

**Second year Higher Secondary Examination**  
**PART III**  
**MATHEMATICS (SCIENCE)**  
 Maximum: 80 (Scores)

1. a) ii) (3,2)

b)  $a \oplus b = a^2 + b^2$

$$3 \oplus 4 = 3^2 + 4 = 13$$

$$a * b = a - b^2$$

$$(3 \oplus 4) * 5 = 13 * 5 = 13 - 5^2 = -12$$

2.  $X + Y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix}$

$$X - Y = \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$$

$$2X = \begin{bmatrix} 8 & 8 \\ 0 & 8 \end{bmatrix}$$

$$X = \begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix}$$

From (1)  $Y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} - X$

$$= \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}$$

$$2X - 3Y = 2 \begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix} + 3 \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 8 \\ 0 & 8 \end{bmatrix} + \begin{bmatrix} 3 & -6 \\ 0 & 15 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 2 \\ 0 & 23 \end{bmatrix}$$

3. a) Area of a circle,  $A = \pi r^2$

$$\frac{dA}{dr} = \pi \cdot 2r = 2\pi \times 10 = 20\pi \text{ cm}^2 / \text{cm}$$

b)  $A = \pi r^2$

$$\frac{dA}{dt} = \pi \cdot 2r \cdot \frac{dr}{dt}$$

$$= 2\pi \times 5 \times 0.7 = 10\pi \times 0.7 = 7\pi \text{ cm}^2 / \text{sec}$$

4. a)  $I = \int \frac{(1 + \log x)^2}{x} dx$

$$= \int (1 + \log x)^2 \cdot \frac{1}{x} dx$$

Put  $1 + \log x = t$

$$\left(0 + \frac{1}{x}\right) dx = dt$$

$$\therefore I = \int t^2 \cdot dt$$

$$= \frac{t^3}{3} + C$$

$$= \frac{1}{3}(1 + \log x)^3 + C$$

b)  $\int \frac{f'(x)}{f(x)} dx = \log|f(x)| + C$

5. Equation of the circle is  $x^2 + y^2 = a^2$

$$\therefore y^2 = a^2 - x^2$$

$$y = \sqrt{a^2 - x^2}$$

$$\text{Area} = 4 \int_0^a y dx$$

$$= 4 \int_0^a \sqrt{a^2 - x^2} dx$$

$$= 4 \left[ \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left( \frac{x}{a} \right) \right]_0^a$$

$$= 4 \left[ \frac{a}{2} \sqrt{a^2 - a^2} + \frac{a^2}{2} \sin^{-1} \left( \frac{a}{a} \right) - (0 + 0) \right]$$

$$= 4 \times \frac{a^2}{2} \sin^{-1}(1)$$

$$= 2a^2 \cdot \frac{\pi}{2} = \pi a^2 \text{ sq. units}$$

6.  $\frac{dy}{dx} = \frac{x+y}{x} \quad \dots(1)$

a) order = 1

b) it is a homogeneous DE.

Put  $y = vx$  and  $\frac{dy}{dx} = v + x \frac{dv}{dx}$

$$(1) \Rightarrow$$

$$v + x \frac{dv}{dx} = \frac{x + vx}{x}$$

$$v + x \frac{dv}{dx} = 1 + v \Rightarrow x \frac{dv}{dx} = 1$$

$$x dv = dx$$

$$dv = \frac{dx}{x} \text{ is in Variable Separable}$$

$$\therefore \int dv = \int \frac{dx}{x}$$

$$v = \log|x| + \log|C|$$

$$\frac{y}{x} = \log|Cx| \Rightarrow y = x \log|Cx|$$

7. Let  $x$  = Machine G and  $y$  = Machine H

- a) Objective function:  $Z = 20x + 30y$
- b) It is a maximization problem.
- c) Constraints are:  $3x + 4y \leq 10$ ;  $5x + 6y \leq 15$ ;  $x \geq 0$ ;  $y \geq 0$

8. a)  $A = \{1,2,3\}$   
 $B = \{4,5,6\}$   
 $f(1) = 5$ ;  $f(2) = 6$ ;  $f(3) = 4$

Since it is one-one as well as onto,  $f$  is bijective.

$$\therefore f = \{(1,5), (2,6), (3,4)\}$$

$$\therefore f^{-1} = \{(5,1), (6,2), (4,3)\}$$

- b) ans: ii) 1,2,3

$$1 * 2 = 2$$

$$2 * 1 = 2$$

$$\text{Further } 3 * 2 = 3$$

$$2 * 3 = 3$$

$\therefore *$  is commutative.

9. a) ii)  $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

b)  $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

c)  $\sin^{-1} x = \frac{3}{4}$

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$$

$$\therefore \cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x = \frac{\pi}{2} - \frac{3}{4}$$

10. a)  $LHL = \lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} (ax + 1) = a \times 3 + 1 = 3a + 1$

$$RHL = \lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} (ax + 3) = 3b + 3$$

$\therefore f(x)$  is continuous at  $x = 3$

$$LHL = RHL$$

$$3a + 1 = 3b + 3$$

$$3a + 1 - 3b - 3 = 0$$

$$3a - 3b - 2 = 0 \text{ in the relation.}$$

b)  $|x|$  is a continuous function everywhere, but it is not differentiable at  $x = 0$ .

Let  $f(x) = |x|$

At  $x = 0$

$$LHL = \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} |x| = \lim_{h \rightarrow 0} |-h| = |-0| = 0$$

$$RHL = \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} |x| = \lim_{h \rightarrow 0} |h| = 0$$

$$\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} |x| = 0$$

$$\therefore LHL = RHL = f(0)$$

$\therefore f(x)$  is continuous at  $x = 0$

Now  $Lf'(x) = \lim_{x \rightarrow 0^-} \frac{f(x-h) - f(x)}{-h}$

$$= \lim_{h \rightarrow 0} \frac{|0-h| - 0}{-h} = -1$$

$$Rf'(x) = \lim_{h \rightarrow 0} \frac{|0+h| - 0}{h}$$

$$= \lim_{h \rightarrow 0} \left( \frac{h}{h} \right) = 1$$

$$\therefore Lf'(x) \neq Rf'(x)$$

$f$  is not differentiable at  $x = 0$ . Hence proved.

[ 1 mark question. So proof is not needed]

11. a)  $y = x^2 - 2x + 7$

$$\frac{dy}{dx} = 2x - 2$$

Slope of the tangent =  $2(2) - 2 = 2$

Equation of the tangent is

$$y - y_1 = m(x - x_1)$$

$$y - 7 = 2(x - 2)$$

$$y - 7 = 2x - 4$$

$$2x - 4 - y + 7 = 0$$

$$2x - y + 3 = 0$$

b)  $f(x) = \sin x + \cos x$

$$f'(x) = \cos x - \sin x$$

$$f''(x) = -\sin x - \cos x$$

For max.,  $f'(x) = 0$

$$\cos x - \sin x = 0$$

$$\cos x = \sin x$$

$$\tan x = 1$$

$$\therefore x = \frac{\pi}{4} \in \left( 0, \frac{\pi}{2} \right)$$























