\Delta x$	$\frac{1}{2}$	
		$=\frac{1}{2\sqrt{x}}\times.6$		
		$=\frac{1}{2\sqrt{36}}\times 0.6=0.05$		
		$\sqrt{36.6}=6+0.05=6.05$	1	
	(b)	Let One no. $=x$, other no. $=8-x$		6
		$S(x)=x^2+(8-x)^2$	1	
		$=x^2+64-16x+x^2$		
		$=2x^2-16x+64$		
		$S'(x)=4x-16$	½	
		$S''(x)=4$	½	
		$S'(x)=0\Rightarrow 4x-16=0\Rightarrow x=4$	½	
		$x=4$, $S''(4)=4>0$		
		$\therefore x=4$ is the point of local minimum.	½	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
22	(a)	$\vec{AB} = (2-1)\hat{i} + (6-2)\hat{j} + (3-7)\hat{k}$		
		$= \hat{i} + 4\hat{j} - 4\hat{k}$	1	
		$\lvert\vec{AB}\rvert = \sqrt{1^2+4^2+(-4)^2} = \sqrt{33}$		
		$\vec{BC} = (3-2)\hat{i} + (10-6)\hat{j} + (-1-3)\hat{k}$		
		$= \hat{i} + 4\hat{j} - 4\hat{k}$	1	
		$\lvert\vec{BC}\rvert = \sqrt{1^2+4^2+(-4)^2} = \sqrt{33}$	1	
		Since $\lvert\vec{AB}\rvert = \lvert\vec{BC}\rvert$		
		A, B, C are collinear		
		Alternate method give full score		6
	(b)	$\vec{a} = 2\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b} = -\hat{i}+2\hat{j}+\hat{k}$		
		$\vec{c} = 3\hat{i}+\hat{j}$		
		Since $\vec{a}+\lambda\vec{b}$ is $\perp$r to $\vec{c}$	1	
		$(\vec{a}+\lambda\vec{b})\cdot\vec{c} = 0$		
		$[(2\hat{i}+2\hat{j}+3\hat{k}) + \lambda(-\hat{i}+2\hat{j}+\hat{k})]\cdot(3\hat{i}+\hat{j}) = 0$	½	
		$(2-\lambda)3 + (2+2\lambda) = 0$	½	
		$6-3\lambda+2+2\lambda = 0$		
		$8-\lambda = 0$		
		$\lambda = 8$	1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score

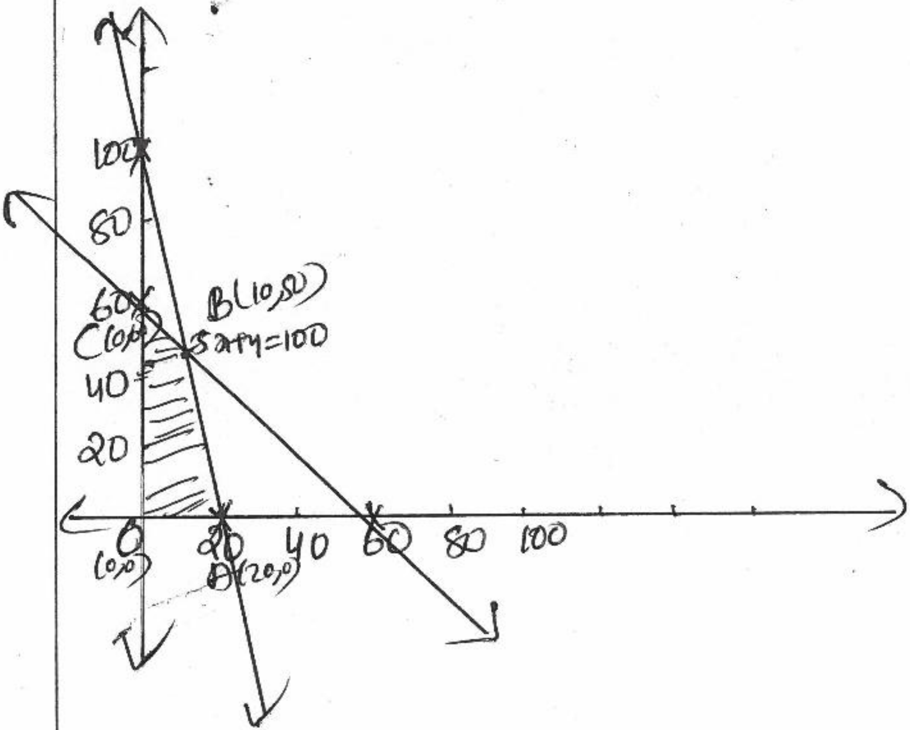
23.

$5x+y=100$ $\qquad$ $x+y=60$

x	0	20
y	100	0

x	0	60
y	60	0

1



3

6

Vertex of Feasible region	Value of Z
O(0,0)	0
A(20,0)	5000
B(10,50)	6250 ← Maximum
C(0,60)	4500

2

Maximum value of Z = 6250 at the point (10,50)

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
24	a)	$\sum P(x) = 1$	$\frac{1}{2}$	
		$0 + k + 2k + 3k + 4k = 1$	$\frac{1}{2}$	
		$10k = 1$		
		$k = 1/10$	1	
	b)			

X	0	1	2	3	4
$P(X)$	0	k	2k	3k	4k
$XP(X)$	0	k	4k	9k	16k
$X^2P(X)$	0	k	8k	27k	64k

$\sum XP(x) = 30 \times \frac{1}{10} = 3$

$\sum X^2P(X)$ $100 \times 1/10 = 10$

Score: 2   Total Score: 6

$\text{Mean} = \sum XP(X) = 3$ 1

$E(X^2) = \sum X^2P(X) = 10$

$\text{Variance} = E(X^2) - (E(X))^2$ ½

$= 10 - 9$

$= 1$ $\frac{1}{2}$