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## MATHEMATICS 81 E MODEL KEY ANSWERS

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## KARNATAKA SCHOOL EXAMINATION \& ASSESSMENT BOARD

## MODEL KEY ANSWERS ANNUAL EXAM-2022-23

1. Answer: (A) 3
2. Answer: (B) 0.25
3. Answer : (D) $2 \prod \mathrm{r}(\mathrm{r}+\mathrm{h})$
4. Answer: (C) 1
5. Answer: (A) $45^{0}$
6. Answer: (C) $\frac{A D}{D B}=\frac{A E}{E C}$
7. Option (D) Parallel lines
8. Option (B) 3 units
9. $80=2^{4} \times 5^{1}$
10. $a=3, b=6$
11. $P Q=10 \mathrm{~cm}$
12. $\mathrm{x}^{2}+2 \mathrm{x}+3=0$
13. $\Delta=\mathrm{b}^{2}-4 \mathrm{ac}$
$\Delta=(-4)^{2}-4 \times 2 \times 3$
$\Delta=16-24$
$\Delta=-8$
Hence No real roots
14. The coordinates of the line joining the midpoints of two vertices are

$$
\begin{aligned}
& \mathrm{P}(\mathrm{x}, \mathrm{y})=\left[\frac{m x 2+n x 1}{2}, \frac{m y 2+n y 1}{2}\right] \\
&=\left[\frac{6+4}{2}, \frac{3+7}{2}\right] \\
&=\left[\frac{10}{2}, \frac{10}{2}\right] \\
&=(5,5)
\end{aligned}
$$

15. Degree is 4
16. Volume of the frustum of a cone is given by $V=\frac{1}{3} \Pi h\left(r_{1}^{2}+r_{2}^{2}+r_{1} r^{2}\right)$
17. 

Solution:
Let us assume that $5+\sqrt{3}$ is a rational number with $p$ and $q$ as coprime integer and $\mathrm{q} \neq 0$
$\Rightarrow 5+\sqrt{3}=\mathrm{p} / \mathrm{q}$
$\Rightarrow \sqrt{3}=\mathrm{p} / \mathrm{q}-5$
$\Rightarrow \sqrt{3}=\mathrm{p} / \mathrm{q}-5$
$\Rightarrow \mathrm{p} / \mathrm{q}-5$ is a rational number
However, $\sqrt{3}$ is in irrational number

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This leads to a contradiction that $5+\sqrt{3}$ is a rational number wrong
Hence $5+\sqrt{3}$ is an irrational number.
OR
Let the three numbers are 72 \& 120
By Euclid's division algorithm, 72)120(1

Hence HCF of 72 \& 120 is 24
18. Consider the given equation.
$3 x+y=12$
$x+y=6$
On subtracting both equation (1) and (2), we get
$2 \mathrm{x}=6$
$\mathrm{x}=3$
Now, put the value of $x$ in equation (2), we get
$3+y=6$
$\mathrm{y}=3$
Hence, the value of $x$ is 3 and $y$ is 3
19. Solution: Given A.P is $4,7,10, \ldots \ldots . .$.
here $a=4, d=3$ we have to find $20^{\text {th }}$ term means $a_{20}=a+19 d$

$$
\begin{aligned}
& =4+19 \times 3 \\
& =4+57 \\
& =61
\end{aligned}
$$

Hence $20^{\text {th }}$ term of this A.P is 61
20.

Given equation is $2 x^{2}-5 x+3=0$
By using formula method,

$$
2 x^{2}-5 x+3=0
$$

Here $a=2, b=-5 \& c=3$
We have $x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}=\frac{-(-5) \pm \sqrt{-5^{2}-4 \times 2 x(3)}}{2 \times 2}=\frac{5 \pm \sqrt{25-24}}{4}=\frac{-5 \pm \sqrt{1}}{4}$

$$
\begin{array}{ll}
x=\frac{5+1}{4} & \text { or } x=\frac{5-1}{4} \\
x=\frac{3}{2} & \text { or } x=1
\end{array}
$$

21. $\operatorname{Sin} \boldsymbol{\theta}=\frac{1}{2}$
$\operatorname{Cos} \alpha=\frac{1}{2}$
22. Solution: possible outcomes: $\{9,10,11,12,13,14,15,16,17,18,19\}$

Among them prime numbers are $\{11,13,17,19\}=4$
Probability is $\frac{n(E)}{n(S)}=\frac{4}{11}$
23. Mark point E on DC, Such that $\mathrm{EC}=6 \mathrm{~cm}$.

In $\triangle \mathrm{ADE}$

$$
\begin{gathered}
\mathrm{AD}^{2}=\mathrm{AE}^{2}+\mathrm{DE}^{2} \\
5^{2}=\mathrm{AE}^{2}+4^{2} \\
25=\mathrm{AE}^{2}+16 \\
25-16=\mathrm{AE}^{2} \\
\mathrm{AE}^{2}=9 \\
\mathrm{AE}=3 \mathrm{~cm}
\end{gathered}
$$

Hence the distance between two parallel lines is 3 cm
24. Solution:

25. Solution:

$$
\begin{array}{r}
3 x-5 \\
x ^ { 2 } + 2 x + 1 \longdiv { 3 x ^ { 3 } + x ^ { 2 } + 2 x + 5 } \begin{array} { l } 
{ 3 x ^ { 3 } + 6 x ^ { 2 } + 3 x }
\end{array} \\
-\quad-\quad-\begin{array}{l}
-5 x^{2}-x+5
\end{array} \\
\begin{array}{l}
-5 x^{2}-10 x-5 \\
+\quad+\quad+
\end{array} \\
\hline \begin{array}{l}
9 x+10
\end{array}
\end{array}
$$

Hence quotient is $3 x-5$ and remainder is $9 x+10$

## OR

Given polynomial is $\mathrm{p}(\mathrm{x})=\mathrm{x}^{2}+7 \mathrm{x}+10$.
Its zeroes are by factorization method,

$$
x^{2}+7 x+10
$$

$$
\begin{aligned}
& x^{2}+5 x+2 x+10 \\
& x(x+5)+2(x+5) \\
& (x+5)(x+2) \\
& \alpha=-5 \& \mathbb{C}=-2
\end{aligned}
$$

Verification: we know that $\mathrm{x}^{2}-(\boldsymbol{\alpha}+$ © $) \mathrm{x}+\boldsymbol{\alpha}$ (C)

$$
\begin{aligned}
& x^{2}-(-5-2)+-7 x-2 \\
& x^{2}+7 x+10
\end{aligned}
$$

Hence the proof
26. Solution:

$$
\begin{aligned}
& \text { We have } \begin{aligned}
& \sqrt{\frac{1+\cos A}{1-\operatorname{Cos} A}}=\sqrt{\frac{1+\cos A(1+\cos A)}{1-\operatorname{Cos} A(1+\cos A)}} \\
= & \sqrt{\frac{(1+\cos A) 2}{(1-\operatorname{Cos} A) 2}} \\
= & \sqrt{\frac{1+\cos A) 2}{\sin 2 A}} \\
= & \frac{1}{\sin A}+\frac{\cos A}{\operatorname{Sin} A} \\
= & \operatorname{Cosec} A+\operatorname{Cot} A
\end{aligned}
\end{aligned}
$$

OR

We have $\frac{\operatorname{Sin} A}{1+\operatorname{Cos} A}+\frac{1+\operatorname{Cos} A}{\operatorname{Sin} A}$

$$
\begin{aligned}
& =\frac{\operatorname{Sin} 2 A+(1+\operatorname{Cos} A) 2}{\operatorname{Sin} A(1+\operatorname{Cos} A)} \\
& =\frac{\operatorname{Sin} 2 A+(1+\operatorname{Cos} 2 A+2 \operatorname{Cos} A)}{\operatorname{Sin} A(1+\operatorname{Cos} A)} \\
& =\frac{\operatorname{Sin} 2 A+\operatorname{Cos} 2 A+1+2 \operatorname{Cos} A}{\operatorname{Sin} A(1+\operatorname{Cos} A)} \\
& =\frac{1+1+2 \operatorname{Cos} A}{\operatorname{Sin} A(1+\operatorname{Cos} A)} \\
& =\frac{2+2 \operatorname{Cos} A}{\operatorname{Sin} A(1+\operatorname{Cos} A)} \\
& =\frac{2(1+\operatorname{Cos} A)}{\operatorname{Sin} A(1+\operatorname{Cos} A)} \\
& =2 \operatorname{Cosec} A
\end{aligned}
$$

27. Solution:

We have to find mean

| C.I | f | x | fx |
| :--- | :--- | :--- | :--- |
| $1-5$ | 4 | 3 | 12 |
| $6-10$ | 3 | 8 | 24 |
| $11-15$ | 2 | 13 | 26 |
| $16-20$ | 1 | 18 | 18 |
| $21-25$ | 5 | 23 | 115 |
|  | $\mathrm{~N}=15$ |  | 195 |

Mean $=\frac{\sum f x}{N}=\frac{195}{15}=13$
OR
We have to find Mode for the following frequency data

| C.I | f |
| :--- | :--- |
| $1-3$ | 9 |
| $3-5$ | $9 \mathrm{f0} 0$ |
| $5-7$ | 15 f 1 |
| $7-9$ | 9 f 2 |
| $9-11$ | 1 |
|  | $\mathrm{~N}=60$ |

## LRL=5, f1=15, f0=9, f2=9 and $\mathrm{h}=3$

We have formula Mode $=L R L+\left\{\begin{array}{c}f 1-f 0 \\ 2 f 1-f 0-f 2\end{array} \mathrm{xh}\right.$

$$
=5+\frac{15-9}{30-9-9} \times 2
$$

$$
=5+\frac{6}{6}
$$

$$
=5+1
$$

## Mode=6

28. Solution:

We have $(-6,10)$ and $(3,-8)$ is divided by $(-4,6)$
By section formula,
$-4=\left(\frac{m(3)+n(-6)}{m+n}\right)$ and $6=\left(\frac{m(-8)+n(10)}{m+n}\right)$
$\mathrm{m}+\mathrm{n}=\left(\frac{3 m-6 n}{-4}\right)$ and $\mathrm{m}+\mathrm{n}=\left(\frac{-8 m+10 n}{6}\right)$
on comparing both we get $\frac{3 m-6 n}{-4}=\frac{-8 m+10 n}{6}$

$$
\begin{aligned}
& 18 \mathrm{~m}-36 \mathrm{n}=32 \mathrm{~m}-40 \mathrm{n} \\
& 4 \mathrm{n}=14 \mathrm{~m} \\
& \mathrm{~m} / \mathrm{n}=2 / 7
\end{aligned}
$$

The ratio is $2: 7$
OR
$(1,-1),(-4,6) \quad \&(-3,-5)$ ( $\mathrm{x} 1, \mathrm{y} 1$ ), $(\mathrm{x} 2, \mathrm{y} 2)$ \& $(\mathrm{x} 3, \mathrm{y} 3)$
Area of the triangle is $A=\frac{1}{2}\{x 1(y 2-y 3)+x 2(y 3-y 1)+x 3(y 1-y 2)\}$

$$
\begin{aligned}
& =\frac{1}{2}\{1(6+5)-4(-5+1)+(-3)(-1-6)\} \\
& =\frac{1}{2}(11+16+21) \\
& =\frac{1}{2}(48) \\
& =24 \text { sq units. }
\end{aligned}
$$

29.Given: PT and PS are tangents from an external point $P$ to the circle with centre O .
To prove: $\mathrm{PT}=\mathrm{PS}$
Construction: Join O to $\mathrm{P}, \mathrm{T}$ and S .


Proof: In $\triangle$ OTP and $\Delta \mathrm{OSP}$.
OT $=$ OS $\ldots$ [radii of the same circle]
$\mathrm{OP}=\mathrm{OP} \ldots$ [common]
$\angle \mathrm{OTP}=\angle \mathrm{OSP} \ldots\left[\right.$ each $90^{\circ}$ ]
$\Delta \mathrm{OTP}=\Delta \mathrm{OSP} \ldots$ [R.H.S.]
$\mathrm{PT}=\mathrm{PS} \ldots$ [c.p.c.t.]
30. Solution:

Area of shaded region= area of circle - area of sector of a circle OPQ----(1) In the given figure, OAB is an equilateral triangle.
Its area is $36 \sqrt{3} \mathrm{sq} \mathrm{cm}$.
We know $A=\frac{\sqrt{3}}{4} \mathrm{a}^{2}$ (area of an equilateral triangle)

$$
36 \sqrt{3}=\frac{\sqrt{3}}{4} \mathrm{a}^{2}
$$

Then side of the triangle is 12 cm , then radius of circle is 6 cm (mid-point)
At 0 angle should be $60^{\circ}$.
Area of sector $=\frac{60}{360} \times \frac{22}{7} \times 6 \times 6$

$$
=\frac{132}{7}
$$

Area of a circle $=\frac{22}{7} \times 6 \times 6$

$$
=\frac{792}{7}
$$

Equation 1 becomes
$\frac{792}{7}-\frac{132}{7}=94.28 \mathrm{sq} \mathrm{cm}$.
31. Solution:


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## 32. Solution :

Let us say, the current average speed of car $=x \mathrm{~km} / \mathrm{h}$.
If it goes $11 \mathrm{~km} / \mathrm{hr}$ more then it would take 1 hour less
Total distance between the two city is 132 km . Therefore, according to question
$(132 / x)-(132 /(x+11))=1$
$132(x+11-x) /(x(x+11))=1$
$132 \times 11 /(x(x+11))=1$
$\Rightarrow 132 \times 11=x(x+11)$
$\Rightarrow x^{2}+11 x-1452=0$
$\Rightarrow x^{2}+44 x-33 x-1452=0$
$\Rightarrow x(x+44)-33(x+44)=0$
$\Rightarrow(x+44)(x-33)=0$
$\Rightarrow x=-44,33$
As we know, Speed cannot be negative.
Therefore, the speed of the car $33 \mathrm{~km} / \mathrm{h}$.

## 33. Solution:

| C.I | Frequency | Coordinates |
| :--- | :--- | :--- |
| $<20$ | 2 | $(20,2)$ |
| $<25$ | 6 | $(25,6)$ |
| $<30$ | 24 | $(30,24)$ |
| $<35$ | 45 | $(35,45)$ |
| $<40$ | 78 | $(40,78)$ |
| $<45$ | 89 | $(45,89)$ |
| $<50$ | 100 | $(50,100)$ |



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34. Solution:

The sum of second and fourth term of the arithmetic progression is 54
$a 2+a 4=54$ and S11=693
$a+d+a+3 d=54$ $2 a+4 d=54---(1)$

$$
\frac{11}{2}(2 a+10 d)=693
$$

$$
\begin{equation*}
2 a+10 d=126 \tag{2}
\end{equation*}
$$

From 1 and 2, subtract above we get
$6 \mathrm{~d}=72$
$\mathrm{d}=12$
thus common diffrence is 12 , put this in equation 1 we get
$2 \mathrm{a}+4(12)=54$
$2 \mathrm{a}=54-48$
$\mathrm{a}=3$
hene first term is 3
A.P is 3, 15, 27 $\qquad$
Its $54^{\text {th }}$ term is $\mathrm{a}+53 \mathrm{~d}=3+53 \times 12=3+636=639$
According to question, 132+639=771 this will be an
an=771
$\mathrm{a}+(\mathrm{n}-1) \mathrm{d}=771$
$3+12 n-12=771$
$12 n=768+12$
$12 n=780$
$\mathrm{n}=65$
then $65^{\text {th }}$ term is 132 more than its $54^{\text {th }}$ term.
OR
Given: $\mathrm{a}=3$ and $\mathrm{l}=253$ and also $\mathrm{a} 20=98$
We know $\mathrm{Sn}=\frac{n}{2}(\mathrm{a}+1) \quad \mathrm{a}+19 \mathrm{~d}=98 \Rightarrow 3+19 \mathrm{~d}=98 \Rightarrow \mathrm{~d}=5$

$$
\mathrm{Sn}=\frac{\bar{n}}{2}(3+253)
$$

$$
\mathrm{Sn}=\frac{n}{2}(256)
$$

$$
\mathrm{Sn}=\mathrm{n} \times 128---\rightarrow(1)
$$

We know $\mathrm{Sn}=\frac{n}{2}(2 \mathrm{a}+(\mathrm{n}-1) \mathrm{d})$
$=\frac{n}{2}(6+5 n-5)$
$=\frac{n}{2}(5 n+1)--\cdots----\rightarrow(2)$
From 1 and 2
$n \times 128=\frac{n}{2}(5 n+1)$
$256=5 n+1$
$5 \mathrm{n}=255$
$\mathrm{n}=51$
Then A.P from last is 253, 248, 243, .......
$\mathrm{a}=253, \mathrm{~d}=-5$

$$
\begin{aligned}
\mathbf{S n} & =\frac{n}{2}(2 a+(n-1) d) \\
& =\frac{10}{2}(506+9(-5)) \\
& =\frac{10}{2}(506-45) \\
& =\mathbf{5 x 4 6 1} \\
& =\mathbf{2 3 0 5}
\end{aligned}
$$

Thus the sum of all 10 terms from last is 2305
35. Given equations are $2 x+y=8$ and $x-y=1$

| $x$ | 0 | 4 |
| :--- | :--- | :--- |
| $y$ | 8 | 0 |

$\mathrm{x}-\mathrm{y}=1$

| x | 0 | 1 |
| :--- | :--- | :--- |
| y | -1 | 0 |

36.' 'If two triangles are equiangular, then their corresponding sides are proportional".


Given: $\angle \mathrm{BAC}=\angle \mathrm{EDF}$
$\angle \mathrm{ABC}=\angle \mathrm{DEF}$
To prove: $\frac{A B}{D E}=\frac{B C}{E F}=\frac{C A}{F D}$
Construction: Mark points G and H on the side AB and AC such that
$\mathrm{AG}=\mathrm{DE}, \mathrm{AH}=\mathrm{DF}$
proof: in triangle AGH and DEF
AG=DE.....by construction
AH=DF ..... by contsruction
$\angle \mathrm{GAH}=\angle \mathrm{EDF}$...Given
therefore,
$\triangle A G H \cong \triangle F E D$ by SAS congruency thus
$\angle \mathrm{AGH}=\angle \mathrm{DEF} . .$. by CPCT
but
$\angle \mathrm{ABC}=\angle \mathrm{DEF}$
$\angle A G H=\angle A B C$
thus
GH\|BC
Now, In triangle ABC
$\frac{A B}{A G}=\frac{B C}{G H}=\frac{C A}{H A}$
Hence,
$\frac{A B}{D E}=\frac{B C}{E F}=\frac{C A}{F D}$
hence proved.
37. In $\triangle \mathrm{OAB}$, take $\mathrm{OB}=\mathrm{x} \mathrm{m}$


Let ' $x$ ' be the distance $b w A B$, then $B D=30-x=A E$ ' $y$ ' be the distance b/W $A B$ ', then $E D=y$.

In $\triangle^{\prime e}$ of $B$ $\tan 30^{\circ}=\frac{A B}{O B}$

$$
\begin{align*}
& \frac{1}{\sqrt{3}}=\frac{y}{x} \\
& x=\sqrt{3} y \tag{1}
\end{align*}
$$

From (1) \& (2)

$$
\begin{aligned}
& \sqrt{3} y=30-\frac{z-9}{\sqrt{3}} \\
& \sqrt{3} y+\frac{z-y}{\sqrt{3}}=30 \\
& \frac{3 y+z-y}{\sqrt{3}}=30 \\
& 2 y+z=30 \sqrt{3}
\end{aligned}
$$

In $\Delta^{\prime e} C A E$
$\tan 60^{\circ}=\frac{C E}{A E}$

$$
\begin{align*}
& \sqrt{3}=\frac{z-y}{30-x} \\
& 30-x=\frac{z-y}{\sqrt{3}} \\
& x=30-\frac{z-y}{\sqrt{3}} \tag{2}
\end{align*}
$$

In $\triangle$ le $O A B$.

$$
\begin{aligned}
& O A^{2}=x^{2}+y^{2} \\
& 20^{2}=x^{2}+y^{2} \\
& 400=x^{2}+y^{2}
\end{aligned}
$$

From (1) E5 (3)

$$
400=4 y^{2}
$$

$$
\frac{400}{x}=a y^{2}
$$

$$
\begin{aligned}
& \therefore y^{2}=100 \\
& y=10 \mathrm{~m}
\end{aligned}
$$

Then eqn (1) becomes

$$
\begin{aligned}
& x=\sqrt{3} y \\
& x=\sqrt{3} \mathrm{~m}
\end{aligned}
$$

Then equation (3) becones

$$
\begin{aligned}
& x=30-\frac{z-9}{\sqrt{3}} \\
& 10 \sqrt{3}=30-\frac{z-10}{\sqrt{3}} \\
& 10 \sqrt{3}+\frac{z-10}{\sqrt{3}}=30 \\
& \begin{array}{l}
\frac{10 \times 3+z-10}{\sqrt{3}}=30 \\
\frac{20+z}{\sqrt{3}}=30 \\
20+z=30 \sqrt{3} . \\
z=30 \sqrt{3}-20 . \quad \text { use } \sqrt{3}=1.73 \\
z=30(1.73)-20 \\
=51.9-20 \\
z=31.9
\end{array}
\end{aligned}
$$

38. Given :

Cone: area $A=38.5 \mathrm{sq} \mathrm{cm}$. we know area of circle $A=\Pi \mathrm{r}^{2}$

$$
\begin{gathered}
38.5=\frac{22}{7} \mathrm{r}^{2} \\
\mathrm{r}^{2}=12.25 \\
\mathrm{r}=3.5 \mathrm{~cm}
\end{gathered}
$$

Then radius of the base of the cone as well as hemisphere is 3.5 cm .
Height of the cone is $15.5-3.5=12 \mathrm{~cm}$, slant height of cone
$\mathrm{l}=\sqrt{122+3.52}$
$=\sqrt{144+12.25}$
$=\sqrt{156.25}$
$\mathrm{l}=12.5 \mathrm{~cm}$
TSA of toy $=$ CSA of cone + CSA of hemisphere

$$
\begin{aligned}
& =\prod \mathrm{rl}+2 \prod \mathrm{r}^{2} \\
& =\prod_{\mathrm{r}} \mathrm{r}(\mathrm{l}+2 \mathrm{r}) \\
& =\frac{22}{7} \times 3.5(12.5+7) \\
& =11(19.5) \\
& =214.5 \mathrm{sq} \mathrm{~cm} .
\end{aligned}
$$

Volume of Toy=volume of cone + volume of hemisphere

$$
\begin{aligned}
& =\frac{1}{3} \Pi \mathrm{r}^{2} \mathrm{~h}+\frac{2}{3} \Pi \mathrm{r}^{3} \\
& =\frac{1}{3} \Pi \mathrm{r}^{2}(\mathrm{~h}+2 \mathrm{r}) \\
& =\frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5(12.5+7) \\
& =\frac{11 \times 3.5}{3}(19.5) \\
& =250.25 \text { cubic } \mathrm{cm} .
\end{aligned}
$$

