Reg. No. : ....

Name : ....

SME-27

SECOND YEAR HIGHER SECONDARY MODEL EXAMINATION, FEBRUARY 2020

Part – III

Time : 2½ Hours

MATHEMATICS (SCIENCE)

Cool-off time : 15 Minutes

Maximum : 80 Scores

General Instructions to Candidates :

- There is a 'Cool-off time' of 15 minutes in addition to the writing time.
- Use the 'Cool-off time' to get familiar with questions and to plan your answers.
- Read questions carefully before answering.
- Read the instructions carefully.
- Calculations, figures and graphs should be shown in the answer sheet itself.
- Malayalam version of the questions is also provided.
- Give equations wherever necessary.
- Electronic devices except non-programmable calculators are not allowed in the Examination Hall.

വിദ്യാർത്ഥികൾക്കുള്ള പൊതുനിർദ്ദേശങ്ങൾ :

- നിർദ്ദിഷ്ട സമയത്തിന് പുറമെ 15 മിനിറ്റ് 'കൂൾ ഓഫ് ടൈം' ഉണ്ടായിരിക്കും.
- 'കൂൾ ഓഫ് ടൈം' ചോദ്യങ്ങൾ പരിചയപ്പെടാനും ഉത്തരങ്ങൾ ആസൂത്രണം ചെയ്യാനും ഉപയോഗിക്കുക.
- ഉത്തരങ്ങൾ എഴുതുന്നതിന് മുമ്പ് ചോദ്യങ്ങൾ ശ്രദ്ധാപൂർവ്വം വായിക്കണം.
- നിർദ്ദേശങ്ങൾ മുഴുവനും ശ്രദ്ധാപൂർവ്വം വായിക്കണം.
- കണക്ക് കൂട്ടലുകൾ, ചിത്രങ്ങൾ, ഗ്രാഫുകൾ, എന്നിവ ഉത്തരപേപ്പറിൽ തന്നെ ഉണ്ടായിരിക്കണം.
- ചോദ്യങ്ങൾ മലയാളത്തിലും നല്കിയിട്ടുണ്ട്.
- ആവശ്യമുള്ള സ്ഥലത്ത് സമവാക്യങ്ങൾ കൊടുക്കണം.
- പ്രോഗ്രാമുകൾ ചെയ്യാനാകാത്ത കാൽക്കുലേറ്ററുകൾ ഒഴികെയുള്ള ഒരു ഇലക്ട്രോണിക് ഉപകരണവും പരീക്ഷാഹാളിൽ ഉപയോഗിക്കുവാൻ പാടില്ല.

Answer any 6 questions from 1 to 8. Each carries 3 scores. **(6 × 3 = 18)**

1. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 1 & 0 \\ 1 & 3 & -2 \end{bmatrix}$

   (i) Which of the following is the order of the matrix A + B ? **(1)**

   (a) 2×2 (b) 3×2

   (c) 2×3 (d) 3×3

   (ii) Find 3A **(1)**

   (iii) Evaluate 3A – B **(1)**

2. (i) If $y = \sin^{-1} x$, find $\frac{dy}{dx}$. **(1)**

   (ii) Hence show that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$. **(2)**

3. (i) Which of the following is the solution of the differential equation $\frac{dy}{dx} + \sin x = 0$? **(1)**

   (a) $y = C \cos x$ (b) $y = \cos x + C$

   (c) $y = \sin x + C$ (d) $y = C \sin x$

   (ii) Form the differential equation representing the family of curves $y = a \sin (x + b)$, where a and b are arbitrary constants. **(2)**

4. Using properties of determinants prove that $\begin{bmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{bmatrix} = 4\,abc$. **(3)**

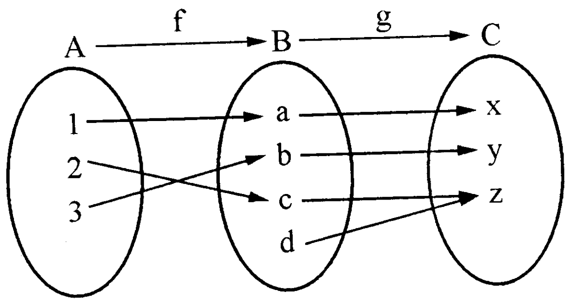
5. (i) Find the maximum and minimum values of f, if any of the function $f(x) = |x| + 3$, $x \in \mathbb{R}$. **(2)**

   (ii) What is the absolute maximum value of the function $f(x) = |x| + 3, x \in [-3, 2]$? **(1)**

6. (i) Write a function which is not continuous at $x = 0$ and justify your answer. **(2)**

(ii) Check the continuity of the function $f(x) = \begin{cases} x + 2, & \text{if } x < 0 \\ -x + 2, & \text{if } x > 0 \end{cases}$ **(1)**

7. Consider the arrow diagram of the functions f and g.



(i) Check whether the functions f and g are bijective. Justify. **(1)**

(ii) Write the function gof. **(1)**

(iii) Is gof a bijective function ? Justify. **(1)**

8. (i) If α, β, γ are the direction angles of a vector, then which the following can be $\alpha + \beta$? **(1)**

(a) 80° (b) 60°

(c) 120° (d) Can't be determined.

(ii) Find a direction consines of the line passing through the points (2, 8, 3) and (4, 5, 9). **(2)**

Answer any 8 questions from 9 to 18. Each carries 4 scores. **(8 × 4 = 32)**

9. (i) If $\tan^{-1} x = \frac{\pi}{10}$, then the value of $\cot^{-1} x$ is **(1)**

(a) $\frac{\pi}{5}$ (b) $\frac{2\pi}{5}$

(c) $\frac{3\pi}{5}$ (d) $\frac{4\pi}{5}$

(ii) Find the value of $\sin\left(2 \tan^{-1}\frac{2}{3}\right) + \cos\left(\tan^{-1}\sqrt{3}\right)$. **(3)**

10. Let A = {–1, 0, 1}

(i) Give reason why the operation defined by $a \otimes b = \frac{a}{b}$ is not a binary operation on A. **(1)**

(ii) (a) Write a binary operation $*$ on A. **(1)**

(b) Find $(-1 * -1) * -1$. **(1)**

(iii) How many binary operations are possible on A ? **(1)**

11. (i) Find the equation of a line L passing through the points (–1, 0, 2) and (2, 1, 3). **(2)**

(ii) If $\vec{c} = \hat{i} + \hat{j} + \lambda\hat{k}$ be a vector perpendicular to the above line, then find λ. **(1)**

(iii) Find the equation of a plane on which the line L lies. **(1)**

12. Find $\frac{dy}{dx}$ of the following :

(i) $y = \sec(\tan x)$ **(1)**

(ii) $x^y = y^x$ **(3)**

13. (i) Find $\int \tan^{-1} x \, dx$. **(2)**

(ii) Hence find the area bounded by the curve $y = \tan^{-1} x$ with X – axis from $x = 0$ and $x = 1$. **(2)**

14. Solve the differential equation $\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$. **(4)**

15. Consider a plane which is equidistant from the points (1, 2, 1) and (3, 4, 7).

(i) Which of the following is a point on the plane ? **(1)**

(a) (1, 3, 1) (b) (4, 2, 3)

(c) (2, 3, 4) (d) (1, 3, 6)

(ii) Find the equation of the above plane. **(3)**

16. Let $\vec{a} = 2\hat{i} + \hat{j} - 3\hat{k}$ and $\vec{b} = 4\hat{i} + \hat{j} + \hat{k}$ be two vectors.

(i) Find a vector $\vec{c}$ perpendicular to $\vec{a}$ and $\vec{b}$. **(1)**

(ii) Find the volume of the parallelepiped with co-initial vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$. **(1)**

(iii) If $\vec{c}$ is rotated in such a way that it makes 60° with its initial direction, then what is the volume of the new parallelepiped formed ? **(2)**

17. (i) Evaluate $\int_0^\pi x \sin x \, dx$. **(2)**

(ii) Hence evaluate the area bounded by the curve $y = x \sin x$ between $x = -\pi$ and $x = \pi$. **(2)**

18. In a ladies hostel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspapers. A student is selected at random.

(i) Find the probability that she reads neither Hindi nor English newspapers. **(2)**

(ii) If she reads Hindi newspaper, find the probability that she reads English newspaper too. **(1)**

(iii) If she reads English newspaper, find the probability that she reads Hindi newspaper too. **(1)**

Answer any 5 questions from 19 to 25. Each carries 6 scores. **(5 × 6 = 30)**

19. (i) Construct a 3 × 3 matrix A, where elements are given by $a_{ij} = 2i - j$. **(2)**

(ii) Verify that $C = A - A'$ is a skew symmetric matrix. **(2)**

(iii) Verify that C^2 is a symmetric matrix. **(2)**

20. Consider the matrix $A = \begin{bmatrix} 1 & 3 & 6 \\ -1 & -1 & 2 \\ 1 & 1 & 5 \end{bmatrix}$

(i) Find $|A|$. **(1)**

(ii) Verify that $A \times \text{adj}A = |A|I$ **(4)**

(iii) Evaluate $|A^{-1}|$. **(1)**

21. Integrate the following with respect to x :

(i) $\dfrac{1}{x^2 - 6x + 13}$ **(3)**

(ii) $\dfrac{\cos x}{(\sin x - 1)(\sin x - 2)}$ **(3)**

22. Let a pair of dice be thrown and the random variable X be the sum of the numbers that appear on the two dice.

(i) Write the probability distribution of X. **(3)**

(ii) Find variance of X. **(3)**

23. (i) Consider the function $f(x) = 2x^3 - 6x^2 + 1$.

(a) Find the equation of tangents parallel to X-axis. **(2)**

(b) Find the intervals in which the function f is decreasing. **(2)**

(ii) The length x of a rectangle is decreasing at the rate of 5 cm/s and the width y is increasing at the rate of 4 cm/s. When $x = 8$ cm and $y = 6$ cm, find the rate of change of the area of the rectangle. **(2)**

24. Let $\vec{a} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} - 8\hat{k}$ be two vectors.

(i) Find a vector $\vec{c}$ representing a diagonal of the parallelogram with $\vec{a}$ and $\vec{b}$ as the adjacent sides. **(2)**

(ii) Find the projection of $\vec{b}$ on $\vec{c}$. **(2)**

(iii) Find the angle between the vectors $\vec{c}$ and $\vec{a}$. **(2)**

25. Consider the following L.P.P.

Maxmise : $Z = 600x + 400y$

Subject to the constraints :

$x + 2y \leq 12$

$2x + y \leq 12$

$4x + 5y \geq 20$

$x \geq 0, y \geq 0$

(i) Draw the feasible region. **(3)**

(ii) Solve the LPP. **(3)**