அரசுப் பொதுத் தேர்வு : 2024

பாடம் : கணிதம் - விடைக்குறிப்பு

மு. மு. மேல்நிலைப்பள்ளி - திருநகர்

PART - I

TYPE - A

1	a	$\frac{8}{3}$
2	d	$\frac{3\pi a^4}{16}$
3	a	10
4	d	2
5	d	8
6	a	$[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = 0$
7	d	$\frac{d^2y}{dx^2} - y = 0$
8	b	$y = 0$
9	d	$\frac{1}{(1+x)^2}dx$
10	a	$x^2 + y^2$
11	c	2
12	d	பரவளையம் / parabola
13	c	$\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$
14	a	45°
15	d	$\frac{1}{\sqrt{5}}$
16	b	$-\frac{q}{r}$
17	a	0
18	c	2
19	d	adj (AB) = (adj A)(adj B)
20	c	−4

PART - II

21

$$\sum_{n=1}^{12} i^n = i + i^2 + \cdots + i^{12}$$

$$= (i^1 + i^2 + i^3 + i^4) + (i^5 + i^6 + i^7 + i^8) + (i^9 + i^{10} + i^{11} + i^{12})$$

$$= 0 + 0 + 0$$

$$= 0$$

22

$$\alpha + \beta = \frac{7}{2}, \quad \alpha\beta = \frac{13}{2}$$

$$\alpha^2 + \beta^2 = \frac{-3}{4}, \quad \alpha^2\beta^2 = \frac{169}{4}$$

$$x^2 + \frac{3}{4}x + \frac{169}{4} = 0$$

$$4x^2 + 3x + 169 = 0$$

23

$$df = (2x+3)dx$$

$x = 3$ எனில் $dx = 0.02$

$$df = (6+3)(0.02)$$

$$= 9(0.02)$$

$$= 0.18$$

24

$$y = mx \quad \text{--- (1)}$$

$$\frac{dy}{dx} = m$$

$$(1) \Rightarrow y = x\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x}$$

25

$$E(X) = \int_1^2 f(x)dx = \int_1^2 2(x-1)dx$$

$$= 2\left[\frac{x^2}{2} - x\right]_1^2$$

$$E(X) = 1$$

சராசரி $\doteq 1$

26

$$(h, k) = (-3, -4), \quad r = 3$$

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x+3)^2 + (y+4)^2 = 3^2$$

$$x^2 + y^2 + 6x + 8y + 16 = 0$$

27 $A = \begin{bmatrix} -1 & 3 \\ 4 & -7 \\ 3 & -4 \end{bmatrix}$, $\rho(A) \leq \{\min 3, 2\} = 2$

$\begin{vmatrix} -1 & 3 \\ 4 & -7 \end{vmatrix} = 7 - 12$

$= -5$

$\neq 0$

ஆகவே $\rho(A) = 2$

28 $\int_0^{\frac{\pi}{2}} \sin^{10} x\, dx = \frac{9}{10} \times \frac{7}{8} \times \frac{5}{6} \times \frac{3}{4} \times \frac{1}{2} \times \frac{\pi}{2}$

$= \frac{63\pi}{512}$

29 $\lim_{x \to 1} \frac{x^2 - 3x + 2}{x^2 - 4x + 3}$

$= \lim_{x \to 1} \left(\frac{2x - 3}{2x - 4} \right)$

$= \frac{2-3}{2-4} = \frac{-1}{-2}$

$= \frac{1}{2}$

30 $[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} 2 & -1 & 3 \\ 1 & -1 & 0 \\ 3 & -1 & 6 \end{vmatrix}$

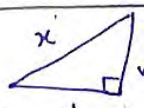
$= 2(-6-0) + 1(6-0) + 3(-1+3)$

$= -12 + 6 + 6$

$= 0$

இவை ஒரு தளத்தில் உள்ளன

PART – III



31 $\cos^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) = \alpha$ எனில்.

$\cot\alpha = \frac{1}{\sqrt{x^2-1}}$

$\sec\alpha = \frac{x}{1} = x$

$\alpha = \sec^{-1} x$

$\cos^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) = \sec^{-1} x$

32 $x^2 + 6x + 4y + 5 = 0$

$(x+3)^2 = -4(y-1)$

$x^2 = -4y$

முதல் கோட்டின் சமன்பாடு

$2x + y + 1 = 0$

இரண்டாம் கோட்டின் சமன்பாடு

$x - 2y - 7 = 0$

33 LHS $= [\vec{a} - \vec{b},\ \vec{b} - \vec{c},\ \vec{c} - \vec{a}]$

$= (\vec{a} - \vec{b}) \cdot ((\vec{b} - \vec{c}) \times (\vec{c} - \vec{a}))$

$= [\vec{a}\ \vec{b}\ \vec{c}] - 0 + 0 - 0 + 0 - [\vec{b}\ \vec{c}\ \vec{a}]$

$= 0$

$=$ RHS

34 $u(x, y) = \frac{x^2 + y^2}{\sqrt{x+y}}$

$u(\lambda x, \lambda y) = \frac{\lambda^2 x^2 + \lambda^2 y^2}{\sqrt{\lambda}\sqrt{x+y}}$

$= \lambda^{2 - \frac{1}{2}} u(x, y)$

$= \lambda^{\frac{3}{2}} u(x, y)$

$u(x, y)$-ன் படி $\frac{3}{2}$ என்பது தொடர்புடையது

நிரூபி: $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$

$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2}u$

35 $x + y = 12$ —①

$y = 12 - x$

$P = xy = x(12 - x) = 12x - x^2$

$\frac{dP}{dx} = 12 - 2x$

$\frac{dP}{dx} = 0 \Rightarrow 12 - 2x = 0$

$x = 6$

① → $y = 12 - 6$

$y = 6$

தேவையான எண்கள் $x = 6,\ y = 6$

36 $I = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{1}{1+\sqrt{\tan x}}\,dx$ —①

$I = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{\sqrt{\tan x}}{1+\sqrt{\tan x}}\,dx$ —②

Adding ①, ②

$I + I = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{1+\sqrt{\tan x}}{1+\sqrt{\tan x}}$

$2I = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} 1\,dx$

$= [x]_{\frac{\pi}{8}}^{\frac{3\pi}{8}}$

$2I = \frac{\pi}{4}$

$I = \frac{\pi}{8}$

37 $\frac{1+i}{1-i} = i, \quad \frac{1-i}{1+i} = -i$

$\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = i^3 - (-i)^3$

$= -i - i$

$= -2i$

38 $\frac{dy}{1+y^2} = \frac{dx}{1+x^2}$

$\int \frac{1}{1+y^2}dx = \int \frac{1}{1+x^2}dx$

$\tan^{-1} y = \tan^{-1} x + c$

$\tan^{-1}\left(\frac{y-x}{1+xy}\right) = c$

$\frac{y-x}{1+xy} = \tan c = C$

39 நிகழ்தகவு பரவல் சார்பு

x	0	1	2	3
$f(x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

$f(x) = \begin{cases} \frac{1}{8}, & x = 0, 3 \\ \frac{3}{8}, & x = 1, 2 \end{cases}$

40 $|\text{adj}(\text{adj } A)| = |A|^{(n-1)^2}$

$\begin{vmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{vmatrix} = -1$

$|\text{adj}(\text{adj } A)| = (-1)^4 = 1$

PART — IV

41 (a) ஒரு பரவளையத்தின் முனை $\left(\frac{3}{2}, \frac{9}{4}\right)$

$y = x^2$

$\frac{dy}{dx} = 2x, \quad m_1 = 3$

$y = (x-3)^2$

$\frac{dy}{dx} = 2(x-3), \quad m_2 = -3$

$\tan\alpha = \left|\frac{3-(-3)}{1-9}\right| = \frac{3}{4}$

$\alpha = \tan^{-1}\left(\frac{3}{4}\right)$

b $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$

$\frac{\frac{x-1}{x-2} + \frac{x+1}{x+2}}{1 - \left(\frac{x-1}{x-2}\right)\left(\frac{x+1}{x+2}\right)} = 1$

$2x^2 - 4 = -3$

$x^2 = \frac{1}{2}$

$x = \pm\frac{1}{\sqrt{2}}$

42 a

சோதனையின் எண்ணிக்கை	2	3	4	5	6
இரண்டாம் எண்ணின் எண்ணிக்கை	1	4	10	12	9

நிகழ்தகவு எண்ணிக்கை

x	2	3	4	5	6
$f(x)$	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{10}{36}$	$\frac{12}{36}$	$\frac{9}{36}$

அடர்வு சார்பு எண்ணிக்கை

$$f(x)=\begin{cases} 0 & ; -\infty < x < 2 \\ \frac{1}{36} & ; 2 \le x < 3 \\ \frac{5}{36} & ; 3 \le x < 4 \\ \frac{15}{36} & ; 4 \le x < 5 \\ \frac{27}{36} & ; 5 \le x < 6 \\ 1 & ; 6 \le x < \infty \end{cases}$$

b $\text{Im}\left(\frac{(2x+1)+2iy}{(1-y)+ix} \times \frac{(1-y)-ix}{(1-y)-ix}\right)=0$

$$\frac{(2x+1)(-x)+2y(1-y)}{(1-y)^2+x^2}=0$$

$$-2x^2-x+2y-2y^2=0$$

$$2x^2+2y^2+x-2y=0$$

43 (a) $12r=5h$

$$r=\frac{5h}{12}$$

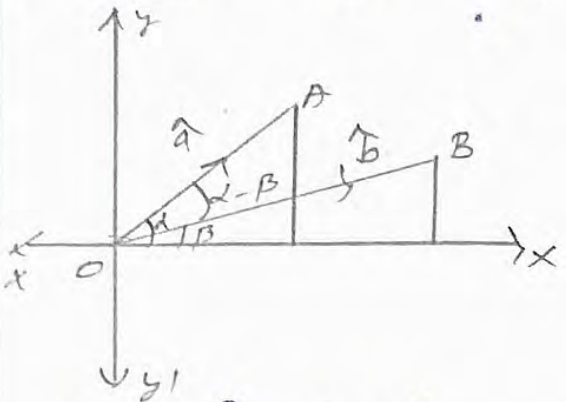
$$V=\frac{1}{3}\pi r^2 h$$

$$V=\frac{25\pi h^3}{\frac{1}{3}\times 144}$$

$$\frac{dv}{dt}=\frac{25\pi}{144}h^2\frac{dh}{dt}$$

$$\frac{dh}{dt}=\frac{10\times 144}{25\pi \times 64}=\frac{9}{10\pi}\ \text{செ.மீ/நிமிடம்}$$

b $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$



$\hat{a}=\cos\alpha\,\hat{i}+\sin\alpha\,\hat{j}$

$\hat{b}=\cos\beta\,\hat{i}+\sin\beta\,\hat{j}$

$\hat{b}\times\hat{a}=(\sin\alpha\cos\beta-\cos\alpha\sin\beta)\hat{k}$ —①

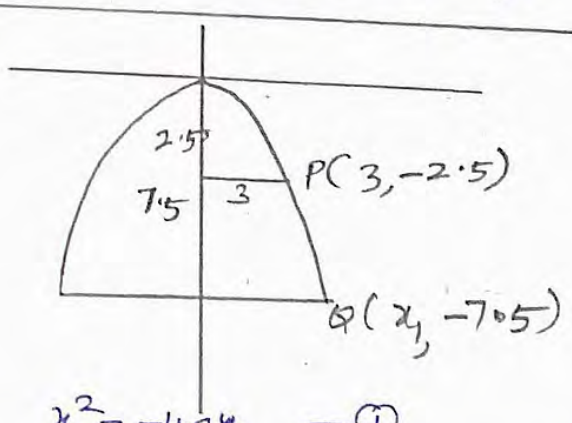
$\hat{b}\times\hat{a}=|\hat{b}||\hat{a}|\sin(\alpha-\beta)\hat{k}$

$=\sin(\alpha-\beta)\hat{k}$ —②

①, ②

$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$

44 (a)



$x^2=-4ay$ —①

$(3,-2\cdot5)\Rightarrow 3^2=-4a(-2\cdot5)$

$4a=\frac{9}{2\cdot5}$

$x^2=-\frac{9}{25}y$ —②

$(x, -7.5)$ ② ⟹

$x_1^2 = 9(3)$

$x_1 = 3\sqrt{3}$ மீ

தரைக்கோட்டிலிருந்து $3\sqrt{3}$ மீ தூரத்திற்கு அப்பால் நீரானது தரையில் விழும்.

(b) $P = \frac{1}{x}, \quad Q = \sin x$

$\int P = \log x, \quad I.F = x$

$y e^{\int P dx} = \int Q e^{\int P dx} dx + c$

$xy = \int \sin x \cdot x \, dx + c$

$xy = -x\cos x + \sin x + c$

$xy + x\cos x = \sin x + c$

45 (a) $\int \frac{dA}{A} = k \int dt$

$A = ce^{kt}$ ——①

$t = 0$ எனில் $A = A_0$

$A = A_0 e^{kt}$ ——②

$t = 5$ எனில் $A = 3A_0$

$e^{5k} = 3$

$t = 10$ எனில்

$A = 9A_0$

(b) $\vec{r} = \vec{a} + s\vec{u} + t\vec{v}$

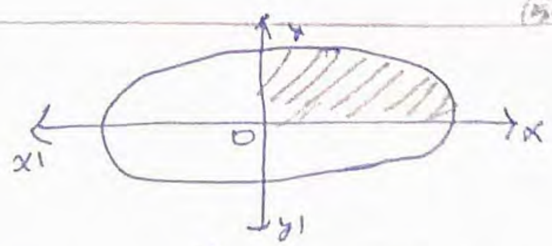
$\vec{r} = (2\hat{i} + 2\hat{j} + \hat{k}) + s(2\hat{i} + 3\hat{j} + 3\hat{k}) + t(3\hat{i} + 2\hat{j} + \hat{k})$

C.E
$$\begin{vmatrix} x-2 & y-2 & z-1 \\ 2 & 3 & 3 \\ 3 & 2 & 1 \end{vmatrix} = 0$$

$3x - 7y + 5z + 3 = 0$

19/03/24

46 (a)

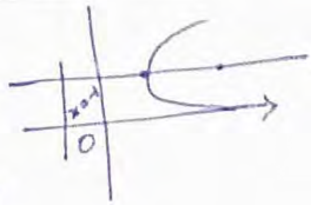


$A = 4\int_0^a \frac{b}{a}\sqrt{a^2 - x^2}\, dx$

$= \pi ab$

(b) முனை : (1, 2)

குவியம் : (3, 2)



இயக்குவரை : $x = -1$

செவ்வகலத்தின் நீளம் $= 8$

47 (a)

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

$((\sim p) \vee q) \wedge ((\sim q) \vee p) = TFFT$

(b) $\Delta = -15, \quad \Delta_1 = -15,$

$\Delta_2 = -5, \quad \Delta_3 = -5$

$x = 1, \quad y = 3, \quad z = 3$

V. GURUNATHAN
P.G. ASST. - Maths
M. M. Hr. S. School
Thirunagar
Madurai - 6.