Reg. No. : ....

Name : ....

SY-227

SECOND YEAR HIGHER SECONDARY EXAMINATION, MARCH 2021

Part – III

MATHEMATICS

Maximum : 60 Scores

Time : 2 Hours

Cool-off time : 20 Minutes

General Instructions to Candidates :

- There is a 'Cool-off time' of 20 minutes in addition to the writing time.
- Use the 'Cool-off time' to get familiar with questions and to plan your answers.
- Read questions carefully before answering.
- Read the instructions carefully.
- Calculations, figures and graphs should be shown in the answer sheet itself.
- Malayalam version of the questions is also provided.
- Give equations wherever necessary.
- Electronic devices except non-programmable calculators are not allowed in the Examination Hall.

വിദ്യാർത്ഥികൾക്കുള്ള പൊതുനിർദ്ദേശങ്ങൾ :

- നിർദ്ദിഷ്ട സമയത്തിന് പുറമെ 20 മിനിറ്റ് 'കൂൾ ഓഫ് ടൈം' ഉണ്ടായിരിക്കും.
- 'കൂൾ ഓഫ് ടൈം' ചോദ്യങ്ങൾ പരിചയപ്പെടാനും ഉത്തരങ്ങൾ ആസൂത്രണം ചെയ്യാനും ഉപയോഗിക്കുക.
- ഉത്തരങ്ങൾ എഴുതുന്നതിന് മുമ്പ് ചോദ്യങ്ങൾ ശ്രദ്ധാപൂർവ്വം വായിക്കണം.
- നിർദ്ദേശങ്ങൾ മുഴുവനും ശ്രദ്ധാപൂർവ്വം വായിക്കണം.
- കണക്ക് കൂട്ടലുകൾ, ചിത്രങ്ങൾ, ഗ്രാഫുകൾ, എന്നിവ ഉത്തരപേപ്പറിൽ തന്നെ ഉണ്ടായിരിക്കണം.
- ചോദ്യങ്ങൾ മലയാളത്തിലും നല്കിയിട്ടുണ്ട്.
- ആവശ്യമുള്ള സ്ഥലത്ത് സമവാക്യങ്ങൾ കൊടുക്കണം.
- പ്രോഗ്രാമുകൾ ചെയ്യാനാകാത്ത കാൽക്കുലേറ്ററുകൾ ഒഴികെയുള്ള ഒരു ഇലക്ട്രോണിക് ഉപകരണവും പരീക്ഷാഹാളിൽ ഉപയോഗിക്കുവാൻ പാടില്ല.

Answer the following questions from 1 to 29 upto a maximum score of 60.

Part - A

Questions from 1 to 10 carry 3 scores each. **(10 × 3 = 30)**

1. Find the values of x for which

$$\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$$ **(3)**

2. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

   (i) Find adj A **(2)**

   (ii) Find A.adj A. **(1)**

3. Find the value of k so that the function

$$f(x) = \begin{cases} kx + 1 & , \text{if } x \leq 5 \\ 3x - 5 & , \text{if } x > 5 \end{cases}$$

is continuous at $x = 5$. **(3)**

4. Verify Rolle's theorem for the function $f(x) = x^2 + 2x - 8, x \in [-4, 2]$. **(3)**

5. Find the rate of change of the area of a circle with respect to its radius r when r = 5 cm. **(3)**

6. Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $7\hat{i} - \hat{j} + 8\hat{k}$. **(3)**

7. Find the equation of a plane passing through the point (1, 4, 6) and the normal to the plane is $\hat{i} - 2\hat{j} + \hat{k}$. **(3)**

8. (i) Which of the following can be the domain of the function $\cos^{-1}x$?

   (a) $(0, \pi)$ (b) $[0, \pi]$

   (c) $(-\pi, \pi)$ (d) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ **(1)**

   (ii) Find the value of $\cos^{-1}(-1/2) + 2\sin^{-1}(1/2)$. **(2)**

9. Find the area of a triangle with vertices (–2, –3), (3, 2) and (–1, –8). **(3)**

10. Find the general solution of the differential equation $\frac{dy}{dx} - y = \cos x$. **(3)**

Part - B

Questions from 11 to 22 carry 4 scores each. **(12 × 4 = 48)**

11. Consider the matrices $A = \begin{bmatrix} 3 & 4 \\ -5 & -1 \end{bmatrix}$ and $3A + B = \begin{bmatrix} 2 & 8 \\ 3 & -4 \end{bmatrix}$

(i) Find the matrix B. **(2)**

(ii) Find AB. **(2)**

12. If $A = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}$ and $B = [1, 3, -6]$

(i) What is the order of AB ? **(1)**

(ii) Verify $(AB)' = B'A'$. **(3)**

13. (i) If $xy < 1$, $\tan^{-1} x + \tan^{-1} y$ = _______.

(a) $\tan^{-1}\frac{x - y}{1 + xy}$ (b) $\tan^{-1}\frac{1 - xy}{x + y}$

(c) $\tan^{-1}\frac{x + y}{1 - xy}$ (d) $\tan^{-1}\frac{x + y}{1 + xy}$ **(1)**

(ii) Prove that $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$. **(3)**

14. Find $\frac{dy}{dx}$

(i) $x^2 + xy + y^2 = 100$. **(2)**

(ii) $y = \sin^{-1}\left(\frac{2x}{1 + x^2}\right), -1 \le x \le 1$. **(2)**

15. Find the intervals in which the function f given by $f(x) = 2x^2 - 3x$ is

(i) increasing

(ii) decreasing **(4)**

16. (i) Find the order and degree of the differential equation $\left(\frac{ds}{dt}\right)^4 + \frac{3\, d^2s}{dt^2} = 0$. **(1)**

(ii) Find the general solution of the differential equation $\frac{dy}{dx} = (1 + x^2)(1 + y^2)$. **(3)**

17. Find a unit vector both perpendicular to the vectors if

$\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$. **(4)**

18. Find the shortest distance between the skew lines

$\vec{r} = \hat{i} + 2\hat{j} + \hat{k} + \lambda(\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu(2\hat{i} + \hat{j} + 2\hat{k})$ **(4)**

19. If $P(A) = 0.8$, $P(B) = 0.5$ and $P(B|A) = 0.4$. Find

(i) $P(A \cap B)$ **(2)**

(ii) $P(A|B)$ **(1)**

(iii) $P(A \cup B)$ **(1)**

20. (i) Let R be a relation on a set A = {1, 2, 3}, defined by R = {(1, 1), (2, 2), (3, 3), (1, 3)}. Then the ordered pair to be added to R to make it a smallest equivalence relation is _______.

(a) (2, 1) (b) (3, 1)

(c) (1, 2) (d) (1, 3) **(1)**

(ii) Determine whether the relation R in the set A = {1, 2, 3, 4, 5, 6} as R = {(x, y) : y is divisible by x} is reflexive, symmetric and transitive. **(3)**

21. Find $\frac{dy}{dx}$

(i) x^x **(2)**

(ii) $x = 2at^2$; $y = at^4$ **(2)**

22. Integrate :

$$\int \frac{x}{(x+1)(x+2)} dx.$$ **(4)**

Part - C

Questions from 23 to 29 carry 6 scores each. **(7 × 6 = 42)**

23. (i) Construct a 3 × 2 matrix A = $[a_{ij}]$ whose elements are given by

$$a_{ij} = 3\hat{i} - \hat{j}$$ **(2)**

(ii) Express $\begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix. **(4)**

24. Solve the following system of equations by matrix method

$3x - 2y + 3z = 8$

$2x + y - z = 1$

$4x - 3y + 2z = 4$ **(6)**

25. (i) Let f : {1, 3, 4} → {1, 2, 5} and g : {1, 2, 5} → {1, 3} be given by

f = {(1, 2), (3, 5), (4, 1)} and g = {(1, 3), (2, 3), (5, 1)}. Write down gof. **(3)**

(ii) Consider f : R → R given by f(x) = 2x + 1. Show that f is invertible. Find the inverse of f. **(3)**

26. (i) Find the slope of the tangent to the curve $y = x^3 - x$ at $x = 2$. **(2)**

(ii) Find the equation of tangent to the above curve. **(2)**

(iii) What is the maximum value of the function $\sin x + \cos x$? **(2)**

27. Integrate :

(i) $\int \sin x \sin(\cos x)\, dx$. **(3)**

(ii) $\int_0^1 \frac{\tan^{-1}x}{1+x^2}\, dx$. **(3)**

28. Solve the following problem graphically

Maximise : $z = 3x + 2y$

Subject to : $x + 2y \leq 10$

$3x + y \leq 15,$

$x, y \geq 0$ **(6)**

29. (i) Find the area of the region bounded by the curve $y^2 = x$ and the lines $x = 1$ and $x = 4$ and the x-axis. **(3)**

(ii) Find the area of the region bounded by two parabolas $y = x^2$ and $y^2 = x$. **(3)**