Code No. 5018

Name : ....

Second Year – March 2017

Time : 2½ Hours
Cool-off time : 15 Minutes

Part – III

MATHEMATICS (SCIENCE)

Maximum : 80 Scores

General Instructions to Candidates :

- There is a 'cool-off time' of 15 minutes in addition to the writing time of 2½ hrs.
- You are not allowed to write your answers nor to discuss anything with others during the 'cool-off time'.
- Use the 'cool-off time' to get familiar with questions and to plan your answers.
- Read questions carefully before answering.
- All questions are compulsory and only internal choice is allowed.
- When you select a question, all the sub-questions must be answered from the same question itself.
- Calculations, figures and graphs should be shown in the answer sheet itself.
- Malayalam version of the questions is also provided.
- Give equations wherever necessary.
- Electronic devices except non-programmable calculators are not allowed in the Examination Hall.

നിർദ്ദേശങ്ങൾ :

- നിർദ്ദിഷ്ട സമയത്തിന് പുറമെ 15 മിനിറ്റ് 'കൂൾ ഓഫ് ടൈം' ഉണ്ടായിരിക്കും. ഈ സമയത്ത് ചോദ്യങ്ങൾക്ക് ഉത്തരം എഴുതാനോ, മറ്റുള്ളവരുമായി ആശയവിനിമയം നടത്താനോ പാടില്ല.
- ഉത്തരങ്ങൾ എഴുതുന്ന തിന് മുമ്പ് ചോദ്യങ്ങൾ ശ്രദ്ധാപൂർവ്വം വായിക്കണം.
- എല്ലാ ചോദ്യങ്ങൾക്കും ഉത്തരം എഴുതണം.
- ഒരു ചോദ്യനമ്പർ ഉത്തരമെഴുതാൻ തെരഞ്ഞെടുത്തു കഴിഞ്ഞാൽ ഉപചോദ്യങ്ങളും അതേ ചോദ്യനമ്പരിൽ നിന്ന് തന്നെ തെരഞ്ഞെടുക്കേണ്ടതാണ്.
- കണക്ക് കൂട്ടലുകൾ, ചിത്രങ്ങൾ, ഗ്രാഫുകൾ എന്നിവ ഉത്തരപേപ്പറിൽ തന്നെ ഉണ്ടായിരിക്കണം.
- ചോദ്യങ്ങൾ മലയാളത്തിലും നൽകിയിട്ടുണ്ട്.
- ആവശ്യമുള്ള സ്ഥലത്ത് സമവാക്യങ്ങൾ കൊടുക്കണം.
- പ്രോഗ്രാമുകൾ ചെയ്യാനാകാത്ത കാൽക്കുലേറ്ററുകൾ ഒഴികെയുള്ള ഒരു ഇലക്ട്രോണിക് ഉപകരണവും പരീക്ഷാഹാളിൽ ഉപയോഗിക്കുവാൻ പാടില്ല.

1. (a) Let R be a relation defined on A = {1, 2, 3} by R = {(1, 3), (3, 1), (2, 2)}. R is

(a) Reflexive (b) Symmetric

(c) Transitive (d) Reflexive but not transitive **(Score : 1)**

(b) Find fog and gof if $f(x) = |x + 1|$ and $g(x) = 2x - 1$. **(Scores : 2)**

(c) Let * be a binary operation defined on N × N by

(a, b) * (c, d) = (a + c, b + d).

Find the identity element for * if it exists. **(Scores : 2)**

2. (a) Principal value of $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ is

(a) $\frac{\pi}{3}$ (b) $-\frac{\pi}{3}$

(c) $\frac{\pi}{6}$ (d) $\frac{2\pi}{3}$ **(Score : 1)**

(b) Solve : $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$. **(Scores : 3)**

3. (a) The value of k such that the matrix $\begin{pmatrix} 1 & k \\ -k & 1 \end{pmatrix}$ is symmetric is

(a) 0 (b) 1

(c) −1 (d) 2 **(Score : 1)**

(b) If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ then prove that $A^2 = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$. **(Scores : 3)**

(c) If $A = \begin{bmatrix} 1 & 3 \\ 4 & 1 \end{bmatrix}$, then find $|3A'|$. **(Scores : 2)**

4. (a) If $A = \begin{bmatrix} a & 1 \\ 1 & 0 \end{bmatrix}$ is such that $A^2 = I$ then a equals

(a) 1 (b) −1

(c) 0 (d) 2 **(Score : 1)**

(b) Solve the system of equations :

$x - y + z = 4$

$2x + y - 3z = 0$

$x + y + z = 2$ using matrix method. **(Scores : 4)**

5. (a) Find the values of a and b such that the function

$$f(x) = \begin{cases} 5a & x \leq 0 \\ a \sin x + \cos x & 0 < x < \frac{\pi}{2} \\ b - \frac{\pi}{2} & x \geq \frac{\pi}{2} \end{cases}$$ is continuous **(Scores : 3)**

(b) Find $\frac{dy}{dx}$ if $(\sin x)^{\cos y} = (\cos y)^{\sin x}$. **(Scores : 3)**

6. (a) Slope of the normal to the curve $y^2 = 4x$ at (1, 2) is

(a) 1 (b) $\frac{1}{2}$

(c) 2 (d) -1 **(Score : 1)**

(b) Find the interval in which $2x^3 + 9x^2 + 12x - 1$ is strictly increasing. **(Scores : 4)**

OR

(a) The rate of change of volume of a sphere with respect to its radius when radius is 1 unit

(a) 4π (b) 2π

(c) π (d) $\frac{\pi}{2}$ **(Score : 1)**

(b) Find two positive numbers whose sum is 16 and the sum of whose cubes is minimum. **(Scores : 4)**

7. Find the following :

(a) $\int \frac{1}{x(x^7 + 1)} dx$ **(Scores : 3)**

(b) $\int_1^4 |x - 2| \, dx$ **(Scores : 3)**

8. Evaluate $\int_0^{\pi/2} \log \sin x \, dx$. **(Scores : 4)**

OR

Evaluate $\int_0^{4} x^2 dx$ as the limit of a sum. **(Scores : 4)**

9. (a) Area bounded by the curves $y = \cos x$, $x = \frac{\pi}{2}$, $x = 0$, $y = 0$ is

(a) $\frac{1}{2}$ (b) $\frac{2}{\pi}$

(c) 1 (d) $\frac{\pi}{2}$ **(Score : 1)**

(b) Find the area between the curves $y^2 = 4ax$ and $x^2 = 4ay$, $a > 0$. **(Scores : 5)**

10. (a) The order of the differential equation $x^4 \frac{d^2y}{dx^2} = 1 + \left(\frac{dy}{dx}\right)^3$ is

(a) 1 (b) 3

(c) 4 (d) 2 **(Score : 1)**

(b) Find the particular solution of the differential equation

$(1 + x^2)\frac{d^2y}{dx^2} + 2xy = \frac{1}{1 + x^2}$, $y = 0$ when $x = 1$. **(Scores : 5)**

11. (a) The projection of the vector $2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\hat{i} + \hat{j} + \hat{k}$ is

(a) $\frac{3}{\sqrt{3}}$ (b) $\frac{7}{\sqrt{3}}$

(c) $\frac{3}{\sqrt{17}}$ (d) $\frac{7}{\sqrt{17}}$ **(Score : 1)**

(b) Find the area of a parallelogram whose adjacent sides are the vectors $2\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} - \hat{j}$. **(Scores : 2)**

12. (a) The angle between the vectors $\hat{i} + \hat{j}$ and $\hat{j} + \hat{k}$ is

(a) 60° (b) 30°

(c) 45° (d) 90° **(Score : 1)**

(b) If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$, find the value of

$\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ **(Scores : 4)**

13. (a) The line $x - 1 = y = z$ is perpendicular to the line

(a) $\frac{x-2}{1} = \frac{y-1}{2} = \frac{z}{-3}$ (b) $x - 2 = y - 2 = z$

(c) $\frac{x-2}{1} = \frac{y-1}{2} = \frac{z}{3}$ (d) $x = y = \frac{z}{2}$ **(Score : 1)**

(b) Find the shortest distance between the lines

$\bar{r} = i + 2j + 3k + \lambda(i + j + k)$

$\bar{r} = i + j + k + \mu(i + j + k)$ **(Scores : 3)**

14. (a) Distance of the point (0, 0, 1) from the plane $x + y + z = 3$.

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{2}{\sqrt{3}}$

(c) $\sqrt{3}$ (d) $\frac{\sqrt{3}}{2}$ **(Score : 1)**

(b) Find the equation of the plane through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y + 4z = 5$ which is perpendicular to $x - y + z = 0$. **(Scores : 3)**

15. Consider the linear programming problem :

Maximize $Z = 50x + 40y$

Subject to the constraints

$x + 2y \geq 10$

$3x + 4y \leq 24$

$x \geq 0, y \geq 0$

(a) Find the feasible region. **(Scores : 3)**

(b) Find the corner points of the feasible region. **(Scores : 2)**

(c) Find the maximum value of Z. **(Score : 1)**

16. (a) If A and B are two events such that $A \subset B$ and $P(A) \neq 0$ then $P(A/B)$ is

(a) $\frac{P(A)}{P(B)}$ (b) $\frac{P(B)}{P(A)}$

(c) $\frac{1}{P(A)}$ (d) $\frac{1}{P(B)}$ **(Score : 1)**

(b) There are two identical bags. Bag I contains 3 red and 4 black balls while Bag II contains 5 red and 4 black balls. One ball is drawn at random from one of the bags.

(i) Find the probability that the ball drawn is red. **(Scores : 2)**

(ii) If the ball drawn is red what is the probability that it was drawn from bag I ? **(Scores : 2)**

OR

Consider the following probability distribution of a random variable X.

X	0	1	2	3	4
P(X)	$\frac{1}{16}$	$\frac{2}{16}$	K	$\frac{5}{16}$	$\frac{1}{16}$

(i) Find the value of K. **(Score : 1)**

(ii) Determine the Mean and Variance of X. **(Scores : 4)**