SECOND TERMINAL EXAMINATION – 2022

MATHAMATICS (SCIENCE) **HSE II**

(Chapters 1 to 10)

Questions : -

Answer any 6 questions rom 1 to 8. Each carries 3 scores. (6x3=18)

1. (i) Let f : R → R be defined as f(x) = 3x, then
   (A) f is one-one onto.
   (B) f is many-one onto.
   (C) f is one-one but not onto
   (D) f is neither one-one nor onto (1)

   (ii) Show that the function f : R → R defined as $f(x) = x^2$ is neither one-one nor onto. (2)

2. Find the values of x,y and z from $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$ (3)

3. If A = $\begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then
   (i) Find |A| (1)
   (ii) Show that |2A| = 4|A| (2)

4. Discuss the continuity of the sine function. (3)

5. Find the absolute maximum value and absolute minimum value of the function: $f(x) = x^3$, $x \in [-2,2]$ (3)

6. Find the general solution of the differential equation: $\frac{dy}{dx} + \frac{y}{x} = x^2$ (3)

7. Find a vector in the direction of $\vec{a} = \hat{\imath} - 2\hat{\jmath}$ that has magnitude 7 units. (3)

8. If $\vec{a} = \hat{\imath} - 7\hat{\jmath} + 7\hat{k}$ and $\vec{b} = 3\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ find
   (i) $\vec{a} \times \vec{b}$ (2)
   (ii) $|\vec{a} \times \vec{b}|$ (1)

Answer any 6 questions from 9 to 16. Each carries 4 scores. (6x4=24)

9. (i) Let R be a relation defined on A = {1, 2, 3) by R= {(1, 3), (3, 1),(2, 2)}, R is
   (A) Reflexive
   (B) Symmetric
   (C) Transitive
   (D) Reflexive but not transitive (1)

   (ii) Show that the relation R in the set *R* of real numbers define as R= {(a, b), a ≤ b^2) is neither reflexive nor symmetric nor transitive. (3)

10. (i) The principal value of $\sin^{-1}\left(\frac{1}{2}\right)$ = ……… (1)

    (ii) Find the value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$ (3)

11. (i) Construct a 2x2 matrix A = $[a_{ij}]$ whose elements are given by $a_{ij} = i/j$. (2)

    (ii) Find X and Y if X+Y = $\begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$, X–Y = $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ (2)

12. (i) A square matrix A is singular if |A|= ……. (1)

    (ii) Find the area of the triangle whose vertices are (3, 8), (–4, 2), (5, 1). (3)

13. Find the intervals in which the function f given by f(x) = sin x + cos x, 0 ≤ x ≤ 2π is strictly
    (i) increasing
    (ii) decreasing. (4)

14. (i) $\int_{-\pi/2}^{\pi/2} \sin^7 x \, dx$ = ……..
    (A) -1 (B) 0 (C) 1 (D) 2

    (ii) Evaluate $\int x \cos x \, dx$ (3)

15. Find the area of the region bounded by the curve $y^2 = x$, x = 1, x=4 and the x-axis in the first quadrant. (4)

16. (i) Write the order and degree of the differential equation: $\left(\frac{ds}{dt}\right)^4 + 3s\frac{d^2s}{dt^2} = 0$ (2)

    (ii) Find the general solution of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ (2)

Answer any 3 questions from 17 to 20. Each carries 6 scores. (3 × 6=18)

17. (i) If A = $\begin{bmatrix} 0 & 3 & a \\ b & 0 & -2 \\ 5 & 2 & 0 \end{bmatrix}$ is a skew symmetric matrix. Find a, b. (2)

    (ii) Express A = $\begin{bmatrix} 7 & 3 & -5 \\ 0 & 1 & 5 \\ -2 & 7 & 3 \end{bmatrix}$ as the sum of a symmetric and skew symmetric matrix (4)

18. Solve the following system of equations by matrix method:
    3x–2y+3z=8
    2x+y–z=1
    4x–3y+2z=4 (6)

19. (i) Find $\frac{dy}{dx}$ if x = a (θ + sinθ), y = a(1 – cosθ) (3)

    (ii) If $y = (\tan^{-1} x)^2$ show that $(x^2 +1)^2 y_2 + 2x(x^2+1)y_1 = 2$ (3)

20. Find
    (i) $\int \frac{dx}{x + x\log x}$ (2)
    (ii) $\int \frac{dx}{x^2 - 6x + 13}$ (2)
    (iii) $\int_0^{\pi/2} \frac{\cos^5 x \, dx}{\sin^5 x + \cos^5 x}$ (2)

Answers : -

1. (i) (A) f is one-one onto.

   (ii) $f(-2) = (-2)^2 = 4$

   $f(2) = 2^2 = 4$

   $\therefore$ f is not one one

   Clearly, -1 in the codomain do not have a pre image. (range=$[0,\infty)$) $\therefore$ f is not onto.

2. Given, $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

   Equating corresponding elements,

   $x + y + z = 9 \longrightarrow (1)$

   $x + z = 5 \longrightarrow (2)$

   $y + z = 7 \longrightarrow (3)$

   $(1) - (2) \Rightarrow y = 4$

   $(1) - (3) \Rightarrow x = 2$

   $(2) \Rightarrow z = 5 - x = 5 - 2 = 3$

   $x = 2, y = 4, z = 3$

3. (i) $|A| = \begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix} = 2 - 8 = -6$

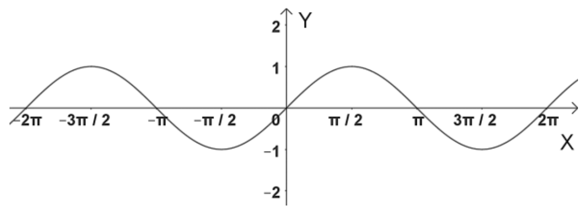
   (ii) $2A = 2\begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 8 & 4 \end{bmatrix}$

   LHS = $|2A| = \begin{vmatrix} 2 & 4 \\ 8 & 4 \end{vmatrix} = 8 - 32 = -24$

   RHS = $4|A| = 4 \times -6 = -24$

   ie, $|2A| = 4|A|$

4.



   Graph of sine function is a continuous curve in its domain. $\therefore$ It is a continuous function.

5. Given, $f(x) = x^3$

   $\therefore f'(x) = 3x^2$

   $f'(x) = 0 \Rightarrow 3x^2 = 0$

   ie, $x^2 = 0$

   ie, $x = 0$

   We have, $f(0) = 0^3 = 0$

   $f(-2) = (-2)^3 = -8$

   $f(2) = 2^3 = 8$

   Absolute minimum value is -8 at x = -2

   Absolute maximum value is 8 at x = 2

6. Given, $\frac{dy}{dx} + \frac{y}{x} = x^2$

   Which is a linear differential equation,

   Here $P = \frac{1}{x}$, $Q = x^2$

   Integrating factor $= e^{\int P\,dx}$

   $= e^{\int \frac{1}{x} dx}$

   $= e^{\log x}$

   $= x$

   The solution is given by, $y \cdot IF = \int (Q.IF)\,dx$

   $y \cdot x = \int (x^2.x)\,dx$

   ie, $xy = \int x^3\,dx$

   ie, $xy = \frac{x^4}{4} + C$

7. Vector in the direction of $\vec{a}$ that has magnitude 7 units $= 7\left(\frac{\vec{a}}{|\vec{a}|}\right) = 7\left(\frac{\hat{\imath} - 2\hat{\jmath}}{\sqrt{1^2+(-2)^2}}\right)$

   $= 7\left(\frac{\hat{\imath} - 2\hat{\jmath}}{\sqrt{5}}\right)$

   $= \frac{7\hat{\imath} - 14\hat{\jmath}}{\sqrt{5}}$

8. (i) $\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{vmatrix}$

   $= \hat{\imath}(-14 + 14) - \hat{\jmath}(2 - 21) + \hat{k}(-2+21)$

   $= 0\hat{\imath} + 19\hat{\jmath} + 19\hat{k}$

   (ii) $|\bar{a} \times \bar{b}| = \sqrt{0^2 + 19^2 + 19^2} = \sqrt{722}$

   $= 19\sqrt{2}$

9. (i) (B) Symmetric

   (ii) Since $\frac{1}{2} \le \left(\frac{1}{2}\right)^2$ is not true $\therefore \left(\frac{1}{2}, \frac{1}{2}\right) \notin R$

   $\therefore$ R is not reflexive

   Clearly $(1,2) \in R \Rightarrow 1 \le 2^2$, but $2 \le 1^2$ is not true

   $\Rightarrow (2,1) \notin R$

   $\therefore$ R is not symmetric.

   Clearly $(6,3) \in R$ and $(3, 2) \in R$

   $\Rightarrow 6 \le 3^2$ and $3 \le 2^2$ but $6 \le 2^2$ is not true.

   ie, $(6, 2) \notin R$

   $\therefore$ R is not transitive

10. (i) $\frac{\pi}{6}$

(ii) $\tan^{-1}(1) = \frac{\pi}{4}$

$\cos^{-1}\left(\frac{-1}{2}\right) = \pi - \cos^{-1}\left(\frac{1}{2}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$\sin^{-1}\left(\frac{-1}{2}\right) = -\sin^{-1}\left(\frac{1}{2}\right) = -\frac{\pi}{6}$

$\tan^{-1}(1) + \cos^{-1}\left(\frac{-1}{2}\right) + \sin^{-1}\left(\frac{-1}{2}\right)$

$= \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6}$

$= \frac{3\pi}{12} + \frac{8\pi}{12} - \frac{2\pi}{12}$

$= \frac{9\pi}{12}$

$= \frac{3\pi}{4}$

11. (i) $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

Given, $a_{ij} = \frac{i}{j}$

$a_{11} = \frac{1}{1} = 1$ $\qquad a_{12} = \frac{1}{2}$

$a_{21} = \frac{2}{1} = 2$ $\qquad a_{22} = \frac{2}{2} = 1$

$A = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{bmatrix}$

(ii) Given, $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} \rightarrow \langle 1 \rangle$

$X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \rightarrow \langle 2 \rangle$

$\langle 1 \rangle + \langle 2 \rangle \Rightarrow 2X = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$= \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix}$

$\therefore X = \frac{1}{2}\begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$

$\langle 1 \rangle \Rightarrow \quad Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} - X$

$= \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

ie, $X = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$,

$Y = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

12. (i) 0

(ii) Take $(x_1, y_1) = (3,8)$,

$(x_2, y_2) = (-4,2)$,

$(x_3, y_3) = (5,1)$

Area of a triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is $\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 5 & 1 & 1 \end{vmatrix}$

$= \frac{1}{2}\{3(2-1) - 8(-4-5) + 1(-4-10)\}$

$= \frac{1}{2}\{3(1) - 8(-9) + 1(-14)\}$

$= \frac{1}{2}\{3 + 72 - 14\}$

$= \frac{61}{2}$ sq units

13. Given, $f(x) = \sin x + \cos x,\ 0 \le x \le 2\pi$

$\therefore f'(x) = \cos x - \sin x$

$f'(x) = 0 \Rightarrow \cos x - \sin x = 0$

ie, $\cos x = \sin x$

ie, $\tan x = 1$

ie $\tan x = \tan\frac{\pi}{4}$

$\Rightarrow \quad x = \frac{\pi}{4}, \pi + \frac{\pi}{4}$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

(The general solution is $x = n\pi + \frac{\pi}{4}$ $\therefore$ The particular solution is given by $x = \frac{\pi}{4}, \frac{5\pi}{4}$)

Consider the intervals,

$[0, \frac{\pi}{4}), (\frac{\pi}{4}, \frac{5\pi}{4}), (\frac{5\pi}{4}, 2\pi]$

Clearly, $f'(x) > 0$ for all x in $[0, \frac{\pi}{4})$

$f'(x) < 0$ for all x in $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$

$f'(x) > 0$ for all x in $(\frac{5\pi}{4}, 2\pi]$

$\therefore$ f(x) is strictly increasing in the intervals $[0, \frac{\pi}{4})$ and $(\frac{5\pi}{4}, 2\pi]$

f(x) is strictly decreasing in the interval $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$

14. (i) 0

(ii) We have,

$\int f_1(x) f_2(x)dx = f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx} f_1(x)\int f_2(x)dx\right\}dx$

$\therefore \int x\cos x\,dx$

$= x\int \cos x\,dx - \int\left\{\frac{d}{dx}(x)\int \cos x\,dx\right\}dx$

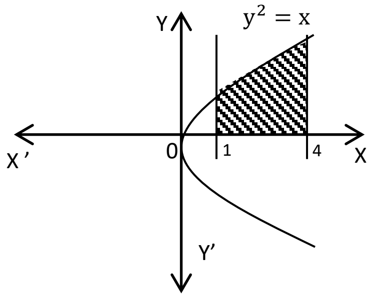
$= x\sin x - \int\{1.\sin x\}\,dx$

$= x\sin x - \int \sin x\,dx$

$= x\sin x + \cos x + C$

15. Given , $y^2 = x$. ie, $y = \sqrt{x}$

and the lines x = 1, x = 4



Required area $= \int_1^4 y\,dx$

$= \int_1^4 \sqrt{x}\,dx = \left[\frac{2}{3}x^{3/2}\right]_1^4$

$= \left(\frac{2}{3}(4)^{3/2}\right) - \left(\frac{2}{3}(1)^{3/2}\right)$

$= \left(\frac{2}{3}(8)\right) - \left(\frac{2}{3}(1)\right)$

$\left[\, 4^{\frac{3}{2}} = \left(4^{\frac{1}{2}}\right)^3 = 2^3 = 8 \,\right]$

$= \frac{16}{3} - \frac{2}{3}$

$= \frac{14}{3}$ sq. units

16. (i) order = 2, degree = 1

(ii) Given, $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

$\therefore \frac{dy}{1+y^2} = \frac{dx}{1+x^2}$, which is a variable separable differential equation.

Integrating, $\int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2}$

$\tan^{-1} y = \tan^{-1} x + C$

17. (i) Given $A = \begin{bmatrix} 0 & 3 & a \\ b & 0 & -2 \\ 5 & 2 & 0 \end{bmatrix}$

$A' = \begin{bmatrix} 0 & b & 5 \\ 3 & 0 & 2 \\ a & -2 & 0 \end{bmatrix}$

$-A = \begin{bmatrix} 0 & -3 & -a \\ -b & 0 & 2 \\ -5 & -2 & 0 \end{bmatrix}$

Since A is skew symmetric A' = – A

ie, $\begin{bmatrix} 0 & b & 5 \\ 3 & 0 & 2 \\ a & -2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -3 & -a \\ -b & 0 & 2 \\ -5 & -2 & 0 \end{bmatrix}$

$\therefore$ a = – 5, b = – 3

(ii) Given , $A = \begin{bmatrix} 7 & 3 & -5 \\ 0 & 1 & 5 \\ -2 & 7 & 3 \end{bmatrix}$

$A^T = \begin{bmatrix} 7 & 0 & -2 \\ 3 & 1 & 7 \\ -5 & 5 & 3 \end{bmatrix}$

Symmetric Part, $P = \frac{1}{2}\left(A + A^T\right)$

$\therefore P = \frac{1}{2}\left(\begin{bmatrix} 7 & 3 & -5 \\ 0 & 1 & 5 \\ -2 & 7 & 3 \end{bmatrix} + \begin{bmatrix} 7 & 0 & -2 \\ 3 & 1 & 7 \\ -5 & 5 & 3 \end{bmatrix}\right)$

$= \frac{1}{2}\begin{bmatrix} 14 & 3 & -7 \\ 3 & 2 & 12 \\ -7 & 12 & 6 \end{bmatrix}$

$= \begin{bmatrix} 7 & 3/2 & -7/2 \\ 3/2 & 1 & 6 \\ -7/2 & 6 & 3 \end{bmatrix}$

Skew symmetric Part, $Q = \frac{1}{2}\left(A - A^T\right)$

$Q = \frac{1}{2}\left(\begin{bmatrix} 7 & 3 & -5 \\ 0 & 1 & 5 \\ -2 & 7 & 3 \end{bmatrix} - \begin{bmatrix} 7 & 0 & -2 \\ 3 & 1 & 7 \\ -5 & 5 & 3 \end{bmatrix}\right)$

$= \frac{1}{2}\begin{bmatrix} 0 & 3 & -3 \\ -3 & 0 & -2 \\ 3 & 2 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0 & 3/2 & -3/2 \\ -3/2 & 0 & -1 \\ 3/2 & 1 & 0 \end{bmatrix}$

We have , A = P + Q

ie, $\begin{bmatrix} 7 & 3 & -5 \\ 0 & 1 & 5 \\ -2 & 7 & 3 \end{bmatrix} = \begin{bmatrix} 7 & 3/2 & -7/2 \\ 3/2 & 1 & 6 \\ -7/2 & 6 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 3/2 & -3/2 \\ -3/2 & 0 & -1 \\ 3/2 & 1 & 0 \end{bmatrix}$

18. Given, 3x – 2y + 3z = 8

2x + y – z = 1

4x – 3y + 2z = 4

ie, $\begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$

ie, AX = B, where $A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$B = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$

We have , $X = A^{-1}B$

$A^{-1} = \frac{1}{|A|}$ (adj A)

$|A| = \begin{vmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{vmatrix}$ = 3(2–3)+2(4+4)+3(–6–4)

= 3(–1) + 2(8) + 3(–10)

= – 3 + 16 – 30

= – 17

To find adj A :–

$A_{11} = \begin{vmatrix} 1 & -1 \\ -3 & 2 \end{vmatrix}$ = 2 – 3 = –1

$A_{12} = -\begin{vmatrix} 2 & -1 \\ 4 & 2 \end{vmatrix}$ = –(4+4) = –8

$A_{13} = \begin{vmatrix} 2 & 1 \\ 4 & -3 \end{vmatrix} = -6 - 4 = -10$

$A_{21} = -\begin{vmatrix} -2 & 3 \\ -3 & 2 \end{vmatrix} = -(-4 + 9) = -5$

$A_{22} = \begin{vmatrix} 3 & 3 \\ 4 & 2 \end{vmatrix} = 6 - 12 = -6$

$A_{23} = -\begin{vmatrix} 3 & -2 \\ 4 & -3 \end{vmatrix} = -(-9 + 8) = 1$

$A_{31} = \begin{vmatrix} -2 & 3 \\ 1 & -1 \end{vmatrix} = 2 - 3 = -1$

$A_{32} = -\begin{vmatrix} 3 & 3 \\ 2 & -1 \end{vmatrix} = -(-3 - 6) = 9$

$A_{33} = \begin{vmatrix} 3 & -2 \\ 2 & 1 \end{vmatrix} = 3 + 4 = 7$

$$\text{adj } A = \begin{bmatrix} -1 & -8 & -10 \\ -5 & -6 & 1 \\ -1 & 9 & 7 \end{bmatrix}^T = \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$$

$\therefore \quad A^{-1} = \frac{1}{|A|}(\text{adj } A)$

$$= \frac{1}{-17}\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{-17}\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}\begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$$

$$= \frac{1}{-17}\begin{bmatrix} -8 - 5 - 4 \\ -64 - 6 + 36 \\ -80 + 1 + 28 \end{bmatrix} = \frac{1}{-17}\begin{bmatrix} -17 \\ -34 \\ -51 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

ie, $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$\therefore$ x = 1, y = 2, z = 3

19. (i) Given, $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$

We have, $\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{\frac{d}{dt}[a(1 - \cos\theta)]}{\frac{d}{dt}[a(\theta + \sin\theta)]}$

$$= \frac{a\frac{d}{dt}(1 - \cos\theta)}{a\frac{d}{dt}(\theta + \sin\theta)}$$

$$= \frac{a(0 - (-\sin\theta))}{a(1 + \cos\theta)} = \frac{\sin\theta}{1 + \cos\theta}$$

(ii) Given, $y = (\tan^{-1} x)^2$

Diff. w.r. to x,

$\frac{dy}{dx} = 2\tan^{-1}x \cdot \frac{1}{1 + x^2}$

$\frac{dy}{dx} = \frac{2\tan^{-1}x}{1 + x^2}$

Multiplying both sides by $(1 + x^2)$, we get

$(1 + x^2)\frac{dy}{dx} = 2\tan^{-1}x$

Diff. again w.r.to x,

$(1 + x^2)\frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \frac{d}{dx}(1 + x^2) = 2 \cdot \frac{1}{(1 + x^2)}$

$(1 + x^2)\frac{d^2y}{dx^2} + \frac{dy}{dx}(2x) = \frac{2}{(1 + x^2)}$

Multiplying both sides by $(1 + x^2)$, we get

ie, $(1 + x^2)^2\frac{d^2y}{dx^2} + 2x(1 + x^2)\frac{dy}{dx} = 2$

ie, $(x^2 + 1)^2 y_2 + 2x(x^2 + 1)y_1 = 2$

20. (i) $\int \frac{dx}{x + x\log x} = \int \frac{dx}{x(1 + \log x)}$

Put t = 1+log x

$\therefore dt = \frac{1}{x}dx$

$\therefore \int \frac{dx}{x(1 + \log x)} = \int \frac{1}{t}\,dt = \log|t| + C$

$= \log|1 + \log x| + C$

(ii) $\int \frac{1}{x^2 - 6x + 13}dx = \int \frac{1}{x^2 - 6x + 3^2 - 3^2 + 13}dx$

$= \int \frac{1}{(x-3)^2 - 9 + 13}dx$

$= \int \frac{1}{(x-3)^2 + 4}dx = \int \frac{1}{(x-3)^2 + 2^2}dx$

$= \frac{1}{2}\tan^{-1}\left(\frac{x-3}{2}\right) + C$

(iii) Let $I = \int_0^{\frac{\pi}{2}} \frac{\cos^5 x}{\sin^5 x + \cos^5 x}dx \rightarrow \langle 1 \rangle$

ie, $I = \int_0^{\frac{\pi}{2}} f(x)\,dx$ where $f(x) = \frac{\cos^5 x}{\sin^5 x + \cos^5 x}$

ie, $I = \int_0^{\frac{\pi}{2}} f(\frac{\pi}{2} - x)\,dx$ $\left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$

$= \int_0^{\frac{\pi}{2}} \frac{\cos^5(\frac{\pi}{2} - x)}{\sin^5(\frac{\pi}{2} - x) + \cos^5(\frac{\pi}{2} - x)}dx$

ie, $I = \int_0^{\frac{\pi}{2}} \frac{\sin^5 x}{\cos^5 x + \sin^5 x}dx \rightarrow \langle 2 \rangle$

$\langle 1 \rangle + \langle 2 \rangle \Rightarrow$

$I + I = \int_0^{\frac{\pi}{2}} \frac{\cos^5 x}{\sin^5 x + \cos^5 x}dx + \int_0^{\frac{\pi}{2}} \frac{\sin^5 x}{\cos^5 x + \sin^5 x}dx$

ie $2I = \int_0^{\frac{\pi}{2}} \frac{\sin^5 x + \cos^5 x}{\sin^5 x + \cos^5 x}dx$

$= \int_0^{\frac{\pi}{2}} 1\,dx = [x]_0^{\pi/2} = \frac{\pi}{2} - 0 = \frac{\pi}{2}$

$\therefore I = \frac{\pi}{4}$