COMMON QUARTERLY EXAMINATION - SEPTEMBER 2019

Standard - 12

Reg No B'12212

PART - III - MATHEMATICS

Time Allowed: 2,30 Hours | Maximum Marks: 90

Instructions: 1. Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

2. Use Blue or Black ink to write and underline and pencil to draw diagrams.

PART - I

Note: i) Answer all the questions. **20x1=20**

ii) Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer.

1) The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}$ is

a) 2 b) 1 c) 3 d) 4

2) If $0 \le \theta \le \pi$ and the system of equations $x+(\sin\theta)y-(\cos\theta)z = 0$, $(\cos\theta)x-y+z = 0$, $(\sin\theta)x+y-z = 0$ has a non trivial solution then θ is

a) $\frac{5\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{2\pi}{3}$ d) $\frac{3\pi}{4}$

3) If $|z_1| = 1$, $|z_2| = 2$, $|z_3| = 3$ and $|9z_1z_2+4z_1z_3+z_2z_3| = 12$ then the value of $|z_1+z_2+z_3|$ is

a) 2 b) 1 c) 4 d) 3

4) Which one of the points i, $-2+i$, 2 and 3 is farthest from the origin?

a) 3 b) $-2+i$ c) i d) 2

5) If α, β and γ are the zeros of x^3+px^2+qx+r, then $\Sigma\frac{1}{\alpha}$ is

a) $\frac{q}{r}$ b) $\frac{-p}{r}$ c) $\frac{-q}{r}$ d) $\frac{-q}{p}$

6) The range of $\sec^{-1}x$ is

a) $[-\pi, \pi]\setminus\left\{\frac{\pi}{2}\right\}$ b) $[0, \pi]\setminus\left\{\frac{\pi}{2}\right\}$ c) $(0, \pi)\setminus\left\{\frac{\pi}{2}\right\}$ d) $(-\pi, \pi)\setminus\left\{\frac{\pi}{2}\right\}$

7) If $P(x, y)$ be any point on $16x^2+25y^2 = 400$ with foci $F_1(3, 0)$ and $F_2(-3, 0)$ then PF_1+PF_2 is

a) 6 b) 8 c) 12 d) 10

8) If the length of the perpendicular from the origin to the plane $2x+3y+\lambda z = 1$, $\lambda > 0$ is $1/5$, then the value of λ is

a) 0 b) 1 c) $2\sqrt{3}$ d) $3\sqrt{2}$

9) The system of linear equations $x+y+z = 2$, $2x+y-z = 3$, $3x+2y+kz = 4$ has a unique solution if

a) $k \ne 0$ b) $-1 < k < 1$ c) $-2 < k < 2$ d) $k = 0$

10) If A is an orthogonal matrix, then $|A|$ is

a) 1 b) -1 c) ± 1 d) 0

11) If z is a complex number such that $Re(z) = Im(z)$, then

a) $Re(z^2) = 0$ b) $Im(z^2) = 0$ c) $Re(z^2) = Im(z^2)$ d) $Re(z^2) = -Im(z^2)$

12) If z is any complex number, then the points $z, iz, -z, -iz$

a) form a square b) form a trapezium

c) are collinear d) lie on a circle $|z| = \sqrt{2}$ with centre (0, 0) and radius $\sqrt{2}$

13) If $\sin\alpha$ and $\cos\alpha$ are the roots of $25x^2+5x-12 = 0$ then the value of $\sin 2\alpha$ is

a) $\frac{12}{25}$ b) $\frac{-12}{25}$ c) $\frac{-24}{25}$ d) $\frac{4}{5}$

14) If a and b are odd integers then the roots of the equation $2ax^2+(2a+b)x+b = 0$ $(a \neq 0)$ are

a) rational b) irrational c) non real d) rational and equal

15) If $4\cos^{-1}x+\sin^{-1}x = \pi$ then the value of x is

a) $\frac{3}{2}$ b) $\frac{1}{\sqrt{2}}$ c) $\frac{\sqrt{3}}{2}$ d) $\frac{2}{\sqrt{3}}$

16) The domain of the function $\cos^{-1}(2x-1)$ is

a) $[0, 1]$ b) $[-1, 1]$ c) $(-1, 1)$ d) $[0, \pi]$

17) If $5x+9 = 0$ is the directrix of the hyperbola $16x^2-9y^2 = 144$ then its corresponding focus is

a) $\left(\frac{-5}{3}, 0\right)$ b) $(5, 0)$ c) $(-5, 0)$ d) $\left(\frac{5}{3}, 0\right)$

18) If $ax^2+by^2+(a+b-4)xy-ax-by-20 = 0$ represent the circle then its centre is

a) $\left(\frac{1}{2}, \frac{1}{2}\right)$ b) $\left(-\frac{1}{2}, -\frac{1}{2}\right)$ c) $(1, 1)$ d) $(-1, -1)$

19) If $\vec{\alpha} = \vec{a}\times\vec{b}$, $\vec{\beta} = \vec{b}\times\vec{c}$, $\vec{\gamma} = \vec{c}\times\vec{a}$ then $\left[\vec{\alpha}\times\vec{\beta},\ \vec{\beta}\times\vec{\gamma},\ \vec{\gamma}\times\vec{\alpha}\right] =$

a) $\left[\vec{a}\ \vec{b}\ \vec{c}\right]^4$ b) $\left[\vec{a}\ \vec{b}\ \vec{c}\right]^2$ c) $2\left[\vec{a}\ \vec{b}\ \vec{c}\right]^2$ d) $4\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

20) The foot of the perpendicular from A(1, 0, 0) to the line $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ is

a) $(3, -4, -2)$ b) $(5, -8, -4)$ c) $(-3, 4, 2)$ d) $(2, -3, 4)$

PART - II

Answer any seven questions. Question no. 30 is compulsory. **7x2=14**

21) If $adj A = \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find A^{-1}.

22) Simplify: $\left(\sin\frac{\pi}{6} + i\cos\frac{\pi}{6}\right)^{18}$

23) Form a polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root.

24) Find the value of $\tan^{-1}\left(\tan\frac{3\pi}{5}\right)$.

25) Find the condition for the line $y = mx+c$ to be a tangent to the circle $x^2+y^2 = a^2$.

26) If $2\hat{i}-\hat{j}+3\hat{k}$, $3\hat{i}+2\hat{j}+\hat{k}$, $\hat{i}+m\hat{j}+4\hat{k}$ are coplanar, find the value of m.

27) Show that $\left(2+i\sqrt{3}\right)^{10}-\left(2-i\sqrt{3}\right)^{10}$ is purely imaginary.

28) Determine the number of positive and negative roots of the equation $x^9-5x^8-14x^7 = 0$.

29) Find the value of $\cot\left(\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{4}{5}\right)$.

30) If the system of linear equation $x+2ay+az = 0$, $x+3by+bz = 0$, $x+4cy+cz = 0$ has a non-trivial solution then show that a, b, c are in H.P.

PART - III

Answer any seven questions. Question No. 40 is compulsory. **7×3=21**

31) Find the inverse of the non-singular matrix $A = \begin{bmatrix} 0 & 5 \\ -1 & 6 \end{bmatrix}$, by Gauss Jordan method.

32) In a competitive examination, one mark is awarded for every correct answer while $\frac{1}{4}$ mark is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly? (Use Cramer's rule to solve the problem)

33) State and prove triangle inequality.

34) Find the condition that the roots of $ax^3+bx^2+cx+d = 0$ are in geometric progression. Assume $a, b, c, d \neq 0$.

35) Find the domain of $\sin^{-1}(2-3x^2)$.

36) Find the equation of the parabola with vertex (−1, −2), axis parallel to y-axis and passing through (3, 6).

37) If the normal at the point 't_1' on the parabola $y^2 = 4ax$ meets the parabola again at the point 't_2', then prove that $t_2 = -\left(t_1+\frac{2}{t_1}\right)$.

38) If $\hat{a}, \hat{b}, \hat{c}$ are three unit vectors such that $\hat{b}$ and $\hat{c}$ are non parallel and $\hat{a}\times(\hat{b}\times\hat{c}) = \frac{1}{2}\hat{b}$, find the angle between $\hat{a}$ and $\hat{c}$.

39) Determine whether the pair of straight lines $\vec{r} = (2\hat{i}+6\hat{j}+3\hat{k})+t(2\hat{i}+3\hat{j}+4\hat{k})$, $\vec{r} = (2\hat{j}-3\hat{k})+s(\hat{i}+2\hat{j}+3\hat{k})$ are parallel. Find the shortest distance between them.

40) Prove that $\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right) = \frac{2b}{a}$.

PART - IV

Answer all the questions: **7×5=35**

41) a) By using Gaussian elimination method, balance the chemical reaction equation: $C_5H_8+O_2 \rightarrow CO_2+H_2O$

(OR)

b) Solve: $6x^4-35x^3+62x^2-35x+6 = 0$

42) a) Find the value of k for which the equations $kx-2y+z = 1$, $x-2ky+z = -2$, $x-2y+kz = 1$ have

(i) no solution (ii) unique solution (iii) Infinitely many solution. **(OR)**

b) Suppose z_1, z_2 and z_3 are the vertices of an equilateral triangle inscribed in the circle $|z| = 2$. If $z_1 = 1+i\sqrt{3}$ then find z_2 and z_3.

43) a) If $z = x+iy$ and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, then show that $x^2+y^2+3x-3y+2 = 0$. **(OR)**

b) If $2+i$ and $3-\sqrt{2}$ are the roots of the equation

$x^6-13x^5+62x^4-126x^3+65x^2+127x-140 = 0$ then find all the roots.

44) a) i) Find the value of $\tan^{-1}(-1)+\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}\left(\frac{-1}{2}\right)$.

ii) If $\cot^{-1}\left(\frac{1}{7}\right) = \theta$, find the value of $\cos\theta$. **(OR)**

b) For the ellipse $4x^2+y^2+24x-2y+21 = 0$, find the centre, vertices, and the foci. Also prove that the length of latus rectum is 2.

45) a) Find the number of solutions of the equation

$\tan^{-1}(x-1)+\tan^{-1}(x)+\tan^{-1}(x+1) = \tan^{-1}(3x)$ **(OR)**

b) Cross section of a Nuclear cooling tower is in the shape of a hyperbola with equation $\frac{x^2}{30^2}-\frac{y^2}{44^2} = 1$. The tower is 150m tall and the distance from the top of the tower to the centre of the hyperbola is half the distance from the base of the tower to the centre of the hyperbola. Find the diameter of the top and base of the tower.

46) a) Find parametric form of vector equation and Cartesian equations of the plane passing through the points (2, 2, 1), (1, −2, 3) and parallel to the straight line passing through the points (2, 1, −3) and (−1, 5, −8). **(OR)**

b) Prove by vector method that the perpendiculars (attitudes) from the vertices to the opposite sides of a triangle are concurrent.

47) a) If $\cos\theta+\cos\phi = \sin\theta+\sin\phi = 0$ then show that

i) $\cos2\theta+\cos2\phi = 2\cos(\pi+\theta+\phi)$ ii) $\sin2\theta+\sin2\phi = 2\sin(\pi+\theta+\phi)$ **(OR)**

b) If a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ makes intercepts h and k on the co-ordinate axes then show that $\frac{a^2}{h^2}+\frac{b^2}{k^2} = 1$.

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PART-II

24. Solve the following systems of linear equations by Cramer's rule : $5x - 2y + 16 = 0,\ x + 3y - 7 = 0$

PART-III

34. Find the rank of the following matrices by row reduction method:

$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{bmatrix}$$

35. Find the least value of the positive integer n for which $\left(\sqrt{3}+i\right)^n$

PART-IV

46. Show that the line $x - y + 4 = 0$ is a tangent to the ellipse $x^2 + 3y^2 = 12$. Also find the coordinates of the point of contact.

(or)

A rod length 1.2m moves with its ends always touching the coordinate axes. The locus of a point P on the rod, which is 0.3m from the end in contact with x-axis is an ellipse. Find the eccentricity.

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