Second year Higher Secondary Examination
PART III
MATHEMATICS (SCIENCE)
Maximum: 80 (Scores)

1. a) ii) (3,2)

   b) $a \oplus b = a^2 + b^2$

   $3 \oplus 4 = 3^2 + 4 = 13$

   $a * b = a - b^2$

   $(3 \oplus 4) * 5 = 13 * 5 = 13 - 5^2 = -12$

2. $X + Y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix}$

   $X - Y = \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$

   $2X = \begin{bmatrix} 8 & 8 \\ 0 & 8 \end{bmatrix}$

   $X = \begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix}$

   From (1) $Y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} - X$

   $= \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix}$

   $= \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}$

   $2X - 3Y = 2\begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix} + 3\begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}$

   $= \begin{bmatrix} 8 & 8 \\ 0 & 8 \end{bmatrix} + \begin{bmatrix} 3 & -6 \\ 0 & 15 \end{bmatrix}$

   $= \begin{bmatrix} 11 & 2 \\ 0 & 23 \end{bmatrix}$

3. a) Area of a circle, $A = \pi r^2$

   $\frac{dA}{dr} = \pi.2r = 2\pi \times 10 = 20\pi\, cm^2 / cm$

   b) $A = \pi r^2$

   $\frac{dA}{dt} = \pi.2r.\frac{dr}{dt}$

   $= 2\pi \times 5 \times 0.7 = 10\pi \times 0.7 = 7\pi\, cm^2 / \sec$

4. a) $I = \int \frac{(1 + \log x)^2}{x} dx$

   $= \int (1 + \log x)^2 . \frac{1}{x} dx$

Put $1+\log x = t$

$$\left(0+\frac{1}{x}\right)dx = dt$$

$$\therefore I = \int t^2 .dt$$

$$=\frac{t^3}{3}+C$$

$$=\frac{1}{3}(1+\log x)^3+C$$

b) $\int \frac{f'(x)}{f(x)}dx = \log|f(x)|+C$

5. Equation of the circle is $x^2+y^2=a^2$

$$\therefore y^2 = a^2 - x^2$$

$$y=\sqrt{a^2-x^2}$$

Area $=4\int_0^a ydx$

$$=4\int_0^a \sqrt{a^2-x^2}dx$$

$$=4\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a$$

$$=4\left[\frac{a}{2}\sqrt{a^2}-a^2+\frac{a^2}{2}\sin^{-1}\left(\frac{a}{a}\right)-(0+0)\right]$$

$$=4\times\frac{a^2}{2}\sin^{-1}(1)$$

$$=2a^2.\frac{\pi}{2}=\pi a^2 \ \ sq.units$$

6. $\frac{dy}{dx}=\frac{x+y}{x} \qquad ....(1)$

a) order = 1

b) it is a homogeneous DE.

Put $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

(1) $\Rightarrow$

$$v+x\frac{dv}{dx}=\frac{x+vx}{x}$$

$$v+x\frac{dv}{dx}=1+v \Rightarrow x\frac{dv}{dx}=1$$

$$xdv = dx$$

$$dv=\frac{dx}{x} \quad \text{is in Variable Separable}$$

$$\therefore \int dv = \int \frac{dx}{x}$$

$v = \log|x| + \log|C|$

$\frac{y}{x} = \log|Cx| \Rightarrow y = x\log|Cx|$

7. Let x = Machine G and y = Machine H

a) Objective function: $Z = 20x + 30y$

b) It is a maximization problem.

c) Constraints are: $3x + 4y \le 10$; $5x + 6y \le 15$; $x \ge 0$; $y \ge 0$

8. a) $A = \{1,2,3\}$
$B = \{4,5,6\}$
$f(1) = 5$; $f(2) = 6$; $f(3) = 4$

Since it is one-one as well as onto, f is bijective.
$\therefore f = \{(1,5),(2,6),(3,4)\}$
$\therefore f^{-1} = \{(5,1),(6,2),(4,3)\}$

b) ans: ii) 1,2,3

$1 * 2 = 2$

$2 * 1 = 2$

Further $3 * 2 = 3$
$2 * 3 = 3$

$\therefore *$ is commutative.

9. a) ii) $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

b) $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

c) $\sin^{-1} x = \frac{3}{4}$

$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$

$\therefore \cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x = \frac{\pi}{2} - \frac{3}{4}$

10. a) $LHL = \lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} (ax+1) = a \times 3 + 1 = 3a + 1$

$RHL = \lim_{x\to 3^+} f(x) = \lim_{x\to 3^-} (ax+3) = 3b + 3$

$\because f(x)$ is continuous at $x = 3$

$LHL = RHL$

$3a + 1 = 3b + 3$

$3a + 1 - 3b - 3 = 0$

$3a - 3b - 2 = 0$ in the relation.

b) $|x|$ is a continuous function everywhere, but it is not differentiable at $x=0$.

Let $f(x)=|x|$

At $x=0$

$$LHL = \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} |x| = \lim_{h\to 0} |-h| = |-0| = 0$$

$$RHL = \lim_{x\to 0^+} f(x) \lim_{x\to 0^+} |x| = \lim_{h\to 0} |+h| = 0$$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} |x| = 0$$

$$\because LHL = RHL = f(0)$$

$\therefore f(x)$ is continuous at $x=0$

$$\text{Now } Lf'(x) = \lim_{x\to 0^-} \frac{f(x-h)-f(x)}{-h}$$

$$= \lim_{h\to 0} \frac{|0-h|-0}{-h} = -1$$

$$R\ f'(x) = \lim_{h\to 0} \frac{|0+h|-0}{h}$$

$$= \lim_{h\to 0}\left(\frac{h}{h}\right) = 1$$

$$\therefore Lf'(x) \neq Rf'(x)$$

f is not differentiable at $x=0$. Hence proved.

[1 mark question. So proof is not needed]

11. a) $y = x^2 - 2x + 7$

$$\frac{dy}{dx} = 2x - 2$$

Slope of the tangent $= 2(2) - 2 = 2$

Equation of the tangent is

$$y - y_1 = m(x - x_1)$$

$$y - 7 = 2(x-2)$$

$$y - 7 = 2x - 4$$

$$2x - 4 - y + 7 = 0$$

$$2x - y + 3 = 0$$

b) $f(x) = \sin x + \cos x$

$$f'(x) = \cos x - \sin x$$

$$f''(x) = -\sin x - \cos$$

For max., $f'(x) = 0$

$$\cos x - \sin x = 0$$

$$\cos x = \sin x$$

$$\tan x = 1$$

$$\therefore x = \frac{\pi}{4} \in \left(0, \frac{\pi}{2}\right)$$

$$f''\left(\frac{\pi}{4}\right) = -\sin\frac{\pi}{4} - \cos\frac{\pi}{4}$$

$$= -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = \frac{-2}{\sqrt{2}} = -\sqrt{2} < 0$$

$\therefore f(x)$ is max.

Max. value of $f(x) = f\left(\frac{\pi}{4}\right)$

$$= \sin\frac{\pi}{4} + \cos\frac{\pi}{4}$$

$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

12. $I = \int \frac{x+2}{2x^2+6x+5} dx$

$$x + 2 = A(4x+6) + B$$

$$4A = 1 \Rightarrow A = \frac{1}{4}$$

$$6A + B = 2$$

$$B = 2 - 6A = 2 - 6 \times \frac{1}{4}$$

$$= \frac{8-6}{4} = \frac{2}{4} = \frac{1}{2}$$

$$x + 2 = \frac{1}{4}(4x+6) + \frac{1}{2}$$

$$\therefore I = \frac{1}{4}\int \frac{4x+6}{2x^2+6x+5} dx + \frac{1}{2}\int \frac{dx}{2x^2+6x+5}$$

$$I = \frac{1}{4}\log\left|2x^2+6x+5\right| + \frac{1}{2}I_1$$

$$Now\, I_1 = \int \frac{dx}{2x^2+6x+5}$$

Let $\frac{1}{2x^2+6x+5} = \frac{1}{2\left(x^2+3x+\frac{5}{2}\right)}$

$$= \frac{1}{2\left[\left(x+\frac{3}{2}\right)^2 + \frac{5}{2} - \left(\frac{3}{2}\right)^2\right]}$$

$$= \frac{1}{2\left[\left(x+\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2\right]}$$

$$\therefore I_1 = \frac{1}{2}\int \frac{dx}{\left(x+\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \frac{1}{2} \times \frac{1}{\frac{1}{2}} \tan^{-1}\left(\frac{x+\frac{3}{2}}{\frac{1}{2}}\right) + C_1$$

$$= \tan^{-1}(2x+3) + C_1$$

$$in(1)$$

$$I = \frac{1}{4}\log\left|2x^2+6x+5\right| + \frac{1}{2}\tan^{-1}(2x+3) + C$$

13. $x\dfrac{dy}{dx} + y = \dfrac{1}{x^2}$

$$\Rightarrow \frac{dy}{dx} + \frac{1}{x}y = \frac{1}{x^3}$$

a) $P = \dfrac{1}{x}; Q = \dfrac{1}{x^3}$

$$\int p dx = \int \frac{1}{x} dx = \log|x|$$

$$e^{\int pdx} = e^{\log|x|} = |x| = x$$

$$\therefore IF = x$$

b) $Q\,e^{\int pdx} = \dfrac{1}{x^3}.x$

$$\int Qe^{\int pdx}.dx = \int \frac{1}{x^2}dx$$

$$= \frac{-1}{x} + C$$

$\therefore$ Soln. is $y e^{\int pdx} = \int Qe^{\int pdx} dx + C$

$$yx = -\frac{1}{x} + C$$

$$\therefore y = -\frac{1}{x^2} + \frac{C}{x}$$

14. $\overrightarrow{PQ} = -3\hat{i} + 4\hat{j} + 4\hat{k}$

$$\overrightarrow{PR} = -5\hat{i} + 2\hat{j} + 4\hat{k}$$

a) $\theta = \cos^{-1}\left(\dfrac{\overrightarrow{PQ}.\overrightarrow{PR}}{\left|\overrightarrow{PQ}\right|\left|\overrightarrow{PR}\right|}\right)$

$$= \cos^{-1}\left[\frac{15+8+16}{\sqrt{9+16+16}\sqrt{25+4+16}}\right]$$

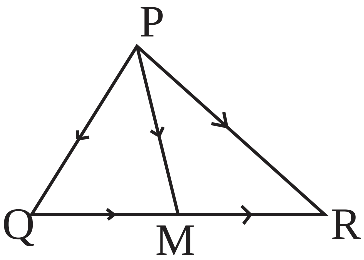
$$= \cos^{-1}\left(\frac{39}{\sqrt{41}\sqrt{45}}\right)$$

$$= \cos^{-1}\frac{39}{\sqrt{9\times5\times41}}$$

$$= \cos^{-1}\left(\frac{39}{3\sqrt{205}}\right)$$

$$= \cos^{-1}\left(\frac{13}{\sqrt{205}}\right)$$

b) $\overrightarrow{QR} = \overrightarrow{PR} - \overrightarrow{PQ}$



$= (-5 - -3)\hat{i} + (2-4)\hat{j} + (4-4)\hat{k}$

$= -2\hat{i} - 2\hat{j} + 0\hat{k}$

$= -2\hat{i} - 2\hat{j}$

Since M is midpoint of QR

$\overrightarrow{MR} = \frac{1}{2}\overrightarrow{QR} = -\vec{i} - \vec{j}$

$\overrightarrow{PM} + \overrightarrow{MR} = \overrightarrow{\mathrm{PR}}$

$\overrightarrow{PM} = \overrightarrow{\mathrm{PR}} - \overrightarrow{MR} = -5\hat{i} + 2\hat{j} + 4\hat{k} - \left(-\hat{i} - \hat{j}\right) = -4\hat{i} + 3\hat{j} + 4\hat{k}$

$\therefore$ length of the median, $PM = \left|\overrightarrow{PM}\right| = \sqrt{16+9+16} = \sqrt{41}$

15. a) $\vec{a} + \vec{b} = 6\hat{i} + 2\hat{j} + 2\hat{k}$

$\vec{a} - \vec{b} = 4\hat{i} - 4\hat{j} - 8\hat{k}$

$\left(\vec{a} + \vec{b}\right).\left(\vec{a} - \vec{b}\right) = 24 - 8 - 16$

$= 24 - 24 = 0$

$\therefore \left(\vec{a} + \vec{b}\right)$ and $\left(\vec{a} - \vec{b}\right)$ are perpendicular.

b) If $\vec{a}, \vec{b}, \vec{c}$ are coplanar, $\left[\vec{a}\,\vec{b}\,\,\vec{c}\right] = 0$

$$\left[\vec{a}\,\vec{b}\,\,\vec{c}\right] = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix}$$

$= 1(15-12) - -2(10 - -4) + 3(-6-3)$

$= 1(3) + 2(14) + 3(-9)$

$= 3 + 28 - 27 = 3 + 1 = 4$

$\because \left[\vec{a}\,\vec{b}\,\vec{c}\right] \neq 0,\ \vec{a}, \vec{b}\ and\ \vec{c}$ are not coplanar.

16. a) Equation of the line is

$$\frac{x-5}{5-0} = \frac{y--2}{-2-0} = \frac{z-3}{3-0}$$

$$\frac{x-5}{5} = \frac{y+2}{-2} = \frac{z-3}{3}$$

b) $\overrightarrow{OP} = \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Given that $|\vec{r}| = 12$

$\vec{r} = |\vec{r}|\left(l\hat{i} + m\hat{j} + n\hat{k}\right)$

$= 12\left(\cos 45\hat{i} + \cos 60\,\hat{j} + \cos\gamma\,\hat{k}\right)$

$\cos^2 45 + \cos^2 60 + \cos^2\gamma = 1$

$$\frac{1}{2}+\frac{1}{4}+\cos^2\gamma=1$$

$$\cos^2\gamma=1-\frac{3}{4}=\frac{1}{4}$$

$$\cos\gamma=\frac{1}{2}$$

$$\therefore \bar{r}=12\left(\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{2}\hat{j}+\frac{1}{2}\hat{k}\right)$$

$$=\frac{12}{\sqrt{2}}\hat{i}+\frac{12}{2}\hat{j}+\frac{12}{2}\hat{k}$$

$$=6\sqrt{2}\hat{i}+6\hat{j}+6\hat{k}$$

$$\therefore P\,is\left(6\sqrt{2},6,6\right)$$

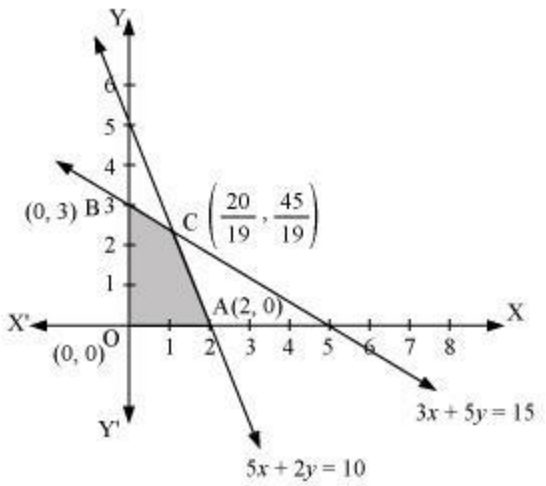
17. $3x+5y=15$

x	0	5
y	3	0

$5x+2y=0$

x	0	2
y	5	0

The feasible region determined by the system of constraints, $3x + 5y \le 15$, $5x + 2y \le 10$, $x \ge 0$, and $y \ge 0$, are as follows.



The corner points of the feasible region are O (0, 0), A (2, 0), B (0, 3), and $C\left(\frac{29}{19},\frac{45}{19}\right)$.

To find B:

$$3x+5y=15 \Rightarrow 6x+10y=30$$

$$5x+2y=10 \Rightarrow \frac{25x+10y=50}{-19x \quad =-20}$$

$$\therefore x \quad =\frac{20}{19}$$

$$5y = 15 - 3x$$

$$= 15 - 3 \times \frac{20}{19}$$

$$= \frac{285 - 60}{19} = \frac{225}{19}$$

$$5y = \frac{225}{19}$$

$$\therefore y = \frac{45}{19}$$

$$\therefore B is \left(\frac{20}{19}, \frac{45}{19} \right)$$

Corner point	Z = 5x + 3y	
0(0, 0)	0	
A(2, 0)	10	
B(0, 3)	9	
$\left(\frac{29}{19}, \frac{45}{19} \right)$	$\frac{235}{19}$	Maximum

$\therefore$ the maximum value of Z is $\frac{235}{19}$ at $C\left(\frac{29}{19}, \frac{45}{19} \right)$.

18. a) $A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$

i) $A' = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$

ii) $\frac{1}{2}(A + A') = \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$,is symmetric.

$\frac{1}{2}(A + A') = \frac{1}{2}\begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix}$,is skew-symmetric.

Now $\frac{1}{2}(A + A') + \frac{1}{2}(A - A')$

$$= \frac{1}{2}\begin{bmatrix} 6+0 & 1+5 & -5+3 \\ 1+-5 & -4+0 & -4+6 \\ -5+-3 & -4+-6 & 4+0 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 6 & 6 & -1 \\ -4 & -4 & 1 \\ -8 & -10 & 2 \end{bmatrix}$$

=A, a square matrix. Hence proved.

b) If $A'\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$

$\therefore A = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$

$A'.A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}\begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$

$= \begin{bmatrix} \cos^2 x + \sin^2 x & -\sin x \cos x + \sin x \cos x \\ -\sin x \cos x + \sin x \cos x & \sin^2 x + \cos^2 x \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

= I, proved.

19. a) $LHS = \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$

$R_1 \to R_1 + R_2$

$= \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$

$= (x+y+z)\begin{vmatrix} 1 & 1 & 1 \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$

$= (x+y+z) \times 0 \qquad (P\ III)$

$= 0$

$= RHS$

b) i) $AX = B$

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

ii) Let $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$

$|A| = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix}$

$= 2(-4 - -4) - -3(-6 - -4) + 5(3-2)$

$= 2(0) + 3(-2) + 5(1)$

$= -6 + 5 = -1 \neq 0$

$$A_{11} = (+)0 = 0$$
$$A_{12} = (-)-2 = 2$$
$$A_{13} = (+)1 = 1$$
$$A_{21} = (-)1 = -1$$
$$A_{22} = (+)-9 = -9$$
$$A_{23} = (-)5 = -5$$
$$A_{31} = (+)2 = 2$$
$$A_{32} = (-)-23 = 23$$
$$A_{33} = (+)13 = 13$$

$$\text{Adj } A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}.adj\ A$$

$$= \frac{1}{-1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

$$X = A^{-1}B$$

$$= \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 + -5 + 6 \\ -22 + -45 + 69 \\ -11 + -25 + 39 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\Rightarrow x = 1; y = 2; z = 3$$

20. a) $x^2 + 2xy + 2y^2 = 1$

Diff w.r.t. x

$$2x + 2\left(x\frac{dy}{dx} + y\right) + 2 \times 2y\frac{dy}{dx} = 0$$

$$x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(x + 2y) = -x - y = -(x + y)$$

$$\frac{dy}{dx} = \frac{-(x + y)}{x + 2y}$$

b) $y^x = 2^x$

taking log on both sides

$\log y^x = \log 2^x$

$x \log y = x \log 2$

$x.\frac{1}{y}.\frac{dy}{dx} + \log y \times 1 = \log 2 \times 1$

$\frac{x}{y}\frac{dy}{dx} = \log 2 - \log y$

$= \log\left(\frac{2}{y}\right)$

$x\frac{dy}{dx} = y \log\left(\frac{2}{y}\right)$

$\therefore \frac{dy}{dx} = \frac{y}{x}\log\left(\frac{2}{y}\right)$

c) $x = \cos\theta; y = \sin\theta$

$\frac{dx}{d\theta} = -\sin\theta$

$\frac{dy}{d\theta} = \cos\theta$

$\frac{dy}{dx} = \frac{dy}{d\theta} / \frac{dx}{d\theta}$

$= \frac{\cos\theta}{-\sin\theta} = -\cot\theta$

$when\, \theta = \frac{\pi}{4}, \frac{dy}{dx} = -\cot\frac{\pi}{4} = -1$

21. a) $I = \int \frac{x}{(x+1)(x+2)}dx$

Let $\frac{x}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$

$x = A(x+2) + B(x+1)$

Put $x = -1$

$-1 = A(-1+2)$

$-1 = 1A \Rightarrow A = -1$

In (1)

$\frac{x}{(x+1)(x+2)} = \frac{-1}{x+1} + \frac{2}{x+2}$

$\therefore I = -\int \frac{dx}{x+1} + 2\int \frac{dx}{x+2}$

$= -\log|x+1| + 2\log|x+2| + C$

$= \log\left|(x+2)^2\right| - \log|x+1| + C$

$= \log\left|\frac{(x+2)^2}{x+1}\right| + C$

b) $I=\int_0^1 x e^{x^2}dx$

Put $x^2=t$

$2x\,dx=dt$

$x\,dx=\frac{1}{2}dt$

When $x=0, t=0^2=0$

When $x=1, t=1^2=1$

$\therefore I=\frac{1}{2}\int_0^1 e^t.dt$

$=\frac{1}{2}\left(e^t\right)_0^1$

$=\frac{1}{2}\left(e^1-e^0\right)$

$=\frac{1}{2}(e-1)$

c) $I=\int_{-5}^{5}|x+2|dx$

$=\int_{-5}^{-2}-(x+2)dx+\int_{-2}^{5}(x+2)dx$

$=-\left[\frac{x^2}{2}+2x\right]_{-5}^{-2}+\left(\frac{x^2}{2}+2x\right)_{-2}^{5}$

$=-\left[\frac{4}{2}+-4-\left(\frac{25}{2}+-10\right)\right]+\left[\frac{25}{2}+10-\left(\frac{4}{2}+-4\right)\right]$

$=-\left[2-4-\frac{25}{2}+10\right]+\left[\frac{25}{2}+10-2+4\right]$

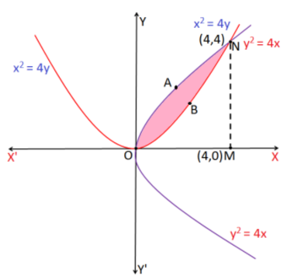
$=-\left[8-\frac{25}{2}\right]+\frac{25}{2}+12$

$=-8+\frac{25}{2}+\frac{25}{2}+12$

$=4+25=29$

22.

a)



b) Two parabolas intersect at O(0,0) and N(4,4)

c) Area of the enclosed region $=\int_0^4\left(2\sqrt{x}-\frac{x^2}{4}\right)dx$

$$= \left[2 \times \frac{2}{3} x^{\frac{3}{2}} - \frac{1}{4} \times \frac{x^3}{3}\right]_0^4 = \left[\frac{4}{3} x^{\frac{3}{2}} - \frac{x^3}{12}\right]_0^4$$

$$= \left[\frac{4}{3}(4)^{\frac{3}{2}} - \frac{4^3}{12}\right] - \left[\frac{4}{3}(0)^{\frac{3}{2}} - \frac{0^3}{12}\right] = \frac{4}{3}(8) - \frac{16}{3} = \frac{16}{3} sq.units$$

23. a) $\vec{b} = \hat{i} - 3\hat{j} + 2\hat{k}$

$\vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{a}_2 = 4\hat{i} + 5\hat{j} + 6\hat{k}$

Since the line are parallel,

Shortest distance $= \left|\frac{\left|\vec{b} \times (\vec{a}_2 - \vec{a}_1)\right|}{\left|\vec{b}\right|}\right|$

$\vec{a}_2 - \vec{a}_1 = 3\hat{i} + 3\hat{j} + 3\hat{k}$

$$\vec{b} \times (\vec{a}_2 - \vec{a}_1) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 3 & 3 & 3 \end{vmatrix}$$

$= \hat{i}(-9-6) - \hat{j}(3-6) + \hat{k}(3--9)$

$= -15\hat{i} - 3\hat{j} + 12\hat{k}$

$\left|\vec{b} \times (\vec{a}_2 - \vec{a}_1)\right| = \sqrt{225+9+144}$

$= \sqrt{378}$

$= 3\sqrt{42}$

$\left|\vec{b}\right| = \sqrt{1+9+4} = \sqrt{14}$

$\therefore d = \frac{3\sqrt{42}}{\sqrt{14}}$

$= \frac{3 \times \sqrt{14} \times \sqrt{3}}{\sqrt{14}} = 3\sqrt{3}$

b) Given that $a = 2, b = 3, c = 4$

Equation of a plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

$\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1$

$6x + 4y + 3z = 12$ || multiplying by 12

$6x + 4y + 3z - 12 = 0$

c) $\perp r$ distance from origin to the above plane is

$$d = \left| \frac{-12}{\sqrt{6^2+4^2+3^2}} \right|$$

$$= \left| \frac{12}{\sqrt{36+16+9}} \right|$$

$$= \left| \frac{12}{\sqrt{61}} \right|$$

$$= \frac{12}{\sqrt{61}}$$

24. a) $n(s) = 36$

$A = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(3,1),(3,2),(3,3),(3,4),$
$(3,5),(3,6),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\}$

$B = \{(1,1),(2,1),(3,1),(4,1),(5,1),(6,1),(1,3),(2,3),(3,3),(4,3)$
$(5,3),(6,3),(1,5),(2,5),(3,5),(4,5),(5,5),(6,5)\}$

$A \cap B = \{(1,1),(1,3),(1,5),(3,1),(3,3),(3,5),(5,1),(5,3),(5,5)\}$

$$P(A \cap B) = \frac{9}{36} = \frac{1}{4}$$

$$P(A).P(B) = \frac{18}{36} \times \frac{18}{36}$$

$$= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$\therefore$ A and B are independent events.

b) Let E_1, E_2 and E_3 be the events of getting the boxes.

$$\therefore P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$

Let E be the event of getting a red ball.

$$P(E/E_1) = \frac{2}{2} = 1$$

$$P(E/E_2) = \frac{0}{2} = 0$$

$$P(E/E_3) = \frac{1}{2}$$

$$P(E_1/E) = \frac{P(E_1)P(E/E_1)}{P(E_1)P(E/E_1)+P(E_2)P(E/E_2)+P(E_3)P(E/E_3)}$$

$$= \frac{\frac{1}{3}\times\frac{2}{2}}{\frac{1}{3}\times\frac{2}{2}+\frac{1}{3}\times\frac{0}{2}+\frac{1}{3}\times\frac{1}{2}} = \frac{2}{2+0+1} = \frac{2}{3}$$

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