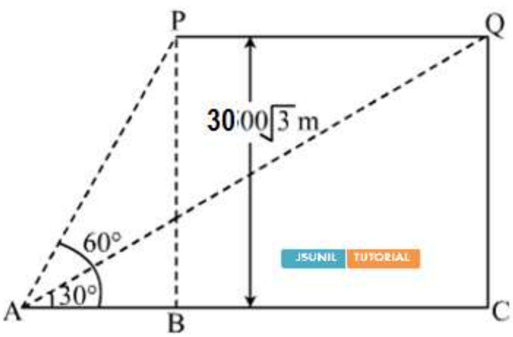
Series HRS Code-30/2 Summative Assessment –II

Subject Mathematics class 10 CBSE Board 2014

SECTION-C

15. The angle of elevation of an areoplane from a point on the ground is 60^0 . After a flight of 30 sec the angle of elevation became 30^0. If areoplane is flying at a constant high $3000\sqrt{3}$ m . Find the speed of areoplane.

Solution:



Let P and Q be the two positions of the plane and A be the point of observation. Let ABC be the horizontal line through A. It is given that angles of elevation of the plane in two positions P and Q from a point A are 60° and 30° respectively.

∴ ∠PAB = 60°, ∠QAB = 30°. It is also given that PB = $3000\sqrt{3}$ m meters

In ΔABP, we have

Tan 60^0 = BP/AB

$\sqrt{3} = 3000\sqrt{3}/ AB$

AB = 3000 m

In ΔACQ,we have

$\tan 30^0$ = CQ/AC

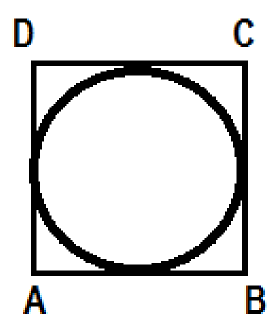
$1/\sqrt{3} = 3000\sqrt{3}/ AC$

AC = 9000 m

∴ Distance = BC = AC – AB = 9000m – 3000m = 6000m

Thus, the plane travels6 in 30 seconds

Hence speed of plane = 6000/30 = 200 m/sec = 720km/h



16. The largest possible sphere is curved out of a wooden solid cube of side 7 cm. find the volume of the wood left.

Solution:

Demeter of sphere curved out = side of cube = 7cm $\Rightarrow$ r =3.5cm

Volume of cube = $a^3 = 7^3 = 343\ cm^3$

Volume of sphere curved out = $4/3\ \pi\ r^3 = 4/3 \times 22/7 \times 7/2 \times 7/2 \times 7/2 = 179.66\ cm^3$

The volume of the wood left = 343 -179.66 = 163.34 cm^3

17. Water in a canal, 6 m wide and 1.5 m deep, is flowing at a speed of 4km/h. How much area will it irrigate in 10 min. , if 8 cm standing water is needed for irrigation.

Solution: Speed = 4km/h = 200/3 m/min

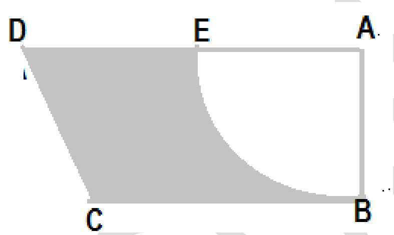
Volume of water irrigate in 10 min = 10 x 6 x 1.5 x 200/3 = $6000m^3$

Volume of water irrigated = base area (of irrigated land) x height = base area x 8cm =base area x 0.08m

6000 = base area x 0.08

Base area = 6000/0.08 =75000 m^2 = 7.5 hectare

18. in fig. 02. , ABCD is a trapezium of area $24.5cm^2$.In it ADII BC, <DAB = 90, AD = 10cm and BC = 4 cm . If ABE is quadrant of circle, find the area of shaded region.



Area of trapezium = $24.5cm^2$

½ [AD + BC] x AB = 24.5 cm^2

½ [10+4] x AB = 24.5

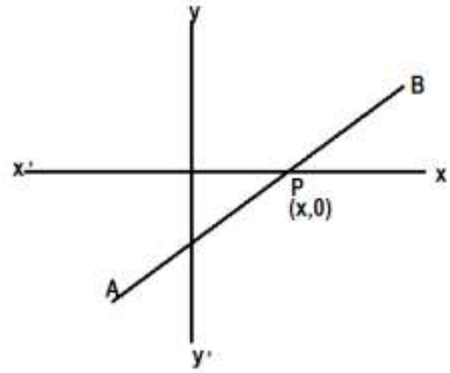
AB = 3.5 cm

r = 3.5 cm

Area of quadrant = $\frac{1}{4}\pi r^2 = 0.25 \times 22/7 \times 3.5 \times 3.5 = 9.625\ cm^2$

The area of shaded region = 24.5 - 9.625 = 14.875 cm^2

19. Find the ratio in which the line segment joining the point A(3,-3)and B(-2,7) is divided by x-axis . Also find the co – ordinate of the point of division.



Solution:

Point p lies on x axis so it's ordinate is 0

Using section formula

Let the ratio be k: 1 Let the coordinate of the point be P(x , 0)

As given A(3,-3) and B(-2,7)

$P_y = (my_2 + ny_1) / (m+n)$

0 = k x 7 +1x -3 / k+1

0(k+1) =7k -3

0 =7k - 3

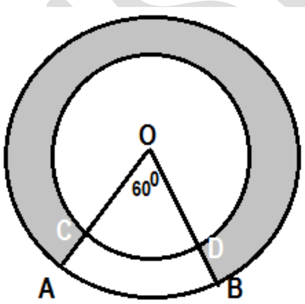
3=7k

k = 3 /7

K:1 = 3 : 7

$P_x = (mx_2 + nx_1) / (m+n)$ = [(3/7 x- 2)+(1x3)]/(3/7 + 1) = 1.5

20.In fig. 03., two concentric circles with centre O , have radii 21 cm and 42 cm . if <AOB = 60^0 , find the area of shaded region.



Solution:

The area of shaded region = Area of ring – Area of ABCD

$= [\pi (R^2 - r^2)] - [\pi(R^2 - r^2)\theta]/360$

$= [\pi (R^2 - r^2)] [1 - (\theta/360)]$

$= [22/7 (42^2 - 21^2)] [1 - (60/360)] = 3465$ cm^2

21. Solve for x

(16/x)-1 = 15/(x+1)

Solution: (16/x)-1 = 15/(x+1)

$\Rightarrow$ (16 - x)/x = 15/(x+1)

$\Rightarrow$ 15 x = 16x + 16 – x^2 – x

$\Rightarrow$ 16 = x^2 $\Rightarrow$ x = 4

22. The sum of 2^{nd} and the 7^{th} terms of an AP is 30. If 15^{th} term is 1 less than twice the 8^{th} term . Find the AP

Solution : The sum of 2^{nd} and the 7^{th} terms of an AP is 30

$\Rightarrow$ a + d + a + 6 d = 30

$\Rightarrow$ 2a + 7d = 30

15^{th} term is 1 less than twice the 8^{th} term

$\Rightarrow$ a + 14d = 2(a + 7d) -1

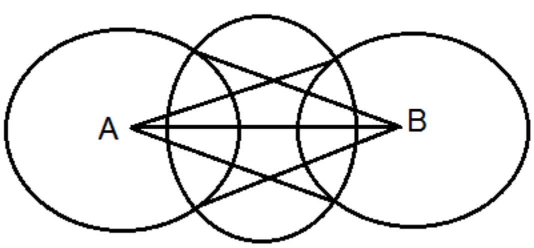
$\Rightarrow$ a + 14d = 2a + 14d -1

$\Rightarrow$ a = 1

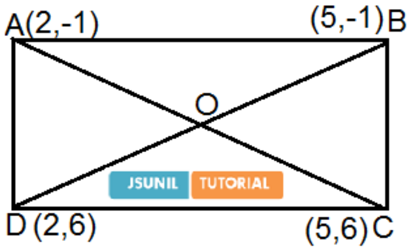
Now, 2x1 + 7d = 30 $\Rightarrow$ d = 4

AP : 1,5,9 ………

23. Draw a line segment AB of length 8 cm. Taking a centre A draw a circle of radius 4 cm and taking B as a centre draw another circle of radius 3 cm. Construct tangent to each circle from the centre of the other circle.



24. Prove that the diagonal of rectangle ABCD, with the vertices A (2,-1), B(5, -1), C(5,6) and D(2,6) are equal and bisect each other.



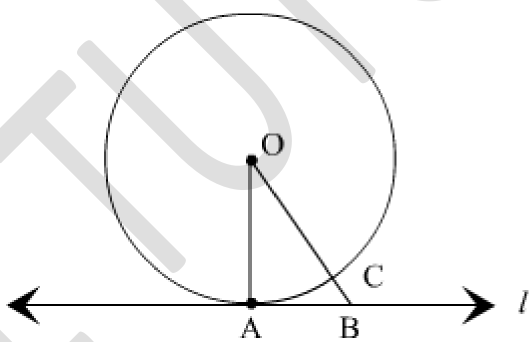
$AC^2 = (5-2)^2 + (6+1)^2 \Rightarrow 9 + 49 = 58$ sq. unit

$BD^2 = (5-2)^2 + (-1-6)^2 \Rightarrow 9 + 49 = 58$ sq. unit

SECTION-D

25. Prove that tangent at any point of circle is perpendicular to the radius through point of contact.

Solution:



Given : A circle C (0, *r*) and a tangent *l* at point A.

To prove : OA ⊥ *l*

Construction : Take a point B, other than A, on the tangent *l*. Join OB. Suppose OB meets the circle in C.

Proof: We know that, among all line segment joining the point O to a point on *l*, the perpendicular is shortest to *l*.

OA = OC (Radius of the same circle)

Now, OB = OC + BC.

∴ OB > OC

⇒ OB > OA

⇒ OA < OB

B is an arbitrary point on the tangent *l*. Thus, OA is shorter than any other line segment joining O to any point on *l*.

Here, OA ⊥ *l*

26. 150 spherical marbles, each of diameter 1.4 cm, are dropped in a cylindrical vessel of diameter 7 cm containing some water, which are completely immersed in water . Find the rise in level of water in the vessel.

Solution:

Volume of 150 spherical marbles, each of diameters 1.4 cm = volume of cylindrical vessel of diameter 7 cm

$150 \times 4/3 \times \pi \times 1.4/2 \times 1.4/2 \times 1.4/2 = \pi \times 7/2 \times 7/2 \times h$

h = 5.6 cm

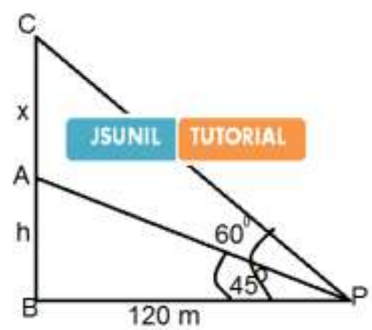
27. A container open at the top, is in the form of a frustum of a cone of height 24 cm with radii of its base 8 cm and 20 cm respectively. Find the cost of milk which can completely fill the container at the rate of Rs.21 per liter.

Solution: Volume of container = $1/3\ \pi h (R^2 + r^2 + Rr) = 1/3 \times 22/7 \times 24[20 \times 20 + 8 \times 8 + 20 \times 8]$

$= 15689.14\ cm^3 = 15.69$ litre

The cost of milk which can completely fill the container at the rate of Rs.21 per liter = Rs(21 x 15.69) = 329.49

28. The angle of elevation of the top of tower at a distance of 120 m from a point A on the ground is 45^0. if the angle of elevation of the top of a flagstaff fixed at the top of tower , at A is 60^0. , then find the height of the flagstaff.



Solution: let AB is the tower of height h meter and AC is flagstaff of height x meter.

$\angle APB = 45^0$ and $\angle BPC = 60^0$

Tan 60^0 = (x+h)/120) ⇒ √3 = (x+h)/120) ⇒ (x+h) = 120√3 ⇒ x = 120√3 –h

Tan 45^0 = h/120 ⇒ 1 = h/120 ⇒ h = 120

the height of the flagstaff = 120√3 – 120 = 120(√3-1) m = 87.6m

29.A motor boat whose speed in steel water is 18 km/h , takes 1 hour more to go 24 km upstream than to return downstream to the same spot . Find the speed of stream.

Solution: Let speed of stream = x km/h

Speed f boat in steel water = 18 km/h

Speed f boat in upstream = (18 –x)km/h

Speed f boat in downstream = (18 + x) km/h

Distance = 24 km

As per question,

24 km /(18 –x) = 24 km/(18 + x)   + 1

$\Rightarrow x^2 + 48x – 324 = 0$

x = 6 or – 54

Hence, the speed of stream = 6 km/h

30. In a school , student decided tonplant tree in and around the school to reduce air pollution. It was decided that the number of tree, that each section of each class will plant , will be double the class in which they are studying . If there are 1 t0 12 classes in the school and each class has two sections. find how many tree were planted by the students. Which value is seen in this question?

Solution

Class 1 plant trees = 2 x class 1 x 2 section = 2x1x2 = 4 x classs= 4x1 = 4 trees

Class 2 plant trees =4 x classs= 4x2 = 8 trees

a = 4

d = 8

n = 12

S_{12} = 12/2[2x4 + 11x4] = 312 trees

Environmental friendly, social etc.

31. Solve for x

(x-3)/(x-4) +(x -5) /(x-6) = 10/3

Solution: (x-3) / (x-4) +(x -5) /(x-6) = 10/3

[(x -3)(x - 6) + (x- 4) (x-5)] /[(x- 4)x (x- 6)] = 10/3

$2[x^2 - 9x + 19]/[x^2 - 10x + 24] = 10/3$

$2x^2 - 23x + 63 = 0$

x = 7 and 9/2

32. All the red face card are removed from a pack of 52 playing card . A card is drawn randomly from the remaining cards , after reshuffling them. Find the probability that the drawn card is

(i) Of red colour

(ii)a queen

(ii) an ace

(iv)a face card

Solution: (i) face card are removed from a pack of 52 playing card = 6

Total favorable outcomes = 52 – 6 = 46

Number of all possible outcomes = 26 -6= 20

P[E] = 20/46 = 0.43

(ii Number of all possible outcomes a queen = 2

P[E] = 2/46= 1/23

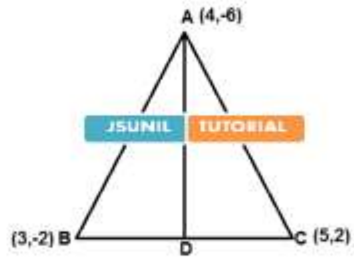
(iii) Number of all possible outcomes an ace = 2

P[E] = 2/46= 1/23

(iv) Number of all possible outcomes = 6

P[E] = 6/46= 3/23

33. A (4,-6),B(3,-2)and C(5,2) are the vertices of a triangle ABC and AD is its median. Prove that the median AD devides Triangle AB into two triangle of equal area.



Solution:

Let co – ordinate of D (x,y) and D is midpoint of BC

x = (3+5)/2 = 4 ; y = (2-2)/2 = 0

Now Area of ∆ ABD = ½ [3(-6-0) +4(0+2) 4(2+6) = 0.5 x [-18 + 8 + 16] = 3 sq unit

and Area of ∆ ACD = ½ [5(-6-0) + 4(0 - 2) + 4(2+6)] = 3 sq unit

Hence, the median AD divides triangle ABC into two triangle of equal area.

34. Prove that opposite side of quadrilateral circumscribing a circle subtend supplementary angle at the centre of circle.

Solution:

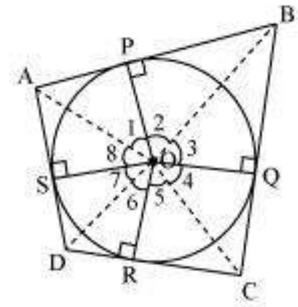
Let ABCD be a quadrilateral circumscribing a circle centered at O such that it touches the circle at point P, Q, R, S. Let us join the vertices of the quadrilateral ABCD to the center of the circle.

Consider ΔOAP and ΔOAS,

AP = AS (Tangents from the same point)

OP = OS (Radii of the same circle)

OA = OA (Common side)



$\Delta OAP \cong \Delta OAS$ (SSS congruence criterion)

Therefore, $A \leftrightarrow A$, $P \leftrightarrow S$, $O \leftrightarrow O$

And thus, $\angle POA = \angle AOS$

$\angle 1 = \angle 8$

Similarly,

$\angle 2 = \angle 3$

$\angle 4 = \angle 5$

$\angle 6 = \angle 7$

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$

$(\angle 1 + \angle 8) + (\angle 2 + \angle 3) + (\angle 4 + \angle 5) + (\angle 6 + \angle 7) = 360^\circ$

$2\angle 1 + 2\angle 2 + 2\angle 5 + 2\angle 6 = 360^\circ$

$2(\angle 1 + \angle 2) + 2(\angle 5 + \angle 6) = 360^\circ$

$(\angle 1 + \angle 2) + (\angle 5 + \angle 6) = 180^\circ$

$\angle AOB + \angle COD = 180^\circ$

Similarly, we can prove that $\angle BOC + \angle DOA = 180^\circ$

Hence, opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.