DIRECTORATE OF GOVERNEMENT EXAMINATION, CHENNAI-6

HSE SECOND YEAR EXAMINATIONS MARCH / APRIL 2023

BUSINESS MATHEMATICS AND STATISTICS - ANSWER KEY

Maximum Marks – 90

General Instructions :

1. Answers written only in **BLACK** or **BLUE** should be evaluated.
2. For objective type questions, award 1 mark for "writing the correct option's code and the corresponding option's answer".
3. Award "0 marks" for one who wrote both "option's code" and "option's answer" with one of them is not correct.
4. Marks should be awarded for suitable alternative method also
5. Mark(s) should not be reduced for the correct answer / stage, if it is written without formula / properties also, 2* means award one mark for the formula.
6. Award full mark directly, if the solution is arrived with no mistakes without giving weightage for the stages.
7. The stage mark is essential, only if the part of the solution is incorrect.
8. Award marks, if the answer is in decimal value and also approximately equal to the key answer
9. **Important Note for Part II, Part III and Part IV**

   For a particular stage in which the stage mark is greater than 1 and one who begins with correct step but reaches with incorrect solution, for such suitable credits should be given by breaking the stage marks.

PART - I

i. **Answer all the questions**
ii. **Choose the most appropriate from the given Four alternatives and write the option code and the corresponding answer**

Q.No	Option	Answer	20×1=20
1	(d)	1	1
2	(a)	6	1
3	(b)	Sin x + c	1
4	(d)	$2\sqrt{e^x}$ + c	1
5	(b)	MC – MR = 0	1
6	(b)	100 - $3x^2$	1
7	(c)	$y_2 + 4y_1 + 5 = 0$	1
8	(d)	xe^{2x}	1
9	(b)	1 + Δ	1
10	(a)	y_2	1
11	(a)	0	1
12	(a)	1	1
13	(d)	De-Moivre	1
14	(b)	poisson	1
15	(b)	a sample	1
16	(b)	Population parameter	1
17	(b)	Geometric mean	1
18	(d)	a year	1
19	(b)	Total supply ≠ Total demand	1
20	(b)	top left corner	1

Q. No.	PART – II **Answer any <u>Seven</u> questions.** **Question no. 30 is compulsory.** **7×2=14**		
21	$\lvert A\rvert = 6 \neq 0$	1	2
	$\rho(A) = 3$	1	
22	$\int(-5x^2 - 13x + 6)dx$	1	2
	$= \frac{-5x^3}{3} - \frac{13x^2}{2} + 6x + c$	1	
23	A $= \int_a^b y\, dx\ (or)\ \int_1^4 (4x+3)dx$	1	2
	= 39 sq.units	1	
24	$\Delta(log ax)$ = log a(x+h) – log ax	1	2
	= log $(1+\frac{h}{x})$	1	
25	$E(X) = \frac{9}{22} + \frac{2}{22}$		2
	$= \frac{11}{22}\ (or)\ \frac{1}{2}$	2*	
26	A Random variable 'X' is said to follow binomial distribution with parameter 'n' and 'p' if it assumes only non-negative value and its probability mass function is given by	1	2
	P(X=x) = p(x) $= \begin{cases} nC_x p^x q^{n-x}, x = 0,1,2,\ldots n, q = 1-p \\ 0,\ otherwise \end{cases}$	1	
27	i. Personal bias is completely eliminated. ii. It is economical as it saves time, money and labour. iii. The method requires minimum knowledge about the population in advance. **(Any 2 points)**		2
28	$\frac{dy}{dx} = (x+1)(y+1)$	1	2
	log (y+1) $= \frac{x^2}{2} + x + c$	1	
29	Minimize z $= \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}x_{ij}$	1	2
	Subject to the constraints $\sum_{i=1}^{n} x_{ij} = 1,\ \ j = 1,2,\ldots n$ $\sum_{j=1}^{n} x_{ij} = 1,\ \ i = 1,2,\ldots n$ and x_{ij}= 0 (or) 1 for all (i,j)	1	

30	**Compulsory :** C.L.I = $\frac{\sum p_1 q_0}{\sum p_0 q_0}$ x 100 = $\frac{3140}{1974}$ ×100	1	2
	= 159.07	1	

Q. No.	**PART - III** **Answer any seven questions.** **Question number 40 is compulsory.** **7×3=21**		
31	[A ,B] = $\begin{bmatrix} 1 & 2 & -3 & -2 \\ 3 & -1 & -2 & 1 \\ 2 & 3 & -5 & k \end{bmatrix}$	1	3
	$[A, B] \sim \begin{bmatrix} 1 & 2 & -3 & -2 \\ 0 & -7 & 7 & 7 \\ 0 & -1 & 1 & 4+K \end{bmatrix} \begin{matrix} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 2R_1 \end{matrix}$		
	$\sim \begin{bmatrix} 1 & 2 & -3 & -2 \\ 0 & -7 & 7 & 7 \\ 0 & 0 & 0 & 21+7K \end{bmatrix} R_3 \to 7R_3 - R_2$	1	
	K = - 3	1	
32	$R = \int MR\, dx + k \quad (or) \quad R = \int (20 - 5x + 3x^2) dx + k$	1	3
	R = $20x - \frac{5x^2}{2} + \frac{3x^3}{3} + k$	1	
	$R = 20x - \frac{5x^2}{2} + x^3$	1	
33	A.E. is $9m^2 - 12m + 4 = 0$	1	3
	$m = \frac{2}{3}, \frac{2}{3}$	1	
	y =$(Ax + B)e^{\frac{2x}{3}}$	1	
34	$\Delta^4 y_0 = 0 \ (or) (E-1)^4 y_0 = 0$	1	3
	$y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$	1	
	$y_3 = 31$ (Any alternative method)	1	

<table>
<tr><td rowspan="3">35</td><td>E$(X)=\frac{4}{5}$</td><td>2*</td><td rowspan="3">3</td></tr>
<tr><td>$E(X^2)=\frac{2}{3}$</td><td></td></tr>
<tr><td>$V(X)=\frac{2}{75}$</td><td>1</td></tr>
<tr><td rowspan="3">36</td><td>λ=$\frac{390}{520}=0.75$</td><td>1</td><td rowspan="3">3</td></tr>
<tr><td>P(X=0) = $e^{-0.75}$</td><td></td></tr>
<tr><td>$\left(P(X=0)\right)^5$=$e^{-3.75}$</td><td>2*</td></tr>
<tr><td rowspan="3">37</td><td>$\sigma=10$, S.E $\bar{X}$=3</td><td>1</td><td rowspan="3">3</td></tr>
<tr><td>S.E. = $\frac{\sigma}{\sqrt{n}}$ = 3</td><td>1</td></tr>
<tr><td>Sample size n = 11</td><td>1</td></tr>
<tr><td rowspan="3">38</td><td colspan="2">
<table>
<tr><th>Commodities</th><th>Price p_0 2007</th><th>p_1 2011</th><th>Weights (V)</th><th>P=$\frac{p_1}{p_0}$X 100</th><th>PV</th><th></th></tr>
<tr><td>A</td><td>350</td><td>400</td><td>40</td><td>114.286</td><td>4571.44</td><td></td></tr>
<tr><td>B</td><td>175</td><td>250</td><td>35</td><td>142.857</td><td>4999.995</td><td></td></tr>
<tr><td>C</td><td>100</td><td>115</td><td>15</td><td>115</td><td>1725</td><td>1</td></tr>
<tr><td>D</td><td>75</td><td>105</td><td>20</td><td>140</td><td>2800</td><td></td></tr>
<tr><td>E</td><td>60</td><td>80</td><td>25</td><td>133.333</td><td>3333.325</td><td></td></tr>
<tr><td></td><td></td><td></td><td>ΣV=135</td><td></td><td>ΣPV= 17429 76</td><td></td></tr>
</table>
</td><td rowspan="3">3</td></tr>
<tr><td>Cost of living Index no. = $\frac{\sum PV}{\sum V}$</td><td>1</td></tr>
<tr><td>= 129.1093</td><td>1</td></tr>
<tr><td rowspan="2">39</td><td>Maxmin = Max (5,7,9,8) = 9 → A_3</td><td>1</td><td rowspan="2">3</td></tr>
<tr><td>Minmax = Min (14,11 11,13) =11→ A_2& A_3</td><td>2</td></tr>
<tr><td rowspan="3">40</td><td>Compulsory:
$\int \frac{1}{\sqrt{x+2}-\sqrt{x+3}} \times \frac{\sqrt{x+2}+\sqrt{x+3}}{\sqrt{x+2}+\sqrt{x+3}}dx$</td><td>1</td><td rowspan="3">3</td></tr>
<tr><td>$=\int \frac{\sqrt{x+2}+\sqrt{x+3}}{-1}dx$</td><td>1</td></tr>
<tr><td>$=-\left[\frac{(x+2)^{\frac{3}{2}}}{\frac{3}{2}}+\frac{(x+3)^{\frac{3}{2}}}{\frac{3}{2}}\right]$+c
(or) $\frac{-2}{3}\left((x+2)^{\frac{3}{2}}+(x+3)^{\frac{3}{2}}\right)+c$</td><td>1</td></tr>
</table>

Q. No.	PART - IV **Answer all the questions.** **7×5=35**		

41 (a)

$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 3 & 2 & -1 \end{vmatrix} = 13 \neq 0$ — 1

$\Delta_x = \begin{vmatrix} 4 & 1 & 1 \\ 1 & -1 & 3 \\ 1 & 2 & -1 \end{vmatrix} = -13$ — 1

$\Delta_y = \begin{vmatrix} 1 & 4 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & -1 \end{vmatrix} = 39$ — 1

$\Delta_z = \begin{vmatrix} 1 & 1 & 4 \\ 2 & -1 & 1 \\ 3 & 2 & 1 \end{vmatrix} = 26$ — 1

x = - 1, y = 3, z = 2 — 1

Total: 5

(OR)

41 (b)

$n = \frac{1946-1941}{10} = 0.5$ — 1

x	y	Δy	Δ^2y	Δ^3y	Δ^4y	Δ^5y
1941	**20**					
		4				
1951	24		**1**			
		5		**1**		
1961	29		2		**0**	
		7		1		**-9**
1971	36		3		-9	
		10		-8		
1981	46		-5			
		5				
1991	51					

2

$y = y_0 + \frac{n}{1!}\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n\ \ 1)(n-2)}{3!}\Delta^3 y_0 + \frac{n(n-1)(n-2)(n-3)}{4!}\Delta^4 y_0$

$$+\frac{n(n-1)(n-2)(n-3)(n-4)}{5!}\Delta^5 y_0$$

= 20 + 2 – 0.125 + 0.0625 – 0.24609

= 21.69 lakhs — 2*

42 (a)	$\int_a^b f(x)dx = \lim_{\substack{n \to \infty \\ h \to 0}} \sum_{r=1}^{n} h\, f(a+rh)$	1	5
	a =1 , b =2 , h=$\frac{b-a}{n} = \frac{2-1}{n} = \frac{1}{n}$	1	
	f(a+rh) = 2 + $\frac{2r}{n}$ + 1 = 3 +$\frac{2r}{n}$	1	
	= $\lim_{n\to\infty}\left[\frac{3}{n}\sum_{r=1}^{n}(1) + \frac{2}{n^2}\sum_{r=1}^{n}(r)\right]$	1	
	$\int_1^2 (2x+1)dx$ = 4 **(OR)**	1	
42 (b)	P = q = $\frac{1}{2}$	1	
	P(x ≥ 6) = P(6) + P(7) +P(8) + P(9) + P(10)	1	
	P(x ≥ 6) = $\left(\frac{1}{2}\right)^{10}$ $[10c_6 + 10c_7 + 10c_8 + 10c_9 + 10c_{10}]$	2 *	
	= $\frac{193}{512}$	1	
43 (a)	At equilibrium, $P_d = P_s$	1	5
	C S = $\int_0^{x_0} f(x)dx - x_0P_0$ = 24 units	2*	
	P S = $x_0P_0 - \int_0^{x_0} g(x)dx$ = 16 units **(OR)**	2*	
43 (b)	Z = $\frac{x-\mu}{\sigma} = \frac{20-12}{4} = 2$	2*	
	P(X≤ 20) = P(Z≤ 2) = 0.9772	1	
	if x = 0 Z =$\frac{0-12}{4}$= - 3, if x = 12 Z =$\frac{12-12}{4}$= 0	1	
	P($0 \leq X \leq 12$) = P($-3 \leq Z \leq 0$) = 0.4987	1	

44 (a)	Auxiliary equation : $m^2 - 2m + 1 = 0$	1	
	C.F = $(Ax + B)e^x$	1	
	$P.I_1 = \frac{e^{2x}}{D^2-2D+1} = e^{2x}$	1	
	$P.I_2 = \frac{e^x}{D^2-2D+1} = \frac{e^x}{(D-1)^2} = \frac{x^2}{2}e^x$	1	
	$Y = (Ax + B)e^x + e^{2x} + \frac{x^2}{2}e^x$ **(OR)**	1	5
44 (b)	n =50, σ = 1.6, $\overline{X}$ = 9 3 , μ = 8.9	1	
	H_0 : μ = 8.9 (Null hypothesis) H_1 : $\mu \neq$ 8.9 (Alternative hypothesis)	1	
	$\|Z\|$ = 1.7676	2*	
	$\|Z\|$< 1 96 , Null hypothesis , H_0 is accepted.	1	
45 (a)	x_0 =5, x_1 =6, x_2 = 9, x_3 = 11 y_0=12, y_1 =13,y_2= 14,y_3 = 16		
	$y = 12\left[\frac{(10-6)(10-9)(10-11)}{(5-6)(5-9)(5-11)}\right]+13\left[\frac{(10-5)(10-9)(10-11)}{(6-5)(6-9)(6-11)}\right]+$ $14\left[\frac{(10-5)(10-6)(10-11)}{(9-5)(9-6)(9-11)}\right]+16\left[\frac{(10-5)(10-6)(10-9)}{(11-5)(11-6)(11-9)}\right]$	3*	
	y= 2 – 4.333 + 11.667+ 5.333	1	
	y = 14.667 **(OR)**	1	5
45 (b)	P = $\frac{1}{x}$ Q = x^3	1	
	I.F = x	1	
	$yx = \int x^4$ dx + c	2*	
	$yx = \frac{x^5}{5}$ + c	1	

46 (a)	(table and sums below)	2	

Commodities	p_0	p_1	q_0	q_1	p_0q_0	p_0q_1	p_1q_0	p_1q_1
A	12	14	18	16	216	192	252	224
B	15	16	20	15	300	225	320	240
C	14	15	24	20	336	280	360	300
D	12	12	29	23	348	276	348	276
					1200	973	1280	1040

$$\sum p_0q_0 = 1200, \sum p_0q_1 = 973,$$
$$\sum p_1q_0 = 1280\,, \sum p_1q_1 = 1040$$

2

i. Laspeyre's index no. — 1

$P_{01}{}^{L} = \frac{\sum p_1q_0}{\sum p_0q_0} \times 100$ = 106.67

ii. Paasche's index no. — 1

$P_{01}{}^{P} = \frac{\sum p_1q_1}{\sum p_0q_1} \times 100$ = 106.89

iii. Fisher's index no — 1

$P_{01}{}^{F} = \sqrt{P_{01}{}^{L} \times P_{01}{}^{P}}$ = 106.78

(OR)

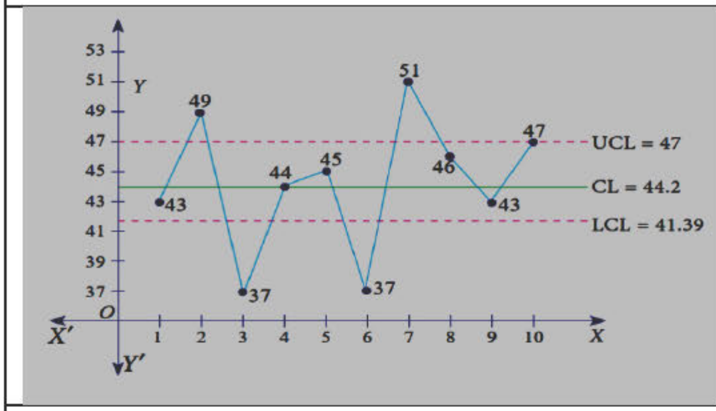
5

46 (b)	(i) $P(X \le 0) = \frac{1}{2}$	1	
	(ii) $P(X < 0) = \frac{1}{4}$	1	
	(iii) $P(\lvert X\rvert \le 2) = P(-2 \le X \le 2) = \frac{1}{2}$	2	
	(iv) $P(0 \le X \le 10) = \frac{3}{4}$	1	

47 (a)

$\bar{\bar{X}} = 44.2, \quad \bar{R} = 5.8$ — 1

UCL = $\bar{\bar{X}} + A_2\bar{R}$ = 7)0
CL = $\bar{\bar{X}}$ = 44.2
LCL = $\bar{\bar{X}} - A_2\bar{R}$ = 41.39 — 1



— 2

u i t f ls o t of the control limits. The process is out of control. — 1

(OR)

5

47 (b)

ta v ia ii y = Total Requirement = 80, This is a a a nsp tion. — 1

Final distribution:

	I	II	III	IV	a_i
A	(6) 5	(6) 1	(17) 3	(5) 3	34
B	(15) 3	3	5	4	15
C	6	4	4	(12) 3	12
D	4	(19) 1	4	5	19
b_j	21	25	17	17	

— 2

ta a s o ti n :ost =
$(6 \times 5) + (6 \times 1) + (17 \times 3) + (5 \times 3) + (15 \times 3)$
$+ (\ 2 \times 3) + (19 \times 1)$
= ₹ 202 — 2