Name : ....

Reg No : ....

SY–HSS

SECOND YEAR HIGHER SECONDARY MODEL EXAMINATION

Part III

Time:2Hours

MATHEMATICS

Cool-off time:15 minutes
Maximum:60 scores

General Instructions to Candidates :

- There is a Cool-off time of 15 minutes in addition to the writing time.
- Use the Cool-off time to get familiar with questions and to plan your answers.
- Read questions carefully before answering.
- Read the instructions carefully.
- Calculations, figures and graphs should be shown in the answer sheet itself.
- Give equations wherever necessary.
- Electronic devices except non-programmable calculators are not allowed in the Examination Hall.

PART-1

A. Answer any five questions from 1 to 9 each carries 1 score.

(1×5 =5)

1. The function $f : R \to R$ defined by $f(x) = x$ .Find $f(f(x)$

Solution: $f(f(x) = f(x) = x$

2. Find the value of $cos(sec^{-1}x + cosec^{-1}x)$

Solution: $cos(\frac{\pi}{2}) = 0$

3. Let a be a square matrix of order 3×3 , then which among the following is the value of $|kA|$

Solution: $k^3|A|$

4. The rate of change of area of a circle with respect to radius , when the radius 6 cm is

> **Solution:** Area $A = \pi r^2$,
> $\dfrac{dA}{dr} = 2\pi r$ at r=6 $\Longrightarrow$ $\dfrac{dA}{dr} = 12\pi$

5. Write the value of the definite integral $\int_{\pi/2}^{\pi/2} sinxdx$

> **Solution:** sinx is an odd function
> $\therefore \int_{-\pi/2}^{\pi/2} sinxdx = 0$

6. Write the degree of the differential equation $\dfrac{d^2y}{dx^2} + y = 0$

> **Solution:** 1

7. if $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = 8\hat{i} + 4\hat{j} + 12\hat{k}$, then $\vec{a} \times \vec{b}$ is

> **Solution:** Here both vectors are parallel so cross product is zero

8. The Cartesion equation of a line is $\frac{x}{2} = \frac{y}{2} = \frac{z}{2}$. write the corresponding vector equation.

> **Solution:** $\vec{r} = \lambda(2\hat{i} + 4\hat{j} + 2\hat{k})$

9. If $A \subset B$, then the value of $P(B/A)$ is ....

> **Solution:** $P(B/A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{P(A)}{P(A} = 1$

B. Answer all questions from 10 to 13 each carries 1 score.(Nonfocus area)
(1×4 =4)

10. Write the principal value of $sin^{-1}\frac{1}{2}$.

> **Solution:** $\dfrac{\pi}{6}$

11. If a is a singular matrix ,then the value of $|A|$ is

Solution: 0

12. $y = e^{logx}$,find $\frac{dy}{dx}$.

Solution: $y = e^{logx} = x$,then $\frac{dy}{dx} = 1$

13. If l,m,n are direction cosines of a line in space then $l^2 + m^2 + n^2$ is .....

Solution: $l^2 + m^2 + n^2 = 1$

PART-II

A. Answer any two questions from 14 to 17 each carries 2 score.(Focus area)
(2×2 =4)

14. Construct a 2×2 matrix $A = [a_{ij}]$ whose elements are given by $a_{ij} = 2i - j$

Solution: $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$

15. Find the intervals in which the function f given by $f(x) = x^2 - 4x + 6$ is strictly increasing.

Solution: Given $f(x) = x^2 - 4x + 6$ then $f'(x) = 2x - 4$,$f(x)$ is strictly increasing in which $f'(x) > 0 \implies 2x - 4 > 0 \implies x > 2$
$\therefore f(x)$ is strictly increasing in $(2, \infty)$

16. Find equation of normal to the curve $y = x^3$ at $(1, 1)$

Solution: $y = x^3 \implies \dfrac{dy}{dx} = 3x^2$
Slope of the tangent at (1,1)=3
$\therefore$ slope of normal $=-\frac{1}{3}$
$\therefore$ equation of normal $y - 1 = -\frac{1}{3}(x - 1)$
$\implies x + 3y - 4 = 0$

17. Form the differential equation corresponding to family of straight lines y=mx where m is an arbitrary constant

Solution: $y = mx$ differentiating $\frac{dy}{dx} = m$ $\therefore$ Differential equation is $y = \frac{dy}{dx}x$

B. Answer any two questions from 18 to 20 each carries 2 score.(Nonfocus area)
(2×2 =4)

18. Find $\frac{dy}{dx}$ if $x - y = \pi$

Solution: $x - y = \pi \implies 1 - \frac{dy}{dx} = 0 \qquad \therefore \frac{dy}{dx} = 1$

19. Solve the differential equation $\frac{dy}{dx} = \frac{x+y}{x}$.

Solution:
$\frac{dy}{dx} = \frac{x+y}{x}$
$\frac{dy}{dx} = 1 + \frac{y}{x}$
$\frac{dy}{dx} - \frac{y}{x} = 1$
integrating factor $e^{\int -1/x} = 1/x$
General solution $\frac{y}{x} = \int 1/x \implies \frac{y}{x} = log|x| + c$

20. Show that the vectors $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = -2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} + 5\hat{k} arecoplanar$

Solution: vectors are coplanar then $[\vec{a} \ \vec{b} \ \vec{c}] = 0 \implies [\vec{a} \ \vec{b} \ \vec{c}] = \begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix} = 0$

$\therefore$ vectors are coplanar

PART-III

A. Answer any three questions from 21 to 24 each carries 3 score.(Focus area)
(3×3 =9)

21. Show that the relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$ is reflexive but neither symmetric nor transitive .

Solution:
Every elements are related to itself $\therefore$ R is reflexiive
$(1,2) \in R$ bt $(2,1) \notin R$ so R is not symmetric
$(1,2),(2,3) \in R$ but $(1,3) \notin R$ so R is not transitive .

22. If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ Find K so that $A^2 = KA - 2I$

Solution: Given $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ so $A^2 = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$
. $$KA - 2I = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix} \quad \therefore k = 1$$

23. Find the area of the parallelogram whose adjacent sides are given by the vectors $\vec{a} = 3\hat{i} + \hat{j} + 4\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$.

Solution: Area of the parallelogram $= |\vec{a} \times \vec{b}|$, $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{vmatrix} = 5\hat{i} + \hat{j} - 4\hat{k}$
$\therefore$ area $|\vec{a} \times \vec{b}| = \sqrt{42}$

24. Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively . If both try to solve the problem independently .Find the probability that exactly one of them solve the problem

Solution:
P(exacly one of them solve the problem)$= P(A \cap \overline{B}) + P(\overline{A} \cap B)$
. $$= P(A).P(\overline{B}) + P(\overline{A}).P(B) = \frac{1}{2} \times \frac{2}{3} + \frac{1}{2} \times \frac{1}{3} = \frac{3}{6} = \frac{1}{2}$$

B. Answer any two questions from 25 to 27 each carries 3 score.(Nonfocus area) (3×2 =6)

25. Consider the binary operation $\wedge$ on the set $\{1,2,3,4,5\}$ defined by $a \wedge b = \min\{a,b\}$. write the operation table of $\wedge$

Solution:

$\wedge$	1	2	3	4	5
1	1	1	1	1	1
2	1	2	2	2	2
3	1	2	3	3	3
4	1	2	3	4	4
5	1	2	3	4	5

26. By using elementary operation , Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

Solution: $A.A^{-1} = I \implies \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}.A^{-1} = I$

$R_2 \to R_2 - 2.R_1 \implies$

$\begin{bmatrix} 1 & 2 \\ 0 & -5 \end{bmatrix}.A^{-1} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$

$R_2 \to R_2 \times -\frac{1}{5} \implies \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}.A^{-1} = \begin{bmatrix} 1 & 0 \\ \frac{2}{5} & \frac{-1}{5} \end{bmatrix}$

$R_1 \to R_1 - 2R_2 \implies \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.A^{-1} = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{-1}{5} \end{bmatrix} \implies A^{-1} = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{-1}{5} \end{bmatrix}$

27. Find $\int_0^2 x^2 dx$ as limit of infinite sum .

Solution: $\int_0^2 x^2 dx = \lim_{h \to 0} h\{f(0) + f(h) + f(h^2) + ... + f((n-1)h^2)\}$; $h = \frac{2}{n}$

$\implies I = \lim_{h \to 0} h\{h^2) + 4h^2 + 9h^2 + ... + f((n-1)^2 h^2)\}$

$\implies I = \lim_{h \to 0} h^3\{1) + 2^2 + 3^2 + ... + ((n-1)^2)\}$

$= \lim_{n \to \infty} \frac{8}{n^3} \dfrac{(n-1)n(2(n-1)+1)}{6} = \dfrac{16}{6} = \dfrac{8}{3}$

PART-IV

A. Answer any 3 questions from 28 to 31 each carries 4 score.(Focus area)
(4×3 =12)

28. Show that:
i) $tan^{-1}\frac{1}{2} + tan^{-1}\frac{2}{11} = tan^{-1}\frac{3}{4}$
ii) $cos^{-1}(4x^3 - 3x) = 3cos^{-1}x, x \in \left[\frac{1}{2}, 1\right]$

Solution:
i)$tan^{-1}\frac{1}{2} + tan^{-1}\frac{2}{11} = tan^{-1}\dfrac{\frac{1}{2} + \frac{2}{11}}{1 - (\frac{1}{2} \times \frac{2}{11})} = tan^{-1}\dfrac{15}{20} = tan^{-1}\dfrac{3}{4}$
ii) put $x = cos\theta$ then $cos^{-1}(4x^3 - 3x) = cos^{-1}cos(3\theta) = 3\theta = 3cos^{-1}x$

29. Verify Mean value theorem for the function $f(x) = x^2$ in the interval $[2, 4]$.

Solution: f(a)=f(2)=4 and f(b)=f(4)=16
$f'(x) = 2x \implies f'(c) = 2c$
by L.M.V.T $f'(c) = 2c = \dfrac{16-4}{4-2} = 6 \implies c = 3 \in [2, 4]$ hence verified

30. Find the area of the region bounded by the curve $y^2 = x$ and the line x=1 and x=4 and the x axis and the first quadrant

Solution: Area $= \int_1^4 \sqrt{x} = \left[\dfrac{x^{3/2}}{3/2}\right]_1^4 = \dfrac{16}{3} - \dfrac{2}{3} = \dfrac{14}{3}$

31. Find the shortest distance between the lines whose vector equations are $\vec{r} = \hat{i}+\hat{j}+\hat{k}+\lambda(2\hat{i}-\hat{j}+\hat{k})$ and $\vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$

Solution: $d = |\dfrac{(\vec{a}_2 - \vec{a}_1).(b_1 \times b_2)}{|b_1 \times b_2|}|$
$\vec{a}_2 - \vec{a}_1 = \hat{i} - 2\hat{j} - 2\hat{k}$ and $(b_1 \times b_2) = 3\hat{i} - \hat{j} - 7\hat{k} \implies d = \dfrac{19}{\sqrt{59}}$

B. Answer any one questions from 32 to 33 each carries 4 score.(Nonfocus area)
(4×1 =4)

32. Random variable X has following probability distribution

X	0	1	2	3	4
P(x)	0.1	k	2k	2k	k

i) Find the value of k
ii)Find $P(X < 3)$

Solution: i)$\sum P(X) = 1 \implies 6k + 0.1 = 1 \therefore k = \dfrac{0.9}{6} = 0.15$
ii)$P(X < 3) = 0.1 + 0.15 + 0.3 = 0.55$

33. Find the cartesion and vector eqation of the plane with intercepts 2,3,4 on the x ,y ,z axis respectively .

Solution:
Equation of plane , Cartesion form
$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \implies \frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1$
ie, $6x + 4y + 3z - 1 = 0 \therefore$ vector form is $\vec{r}.(6\hat{i} + 4\hat{j} + 3\hat{k}) = 1$

PART-V

A. Answer any two questions from 34 to 36 each carries 2 score.(Focus area)
(6×2 =12)

34. Solve the following system of equations by matrix method
$3x - 2y + 3z = 8$
. $2x + y - z = 1$
. $4x - 3y + 2z = 4$

Solution: $3x - 2y + 3z = 8$
. $2x + y - z = 1$
. $4x - 3y + 2z = 4$

$$\implies \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix} . \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix} \text{ ie , AX= B form}$$

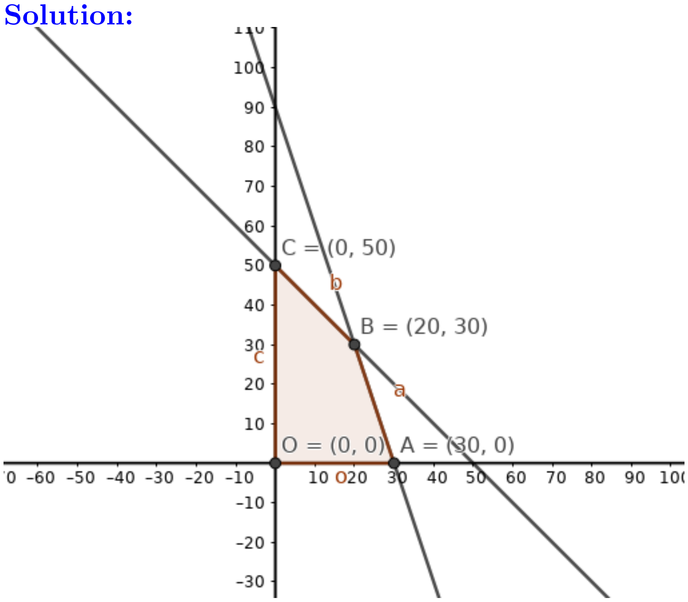
$$|A| = -17 \text{ and } adj(A) = \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$$

$$X = A^{-1}B \implies X = \frac{-1}{17} \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix} . \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

. x=1 ,y=2 , z=3

35. Solve the following linear programing problem graphically :
Maximise $Z = 4x + y$ subject to the constraints $x + x \leq 50$, $3x + y \leq 90$, $x \geq 0$ $y \geq 0$.

Solution:



corner point	Z
(0,0)	0
(30,0)	120
(20,30)	110
(0,50)	50

Maximum $Z = 120$ at (30,0)

36. Find :

i)$\int \dfrac{sin(tan^{-1}x)}{1+x^2}dx$ ii)$\int_0^{\frac{\pi}{2}} \dfrac{\sqrt{sinx}}{\sqrt{sinx}+\sqrt{cosx}}dx$ iii)$\int \dfrac{dx}{x^2-16}dx$

Solution:

i) $tan^{-1}x = t \implies \dfrac{1}{1+x^2}dx = dt$

$\int \dfrac{sin(tan^{-1}x)}{1+x^2}dx = \int sintdt = -cost = -cos(tan^{-1}x)$ ii)Let I=$\int_0^{\frac{\pi}{2}} \dfrac{\sqrt{sinx}}{\sqrt{sinx}+\sqrt{cosx}}dx$

Unig property I=$\int_0^{\frac{\pi}{2}} \dfrac{\sqrt{cosx}}{\sqrt{cosx}+\sqrt{sinx}}dx$

$\therefore 2I = \int_0^{\frac{\pi}{2}} dx = \dfrac{\pi}{2} \implies I = \dfrac{\pi}{4}$

iii)$\int \dfrac{dx}{x^2-16} = \dfrac{1}{2\times 4}log\dfrac{x-4}{x+4} + c$