IIT-JEE-2013

Objective Mathematics

{Mains & Advance}

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Er. L.K.Sharma an engineering graduate from NIT, Jaipur (Rajasthan), {*Gold medalist, University of Rajasthan*} is a well known name among the engineering aspirants for the last 15 years. He has been honored with "*BHAMASHAH AWARD*" two times for the academic excellence in the state of Rajasthan. He is popular among the student community for possessing the excellent ability to communicate the mathematical concepts in analytical and graphical way.

He has worked with many IIT-JEE coaching institutes of Delhi and Kota, {presently associated with *Guidance, Kalu Sarai, New Delhi* as senior mathematics faculty}. He has been a senior mathematics {IIT-JEE} faculty at Delhi Public School, RK Puram for five years.

He is actively involved in the field of online teaching to the engineering aspirants and is associated with *iProf Learning Solutions India (P) Ltd* for last 3 years. As a premium member of www.wiziq.com (an online teaching and learning portal), he has delivered many online lectures on different topics of mathematics at IIT-JEE and AIEEE level.{some of the free online public classes at wizIQ can be accessed at http://www.wiziq.com/LKS }.

Since last 2 years many engineering aspirants have got tremendous help with the blog "*mailtolks.blogspot.com*" and with launch of the site "mathematicsgyan.weebly.com", engineering aspirants get the golden opportunity to access the best study/practice material in mathematics at school level and IIT-JEE/AIEEE/BITSAT level. The best part of the site is availability of e-book of "OBJECTIVE MATHEMATICS for JEE- 2013" authored by Er. L.K.Sharma, complete book with detailed solutions is available for free download as the PDF files of different chapters of JEE-mathematics.

Contents

Chapter 1

Quadratic Equations

Exercise No. (1)

Multiple choice questions with ONE correct answer : (Questions No. 1-25)

1. If the equation $|x - n| = (x + 2)^2$ is having exactly three distinct real solutions , then exhaustive set of values of $'n'$ is given by :

(a) $\left[-\frac{5}{2}, -\frac{3}{2}\right]$ (b) $\left\{-\frac{5}{2}, -2, -\frac{3}{2}\right\}$

(c) $\left\{-\frac{5}{2}, -\frac{3}{2}\right\}$ (d) $\left\{-\frac{9}{4}, -2, -\frac{7}{4}\right\}$

2. Let a , b , c be distinct real numbers , then roots of $(x - a)(x - b) = a^2 + b^2 + c^2 - ab - bc - ac$, are :

(a) real and equal (b) imaginary

(c) real and unequal (d) real

3. If $2x^3 - 12x^2 + 3\lambda x - 16 = 0$ is having three positive real roots , then $'\lambda'$ must be :

(a) 4 (b) 8

(c) 0 (d) 2

4. If a , b , c are distinct real numbers , then number of real roots of equation

$$\frac{(x-a)(x-b)}{(c-a)(c-b)} + \frac{(x-b)(x-c)}{(a-b)(a-c)} + \frac{(x-c)(x-a)}{(b-c)(b-a)} = 1$$

is/are :

(a) 1 (b) 4

(c) finitely many (d) infinitely many

5. If $ax^2 + 2bx + c = 0$ and $a_1x^2 + 2b_1x + c_1 = 0$ have a common root and $\frac{a}{a_1}, \frac{b}{b_1}, \frac{c}{c_1}$ are in A.P. , then a_1 , b_1 , c_1 are in :

(a) A.P. (b) G.P.

(c) H.P. (d) none of these

6. If all the roots of equations

$$(a-1)(1+x+x^2)^2 = (a+1)(x^4+x^2+1)$$

are imaginary , then range of $'a'$ is :

(a) $(-\infty , -2]$ (b) $(2 , \infty)$

(c) $(-2 , 2)$ (d) $(-2 , \infty)$

7. Total number of integral solutions of inequation

$$\frac{x^2(3x-4)^3(x-2)^4}{(x-5)^5(7-2x)^6} \le 0 \text{ is/are :}$$

(a) four (b) three

(c) two (d) only one

8. If exactly one root of $5x^2 + (a + 1)x + a = 0$ lies in the interval $x \in (1 , 3)$, then

(a) $a > 2$

(b) $-12 < a < -3$

(c) $a > 0$

(d) none of these

9. If both roots of $4x^2 - 20px + (25p^2 + 15p - 66) = 0$ are less than 2 , then $'p'$ lies in :

(a) $\left(\frac{4}{5}, 2\right)$ (b) $(2 , \infty)$

(c) $\left(-1 , \frac{4}{5}\right)$ (d) $(-\infty , -1)$

10. If $x^2 - 2ax + a^2 + a - 3 \ge 0 \ \forall \ x \in R$, then $'a'$ lies in

(a) $[3 , \infty)$ (b) $(-\infty , 3]$

(c) $[-3 , \infty)$ (d) $(-\infty , -3]$

11. If $x^3 + ax + 1 = 0$ and $x^4 + ax^2 + 1 = 0$ have a common root , then value of $'a'$ is

(a) 2 (b) –2

(c) 0 (d) 1

12. If $x^2 + px + 1$ is a factor of $ax^3 + bx + c$, then

(a) $a^2 + c^2 + ab = 0$

(b) $a^2 - c^2 + ab = 0$

(c) $a^2 - c^2 - ab = 0$

(d) $a^2 + c^2 - ab = 0$

13. If expression $a^2(b^2-c^2)x^2 + b^2(c^2-a^2)x + c^2(a^2-b^2)$ is a perfect square of one degree polynomial of x , then a^2 , b^2 , c^2 are in :

(a) A.P. (b) G.P.

(c) H.P. (d) none of these

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14. The value of α for which the quadratic equation
$$x^2-(\sin\alpha-2)\,x-(1+\sin\alpha)=0$$
has roots whose sum of squares is least , is :

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{6}$

15. If $\cos\theta$, $\sin\phi$, $\sin\theta$ are in G.P. , then roots of $x^2+2(\cot\phi)x+1=0$ are :

(a) equal (b) real

(c) imaginary (d) greater than 1

16. If $-3<\dfrac{x^2+ax-2}{x^2+x+1}<2$ holds $\forall\ x\in R$, then $'a'$ belongs to :

(a) $[-2\ ,\ 1)$ (b) $(-2\ ,\ 1)$

(c) $R-[-2\ ,\ 2]$ (d) $(-2\ ,\ 2)$

17. The number of real solutions of the equation $\sqrt{2x+\sqrt{2x+4}}=4$ is/are :

(a) 0 (b) 1

(c) 2 (d) 4

18. Let α , β be the roots of quadratic equation $ax^2+bx+c=0$, then roots of the equation $ax^2-bx\,(x-1)+c(x-1)^2=0$ are :

(a) $\dfrac{\alpha}{1-\alpha}$, $\dfrac{\beta}{1-\beta}$ (b) $\dfrac{\alpha}{\alpha+1}$, $\dfrac{\beta}{\beta+1}$

(c) $\dfrac{1-\alpha}{\alpha}$, $\dfrac{1-\beta}{\beta}$ (d) $\dfrac{1+\alpha}{\alpha}$, $\dfrac{1+\beta}{\beta}$

19. If the equation $x^5-10a^3x^2+b^4x+c^5=0$ has 3 equal roots , then :

(a) $b^4=5a^3$ (b) $2c^5+a^2b^3-5=0$

(c) $c^5+6a^5=0$ (d) $2b^2-5a^3c=0$

20. If a , b and c are not all equal and α , β are the roots of $ax^2+bx+c=0$, then value of $(1+\alpha+\alpha^2)(1+\beta+\beta^2)$ is :

(a) zero (b) positive

(c) negative (d) non-negative

21. The equation $(x)^{\frac{3}{4}(\log_2 x)^2+(\log_2 x)-\frac{5}{4}}=\sqrt{2}$ has :

(a) exactly two real roots (b) no real root

(c) one irrational root (d) three rational roots

22. If real polynomial $f(x)$ leaves remainder 15 and $(2x+1)$ when divided by $(x-3)$ and $(x-1)^2$ respectively , then remainder when $f(x)$ is divided by $(x-3)(x-1)^2$ is :

(a) $2x-1$ (b) $3x^2+2x-4$

(c) $2x^2-2x+3$ (d) $3x+6$

23. Let $a\in R^+$ and equation $3x^2+ax+3=0$ is having one of the root as square of the another root , then $'a'$ is equal to :

(a) 2/3 (b) –3

(c) 3 (d) 1/3

24. If the quadratic equation
$$a^2(x+1)^2+b^2(2x^2-x+1)-5x^2-3=0$$
is satisfied for all $x\in R$, then number of ordered pairs $(a\ ,\ b)$ which are possible is/are :

(a) 0 (b) 1

(c) finitely many (d) infinitely many

25. The smallest value of $'k'$ for which both the roots of the equation $x^2-8kx+16(k^2-k+1)=0$ are real and distinct and have values at least 4 , is :

(a) 1 (b) 2

(c) –1 (d) 3

26. Let $f(x)=(x-3k)(x-k-3)$ be negative for all $x\in[1\ ,\ 3]$, where $k\in R$, then complete set of values of $'k'$ belong to :

(a) $\left(-\frac{1}{2}\ ,\ \frac{1}{2}\right)$ (b) $\left(0\ ,\ \frac{1}{3}\right)$

(c) $\left(\frac{1}{3}\ ,\ 3\right)$ (d) $(-3\ ,\ 0)$

27. Let $A=\{y:4\le y<150\ ,\ y\in N\}$ and $\alpha\in A$, then total number of values of $'\alpha'$ for which the equation $x^2-3x-\alpha=0$ is having integral roots , is equal to :

(a) 8 (b) 12

(c) 9 (d) 10

28. Let $\alpha\ ,\beta\ ,\gamma\in R^+$ and $(ln\ 3)^\alpha$, $(ln\ 3)^\beta$, $(ln\ 3)^\gamma$ form a geometric sequence. If the quadratic equation $\alpha x^2+\beta x+\gamma=0$ has real roots , then absolute value of $\left\{\sqrt{\dfrac{\alpha}{\gamma}}-\sqrt{\dfrac{\gamma}{\alpha}}\right\}$ is not less than :

(a) 4 (b) $2\sqrt{3}$

(c) $3\sqrt{2}$ (d) $2\sqrt{2}$

29. Let $a, b, c \in R$ and $f(x) = ax^2 + bx + c$, where the equation $f(x) = 0$ has no real root. If $y + k = 0$ is tangent to the curve $y = f(x)$, where $k \in R^+$, then :

(a) $a - b + c > 0$ (b) $c = 0$

(c) $4a - 2b + c \geq 0$ (d) $a - 2b + 4c < 0$

30. Let a, b, c be the sides of a scalene triangle and $\lambda \in R$. If the roots of the equation $x^2 + 2(a+b+c)x + 3\lambda(ab+bc+ac) = 0$ are real, then :

(a) maximum positive integral value of λ is 2

(b) minimum positive integral value of λ is 2

(c) values of λ lies in $\left[-\frac{2}{3}, \frac{2}{3}\right]$

(d) $\lambda \in (-\infty, 4/3)$

Multiple choice questions with MORE than ONE correct answer : (Questions No. 31-35)

31. Let $|a| < |b|$ and a, b are the real roots of equation $x^2 - |\alpha|x - |\beta| = 0$. If $1 + |\alpha| < b$, then the equation $log_{|a|}\left(\frac{x}{b}\right)^2 = 1$ has

(a) one root in $(-\infty, a)$ (b) one root in (b, ∞)

(c) one root in (a, b) (d) no root in (a, b)

32. Let $p, q \in Q$ and $\cos^2\frac{\pi}{8}$ be a root of the equation $x^2 + px + q^2 = 0$, then :

(a) $[|\sin\theta| + |\cos\theta|] + p = 0$ for all $\theta \in R$, where [.] represents the greatest integer function.

(b) Value of $log_2|q| = -\frac{3}{2}$

(c) $8q^2 - 4p = 0$

(d) $[|\sin\theta| + |\cos\theta|] + 2p = 0$ for all $\theta \in R$, where [.] represents the greatest integer function.

33. Let $S = \{\alpha : \alpha^2 - 5\alpha - 6 \leq 0, \alpha \in R\}$ and $a, b \in S$.

If the equation $x^2 + 7 = 4x - 3\sin(ax + b)$ is satisfied for at least one real value of x, then

(a) minimum possible value of $2a + b$ is $-\pi/2$

(b) maximum possible value of $2a + b$ is $7\pi/2$

(c) minimum possible value of $2a + b$ is $\pi/2$

(d) maximum possible value of $2a + b$ is $11\pi/2$

34. If all the four roots of the bi-quadratic equation $x^4 - 12x^3 + \alpha x^2 + \beta x + 81 = 0$ are positive in nature, then :

(a) value of α is 45

(b) value of β is 108

(c) value of $2\alpha + \beta = 0$

(d) value of $\frac{\beta}{\alpha} log_{0.5} 5 = log_2 25$

35. Let α, β be the real roots of the quadratic equation $x^2 - ax - b = 0$, where $a, b \in R$.

If $A = \{x : x^2 - 4 < 0 ; x \in R\}$ and $\alpha, \beta \in A$, then which of the following statements are incorrect :

(a) $|a| > 2 - \frac{b}{2}$

(b) $|a| < 2 - \frac{b}{2}$

(c) $|a| < 4$

(d) $|a|^2 + 4b > 0$

Assertion Reasoning questions : (Questions No. 36-40)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements , Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers , only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

36. Let $a, b, c \in R$, $a \neq 0$, $f(x) = ax^2 + bx + c$, where $\Delta = b^2 - 4ac$. If $f(x) = 0$ has α, β as two real and distinct roots and $f(x+k) + \lambda f(x) = 0$, $\lambda, k \in R$, has exactly one real root between α and β, then

Statement 1 : $0 < |ak| < \sqrt{\Delta}$

because

Statement 2 : the values of $'k'$ don't depend upon the values of $'\lambda'$.

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37. Statement 1 : If $a, b, c \in R$, then at least one of the following equations ..... (1), (2), (3) has a real solution

$x^2 + (a-b)x + (b-c) = 0$ .... (1)

$x^2 + (b-c)x + (c-a) = 0$ .... (2)

$x^2 + (c-a)x + (a-b) = 0$ .... (3)

because

Statement 2 : The necessary and sufficient condition for at least one of the three quadratic equations, with discriminant $\Delta_1, \Delta_2, \Delta_3$, to have real roots is $\Delta_1 \Delta_2 \Delta_3 \geq 0$.

38. Statement 1 : If the equation

$x^2 + \lambda x - \frac{\pi}{2} + \sin^{-1}(x^2 - 6x + 10) + \cos^{-1}(x^2 - 6x + 10) = 0$

is having real solution, then value of 'λ' must be $2 \log_{\frac{1}{2}} 8$

because

Statement 2 : $\sin^{-1}(x) + \cos^{-1}(x) - \frac{\pi}{2} = 0$ for all $x \in [-1, 1]$.

39. Statement 1 : If equation $x^2 - (\lambda + 1)x + \lambda - 1 = 0$ is having integral roots, then there exists only one integral value of 'λ'

because

Statement 2 : $x = 2$ is the only integral solution of the equation $x^2 - (\lambda + 1)x + \lambda - 1 = 0$, if $\lambda \in I$.

40. Let $f(x) = ax^2 + bx + c$, $a, b, c \in R$ and $a \neq 0$.

Statement 1 : If $f(x) = 0$ has distinct real roots, then the equation $(f'(x))^2 - f(x).f''(x) = 0$ can never have real roots

because

Statement 2 : If $f(x) = 0$ has non-real roots, then they occur in conjugate pairs.

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Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1-3)

Let $a, b \in R-\{0\}$ and α, β, γ be the roots of the equation $x^3+ax^2+bx-b=0$. If $\frac{2}{\beta}=\frac{1}{\alpha}+\frac{1}{\gamma}$, then answer the following questions.

1. The value of $2b+9a+30$ is equal to :

(a) 2 (b) – 5
(c) 3 (d) –2

2. The minimum value of $\frac{(\alpha\beta)^2+(\beta\gamma)^2+(\alpha\gamma)^2}{(\alpha\beta\gamma)^2}$ is equal to :

(a) $\frac{1}{2}$ (b) $\frac{1}{9}$
(c) $\frac{1}{8}$ (d) $\frac{1}{3}$

3. The minimum value of $\frac{a+b}{b}$ is equal to :

(a) $\frac{2}{3}$ (b) $\frac{3}{4}$
(c) $\frac{1}{3}$ (d) $\frac{3}{8}$

Comprehension passage (2)
(Questions No. 4-6)

Let α, β be the roots of equation $x^2+ax+b=0$, and γ, δ be the roots of equation $x^2+a_1x+b_1=0$. If $S=\left\{x : x^2+a_1x+b_1=0 , x\in R\right\}$ and $f : R-S \to R$ is a function which is defined as $f(x)=\frac{x^2+ax+b}{x^2+a_1x+b_1}$, then answer the following question.

4. If $\alpha, \beta, \gamma, \delta \in R$ and $\beta > \delta > \alpha > \gamma$, then

(a) $f(x)$ is increasing in (γ, δ)
(b) $f(x)$ is increasing in (α, β)
(c) $f(x)$ is decreasing in (δ, β)
(d) $f(x)$ is increasing in $(-\infty, \alpha)$

5. If $\alpha, \beta, \gamma, \delta \in R$ and $\gamma < \delta < \alpha < \beta$, then :

(a) $f'(x) > 0 \ \forall \ x \in R-\{\gamma, \delta\}$.
(b) $f(x)$ has local maxima in (γ, δ) and local minima in (α, β).
(c) $f(x)$ has local minima in (γ, δ) and local maxima in (α, β).
(d) $f'(x) < 0 \ \forall \ x \in R-\{\gamma, \delta\}$

6. If $\alpha, \beta, \gamma, \delta$ are the non-real values and $f(x)$ is defined $\forall \ x \in R$, then :

(a) $f'(x)=0$ has real and distinct roots.
(b) $f'(x)=0$ has real and equal roots.
(c) $f'(x)=0$ has imaginary roots.
(d) nothing can be concluded in general for $f'(x)$.

Comprehension passage (3)
(Questions No. 7-9)

Consider the function

$f(x)=(1+m)x^2-2(3m+1)x+(8m+1)$,

where $m \in R-\{-1\}$

7. If $f(x) > 0$ holds true $\forall \ x \in R$, then set of values of $'m'$ is :

(a) (0 , 3) (b) (2 , 3)
(c) (–1 , 3) (d) (–1 , 0)

8. The set of values of $'m'$ for which $f(x)=0$ has at least one negative root is :

(a) $(-\infty, -1)$ (b) $\left(-\frac{1}{8}, \infty\right)$
(c) $\left(-1, -\frac{1}{8}\right)$ (d) $\left(-\frac{1}{8}, 3\right)$

9. The number of real values of $'m'$ such that $f(x)=0$ has roots which are in the ratio 2 : 3 is /are :

(a) 0 (b) 2
(c) 4 (d) 1

Questions with Integral Answer :
(Questions No. 10-15)

10. Let α, β be the roots of the quadratic equation $m^2(x^2-x)+2mx+3=0$, where $m \neq 0$ & m_1, m_2 are two values of m for which $\left\{\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right\}$ is equal to $\frac{4}{3}$.

If $P=\frac{m_1^2}{m_2}+\frac{m_2^2}{m_1}$, then value of $\left\{-\frac{3P}{17}\right\}$ is equal to ....

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11. Let a, b, c, d be distinct real numbers, where the roots of $x^2 - 10cx - 11d = 0$ are a and b. If the roots of $x^2 - 10ax - 11b = 0$ are c and d, then value of $\frac{1}{605}(a+b+c+d)$ is ....

12. If a, b are complex numbers and one of the roots of the equation $x^2 + ax + b = 0$ is purely real where as the other is purely imaginary, then value of $\left\{\frac{a^2 - (\bar{a})^2}{2b}\right\}$ is equal to ....

13. If the equation $x^4 - (a+1)x^3 + x^2 + (a+1)x - 2 = 0$ is having at least two distinct positive real roots, then the minimum integral value of parameter $'a'$ is equal to ....

14. If the equations $ax^3 + 2bx^2 + 3cx + 4d = 0$ and $ax^2 + bx + c = 0$ have a non-zero common root, then the minimum value of $(c^2 - 2bd)(b^2 - 2ac)$ is equal to ....

15. If $n \in I$ and the roots of quadratic equation $x^2 + 2nx - 19n - 92 = 0$ are rational in nature, then minimum possible value of $|n|$ is equal to ....

Matrix Matching Questions :
(Questions No. 16-18)

16. Match the following columns (I) and (II)

Column (I)	Column (II)
(a) If roots of $x^2 - bx + c = 0$ are two consecutive integers, then $(b^2 - 4c)$ is	(p) -2
(b) If $x \in [2, 4]$, then least value of the expression $(x^2 - 6x + 7)$ is :	(q) 0
(c) Number of solutions of equation $\left\| \|x^2 - 1\| - 3 \right\| = 4$ is /are	(r) 2
(d) Minimum value of $f(x) = \|2x - 4\| + \|6 - 4x\|$ is :	(s) 1

17. Match the following columns (I) and (II)

Column (I)	Column (II)
(a) If $(\lambda^2 + \lambda - 2)x^2 + (\lambda + 2)x < 1 \ \forall \ x \in R$, then λ belongs to the interval	(p) $(0, 4)$
(b) If sum and product of the quadratic equation $x^2 - (\lambda^2 - 5\lambda + 5)x + (2\lambda^2 - 3\lambda - 4) = 0$ are both less than one, then set of possible values of λ is	(q) $\left(-2, \frac{2}{5}\right)$
	(r) $\left(1, \frac{5}{2}\right)$
(c) If $5^x + (2\sqrt{3})^{2x} - 169$ is always positive then set of x is	
(d) If roots of equation $2x^2 - (a^2 + 8a + 1)x + a^2 - 4a = 0$ are opposite in sign, then set of values of a is	(s) $(2, \infty)$

18. Let $f(x)=ax^2+bx+c$, $a\neq 0$, a , b , $c\in R$. If column (I) represents the conditions on a , b , c and column (II) corresponds to the graph of $f(x)$, where $D=(b^2-4ac)$, then match columns (I) and (II).

Column (I)

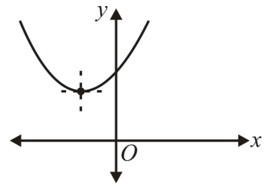
(a) a , b , $c\in R^+$ and $D>0$

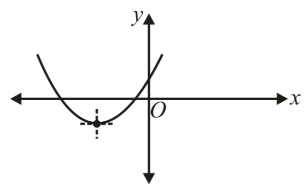
(b) a , $c\in R^-$ and $b\in R^+$, $D>O$

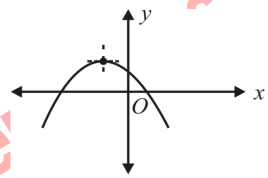
(c) a , b , $c\in R$ and $D>O$

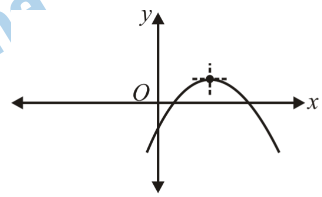
(d) a , $b\in R^+$, $c\in R^+$ and $D<0$

Column (II)

(p) 

(q) 

(r) 

(s) 

ANSWERS — Exercise No. (1)

1. (d)	**2.** (c)	**3.** (b)	**4.** (d)	**5.** (b)
6. (c)	**7.** (a)	**8.** (b)	**9.** (d)	**10.** (a)
11. (b)	**12.** (c)	**13.** (c)	**14.** (c)	**15.** (b)
16. (b)	**17.** (b)	**18.** (b)	**19.** (c)	**20.** (b)
21. (c)	**22.** (c)	**23.** (c)	**24.** (c)	**25.** (b)
26. (b)	**27.** (d)	**28.** (b)	**29.** (d)	**30.** (d)
31. (a , b , d)	**32.** (a , b)	**33.** (a , d)	**34.** (c , d)	**35.** (b , c , d)
36. (b)	**37.** (c)	**38.** (d)	**39.** (c)	**40.** (b)

ANSWERS — Exercise No. (2)

1. (c)	**2.** (d)	**3.** (a)	**4.** (a)	**5.** (b)
6. (a)	**7.** (d)	**8.** (b)	**9.** (a)	**10.** (4)
11. (2)	**12.** (2)	**13.** (2)	**14.** (0)	**15.** (8)

16. (a) $\to$ s
(b) $\to$ p
(c) $\to$ r
(d) $\to$ s

17. (a) $\to$ q
(b) $\to$ r
(c) $\to$ s
(d) $\to$ p

18. (a) $\to$ q
(b) $\to$ s
(c) $\to$ q , r , s
(d) $\to$ p

Chapter 2

Sequences and Series

Exercise No. (1)

Multiple choice questions with ONE correct answer : (Questions No. 1-25)

1. If sum of $'n'$ terms of a sequence is given by $S_n = \sum_{r=1}^{n} T_r = n(n+1)(n+2)$, then $\sum_{r=1}^{12} \frac{1}{T_r}$ is equal to :

(a) $\frac{4}{13}$ (b) $\frac{2}{13}$

(c) $\frac{5}{67}$ (d) $\frac{4}{39}$

2. Let a, b, c be distinct non-zero real numbers such that a^2, b^2, c^2 are in harmonic progression and a, b, c are in arithmetic progression , then :

(a) $2b^2 + ac = 0$ (b) $4b^2 + ac = 0$

(c) $2b^2 - ac = 0$ (d) $4b^2 - ac = 0$

3. Let a, b, c are in A.P. and a^2, b^2, c^2 are in G.P. , if $a < b < c$ and $a + b + c = 3/2$, then value of $'a'$ is :

(a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{2\sqrt{3}}$

(c) $\frac{1}{2} - \frac{1}{\sqrt{3}}$ (d) $\frac{1}{2} - \frac{1}{\sqrt{2}}$

4. If $a, b, c \in R^+$, then maximum value of $\left\{\frac{bc}{b+c} + \frac{ac}{a+c} + \frac{ab}{a+b}\right\}$ is

(a) $\frac{1}{2}(a+b+c)$ (b) $\frac{1}{3}\sqrt{abc}$

(c) $\frac{1}{3}(a+b+c)$ (d) $\frac{1}{2}\sqrt{abc}$

5. If the sum of first n terms of an A.P. is cn^2, then the sum of squares of these n terms is :

(a) $\frac{n(4n^2-1)c^2}{6}$ (b) $\frac{n(4n^2+1)c^2}{3}$

(c) $\frac{n(4n^2-1)c^2}{3}$ (d) $\frac{n(4n^2+1)c^2}{6}$

6. Let $\alpha \in R^+ - \{1\}$ and $(ln\alpha)^p, (ln\alpha)^q, (ln\alpha)^r, (ln\alpha)^s$ be in G.P. , then pqr, pqs, prs, qrs are in :

(a) A.P. (b) G.P.

(c) H.P. (d) A.G.P.

7. Let $T_1 = \frac{1}{2}$, $T_{r+1} = T_r + T_r^2 \ \forall \ r \in N$ and

$S_n = \frac{1}{1+T_1} + \frac{1}{1+T_2} + \frac{1}{1+T_3} + ... + \frac{1}{T_n+1}$, then

(a) $S_{100} \geq 4$ (b) $S_{100} > 2$

(c) $1 < S_{100} < 2$ (d) $0 < S_{100} < 1$

8. Let $S_n = \sum_{r=1}^{n} r^4$, then $\sum_{r=1}^{n} (2r-1)^4$ is given by :

(a) $S_{2n} - 8S_n$ (b) $S_{4n} - 24S_{2n}$

(c) $S_{2n} - 16S_n$ (d) $S_{4n} - 16S_n$

9. Let $\{x_n\}$ represents G.P. with common ratio $'r'$ such that $\sum_{k=1}^{n} x_{2k-1} = \sum_{k=1}^{n} x_{2k+2} \neq 0$, then number of possible values for $'r'$ is/are :

(a) 1 (b) 2

(c) 3 (d) 4

10. Let x, y be non-zero real numbers and the expression $x^{12} + y^{12} - 48x^4y^4$ is not less than $'k'$, then value of $'k'$ is equal to :

(a) -2^{12} (b) 2^{12}

(c) 2^8 (d) -2^8

11. Let 10 A.M.'s and 10 H.M.'s be inserted in between 2 and 3. If $'A'$ be any A.M. and $'H'$ be the corresponding H.M. , then $H(5-A)$ is equal to :

(a) 6 (b) 10

(c) 11 (d) 8

12. Let $a, b, c \in R^+$ and the inequality $bx^2 + (\sqrt{(a+c)^2 + 4b^2})x + (a+c) \geq 0$ holds true for all real value of $'x'$, then $e^{a+1}, e^{b+1}, e^{c+1}$ are in :

(a) A.P. (b) G.P.
(c) H.P. (d) none of these.

13. Let $'A_n'$ denotes the sum of n terms of an A.P. and $A_{2n} = 3A_n$, then $\frac{A_{3n}}{A_n}$ is equal to :

(a) 4 (b) 6
(c) 8 (d) 10

14. If $a \neq 0$, roots of equation $ax^3 + bx^2 + cx + d = 0$ are in G.P., then :

(a) $ac^3 = db^3$ (b) $a^3c = d^3b$
(c) $a^3b = c^3d$ (d) $ab^3 = cd^3$

15. Let a, b, c be non-zero real numbers and $4a^2 + 9b^2 + 16c^2 = 2(3ab + 6bc + 4ac)$, then a, b, c are in :

(a) A.P. (b) G.P.
(c) H.P. (d) A.G.P.

16. In a set of four numbers, if first three terms are in G.P. and the last three terms are in A.P. with common difference 6, then sum of the four numbers, when the first and the last terms are equal, is given by :

(a) 20 (b) 14
(c) 16 (d) 18

17. Let the real numbers α, β, γ be in A.P. and satisfy the equation $x^2(x-1) + px + q = 0$, then :

(a) $p \in \left[\frac{1}{3}, 3\right]$ (b) $q \in \left[-\frac{1}{27}, \infty\right)$
(c) $p \in \left[\frac{1}{3}, \infty\right)$ (d) $q \in \left(-\infty, \frac{1}{27}\right]$

18. In ΔABC, if all the sides are in A.P., then the corresponding ex-radii are in :

(a) A.P. (b) G.P.
(c) H.P. (d) none of these.

19. Let $S = \sum_{r=1}^{n} \frac{8r}{4r^4 + 1}$, then $\lim_{n \to \infty}(S)$ is equal to :

(a) 4 (b) 2
(c) 1 (d) 0

20. In a sequence of $(4n + 1)$ terms, the first $(2n + 1)$ terms are in A.P. whose common difference is 2, and the last $(2n + 1)$ terms are in G.P. whose common ratio is 1/2. If the middle terms of the A.P. and G.P. are equal, then the middle term of sequence is :

(a) $\frac{n.2^n}{2^n + 1}$. (b) $\frac{(n+1)2^n}{2^n + 1}$.
(c) $\frac{n.2^{n+1}}{2^n - 1}$. (d) $\frac{(n+1)2^{n+1}}{2^n - 2}$.

21. Let $a_1, a_2, a_3, \ldots\ldots, a_{50}$ be 50 distinct numbers in A.P., and $\sum_{r=1}^{50} (-1)^{r+1} (a_r)^2 = \left(\sqrt{\frac{5}{7}}\right)^{n/2} (a_1^2 - a_{50}^2)$, where $n \in N$, then value of n is equal to :

(a) 4 (b) 2
(c) 8 (d) 10

22. Let three numbers be removed from the geometric sequence $\{a_n\}$ and the geometric mean of the remaining terms is $\sqrt[5]{2^{37}}$. If $a_n = \left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\ldots\ldots + \infty\right)^n$, then value of $'n'$ can be :

(a) 10 (b) 8
(c) 20 (d) 13

23. Let $x, y \in R^+$ and $x^2y^3 = 6$, then the least value of $3x + 4y$ is equal to :

(a) 12 (b) 10
(c) 8 (d) 20

24. Let $S_n = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots.. \; n$ terms and $S = \lim_{n \to \infty}(S_n)$, if $S - S_n < \frac{1}{1000}$, then least value of $'n'$ is :

(a) 11 (b) 10 (c) 12 (d) 6

25. Let the sides of a triangle be in arithmetic progression. If the greatest angle of triangle is double the smallest angle, then the cosine value of the smallest angle is equal to :

(a) $\frac{3}{8}$ (b) $\frac{3}{4}$
(c) $\frac{4}{5}$ (d) $\frac{1}{4}$

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Multiple choice questions with MORE than ONE correct answer : (Questions No. 26-30)

26. If a , $b \in R^+$, where a , A_1 , A_2 , b are in arithmetic progression , a , G_1 , G_2 , b are in geometric progression and a , H_1 , H_2 , b are in harmonic progression , then which of the following relations are correct ?

(a) $G_1G_2\left(G_1+G_2\right)=\dfrac{A_1+A_2}{H_1+H_2}$

(b) $\dfrac{H_1H_2}{G_1G_2}=\dfrac{H_1+H_2}{A_1+A_2}$

(c) $\dfrac{G_1G_2}{H_1H_2}=\dfrac{(2a+b)(2b+a)}{9ab}$

(d) $\dfrac{A_1+A_2}{H_1+H_2}=\dfrac{(2a-b)(2b-a)}{9ab}$

27. Let four consecutive integers form an increasing arithmetic progression and one of these numbers is equal to the sum of the squares of the other three numbers , then :

(a) the smallest number is 0.

(b) the largest number is 2.

(c) sum of all the four numbers is 2.

(d) product of all the four numbers is 0.

28. For two distinct positive numbers , let A_1, G_1, H_1 denote the AM , GM and HM respectively. For $n \geq 2$, $n \in N$, if A_{n-1} and H_{n-1} has arithmetic , geometric and harmonic means as A_n , G_n , H_n respectively , then :

(a) $A_1 > A_2 > A_3 > A_4 > \ldots\ldots\ldots$

(b) $G_1 < G_2 < G_3 > G_4 < \ldots\ldots\ldots$

(c) $H_1 > H_2 > H_3 > H_4 > \ldots\ldots\ldots$

(d) $G_1 = G_2 = G_3 = G_4 = \ldots\ldots\ldots$

29. Let $\{a_n\}$ represents the arithmetic sequence for which $a_1 = |x|$, $a_2 = |x-1|$ and $a_3 = |x+1|$, then :

(a) $a_n - a_{n-1} = \dfrac{1}{2}$ (b) $a_1 = 2$

(c) $\displaystyle\sum_{n=1}^{10} a_n = 25$ (d) $a_n - a_{n-1} = \dfrac{1}{4}$

30. Let $a_n = \dfrac{3}{4} - \left(\dfrac{3}{4}\right)^2 + \left(\dfrac{3}{4}\right)^3 \ldots\ldots\ldots\ldots (-1)^{n-1}\left(\dfrac{3}{4}\right)^n$ and $b_n + a_n = 1$. If $b_n > a_n$ for all $n > n_0$, where $n \in N$, then possible values of natural number 'n_0' can be :

(a) 4 (b) 6

(c) 8 (d) 2

Assertion Reasoning questions : (Questions No. 31-35)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements , Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers , only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

31. Statement 1 : Let three positive numbers in geometric progression represent the sides of a triangle , then the common ratio of the G.P. can be $\dfrac{1}{2}\sin\left(\dfrac{\pi}{5}\right)$

because

Statement 2 : the common ratio of the G.P. in consideration lies in between $\dfrac{1}{2}\sin\left(\dfrac{\pi}{10}\right)$ and $\dfrac{1}{2}\sin\left(\dfrac{3\pi}{10}\right)$.

32. Statement 1 : In a triangle ABC , if $\cot A$, $\cot B$, $\cot C$ forms an $A.P.$, then $\dfrac{1}{b+a}$, $\dfrac{1}{c+b}$, $\dfrac{1}{a+c}$ also form an $A.P.$

because

Statement 2 : $\dfrac{1}{a^2}$, $\dfrac{1}{b^2}$, $\dfrac{1}{c^2}$ form a $H.P.$

33. Statement 1 : If [.] and {.} denote the greatest integer function and the fractional part , then x , $[x]$, $\{x\}$ can never form a geometric progression for any positive rational value of x

because

Statement 2 : x , $[x]$, $\{x\}$ can form a G.P. for $x \in R^+$, only if $x = \dfrac{1}{2}\sin\left(\dfrac{7\pi}{10}\right)$.

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34. **Statement 1 :** If $a\ ,b\ ,c \in R^+$, then the minimum value of $\{a(b^2+c^2)+b(c^2+a^2)+c(a^2+b^2)\}$ is equal to $6abc$

because

Statement 2 : for $a_1\ ,a_2\ ,a_3\ ,a_4\ ,..........a_n \in R^+$, $(AM)(HM)=(GM)^2 \quad \forall \quad n \in N-\{1\}$

35. **Statement 1 :** Let $S_n = 1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+........+\frac{1}{n}$, $n \in N$, then $S_n > ln(n+1)$

because

Statement 2 : $ln\,(n+1) > ln\,(n) \quad \forall \quad n \in N$

Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1-3)

Let V_r denote the sum of the first r terms of an arithmetic progression (A.P.) whose first term is r and the common difference is $(2r-1)$.

Let $T_r = V_{r+1} - V_r - 2$ and $Q_r = T_{r+1} - T_r$ for $r = 1, 2, \ldots$

1. The sum $V_1 + V_2 + \ldots + V_n$ is :

(a) $\frac{1}{12}n(n+1)(3n^2-n+1)$

(b) $\frac{1}{12}n(n+1)(3n^2+n+2)$

(c) $\frac{1}{2}n(2n^2-n+1)$

(d) $\frac{1}{3}(2n^3-2n+3)$

2. T_r is always :

(a) an odd number (b) an even number

(c) a prime number (d) a composite number

3. Which one of the following is a correct statement ?

(a) $Q_1, Q_2, Q_3, \ldots$ are in A.P. with common difference 5

(b) $Q_1, Q_2, Q_3, \ldots$ are in A.P. with common difference 6

(c) $Q_1, Q_2, Q_3, \ldots$ are in A.P. with common difference 11

(d) $Q_1 = Q_2 = Q_3 = \ldots$

Comprehension passage (2)
(Questions No. 4-6)

Let P and Q be two sets each of which consisting of three numbers in $A.P.$ and $G.P.$ respectively. Sum of the elements of set P is 12 and product of the elements of set Q is 8 , where the common difference and the common ratio of $A.P.$ and $G.P.$ are represented by 'd' and 'r' respectively. If sum of the squares of the terms of $A.P.$ is 8 times the sum of the terms of $G.P.$, where $d = r$, and d , $r \in I^+$, then answer the following questions.

4. Total number of terms in the set of $P \cap Q$ is/are :

(a) 0 (b) 2

(c) 1 (d) 3

5. Let $Q = \{a, b, c\}$, where $a < b < c$, then the roots of the quadratic equation $ax^2 + bx + c = 0$ are :

(a) real (b) real and unequal

(c) real and equal (d) non-real

6. Sum of all the elements of set $P \cup Q$ is equal to :

(a) 56 (b) 13

(c) 19 (d) 25

Questions with Integral Answer :
(Questions No. 7-10)

7. Let x and y be two real numbers such that the k^{th} mean between x and $2y$ is equal to the k^{th} mean between $2x$ and y when n arithmetic means are placed between them in both the situations. The value of expression $\left\{\frac{n+1}{k} - \frac{y}{x}\right\}$ is equal to ....

8. Let $S_n = \sum_{r=1}^{n} \frac{1}{r}$ and

$$S'_n = \frac{n+1}{2} - \left\{\frac{1}{n(n-1)} + \frac{2}{(n-1)(n-2)} + \ldots \frac{(n-2)}{6}\right\},$$

then value of $\left\{\frac{S'_n}{S_n}\right\}$ is equal to ....

9. Let an $A.P.$ and a $G.P.$ each has α as the first term and β as the second term , where $\alpha > \beta > 0$. If sum of infinite terms of $G.P.$ is 4 and the sum of first n terms of $A.P.$ can be written as $n\alpha - \frac{n(n-1)\alpha^2}{k}$, then value of 'k' is equal to ....

10. Let sum of the squares of three distinct real number in geometric progression be S^2 and their sum is $\frac{\sqrt{p}}{2}(S)$. If $p \in R^+$, then total number of possible integral values of 'p' is/are ....

11. Let $a, b, c, d, e \in R^+$ and $s = a + b + c + d + e$, if minimum value of $\left\{\frac{(s-a)(s-b)(s-c)(s-d)(s-e)}{abcde}\right\}$ is 4^n , then value of n is ....

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Matrix Matching Questions :
(Questions No. 12-15)

12. Match the following columns (I) and (II)

Column (I)	Column (II)
(a) Let $\sum_{r=1}^{2009}\left(1+\frac{1}{r^2}+\frac{1}{(r+1)^2}\right)^{\frac{1}{2}}=\alpha+\frac{\alpha}{\beta}$, then sum of all the digits of the number 'β' is	(p) 1
(b) The largest positive term of the harmonic progression whose first two terms are $\frac{2}{5}$ and $\frac{12}{23}$, is equal to	(q) 4
(c) If $I_n=\int_0^{\pi/4}\tan^n x\,dx$, where $n\in N$, and $\frac{1}{I_2+I_4}, \frac{1}{I_3+I_5}, \frac{1}{I_4+I_6}$ ..... form an *A.P.*, then common difference of this *A.P.* is	(r) 3
	(s) 11
(d) Value of $(0.16)^{\log_{\frac{5}{2}}\left(\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\ldots\infty\right)}$ is equal to	(t) 6

13. Match the following columns (I) and (II).

Column (I)	Column (II)
(a) If p is prime number and $x\in N$, where $log_p\left(\sqrt{x}+\sqrt{x+p}\right)=1$, then first three smallest possible values of x are	(p) in arithmetic progression
(b) If a_1, a_2, a_3, a_4, a_5 are five non-zero distinct numbers such that a_1, a_2, a_3 are in *A.P.*, a_2, a_3, a_4 are in *G.P.* and a_3, a_4, a_5 are in *H.P.*, then a_1, a_3, a_5 are	(q) in geometric progression
	(r) in harmonic progression
(c) $\tan 70^\circ$, $\tan 50^\circ+\tan 20^\circ$ and $\tan 20^\circ$ are	
(d) If a, b are positive distinct real number and α, β, γ are three roots of $\frac{x-a}{b}+\frac{x-b}{a}=\frac{b}{x-a}+\frac{a}{x-b}$ such that $\alpha>\beta>\gamma$ and $\alpha-\beta-\gamma=c$, then a, b, c are	(s) not is arithmetic progression
	(t) not in geometric progression

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14. Match the following columns (I) and (II).

Column (I)	Column (II)
(a) If sum of first n positive integers is $\frac{1}{5}$ times the sum of their squares , then n is	(p) 3
(b) If $\sum n$, $\frac{\sqrt{10}}{3}\sum n^2$, $\sum n^3$ are in G.P. , then the value of n is	(q) 7
(c) If $log_3 2$, $log_3(2^x - 5)$ and $log_3\left(2^x - \frac{7}{2}\right)$ are in A.P. , then value of x is	(r) 4
(d) Let $S_1 , S_2 , S_3 , ...$ be squares such that for each $n \geq 1$, length of side of S_n equals the length of diagonal of S_{n+1} . If length of S_1 is 1.5 cm , then for which values of n is the area of S_n less than 1 sq. cm.	(s) 6
	(t) 2

15. Match the following columns (I) and (II).

Column (I)	Column (II)
(a) If altitudes of a triangle are in A.P. , then sides of triangle are in	(p) A.P.
(b) If $\begin{vmatrix} a & b & a\alpha - b \\ b & c & b\alpha - c \\ 2 & 1 & 0 \end{vmatrix} = 0$ and $\alpha \neq \frac{1}{2}$, then a , b , c are in	(q) G.P.
(c) If $\frac{a_2\, a_3}{a_1\, a_4} = \frac{a_2 + a_3}{a_1 + a_4} = 3\left(\frac{a_2 - a_3}{a_1 - a_4}\right)$, then a_1 , a_2 , a_3 , a_4 are in	(r) H.P.
(d) If $(y - x)$, $2(y - a)$ and $(y - z)$ are in H.P. , then $(x - a)$, $(y - a)$, $(z - a)$ are in	(s) A.G.P.

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ANSWERS — Exercise No. (1)

1. (a)	**2.** (a)	**3.** (d)	**4.** (a)	**5.** (c)
6. (c)	**7.** (c)	**8.** (c)	**9.** (c)	**10.** (a)
11. (a)	**12.** (b)	**13.** (b)	**14.** (a)	**15.** (c)
16. (b)	**17.** (b)	**18.** (c)	**19.** (b)	**20.** (c)
21. (c)	**22.** (d)	**23.** (b)	**24.** (a)	**25.** (b)
26. (b , c)	**27.** (b , c ,d)	**28.** (a , d)	**29.** (a , c)	**30.** (b , c)
31. (a)	**32.** (d)	**33.** (a)	**34.** (c)	**35.** (b)

ANSWERS — Exercise No. (2)

1. (b)	**2.** (d)	**3.** (b)	**4.** (b)	**5.** (d)
6. (b)	**7.** (1)	**8.** (1)	**9.** (8)	**10.** (9)
11. (5)				

12. (a) $\rightarrow$ r	**13.** (a) $\rightarrow$ s , t	**14.** (a) $\rightarrow$ q	**15.** (a) $\rightarrow$ r
(b) $\rightarrow$ t	(b) $\rightarrow$ q , s	(b) $\rightarrow$ r	(a) $\rightarrow$ q
(c) $\rightarrow$ p	(c) $\rightarrow$ p , t	(c) $\rightarrow$ p	(a) $\rightarrow$ r
(d) $\rightarrow$ q	(d) $\rightarrow$ r , s , t	(d) $\rightarrow$ p , q , r , s	(a) $\rightarrow$ q

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Chapter 3

Complex Numbers

Exercise No. (1)

Multiple choice questions with ONE correct answer : (Questions No. 1-25)

1. If $A(z_1)$, $B(z_2)$ and $C(z_3)$ are the vertices of an equilateral triangle in the clockwise direction , then $arg\left(\dfrac{z_2+z_3-2z_1}{z_3-z_2}\right)$ is :

(a) $\dfrac{\pi}{4}$ (b) $\dfrac{\pi}{3}$

(c) $\dfrac{\pi}{6}$ (d) $\dfrac{\pi}{2}$

2. Let complex numbers z_1 and z_2 satisfy the conditions $|z+6i|=2$ and $|z-4i|=\left(\dfrac{z-\bar{z}}{2i}\right)$ respectively, then minimum value of $|z_1-z_2|$ is :

(a) 8 (b) 6

(c) 4 (d) 2

3. For non-zero complex number 'z' , if $|z-2-2i|+2\sqrt{2}=|z|$, then $arg\,(i\,\bar{z})$ is equal to :

(a) $\dfrac{3\pi}{4}$ (b) $\dfrac{\pi}{4}$

(c) $\dfrac{5\pi}{4}$ (d) $\dfrac{7\pi}{4}$

4. If α and β are complex numbers , then maximum value of $\dfrac{|\alpha\bar{\beta}+\bar{\alpha}\beta|}{|\alpha\beta|}$ is :

(a) 1 (b) 2

(c) $\dfrac{1}{2}$ (d) 4

5. If α , β , γ are the roots of cubic equation $x^3 - 3x^2 + 3x + 7 = 0$, 'ω' is non-real cube root of unity , then $\left(\dfrac{\alpha-1}{\beta-1}+\dfrac{\beta-1}{\gamma-1}+\dfrac{\gamma-1}{\alpha-1}\right)$ is :

(a) $\dfrac{8}{\omega}$ (b) ω^2 (c) $2\omega^2$ (d) $3\omega^2$

6. $f(z)$ is non-real function of complex number 'z' and when $f(z)$ is divided by $(z-i)$ and $(z+i)$ the remainders are i and $1+i$ respectively, then the remainder when $f(z)$ is divided by (z^2+1) is equal to :

(a) $\dfrac{1}{2}+i+z$ (b) $\dfrac{1}{2}iz+\dfrac{1}{2}+i$

(c) $iz+1+i$ (d) $\dfrac{i}{2}+iz$

7. If $|\alpha_k|<3\ \forall\ 1\le k\le n,\ k\in N$, and complex number 'z' satisfy $1+\alpha_1 z+\alpha_2 z^2+.......\alpha_n z^n=2$, then :

(a) $|z|>\dfrac{1}{4}$ (b) $|z|<\dfrac{1}{4}$

(c) $|z|=\dfrac{1}{4}$ (d) $\dfrac{1}{3}<|z|<\dfrac{1}{2}$

8. If $\left|\dfrac{z_1-3z_2}{3-z_1\bar{z}_2}\right|=1$ and $|z_2|\ne 1$, then $|z_1|$ is equal to :

(a) 3 (b) 1 (c) 2 (d) 4

9. A particle P starts from the point $z_0=1+2i$, where $i=\sqrt{-1}$. It moves first horizontally away from origin by 5 units and then vertically away from origin by 3 units to reach a point z_1 . From z_1 the particle moves $\sqrt{2}$ units in the direction of the vector $\hat{i}+\hat{j}$ and then it moves through an angle 90º in anticlockwise direction on a circle with centre at origin to reach a point z_2 . The point z_2 is given by :

(a) $6+7i$ (b) $-7+6i$

(c) $7+6i$ (d) $-6+7i$

10. Consider a square $OABC$, where O is origin and $A(z_0)$, $B(z_1)$, $C(z_2)$ are in anticlockwise sense , then equation of circle inscribed in the square is :

(a) $|z-z_0(1+i)|=2|z_0|$

(b) $\left|z-\dfrac{1}{2}(1-i)z_0\right|=|z_0|$

(c) $2\left|z-\dfrac{1}{2}(1+i)z_0\right|=|z_0|$

(d) $2\left|z-(1+i)z_0\right|=|z_0|$

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11. If $A(z_1)$, $B(z_2)$ and $C(z_3)$ are the vertices of a triangle ABC inscribed in the circle $|z| = 1$ and internal angle bisector of $\angle A$ meet the circumference at $D(z_4)$, then

(a) $z_4^{\ 2} = z_2 z_3$ (b) $z_4 = \frac{z_2 z_3}{z_1}$

(c) $z_4 = \frac{z_1 z_2}{z_3}$ (d) $z_4 = \frac{z_1 z_3}{z_2}$

12. Centre of the arc represented by $arg\left(\frac{z-3i}{z-2i+4}\right) = \frac{\pi}{4}$ is given by :

(a) $\frac{1}{2}(5+5i)$ (b) $\frac{1}{2}(5i-5)$

(c) $\frac{1}{2}(9i+5)$ (d) $\frac{1}{2}(9i-5)$

13. If a, b, c are integers not all equal and ω is cube root of unity $(\omega \neq 1)$, then minimum value of the expression $|a+b\omega+c\omega^2|$ is :

(a) 0 (b) 1

(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2}$

14. Let $z_1 = 10 + 6i$ and $z_2 = 4 + 6i$. If z is any complex number such that $arg\left(\frac{z-z_1}{z-z_2}\right) = \frac{\pi}{4}$, then

(a) $|z-7+9i| = 3\sqrt{2}$ (b) $|z-7-9i| = 2\sqrt{3}$

(c) $|z+7+9i| = 3\sqrt{2}$ (d) $|z-7-9i| = 3\sqrt{2}$

15. If $A(z_1)$, $B(z_2)$ and $C(z_3)$ form an isosceles right angled triangle and $\angle A = \frac{\pi}{2}$, then

(a) $(z_1-z_2)^2 = 2(z_2-z_3)(z_3-z_2)$

(b) $(z_1-z_2)^2 = 2(z_1-z_3)(z_3-z_2)$

(c) $(z_3-z_2)^2 = 2(z_1-z_3)(z_2-z_1)$

(d) $(z_3-z_2)^2 = 2(z_2-z_1)(z_3-z_1)$

16. If complex number 'z' satisfy $|z + 13i| = 5$, then complex number having magnitude-wise minimum argument is :

(a) $-\frac{12}{13}(12+5i)$ (b) $\frac{12}{13}(5+12i)$

(c) $-\frac{12}{13}i(12+5i)$ (d) $\frac{12}{13}i(12-5i)$

17. Let $|z_1| = 30$ and $|z_2 + 5 + 12i| = 13$, then minimum value of $|z_2 - z_1|$ is :

(a) 2 (b) 6

(c) 4 (d) none of these

18. Area of region on the complex plane which is bounded by the curve $|z+2i| + |z-2i| = 8$ is :

(a) $3\sqrt{8}\pi$ (b) $4\sqrt{12}\,\pi$

(c) $16\pi\sqrt{3}$ (d) none of these

19. If z and w are two non-zero complex numbers such that $|zw| = 1$ and $arg\left(\frac{z}{w}\right) = \frac{\pi}{2}$, then $\bar{z}w$ is equal to:

(a) 1 (b) –1

(c) i (d) $-i$

20. Let $x = e^{i\alpha}$, $y = e^{i\beta}$ and $z = e^{i\gamma}$ and $x+y+z=0$, then which one of the following is not correct :

(a) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=0$

(b) $xy+yz+zx=0$

(c) $x^2+y^2+z^2=1$

(d) $x^3+y^3+z^3=3xyz$

21. Let $z = x+iy$ be a complex number where x and y are integers , then the area of the rectangle whose vertices are roots of the equation $(\bar{z})z^3 + z(\bar{z})^3 = 350$ is :

(a) 48 (b) 32

(c) 40 (d) 80

22. Let $z = \cos\theta + i\sin\theta$, then the value of summation $\sum_{r=1}^{15} Im\left(z^{2r-1}\right)$ at $\theta = 2^{\circ}$ is equal to :

(a) $\frac{1}{\sin 2^{\circ}}$ (b) $\frac{1}{3\sin 2^{\circ}}$

(c) $\frac{1}{2\sin 2^{\circ}}$ (d) $\frac{1}{4\sin 2^{\circ}}$

23. Let $A(z_1)$, $B(z_2)$ and $C(z_3)$ form triangle ABC on the argand plane such that $\frac{z_1-z_2}{z_3-z_2} = \frac{1-i}{\sqrt{2}}$, then ΔABC is :

(a) equilateral (b) right angled

(c) isosceles (d) scalene

24. If moving complex number $'z'$ satisfy the conditions, $1 \le |z-1+i| \le 2$ and $\frac{\pi}{12} \le \arg(z+i-1) \le \frac{5\pi}{12}$, then area of region which is represented by $'z'$ is :

(a) π (b) $\frac{\pi}{2}$

(c) 2π (d) $\frac{\pi}{3}$

25. A man walks a distance of 3 units from the origin towards the north-east (N 45° E) direction. From there, he walks a distance of 4 units towards the north-west (N 45° W) direction to reach a point P, then the position of P in the argand plane is :

(a) $3e^{i\pi/4}+4i$ (b) $(3-4i)e^{i\pi/4}$

(c) $(4+3i)e^{i\pi/4}$ (d) $(3+4i)e^{i\pi/4}$

Multiple choice questions with MORE than ONE correct answer : (Questions No. 26-30)

26. Let z_r, where $r \in \{1, 2, 3, \ldots, n\}$, be the $'n'$ distinct roots of the equation $\sum_{r=1}^{n} {}^nC_r x^r = 1$. If there exists some z_r for which $arg\left(\frac{z_r-(\sqrt{2}\,i-1)}{-1-(\sqrt{2}\,i-1)}\right)=\frac{\pi}{4}$, then $'n'$ can be :

(a) 4 (b) 8

(c) 12 (d) 16

27. Let $2+3i$ and $-2+3i$ be the two vertices of an equilateral triangle on the complex plane, then the third vertex of triangle can be given by :

(a) $(-3+2\sqrt{3})i$ (b) $(-3-2\sqrt{3})i$

(c) $(3+2\sqrt{3})i$ (d) $(3-2\sqrt{3})i$

28. Let α, β, γ be the complex numbers, and $\alpha z^2+\beta z+\gamma=0$, where $z \in C$. If the quadratic equation in $'z'$ is having

(a) both roots real, then $\frac{\alpha}{\bar{\alpha}}=\frac{\beta}{\bar{\beta}}=\frac{\gamma}{\bar{\gamma}}$.

(b) both roots purely imaginary, then $\frac{\bar{\alpha}}{\alpha}=\frac{-\bar{\beta}}{\beta}=\frac{\bar{\gamma}}{\gamma}$.

(c) both roots real, then $\frac{\alpha}{\bar{\alpha}}=\frac{\beta}{\bar{\beta}}=\frac{\gamma}{\bar{\gamma}}$.

(d) both roots purely imaginary, then $\frac{\alpha}{\bar{\alpha}}=\frac{\beta}{\bar{\beta}}=\frac{-\gamma}{\bar{\gamma}}$.

29. Let $A(z_1)$, $B(z_2)$ and $C(z_3)$ be the vertices of ΔABC on the complex plane, where the triangle ABC is inscribed in circle $|z|=1$. If altitude through A meets the circle $|z|=1$ at D and image of D about BC is E, then

(a) complex point $'E'$ is $z_1+z_2+z_3$.

(b) complex point $'D'$ is $-\frac{z_2z_3}{z_1}$

(c) complex point $'E'$ is $2(z_1+z_2+z_3)$.

(d) complex point $'D'$ is $-\frac{z_1z_2}{z_3}$

30. Let P, Q, R be three sets of complex numbers as defined below:

$P=\{z : \mathrm{Re}(z(1-i))=\sqrt{2}\}$
$Q=\{z : |z-i-2|=3\}$
$R=\{z : Im(z)\ge 1\}$

In the context of given sets, which of the following statements are correct ?

(a) number of elements in the set $P\cap Q\cap R$ are infinite.

(b) If $'z'$ be any point in $P\cap Q\cap R$, then $|z-5-i|^2+|z+1-i|^2=36$

(c) number of elements in the set $P\cap Q\cap R$ is one.

(d) number of elements in the set $P\cap Q$ are two.

Assertion Reasoning questions : (Questions No. 31-35)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements, Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers, only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

31. Statement 1 : Let $'z'$ be the moving complex point on argand plane for which

$$|z-3-2i| = \left| |z| \sin\left(\frac{\pi}{4} - arg\ (z)\right) \right| ,$$

then the locus of $'z'$ is part of an ellipse

because

Statement 2 : Ellipse is the locus of a point for which sum of its distances from two distinct fixed points is always constant , where the constant sum is more than the distance between the fixed points.

32. Statement 1 : If $i^2+1=0$, then value of $\cos^{-1}\left\{\sin(ln(i)^i)\right\}$ is equal to π

because

Statement 2 : $\cos^{-1}(\cos x) = 2\pi - x \quad \forall \quad x \in [\pi\ ,\ 2\pi]$

33. Let the equations $arg(z+4-3i) = -\dfrac{\pi}{3}$ and $arg(z-2+3i) = \dfrac{5\pi}{6}$ be represented by the curves C_1 and C_2 respectively on the complex plane , then

Statement 1 : The number of points of intersection of C_1 and C_2 is only one

because

Statement 2 : Two non-parallel lines always intersect at only one point in 2-dimensional plane.

34. Let $z_1 = 5+8i$ and z_2 satisfy $|z+2+3i| \le 2$, then

Statement 1 : minimum value of $|iz_2 + z_1|$ is equal to 8

because

Statement 2 : maximum value of $|z_2|$ is $2+\sqrt{13}$

35. Statement 1 : Let $m\ ,\ n \in N$ and the equations $z^m - 1 = 0$ and $z^n - 1 = 0$ is having only one common root , then m and n must be different prime numbers

because

Statement 2 : the common root for the equations $z^m - 1 = 0$ and $z^n - 1 = 0$ is 1 if m and n are different prime numbers.

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Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1-3)

Let $1, \alpha_1, \alpha_2, \alpha_3, ... \alpha_{n-1}$ be the n^{th} roots of unity, then $\alpha_k = \cos\frac{2k\pi}{n} + i\sin\frac{2k\pi}{n}$, where $k = 0, 1, 2, 3, 4 ...., n-1$, further $x^n - 1 = 0$ can be expressed as $x^n - 1 = (x-1)\prod_{k=1}^{(n-1)}(x-\alpha_k)$. Now answer the following questions based on above information

1. Value of $\sum_{k=0}^{17}\left(\cos\frac{k\pi}{8} + i\sin\frac{k\pi}{8}\right)$ is equal to :

(a) 0 (b) 1

(c) $2\cos\frac{\pi}{16}.\left(e^{i\frac{\pi}{16}}\right)$ (d) $2\cos\frac{\pi}{8}.\left(e^{i\frac{\pi}{8}}\right)$

2. Value of $\sum_{k=1}^{n-1}\frac{1}{(4-\alpha_k)}$ is equal to :

(a) $\frac{1+4^{n-1}(3n-4)}{3(4^n+1)}$

(b) $\frac{4+4^n(3n-4)}{12(4^n-1)}$

(c) $\frac{1+(3n-2)4^{n-1}}{4^n+1}$

(d) $\frac{1+4^n(3n-4)}{12(4^n-1)}$

3. If $1, \alpha_1, \alpha_2, ... \alpha_{n-1}$ forms a polygon on the complex plane, then area of the circle inscribed in the polygon is given by :

(a) $\pi\sin^2\left(\frac{\pi}{n}\right)$

(b) $\frac{\pi}{2}\left(1+\cos\frac{2\pi}{n}\right)$

(c) $\pi\left(\cos\frac{2\pi}{n}+1\right)$

(d) $2\pi\left(\cos^2\frac{\pi}{n}\right)$

Comprehension passage (2)
(Questions No. 4-6)

If complex number $'z_1'$ satisfy $|z-2-2i| = \frac{z+\bar{z}}{2}$ and complex number $'z_2'$ satisfy $|z+4-2i| = 2$, then answer the following questions.

4. Minimum value of $|z_1 - z_2|$ is :

(a) 2 (b) 1 (c) 3 (d) 5

5. If magnitude of $arg(z_2)$ is minimum then $|z_2|$ is :

(a) $5\sqrt{2}$ (b) $4\sqrt{2}$

(c) 4 (d) $\sqrt{18}$

6. Maximum possible value of $|z_2|$ is :

(a) $1+\sqrt{5}$ (b) $2(1+\sqrt{5})$

(c) $3(\sqrt{5}+1)$ (d) $2(\sqrt{5}-1)$

Comprehension passage (3)
(Questions No. 7-9)

Let $P(z_1)$, $Q(z_2)$ and $R(z_3)$ represent the vertices of an isosceles triangle PQR on the argand plane, where $RQ = PR$ and $\angle QPR = \alpha$. If incentre of ΔPQR is given by $I(z_4)$, then answer the following questions.

7. The value of $\left\{\left(\frac{PR}{PQ}\right)\left(\frac{PQ}{PI}\right)^2\right\}$ is equal to :

(a) $\frac{(z_1-z_2)(z_1-z_3)}{(z_1-z_4)^2}$

(b) $\frac{(z_1-z_2)(z_3-z_2)}{(z_1-z_4)^2}$

(c) $\frac{(z_1-z_3)(z_2-z_3)}{(z_2-z_4)^2}$

(d) $\frac{(z_1-z_2)(z_3-z_1)}{(z_3-z_4)^2}$

8. The value of $\left\{(z_1-z_2)^2\tan\alpha.\tan\left(\frac{\alpha}{2}\right)\right\}$ is equal to :

(a) $(z_1+z_2-2z_3)(z_1+z_2-2z_4)$

(b) $(z_1+z_2-z_3)(z_1+z_2-z_4)$

(c) $(2z_3-z_1-z_2)(z_1+z_2-2z_4)$

(d) $(z_1+z_2+z_3)(z_2+z_3-z_4)$

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9. The value of $\left\{(z_1-z_4)^2.\left(\frac{1+\cos\theta}{\cos\theta}\right)\right\}$ is equal to :

(a) $(z_2-z_1)(z_3-z_1)$

(b) $\frac{(z_2-z_1)(z_3-z_1)}{(z_4-z_1)}$

(c) $\frac{(z_2-z_1)(z_3-z_1)}{(z_4-z_1)^2}$

(d) $(z_2-z_1)(z_3-z_1)^2$

Questions with Integral Answer :
(Questions No. 10-14)

10. Let moving complex number $'z_o'$ lies on the curve C_1 on argand plane , where

$$arg\left(\frac{z_o-1-i\tan\left(\frac{7\pi}{8}\right)}{z_o-\tan\left(\frac{15\pi}{8}\right)-i}\right)=2\tan^{-1}(\sqrt{2}-1).$$

If the curve C_2 on argand plane is represented by $|z|=2$, then area of the region bounded by the curves C_1 and C_2 is equal to ....

11. Let moving complex point $A\,(z_0)$ satisfy the condition $|z_o-3+2i|+|z_o-3-6i|=10$, and complex points B , C are represented by $3+6i$ and $3-2i$ respectively. If the area of triangle ABC is maximum , then three times the in-radius of triangle ABC is ....

12. Let z be uni-modular complex number , then value of $\frac{arg(z^2+\bar{z}.z^{1/3})}{arg(z^{1/3})}$, where $arg(z)\in\left(0\,,\frac{3\pi}{8}\right)$, is equal to ....

13. Let $A(z_1)$, $B(z_2)$, $C(z_3)$ form a triangle ABC, where

$$\angle ABC=\angle ACB=\frac{1}{2}(\pi-\alpha).$$

If $\left\{\frac{(z_3-z_2)^2}{(z_3-z_1)(z_1-z_2)}\right\}\text{cosec}^2\frac{\alpha}{2}=k$, then value of $'k'$ is equal to ....

14. Let z_1 , z_2 , z_3 be three distinct complex numbers , where $2|z_1|=|z_3|=4$, $|z_2|=|z_1|+1$ and $|2z_1+3z_2+4z_3|=4.$ If $|8z_2z_3+27z_3z_1+64z_1z_2|$ is equal to $'k'$, then value of $\frac{k}{16}$ is equal to ....

Matrix Matching Questions :
(Questions No. 15-16)

15. Match the following Columns (I) and (II).

Column (I)	**Column (II)**
(a) Let $\theta\in R$ and $'z'$ be any complex number such that $\lvert 2z\cos\theta+z^2\rvert=3$, then minimum value of $\lvert z\rvert$ is :	(p) 1
(b) Let $z=x+iy$, where x , $y\in I$. Area of the octagon whose vertices are the roots of the equation $(z\bar{z})\lvert z^2-\bar{z}^2\rvert=1200$ is :	(q) 27
(c) Let z be complex number such that $(z+\bar{z})(4+i)-(3+i)(z-\bar{z})+26i=0$, then value of $z\bar{z}$ is :	(r) 14
	(s) 62
(d) Let $\lvert z_1\rvert=\lvert z_2\rvert=\lvert z_3\rvert=3$, then minimum value of $\lvert z_1+z_2\rvert^2+\lvert z_2+z_3\rvert^2+\lvert z_3+z_1\rvert^2$ is :	(t) 17

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16. Match the following columns (I) and (II).

Column (I)	Column (II)
(a) The roots of the equation $z^4+z^3+z+1=0$ on the complex plane are represented by the vertices of :	(p) an ellipse
(b) If variable complex number 'z' satisfy the condition $\|z-\bar{z}\|+\|z+\bar{z}\|=4$, then locus of z is given by :	(q) a square
	(r) a trapezium
(c) The roots of the equation $z^4+z^3+z^2+z+1=0$ on the complex plane are represented by the vertices of :	(s) a hexagon
(d) The roots of the equation $z^6+z^4-z^2-1=0$ on the complex plane are represented by the vertices of :	(t) an equilateral triangle

ANSWERS — Exercise No. (1)

1. (d)	**2.** (b)	**3.** (b)	**4.** (b)	**5.** (d)
6. (b)	**7.** (a)	**8.** (a)	**9.** (d)	**10.** (c)
11. (a)	**12.** (d)	**13.** (b)	**14.** (d)	**15.** (c)
16. (c)	**17.** (c)	**18.** (b)	**19.** (d)	**20.** (c)
21. (a)	**22.** (d)	**23.** (c)	**24.** (b)	**25.** (d)
26. (b ,d)	**27.** (c , d)	**28.** (b , c)	**29.** (a , b)	**30.** (b , c , d)
31. (d)	**32.** (b)	**33.** (b)	**34.** (b)	**35.** (d)

ANSWERS — Exercise No. (2)

1. (c)	**2.** (b)	**3.** (b)	**4.** (c)	**5.** (c)
6. (b)	**7.** (a)	**8.** (c)	**9.** (a)	**10.** (2)
11. (4)	**12.** (2)	**13.** (4)	**14.** (6)	

15. (a) $\rightarrow$ p
(b) $\rightarrow$ s
(c) $\rightarrow$ t
(d) $\rightarrow$ q

16. (a) $\rightarrow$ t
(b) $\rightarrow$ q
(c) $\rightarrow$ r
(d) $\rightarrow$ q

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Chapter 4

Binomial Theorem

Exercise No. (1)

Multiple choice questions with ONE correct answer : (Questions No. 1-20)

1. Maximum value of the term independent of x in the expansion of $\left(x\sin\alpha+\frac{\cos\alpha}{x}\right)^{10}$, where $\alpha \in R$, is :

(a) $\frac{10!}{(5!)^2}$ (b) $\frac{10!}{32(5!)^2}$

(c) $\frac{10!}{1024(5!)^2}$ (d) ${}^{10}C_8$

2. Sum of the series, ${}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + \ldots\ldots\ldots\ldots + {}^{20}C_{10}$ is equal to :

(a) $2^{20} + {}^{20}C_{10}$ (b) $2^{19} + {}^{20}C_{10}$

(c) $2^{19} - \frac{1}{2}.{}^{20}C_{10}$ (d) $2^{19} + {}^{19}C_9$

3. Coefficient of x^5 in the expansion of the product $(1+2x)^6(1-x)^7$ is :

(a) 172 (b) 171

(c) 170 (d) 160

4. If the binomial coefficients of three consecutive terms in the expansion of $(1+x)^n$ are in the ratio 1 : 7 : 42, then value of $'n'$ is :

(a) 32 (b) 65

(c) 55 (d) 50

5. Coefficient of x^5 in $\left\{(1+x)^{21}+(1+x)^{22}+\ldots\ldots+(1+x)^{30}\right\}$ is :

(a) ${}^{31}C_6 - {}^{21}C_5$ (b) ${}^{31}C_6 - {}^{21}C_6$

(c) ${}^{32}C_5 - {}^{20}C_4$ (d) ${}^{32}C_6 + {}^{20}C_5$

6. Let ${}^{16}C_r = a_r$, then sum of the series, $3a_0^2 - 7a_1^2 + 11a_2^2 - 15a_3^2 + \ldots\ldots + 67a_{16}^2$, is equal to :

(a) $-35\,a_8$ (b) $70\,a_8$

(c) $35\,a_8$ (d) $-70\,a_8$

7. Let $(1+x)^n = \sum_{r=0}^{n} {}^nC_r\, x^r$, then value of $\sum_{r=0}^{n}(-1)^r.{}^nC_r.\frac{1+r\,ln10}{(1+ln10^n)^r}$ is equal to :

(a) 1 (b) 2

(c) 0 (d) –1

8. Coefficient of x^4 in expansion of $(1+x+x^2+x^3)^{11}$ is :

(a) 605 (b) 810

(c) 990 (d) 1020

9. If ${}^nC_r = \binom{n}{r}$, then $\sum_{r=2}^{n+1}\binom{n+2}{r-1}$ is equal to :

(a) $2^{n+2}-2$ (b) $2^{n+2}-n+1$

(c) $4(2^n-1)-n$ (d) $4(2^n+1)-2n$

10. $\sum_{r=0}^{10}(-1)^r.{}^{10}C_r\left(\frac{1}{2^r}+\frac{3^r}{2^{2r}}+\frac{7^r}{2^{3r}}+\ldots\ldots\ldots\infty\right)$ is equal to :

(a) $\frac{1}{255}$ (b) $\frac{1}{1023}$

(c) $\frac{1}{511}$ (d) $\frac{1}{2047}$

11. The value of $\sum\sum_{0\le i<j\le n} j.{}^nC_i$ is equal to :

(a) $n(3n+1)2^{n-3}$ (b) $n(n+3).2^{n-3}$

(c) $(n+3).2^{n-3}$ (d) $n.2^{n-3}$

12. Let $n\in N$ and $(1+x+x^2)^n = \sum_{r=0}^{2n} a_r x^r$; then value of $\sum_{r=0}^{2n}(-1)^r.a_r^2$ is equal to :

(a) a_n^2 (b) $3a_n$

(c) a_n (d) $2a_n^2$

13. Let $n \in I^+ - \{1, 2\}$ and the digits at the unit's place and ten's place of 3^n are 9 and 0 respectively, then $(n-2)$ must be divisible by :

(a) 16 (b) 6
(c) 10 (d) 18

14. Let T_r denotes the r^{th} term in the expansion of $(1+x)^n$ and T_n is the only term which is numerically greatest exactly for three natural values of 'x', then 'n' can be:

(a) 5 (b) 10
(c) 7 (d) 8

15. Let $n_1 + n_2 = 40$, where $n_1, n_2 \in N$ and the value of $\sum_{r=0}^{n} {}^{n_1}C_{n-r} \cdot {}^{n_2}C_r$ is maximum, then value of 'n' must be :

(a) 25 (b) 15
(c) 20 (d) 22

16. Value of $\sum_{\alpha=0}^{n} {}^nC_\alpha (\sin \alpha x)$ is equal to :

(a) $2^n.\cos^n \frac{x}{2}.\sin\frac{nx}{2}$ (b) $2^n.\sin^n \frac{x}{2}.\cos\frac{nx}{2}$
(c) $2^{n+1}.\cos^n \frac{x}{2}.\sin\frac{nx}{2}$ (d) $2^{n+1}.\sin^n \frac{x}{2}.\cos\frac{nx}{2}$

17. If [.] represents the greatest integer function and $\alpha = (\sqrt{3}+2)^n$, then value of $\alpha[\alpha]+\alpha-\alpha^2$ is equal to :

(a) 0 (b) 1
(c) 2 (d) –1

18. For natural number m, n if $(1-y)^m (1+y)^n = 1 + a_1 y + a_3 y^2 + \ldots..$, and $a_1 = a_2 = 10$ then (m, n) ordered pair is :

(a) (35 , 45) (b) (20 , 45)
(c) (35 , 20) (d) (45 , 35)

19. The coefficient of x^8 in $\left\{\sum_{r=0}^{\infty}(r+1)x^r\right\}^{-5}$, where $|x|<1$, is equal to :

(a) – 50
(b) – 45
(c) 50
(d) 45

20. Let T_β be the term which is independent of 'α' in the binomial expansion of $\left(\frac{\alpha-1}{\alpha-\alpha^{1/2}} - \frac{\alpha+1}{\alpha^{2/3}-\alpha^{1/3}+1}\right)^{10}$, then T_β is equal to :

(a) 300 (b) 210
(c) 420 (d) 500

Multiple choice questions with MORE than ONE correct answer : (Questions No. 21-25)

21. Let $a_n = \frac{(10)^{3n}}{n!}$, $n \in N$, and the value of a_n is greatest , then :

(a) $n = 998$
(b) $n = 999$
(c) $n = 1000$
(d) $n = 1001$

22. Let $n \in I^+$, $(5+3\sqrt{3})^{2n+1} = \alpha + \beta$, where α is an integer and $\beta \in (0, 1)$, then :

(a) $(\alpha+\beta)^2$ is divisible by 2^{2n+1}
(b) $\alpha\beta = 2(4)^n$
(c) α is divisible by 10
(d) α is an odd integer

23. Let $A_n = \sum_{r=0}^{n} {}^nC_r.\cos\left(\frac{2r\pi}{n}\right)$ & $B_n = \sum_{r=0}^{n-1} {}^{n-1}C_r.\cos\left(\frac{2r\pi}{n}\right)$, where ${}^nC_r = \frac{n!}{r!.(n-r)!}$, then which of the following statements are correct :

(a) $A_n = 2B_{n+2}$ (b) $A_n = 2B_n$
(c) $B_8 = -\frac{27}{2}$ (d) $A_6 = -27$

24. If $S = \sum_{m=1}^{n+1} \frac{(1+x)^{n+m-1}}{2^{m-1}}$, where $x \in (-1, 1)$, then the correct statements are :

(a) coefficient of x^n in S is $2^{n+1} - 2^{n-1}$
(b) $\lim_{p\to\infty} \frac{2^n(S)}{1+x+x^2+\ldots\ldots+x^p} = (1+x)^n - (1+x)^{2n+1}$
(c) coefficient of x^n in S is 2^n
(d) value of $\sum_{r=0}^{n} {}^{n+r}C_r \left(\frac{1}{2}\right)^{n+r}$ is 1

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25. Let T_r denotes the r^{th} term in the binomial expansion of $(1 + x)^n$, where T_{n-1} and T_n are equal for at least one integral value of x, then value of 'n' can be :

(a) 11 (b) 7

(c) 12 (d) 8

Assertion Reasoning questions :
(Questions No. 26-30)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements, Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers, only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

26. Statement 1 : Total number of distinct terms in the expansion of $\{(x+y^2)^{13}+(x^2+y)^{14}\}$ is 28,

because

Statement 2 : Total number of common terms in the expansion of $(x + y^2)^{13}$ and $(x^2 + y)^{14}$ are 2.

27. Statement 1 : If the binomial expansion of $(\sqrt{2}+\sqrt[3]{7})^n$ contains only two rational terms, then value of 'n' can be 10

because

Statement 2 : The applicable natural values of 'n' are 6, 8, 10, which are all even in nature.

28. Statement 1 : The coefficient of term containing x^o in the expansion of $\left(x^2+\dfrac{1}{x^2}+2\right)^{23}$ is ${}^{46}C_{23}$

because

Statement 2 : ${}^nC_{\frac{n}{2}}$ is maximum, if n is even natural number.

29. Let a, b, c denote the sides of a triangle ABC opposite to the vertices A, B and C respectively, then

Statement 1 :

Value of $\sum_{r=0}^{n} {}^nC_r(a)^r.(b)^{n-r}.\cos\big((n-r)A-rB\big)$ is equal to zero

because

Statement 2 : In any triangle ABC, $(a\cos B + b\cos A)^n = c^n$ for all $n \in R$.

30. Statement 1 : If ${}^{50}C_{25}$ is divisible by $(18)^n$, where $n \in N$, then maximum value of n can be 2

because

Statement 2 : ${}^{2n}C_n=\dfrac{2^n}{n!}\left\{\prod_{r=1}^{n}(2r-1)\right\}$ for all $n \in N$.

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Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1-3)

Let $f(x) = (1 + x + x^2)^n = a_0 + a_1 x + a_2 x^2 + ... + a_{2n} x^{2n}$,
and $g(x) = b_0 + b_1 x + b_2 x^2 + b_3 x^3 + ... + b_{2n} x^{2n}$,
where $b_k = 1 \ \forall \ k \geq n, n \in N$. Answer the following questions based on the given information.

1. If $f(x) = g(x+1)$, then value of a_n is equal to :

(a) ${}^{2n+2}C_{n+1}$ (b) ${}^{2n-1}C_n$
(c) ${}^{2n+1}C_n$ (d) ${}^{2n}C_n$

2. In $f(x)$, if n is even positive integer, then value of $\left\{(a_0 - a_2 + a_4 - a_6 + a_8 .....)^2 + (a_1 - a_3 + a_5 - a_7 .....)^2\right\}$ is equal to :

(a) 1 (b) 2
(c) 0 (d) 4

3. In $f(x)$, if n is positive integral multiple of 3, then $\sum_{r=0}^{n} (-1)^r . a_r . {}^nC_r$ is equal to :

(a) ${}^{3n}C_{n/3}$ (b) ${}^{n}C_{2n/3}$
(c) ${}^{2n}C_{n/3}$ (d) ${}^{n+1}C_{n/3}$

Comprehension passage (2)
(Questions No. 4-6)

Let $m, n \in N$ and ${}^nS_m = \sum_{r=1}^{n} (r)^m$, if

$$P(m,n) = m!\left\{\binom{m}{m} + \binom{m+1}{m} + \binom{m+2}{m} + ... + \binom{n+m-1}{m}\right\},$$

where $\binom{p}{q} = {}^pC_q$, then answer the following questions.

4. Value of $\lim_{n\to\infty} \left\{\frac{{}^nS_6}{n^7}\right\}$ is equal to :

(a) 0 (b) 1/7
(c) 1/6 (d) 1/14

5. Value of ${}^nS_2 + {}^nS_1$ is equal to :

(a) $\frac{1}{2}P(2, n)$
(b) $P(2, n)$
(c) $\frac{1}{3}P(2, n)$
(d) $\frac{1}{6}P(2, n)$

6. Value of ${}^nS_3 + 3\,{}^nS_2$ is equal to :

(a) $P(3, n) - 2P(2, n)$
(b) $P(3, n) + 2P(1, n)$
(c) $P(3, n) - 2P(1, n)$
(d) $P(3, n) + 2P(2, n)$

Questions with Integral Answer :
(Questions No. 7-10)

7. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + ... C_n x^n$, where $n \in N$, and $\sum_{r=1}^{n} \left\{ r^3 \left(\frac{C_r}{C_{r-1}} \right)^2 \right\} = 540$, then value of n is equal to ....

8. Let the binomial coefficients of the 3rd, 4th, 5th and 6th terms in the expansion of $(1+x)^{100}$ be a, b, c and d respectively. If α, β are relatively prime numbers and $\frac{b^2 - ac}{c^2 - bd} = \frac{\alpha a}{\beta c}$, then value of $(\alpha - \beta)$ is equal to ....

9. Let $n \in N$ and ${}^{n+1}C_{n-2} \leq 100 + {}^{n-1}C_{n-2}$, then number of possible values of 'n' is equal to ....

10. If $\left\{\frac{1}{2}\,{}^{30}C_1 - \frac{2}{3}\,{}^{30}C_2 + \frac{3}{4}\,{}^{30}C_3 \,........ - \frac{30}{31}\,{}^{30}C_{30}\right\}$ is equal to $(10\lambda + 1)^{-1}$, then value of 'λ' is equal to ....

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Matrix Matching Questions :
(Questions No. 11-12)

11. Match the following columns (I) and (II)

Column (I)	Column (II)
(a) $\dfrac{{}^{20}C_0}{3} - \dfrac{{}^{20}C_1}{4} + \dfrac{{}^{20}C_2}{5}$ .... upto 21 terms.	(p) 465
(b) $\dfrac{{}^{30}C_1}{{}^{30}C_0} + 2\dfrac{{}^{30}C_2}{{}^{30}C_1} + 3\dfrac{{}^{30}C_3}{{}^{30}C_2} + ...$ upto 30 terms.	(q) 0
(c) ${}^{10}C_0^2 + 2\,{}^{10}C_1^2 + 3\,{}^{10}C_2^2 + ...$ upto 11 terms.	(r) $\dfrac{6}{5}\left(\dfrac{(19!)}{(9!)^2}\right)$
(d) $\displaystyle\sum_{r=1}^{10} \frac{(-1)^r\,{}^{10}C_r}{(4r+1)}$	(s) $\dfrac{1}{11\,.\,21\,.\,23}$
	(t) $\displaystyle\int_0^1 (1-x^4)^{10}\,dx$

12. Match the following columns (I) and (II).

Column (I)	Column (II)
(a) If the sixth term in the binomial expansion of $\left(3^{\log_3\sqrt{9^{x-1}+7}} + \dfrac{1}{5^{(1/5)\log_5(3^{x-1}+1)}}\right)^7$ is 84, then values of $'x'$ can be	(p) 5
	(q) 1
	(r) 4
(b) The second last digit of number 7^{283} is equal to	
(c) The coefficient of x^{10} in the expansion of $(1+x^2-x^3)^8$ is not divisible by	(s) 3
(d) The positive integer which is greater than $(1+0.00001)^{10^5}$ can be	(t) 2

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ANSWERS Exercise No. (1)

1. (b)	**2.** (d)	**3.** (b)	**4.** (c)	**5.** (b)
6. (c)	**7.** (c)	**8.** (c)	**9.** (c)	**10.** (b)
11. (a)	**12.** (c)	**13.** (c)	**14.** (c)	**15.** (c)
16. (a)	**17.** (b)	**18.** (a)	**19.** (d)	**20.** (b)
21. (b , c)	**22.** (a , c)	**23.** (b , d)	**24.** (c , d)	**25.** (a , b)
26. (a)	**27.** (c)	**28.** (b)	**29.** (d)	**30.** (b)

ANSWERS Exercise No. (2)

1. (c)	**2.** (a)	**3.** (b)	**4.** (b)	**5.** (b)
6. (c)	**7.** (8)	**8.** (2)	**9.** (7)	**10.** (3)

11. (a) $\to$ s
(b) $\to$ p
(c) $\to$ r
(d) $\to$ t

12. (a) $\to$ q , t
(b) $\to$ r
(c) $\to$ p , s
(d) $\to$ p , r , s

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Chapter 5

Permutation & Combination

Exercise No. (1)

Multiple choice questions with ONE correct answer : (Questions No. 1-20)

1. The letters of the word 'GHAJINI' are permuted and all the permutations are arranged in a alphabetical order as in an English dictionary , then total number of words that appear after the word 'GHAJINI' is given by :

(a) 2093 (b) 2009

(c) 2092 (d) 2091

2. If John is allowed to select at most $(n + 1)$ chocolates from a collection of $(2n + 2)$ distinct chocolates , then total number of ways by which John can select at least two chocolates are given by :

(a) $(4)^n + 4.\,{}^{2n+1}C_n - 2n + 1$

(b) $2(4)^n + 4.\,{}^{2n+1}C_n - 2n + 3$

(c) $2(4)^n - {}^{2n+1}C_n - 2n - 3$

(d) $2(4)^n + {}^{2n+1}C_n - 2n - 3$

3. The coefficient of x^{1502} in the expansion of $\left\{(1+x+x^2)^{2007}.(1-x)^{2008}\right\}$ is

(a) ${}^{2007}C_{501} - {}^{2006}C_{500}$

(b) ${}^{2006}C_{500} - {}^{2006}C_{501}$

(c) ${}^{2007}C_{498} - {}^{2006}C_{499}$

(d) ${}^{2007}C_{501} - {}^{2007}C_{1506}$

4. X and Y are any 2 five digits numbers , total number of ways of forming X and Y with repetition , so that these numbers can be added without using the carrying operation at any stage , is equal to :

(a) $45(55)^4$ (b) $36(55)^4$

(c) $(55)^5$ (d) $51(55)^4$

5. A team of four students is to be selected from a total of 12 students , total number of ways in which team can be selected if two particular students refuse to be together and other two particular students wish to be together only , is equal to :

(a) 226 (b) 182

(c) 220 (d) 300

6. If the L.C.M. of $'\alpha'$ and $'\beta'$ is $p^2 q^4 r^3$, where p , q , r are prime numbers and $\alpha , \beta \in I^+$, then the number of ordered pairs (α , β) are :

(a) 225 (b) 420 (c) 315 (d) 192

7. Total number of non-negative integral solutions of $18 < x_1 + x_2 + x_3 \le 20$, is given by :

(a) 1245 (b) 685

(c) 1150 (d) 441

8. If Mr. and Mrs. Rustamji arrange a dinner party of 10 guests and they are having fixed seats opposite one another on the circular dinning table , then total number of arrangements on the table , if Mr. and Mrs. Batliwala among the guests don't wish to sit together , are given by :

(a) 148 (8!) (b) 888 (8!)

(c) 74 (8!) (d) 164 (8!)

9. If 10 identical balls are to be placed in identical boxes , then the total number of ways by which this placement is possible , if no box remains empty , is given by :

(a) 2^{10} (b) 11 (c) 9 (d) 5

10. Total number of ways by which the word 'HAPPYNEWYEAR' can by arranged so that all vowels appear together and all consonants appear together , is given by :

(a) 12(7!) (b) 6(8!)

(c) 8 (7!) (d) 3 (8!)

11. The number of seven digit integers , with sum of the digits equal to 10 and formed by using the digits 1 , 2 and 3 only , is :

(a) 55 (b) 66 (c) 77 (d) 88

12. Let $\vec{r}$ be a variable vector and $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ such that scalar values $(\vec{r}.\hat{i})$, $(\vec{r}.\hat{j})$ and $(\vec{r}.\hat{k})$ are positive integers. If $\vec{r}.\vec{a}$ is not greater than 10 , then total numbers of possible $\vec{r}$ are given by :

(a) 80 (b) 120 (c) 240 (d) 100

13. Let three lines L_1, L_2, L_3 be given by $2x+3y=2$, $4x+6y=5$ and $6x+9y=10$ respectively. If line L_r contains 2^r different points on it, where $r \in \{1, 2, 3\}$, then maximum number of triangles which can be formed with vertices at the given points on the lines, are given by :

(a) 320 (b) 304

(c) 364 (d) 360

14. Let function $'f'$ be defined from set A to set B, where $A = B = \{1, 2, 3, 4\}$. If $f(x) \neq x$, where $x \in A$, then total number of functions which are surjective is given by :

(a) 12 (b) 10

(c) 9 (d) 8

15. Total number of five digit numbers that can be formed, having the property that every succeeding digit is greater than the preceding digit, is equal to :

(a) ${}^{9}P_5$ (b) ${}^{9}C_4$

(c) ${}^{10}C_5$ (d) ${}^{10}P_5$

16. An n-digit number is a positive number with exactly n digits. Nine hundred distinct n-digit numbers are to be formed using only the three digits 2, 5 and 7. The smallest value of n for which this is possible, is :

(a) 6 (b) 7

(c) 8 (d) 9

17. Consider n boxes which are numbered by n consecutive natural numbers starting with the number m. If the box with labelled number k, $k \geq m$, contains k distinct books, then total number of ways by which m books can be selected from any one of the boxes, are :

(a) ${}^{n}C_{m+1}$ (b) ${}^{n+m}C_m$

(c) ${}^{n}C_{m+1}$ (d) ${}^{n+m}C_{n-1}$

18. Total number of triplets (x, y, z) which can be formed, selecting x, y, z from the set $\{1, 2, 3, 4, ... 100\}$ such that $x \leq y < z$, is equal to :

(a) ${}^{100}C_3$ (b) ${}^{101}C_3$

(c) ${}^{102}C_3$ (d) ${}^{100}C_2$

19. Total number of ways in which a group of 10 boys and 2 girls can be arranged in a row such that exactly 3 boys sit in between 2 girls, is equal to :

(a) 1440(8!) (b) 720(8!)

(c) 10(9!) (d) 180(8!)

20. Total number of ways of selecting two numbers from the set of $\{1, 2, 3, 4, ..., 3n\}$ so that their sum is divisible by 3 is equal to :

(a) $3n^2 - n$ (b) $\dfrac{3n^2 - n}{2}$

(c) $\dfrac{2n^2 - n}{2}$ (d) $2n^2 - n$

Multiple choice questions with MORE than ONE correct answer : (Questions No. 21-25)

21. Total number of four letters words that can be formed from the letters of the word 'DPSRKPURAM', is given by

(a) ${}^{10}C_4.(4!)$

(b) 2190

(c) Coefficient of x^4 in $\left\{4!.\left(1+x+x^2\right)\left(1+x\right)^6\right\}$

(d) Coefficient of x^4 in $\left\{3!.\left(1+x\right)^6\left(1+(x+1)^2\right)^2\right\}$

22. Consider seven digit number $x_1x_2x_3x_4 .... x_7$, where $x_1, x_2, ... x_7 \neq 0$, having the property that x_4 is the greatest digit and digits towards the left and right of x_4 are in decreasing order, then total number of such numbers in which all digits are distinct is given by :

(a) ${}^{9}C_7.{}^{6}C_3$ (b) ${}^{9}C_2 . {}^{6}C_4$

(c) $3. {}^{9}C_7 . {}^{5}C_1$ (d) $2. {}^{9}C_2 . {}^{5}C_2$

23. Consider $xyz = 24$, where $x, y, z \in I$, then

(a) Total number of positive integral solutions for x, y, z are 81

(b) Total number of integral solutions for x, y, z are 90

(c) Total number of positive integral solutions for x, y, z are 30

(d) Total number of integral solutions for x, y, z are 120

24. If ${}^{n}C_{r+1} = (m^2 - 8).{}^{n-1}C_r$; then possible value of $'m'$ can be :

(a) 4 (b) 2

(c) 3 (d) –5

25. Let 10 different books are to be distributed among four students A, B, C and D. If A and B get 2 books each C and D get 3 books each, then total number of ways of distribution are equal to :

(a) ${}^{10}C_4$ (b) 25200

(c) 12600 (d) $\dfrac{10!}{(2!)^2(3!)^2}$

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Assertion Reasoning questions :
(Questions No. 26-30)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements , Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers , only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

26. Statement 1 : If $n , m \in I^+$, then $N = \dfrac{(mn)!}{(n!)^m.(m!)}$ is always an integral value

because

Statement 2 : $'N'$ represents the total numbers of ways of equal distribution of (mn) distinct objects among $'m'$ persons.

27. Statement 1 : From a group of 5 teachers and 5 students , if a team of 5 persons is to be formed having at least two teachers then total number of ways be which team can be formed is given by $\left({}^5C_2\right).\left({}^8C_3\right)$

{i.e. , selection of 2 teachers from 5 and 3 more persons from remaining 8}

because

Statement II : The team may have 5 teachers , or 4 teachers and 1 student , or 3 teachers and 2 students , or 2 teachers and 3 students.

28. Statement 1 : Total number of polynomials of the form $x^3 + ax^2 + bx + c$ which are divisible by $x^2 + 1$, where $a , b , c \in \{1 , 2 , 3 , ...10\}$ must be 10

because

Statement 2 : value of $'b'$ can be selected in 10 ways from the set of first 10 natural number and $a = c = 1$.

29. Statement 1 : If $a , b \in N$ and $x = 7^a.5^b$, where x and $7x$ is having 12 and 15 positive divisors respectively , then the number of positive divisors of $5x$ is 16

because

Statement 2 : Sum of all the positive divisors of $(\alpha)^a.(\beta)^b$, where $a , b \in N$, is equal to $\dfrac{(1-\alpha^{a+1})(1-\beta^{b+1})}{1-\alpha-\beta+\alpha\beta}$, provided α and β are the prime numbers.

30. Statement 1 : Let $A_1 , A_2 ... , A_{30}$ be thirty sets each with five elements and $B_1 , B_2 , ... , B_n$ be n sets each with three elements such that $\bigcup_{i=1}^{30} A_i = \bigcup_{i=1}^{n} B_i = S$. If each element of S belongs to exactly ten of the A_i's and exactly nine of the B_j's , then the value of n is 45

because

Statement 2 : $n\left(\bigcup_{i=1}^{n} A_i\right) \le \sum_{i=1}^{n} n(A_i)$, where $n(A)$ represent the number of elements of set A.

Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1 - 3)

Consider the letters of the word 'MATHEMATICS' , some of them are identical and some are distinct. Letters are classified as repeating and non-repeating , such as $\{M , A , T\}$ is repeating set of letters and $\{H , E , I , C , S\}$ is non-repeating set of letters , answer the following questions based on given information.

1. Total numbers of words , taking all letters at a time , such that at least one repeating letter is at odd position in each word is given by

(a) $\frac{9!}{8}$ (b) $\frac{11!}{8}$

(c) $\frac{9!}{4}$ (d) $\frac{11!}{8}-\frac{9!}{4}$

2. Total number or words , taking all letters at a time , in which no vowel is together , is given by

(a) $\frac{7!}{(2!)^2}.{}^8C_4\left(\frac{4!}{2!}\right)$ (b) $\frac{7!}{2!}.{}^8C_4.\left(\frac{4!}{2!}\right)$

(c) $7!.{}^8C_4\left(\frac{4!}{2!}\right)$ (d) $\frac{7!}{8}.{}^8C_4.\left(\frac{4!}{2!}\right)$

3. Total number of words , taking all letters at a time , such that each word contains both M's together and both T's together but both A's are not together , is given by

(a) $\left({}^8C_2\right).7!$ (b) $\frac{11!}{8}-\frac{10!}{4}$

(c) 6(6!) (d) 9(7!)

Comprehension passage (2)
(Questions No. 4 - 6)

Let B_1 , B_2 and B_3 are three different boxes which contains y_1 , y_2 and y_3 distinct balls respectively , where $y_i \geq 1 \ \forall \ i = \{1 , 2 , 3\}$, $\sum_{i=1}^{3} y_i = 20$ and $y_2 = y_1 + 2$. If total number of ways by which John can select exactly 2 balls from the boxes is 'N' and he is not allowed to select two balls from the same box , then answer the following questions

4. If $y_3 = 14$, then value of N is equal to :

(a) 90 (b) 112

(c) 140 (d) 92

5. If N assumes its maximum value , then which one of the following is correct :

(a) $y_1 = y_3 = 5$ (b) $y_1 = y_3 = 8$

(c) $y_2 = 8$ (d) $y_2 = 6$

6. Maximum value of N is equal to :

(a) 131 (b) 140

(c) 132 (d) 130

Comprehension passage (3)
(Questions No. 7 - 9)

Let $A = \{ 1 , 2 , 3 , 4 , ... , n \}$ be the set of first n natural numbers , where $S \subseteq A$. If the number of elements in set S is represented by $\eta(S)$ and the least number in the set S is denoted by S_{min} , then answer the following questions.

7. If any of the subset S of set A is having $\eta(S) = r$, where $1 \leq r \leq n$, then maximum value of S_{min} which can occur is equal to :

(a) r (b) $n-r$

(c) $n-r+1$ (d) $r+1$

8. The number of subsets $'S'$ with $S_{min} = m$ and $\eta(S) = r$, is equal to :

(a) $m\left({}^{n-m}C_{r-1}\right)$ (b) ${}^{n-m}C_r$

(c) ${}^nC_{r-1}$ (d) ${}^{n-m}C_{r-1}$

9. Let $\eta(S) = r$ and $S_{min} = m$, where $r < n-m$, then sum of all the S_{min} for possible subsets $'S'$ is equal to :

(a) $m\left({}^{n-m}C_{r-1}\right)$

(b) $n\left({}^{n-m}C_{r-1}\right)$

(c) $(n+1)\,{}^{n-m}C_{r-1} - r\left({}^{n-m+1}C_r\right)$

(d) $m\left({}^{n-m}C_{r-1}\right) + n\left({}^{n-m+1}C_r\right)$

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Questions with Integral Answer :
(Questions No. 10-14)

10. Let $'N'$ triangles can be formed by joining the vertices of a regular decagon in which no two consecutive vertices are selected , then value of $\left\{\frac{N}{10}\right\}$ is equal to ....

11. Let in ${}^{\alpha}C_{\beta}$ number of ways four tickets can be selected from 35 tickets numbered from 1 to 35 so that no two consecutive numbered tickets are selected , then the value of $\left\{\frac{\alpha}{\beta}\right\}$ is equal to ....

12. Let all the letters of the word SACHHABACHHA be arranged in a matrix of order 4×3 , and at least one of the row of matrix is having all the identical elements. If the total number of arrangements are $'N'$, then least prime number dividing the number $'N'$ is equal to ....

13. Let $P(n)$ denotes the sum of the even digits of the number $'n'$, for example : $P(8592)=8+2=10$, then value of of $\dfrac{\left\{\sum_{r=1}^{100} P(r)\right\}}{100}$ is equal to ....

14. Let 16 people are to be arranged around a regular octagonal frame such that people can either sit at the corner or at the mid of the side. If the number of ways in which the arrangement is possible is $\lambda(15!)$, then value of $'\lambda'$ is equal to ....

Matrix Matching Questions :
(Questions No. 15-16)

15. Consider a set $'A'$ containing 8 different elements from which a subset $'P'$ is chosen and the set A is reconstructed by replacing the elements of P. From set A if another subset Q is chosen , then match the following columns for the number of ways of choosing P and Q in column (II) with the conditions in column (I)

Column (I)	Column (II)
(a) $P\cap Q$ contains exactly one element	(p) 6561
(b) Q is subset of P	(q) 24
(c) $P\cup Q$ contains exactly one element	(r) 256
(d) $P\cup Q=A$	(s) 17496
	(t) 2187

16. Consider all possible permutations of the letters of the word ENDEANOEL. Match the statements in column I with the statements in column II .

Column (I)	Column (II)
(a) The number of permutations containing the word ENDEA	(p) 120
(b) The number of permutations in which the letter E occurs in the first and the last positions	(q) 240
	(r) 840
(c) The number of permutations in which none of the letters D , L , N occurs in the last five positions	(s) 2520
(d) The number of permutations in which the letters A , E , O occur only in odd positions	(t) 420

ANSWERS — Exercise No. (1)

1. (c)	**2.** (d)	**3.** (d)	**4.** (b)	**5.** (a)
6. (c)	**7.** (d)	**8.** (c)	**9.** (d)	**10.** (d)
11. (c)	**12.** (b)	**13.** (b)	**14.** (c)	**15.** (b)
16. (b)	**17.** (d)	**18.** (b)	**19.** (a)	**20.** (b)
21. (b , d)	**22.** (a , d)	**23.** (c , d)	**24.** (a , c , d)	**25.** (b , d)
26. (c)	**27.** (d)	**28.** (c)	**29.** (b)	**30.** (b)

ANSWERS — Exercise No. (2)

1. (b)	**2.** (a)	**3.** (a)	**4.** (d)	**5.** (c)
6. (c)	**7.** (c)	**8.** (d)	**9.** (a)	**10.** (5)
11. (8)	**12.** (2)	**13.** (2)	**14.** (2)	

15. (a) $\rightarrow$ s
(b) $\rightarrow$ p
(c) $\rightarrow$ q
(d) $\rightarrow$ p

16. (a) $\rightarrow$ p
(b) $\rightarrow$ s
(c) $\rightarrow$ q
(d) $\rightarrow$ q

Chapter 6

Probability

Exercise No. (1)

Multiple choice questions with ONE correct answer :
(Questions No. 1-25)

1. Let A, B, C be pair-wise independent events, where $P(A \cap B \cap C) = 0$ and $P(C) > 0$, then $P\left(\frac{\overline{A} \cap \overline{B}}{C}\right)$ is equal to :

(a) $P(\overline{A}) + P(\overline{B})$ (b) $P(\overline{A}) - P(\overline{B})$

(c) $P(\overline{A}) - P(B)$ (d) $P(A) - P(\overline{B})$

2. If three identical dice are rolled, then probability that the same number appears on each of them is :

(a) $\frac{1}{6}$ (b) $\frac{1}{36}$

(c) $\frac{3}{28}$ (d) $\frac{1}{18}$

3. If A, B, C are three mutually independent events, where $P(A \cup B \cup C) = 3P(A \cup B \cap \overline{C}) = \frac{1}{2}$ and $P(A \cap C) = P(\overline{A} \cap B \cap C) = \frac{1}{12}$, then $P(\overline{A} \cap C \cap \overline{B})$ is equal to :

(a) $\frac{1}{12}$ (b) $\frac{5}{6}$

(c) $\frac{1}{6}$ (d) $\frac{1}{24}$

4. An unbiased die is thrown and the number shown on the die is put for 'p' in the equation $x^2 + px + 2 = 0$, probability of the equation to have real roots is :

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$

(c) $\frac{2}{3}$ (d) $\frac{1}{4}$

5. Minimum number of times a fair coin must be tossed so that the probability of getting at least one head is at least 0.95 is

(a) 4 (b) 5

(c) 6 (d) 7

6. Let 'A' and 'B' be two events such that $P(A) = 0.70$, $P(B) = 0.40$ and $P(A \cap \overline{B}) = 0.5$, then $P\left(\frac{B}{A \cup \overline{B}}\right)$ is equal to :

(a) 0.20 (b) 0.25

(c) 0.40 (d) 0.895

7. Three numbers are chosen at random without replacement from $\{1, 2, 3, \ldots, 10\}$. Probability that the minimum of the chosen number is 3 or their maximum is 7, is given by :

(a) 3/10 (b) 11/40

(c) 11/50 (d) 27/40

8. If $a, b, c, d \in \{0, 1\}$, then the probability that system of equations $ax + by = 2$; $cx + dy = 4$ is having unique solution is given by :

(a) $\frac{5}{8}$ (b) $\frac{3}{8}$

(c) 1 (d) $\frac{1}{2}$

9. For a student to qualify, he must pass at least two out of the three exams. The probability that he will pass the first exam is $\frac{1}{2}$, if he fails in one of the exams then the probability of his passing in the next exam is $\frac{1}{4}$ otherwise it remains the same. The probability that student will pass the exam is :

(a) $\frac{4}{5}$ (b) $\frac{3}{8}$

(c) $\frac{1}{4}$ (d) $\frac{3}{4}$

10. Let eight players $P_1, P_2, P_3, \ldots\ldots\ldots P_8$ be paired randomly in each round for a knock-out tournament. If the player P_i wins if $i > j$, then the probability that player P_6 reaches the final round is :

(a) $\frac{2}{35}$ (b) $\frac{8}{35}$

(c) $\frac{10}{17}$ (d) none of these

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11. Let John appears in the exams of physics , chemistry and mathematics and his respective probability of passing the exams is p , c and m. If John has 80% chance of passing in at least one of the three exams , 55% chance of passing in at least two exams , and 35% chance of passing in exactly two of the exams , then $p + c + m$ is equal to :

(a) $\frac{31}{20}$ (b) $\frac{18}{31}$

(c) $\frac{17}{20}$ (d) $\frac{45}{32}$

12. Let one hundred identical coins , each with probability $'p'$ of showing up head are tossed once. If $0 < p < 1$ and the probability that head turns up on 50 coins is equal to the probability that head turns up on 51 coins, then the value of $'p'$ is :

(a) $\frac{50}{101}$ (b) $\frac{49}{101}$

(c) $\frac{51}{101}$ (d) $\frac{52}{101}$

13. In a set of four bulbs it is known that exactly two of them are defective. If the bulbs are tested one by one in random order till both the defective bulbs are identified , then the probability that only two tests are needed is given by :

(a) $\frac{1}{6}$ (b) $\frac{1}{2}$

(c) $\frac{1}{4}$ (d) $\frac{1}{3}$

14. Let 3 faces of an unbiased die are red , 2 faces are yellow and 1 face is green. If the die is tossed three times , then the probability that the colors red , yellow and green appear in the first , second and the third tosses respectively is :

(a) $\frac{1}{18}$ (b) $\frac{1}{36}$

(c) $\frac{7}{36}$ (d) $\frac{1}{9}$

15. Let one Indian and four American men and their wives are to be seated randomly around a circular table. If each American man is seated adjacent to his wife , then the probability that Indian man is also seated adjacent to his wife is given by :

(a) $\frac{1}{5}$ (b) $\frac{1}{3}$

(c) $\frac{2}{5}$ (d) $\frac{1}{2}$

16. Let $'A'$ and $'B'$ be two independent events. The probability that both A and B happen is $\frac{1}{12}$ and the probability that neither A nor B happen is $\frac{1}{2}$, then $\{3P(A) - 4P(B)\}$ may be

(a) 1 or 0 (b) $\frac{7}{12}$ or 0

(c) 0 or $-\frac{7}{12}$ (d) $-\frac{7}{12}$ or 1

17. An urn contains 2 white and 2 black balls , a ball is drawn at random , if it is white it is not replaced into the urn , otherwise it is dropped along with one another ball of same color. The process is repeated , probability that the third drawn ball is black , is :

(a) $\frac{31}{60}$ (b) $\frac{41}{60}$

(c) $\frac{29}{30}$ (d) none of these

18. An experiment has ten equally likely outcomes. Let A and B be two non-empty events of the experiment. If A consists of 4 outcomes , then number of outcomes that B must have so that A and B are independent , is :

(a) 2 , 4 or 8 (b) 3 , 6 or 9

(c) 4 or 8 (d) 5 or 10

19. A fair die is tossed repeatedly until a six is obtained , if $'k'$ denotes the number of tosses required , then the conditional probability that $'k'$ is not less than six when it is given that $'k'$ is greater than 3 , is equal to :

(a) $\frac{5}{36}$ (b) $\frac{125}{216}$

(c) $\frac{25}{36}$ (d) $\frac{25}{216}$

20. A box contain 15 coins , 8 of which are fair and the rest are biased. The probability of getting a head on fair coin and biased coin is $\frac{1}{2}$ and $\frac{2}{3}$ respectively. If a coin is drawn randomly from the box and tossed twice , first time it shows head and the second time it shows tail , then the probability that the coin drawn is fair , is given by :

(a) $\frac{5}{8}$ (b) $\frac{9}{16}$

(c) $\frac{3}{8}$ (d) $\frac{5}{16}$

21. A person goes to office either by car , scooter , bus or train probability of which being $\frac{1}{7}, \frac{3}{7}, \frac{2}{7}$ and $\frac{1}{7}$ respectively. Probability that he reaches office late , if he takes car , scooter , bus or train is $\frac{2}{9}, \frac{1}{9}, \frac{4}{9}$ and $\frac{1}{9}$ respectively. If it is given that he reached office in time then the probability that he travelled by car is :

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$

(c) $\frac{3}{7}$ (d) $\frac{4}{7}$

22. Let $'K'$ be the integral values of x for which the inequation $x^2 - 9x + 18 < 0$ holds. If three fair dice are rolled together , then the probability that the sum of the numbers appearing on the dice is K, is given by :

(a) $\frac{41}{216}$ (b) $\frac{5}{24}$

(c) $\frac{1}{24}$ (d) $\frac{31}{216}$

23. Let set $'S'$ contains all the matrices of 3×3 order in which all the entries are either 0 or 1. If a matrix is selected randomly from set $'S'$ and it is found that it contains exactly five of the entries as 1 , then the probability that the matrix is symmetric , is given by :

(a) $\frac{63}{256}$ (b) $\frac{3}{128}$

(c) $\frac{2}{21}$ (d) $\frac{7}{512}$

24. Let two positive real numbers $'x'$ and $'y'$ are chosen randomly , where $x \in [0\ ,1]$ and $y \in [0\ ,1]$. The probability that $x + y \le 1$, given that $x^2 + y^2 \ge \frac{1}{4}$, is :

(a) $\frac{8+\pi}{16+\pi}$ (b) $\frac{\pi-8}{\pi-16}$

(c) $\frac{4-\pi}{8+\pi}$ (d) $\frac{\pi+2}{16-\pi}$

25. Let a natural number $'N'$ be selected at random from the set of first hundred natural numbers. The probability that $N + \frac{225}{N}$ is not greater than 30 is given by :

(a) 0.01 (b) 0.05

(c) 0.25 (d) 0.025

Multiple choice questions with MORE than ONE correct answer : (Questions No. 26-30)

26. For two events A and B , if $P\left(\frac{B}{A}\right) = \frac{1}{2}$, $P(A) = P\left(\frac{A}{B}\right) = \frac{1}{4}$, then the correct statements are :

(a) $P(A\cap B) = \frac{3}{8}$ (b) $P(\bar{A}\cup\bar{B}) = \frac{7}{8}$

(c) $P\left(\frac{\bar{A}}{\bar{B}}\right) = \frac{3}{4}$ (d) $P\left(\frac{\bar{B}}{\bar{A}}\right) = \frac{1}{2}$

27. Let A , B , C be three independent events , where $3P(A) = 2P(B) = 4P(C) = 1$, then :

(a) probability of occurrence of exactly 2 of the three events is $\frac{1}{4}$.

(b) probability of occurrence of at least one of the three events is $\frac{3}{4}$.

(c) probability of occurrence of all the three events is $\frac{1}{24}$.

(d) probability of occurrence of exactly one of the three events is $\frac{11}{24}$.

28. Let a bag contain 15 balls in which the balls can have either black colour or white colour. If B_n is the event that bag contains exactly n black balls and its probability is proportional to n^2 , and E is the event of getting a black ball when a ball is drawn randomly from the bag , then :

(a) $\sum_{n=0}^{15} P(B_n) = 1$

(b) $P(E) = \frac{24}{31}$

(c) $P\left(\frac{B_5}{E}\right) = \frac{5}{376}$

(d) $P\left(\frac{B_5}{E}\right) = \frac{5}{576}$

29. Let the events $'A'$ and $'B'$ be mutually exclusive and exhaustive in nature , then :

(a) $P(A) \le P(\bar{B})$

(b) $P(\bar{A}\cap\bar{B}) = 0$

(c) $P(A\cup B) = P(A) + P(B)$

(d) $P(A\cup B) = 1 - P(\bar{A})P(\bar{B})$

30. There are four boxes B_1, B_2, B_3 and B_4. Box B_i contain i cards and on each card a distinct number is printed, the printed number varies from 1 to i for box B_i. If a box is selected randomly, then probability of occurrence of box B_i is given by $\left(\frac{i}{10}\right)$ and if a card is drawn randomly from it then E_i represents the event of occurrence of number i on the card, then :

(a) value of $P(E_1)$ is $\frac{2}{5}$

(b) inverse probability $P\left(\frac{B_3}{E_2}\right)$ is $\frac{1}{3}$

(c) conditional probability $P\left(\frac{E_3}{B_2}\right)$ is zero

(d) value of $P(E_3)$ is $\frac{1}{4}$

Assertion Reasoning questions :
(Questions No. 31-35)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements, Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers, only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

31. Statement 1 : Let any two digit number is raised with power $4K + 2$, where $K \in N$, then the probability that unit's place digit of the resultant number is natural multiple of 3 is 1/3

because

Statement 2 : If any two digit number is raised with power $4K + 2$, $K \in N$, then digit at units place can be $0, 1, 4, 5, 6, 9$.

32. Statement 1 : Let 'A' and 'B' be two dependent events and if $\{P(A \cup B)\}^2 = P(\bar{B})$, then least value of $P(A \cup B)$ is $2\sin\left(\frac{\pi}{10}\right)$,

because

Statement 2 : $P\left(\frac{\bar{A}}{\bar{B}}\right) = \frac{P(\bar{A} \cap \bar{B})}{P(\bar{B})}$, where $P(\bar{B}) \neq 0$

33. Statement 1 : In a binomial probability distribution $B(n, p = 1/4)$, if the probability of at least one success is not less than 0.90, then value of 'n' can be $\log_{\frac{2}{\sqrt{3}}} 12$

because

Statement 2 : In the given binomial probability distribution 'n' is greater than or equal to $\log_{\frac{4}{3}} 10$

34. Statement 1 : Let A and B be any two events of a random experiment, where $P(A) = \frac{4}{5}$ and $P(B) = \frac{1}{3}$, then the value of $P(A \cap B)$ lies in $\left[\frac{2}{15}, \frac{1}{3}\right]$

because

Statement 2 : For any two events A and B, $\max\{P(A), P(B)\} \le P(A \cup B) \le 1$ and $P(A \cap B) \le \min\{P(A), P(B)\}$.

35. Statement 1 : Let an ellipse of eccentricity $\frac{4}{5}$ be inscribed in a circle and a point within the circle be chosen randomly, then the probability that the point lies outside the ellipse is $\frac{2}{5}$

because

Statement 2 : The area of an ellipse of eccentricity 'e' is given by $\pi a^2\sqrt{1-e^2}$ square units, where 'a' represents the radius of auxiliary circle of the ellipse.

Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1-3)

For a biased coin , let the probability of getting head be $\frac{2}{3}$ and that of tail be $\frac{1}{3}$. If A_n denotes the event of tossing the coin till the difference of the number of heads and tails become 'n' , then answer the following questions.

1. If $n = 2$, then the probability that the experiment ends with more number of heads than tails , is equal to :

(a) $\frac{3}{5}$ (b) $\frac{4}{5}$

(c) $\frac{5}{9}$ (d) $\frac{4}{9}$

2. If it is given that the experiment ends with a head for $n = 2$, then the probability that the experiment ends in minimum number of throws , is equal to :

(a) $\frac{3}{5}$ (b) $\frac{4}{9}$ (c) $\frac{3}{8}$ (d) $\frac{5}{9}$

3. If E is the event that the last two throws show either two consecutive heads or tails , then the value of $P\left(\frac{E}{A_n}\right)$ is equal to :

(a) 1 (b) $1-\left(\frac{5}{9}\right)^n$

(c) $1-\left(\frac{4}{9}\right)^n$ (d) 0

Comprehension passage (2)
(Questions No. 4-6)

Consider a bag containing six different balls of three different colours. If it is known that the colour of the balls can be white , green or red , then answer the following questions.

4. The probability that the bag contains 2 balls of each colour is :

(a) $\frac{1}{10}$ (b) $\frac{1}{7}$ (c) $\frac{1}{9}$ (d) $\frac{1}{8}$

5. If three balls are picked up at random from the bag and all the balls are found to be of different colour , then the probability that bag contained 4 white balls , is :

(a) $\frac{7}{25}$ (b) $\frac{1}{7}$

(c) $\frac{1}{14}$ (d) $\frac{1}{10}$

6. If three balls are picked up at random and found to be one of each colour , then the probability that bag contained equal number of white and green balls is equal to :

(a) $\frac{3}{14}$ (b) $\frac{1}{10}$

(c) $\frac{7}{25}$ (d) $\frac{2}{7}$

Comprehension passage (3)
(Questions No. 7-9)

A fair die is tossed repeatedly until a six is obtained. If X denote the number of tosses required , then answer the following questions.

7. The probability that $X = 3$ equals

(a) $\frac{25}{216}$ (b) $\frac{25}{36}$

(c) $\frac{5}{36}$ (d) $\frac{125}{216}$

8. The probability that $X \geq 3$ equals

(a) $\frac{125}{216}$ (b) $\frac{25}{36}$

(c) $\frac{5}{36}$ (d) $\frac{25}{216}$

9. The conditional probability that $X \geq 6$ given $X > 3$ equals

(a) $\frac{125}{216}$ (b) $\frac{25}{216}$

(c) $\frac{5}{36}$ (d) $\frac{25}{36}$

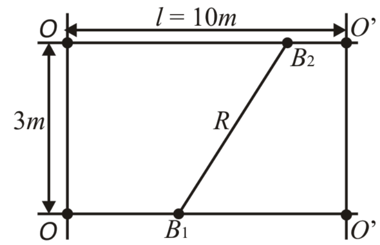
Questions with Integral Answer :
(Questions No. 10-14)

10. If the papers of 4 students can be checked by any one of the 7 teachers. If the probability that all the 4 papers are checked by exactly 2 teachers is P, then the value $49P$ is equal to ....

11. A bag contain 3 black and 3 white balls , from the bag John randomly pick three balls and then drop 3 balls of red colour into the bag. If now John randomly pick three balls from the bag and the probability of getting all the three balls of different colour is p, then value of $\frac{100}{3}p$ is ....

12. Let a cubical die has four blank faces , one face marked with 2 , another face marked with 3 , if the die is rolled and the probability of getting a sum of 6 in 3 throws is p, then value of $\frac{432}{13}p$ is equal to ....

13. There are two parallel telephone lines of length $l = 10m$ which are $3m$ apart as showin figure. It is known that there is a break in each of them , the location of the break being unknown , if the probability that the distance $'R'$ between the breaks is not larger than $5m$ is p, then $\frac{25}{2}p$ is ....



14. A person while dialing a telephone number forgets the last three digits of the number but remembers that exactly two of them are same. He dials the number randomly , if the probability that he dialed the correct number is P, then value of $(1080P)$ is ....

Matrix Matching Questions :
(Questions No. 15-16)

15. Consider a cube having the vertex points A, B, C, D, E, F, G, and H. If randaonly three corner points are selected to form a triangle then match the following columns for the probability of the nature of triangle.

Column (I)	Column (II)
(a) Probability that the triangle is scalene	(p) $\frac{6}{7}$
(b) Probability that the triangle is right-angled	(q) $\frac{4}{7}$
(c) Probability that the triangle is isosceles with exactly	(r) $\frac{1}{7}$
two equal sides	(s) $\frac{3}{14}$
(d) Probability that the triangle is equailateral	(t) $\frac{3}{7}$

16. Five unbiased cubical dice are rolled simultaneously. Let m and n be the smallest and the largest number appearing on the upper faces of the dice , then match the probabilitiy given in the column II corresponding to the events given in the column I :

Column (I)	**Column (II)**
(a) $m=3$	(p) $\left(\frac{2}{3}\right)^5$
(b) $n=4$	(q) $\left(\frac{2}{3}\right)^5+\left(\frac{1}{3}\right)^5-\left(\frac{1}{2}\right)^4$
(c) $2\le m\le 4$	(r) $\left(\frac{5}{6}\right)^5-\left(\frac{1}{3}\right)^5$
(d) $m=2$ and $n=5$	(s) $\left(\frac{2}{3}\right)^5-\left(\frac{1}{2}\right)^5$

ANSWERS Exercise No. (1)

1. (c)	**2.** (c)	**3.** (c)	**4.** (c)	**5.** (b)
6. (b)	**7.** (b)	**8.** (b)	**9.** (b)	**10.** (d)
11. (a)	**12.** (c)	**13.** (d)	**14.** (b)	**15.** (c)
16. (c)	**17.** (d)	**18.** (d)	**19.** (c)	**20.** (b)
21. (a)	**22.** (c)	**23.** (c)	**24.** (b)	**25.** (a)
26. (b , c , d)	**27.** (a , b , c , d)	**28.** (a , b , d)	**29.** (a , b , c)	**30.** (a , b , c)
31. (d)	**32.** (a)	**33.** (a)	**34.** (a)	**35.** (b)

ANSWERS Exercise No. (2)

1. (b)	**2.** (d)	**3.** (a)	**4.** (a)	**5.** (c)
6. (a)	**7.** (a)	**8.** (b)	**9.** (d)	**10.** (6)
11. (9)	**12.** (2)	**13.** (8)	**14.** (4)	

15. (a) $\to$ t
(b) $\to$ p
(c) $\to$ p
(d) $\to$ r

16. (a) $\to$ s
(b) $\to$ s
(c) $\to$ r
(d) $\to$ q

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Chapter 7

Matrices

Exercise No. (1)

Multiple choice questions with ONE correct answer : (Questions No. 1-15)

1. Let $A=\left[a_{ij}\right]_{3\times3}$; $a_{ij}=\sin^3(i-j)$, then

(a) det $(A) = \sin 1$ (b) det $(A) = 0$

(c) det $(A) > 0$ (d) det $(A) < 0$

2. Let $A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$, then $A+A^T=I$ if the values of $'\alpha'$ belong to :

(a) $2n\pi\pm\frac{\pi}{6}$; $n\in I$ (b) $(2n+1)\pi\pm\frac{\pi}{3}$; $n\in I$

(c) $(2n+1)\pi\pm\frac{2\pi}{3}$; $n\in I$ (d) $2n\pi\pm\frac{2\pi}{3}$; $n\in I$

3. Let $A=\left[a_{ij}\right]_{3\times3}$, $B=\left[b_{ij}\right]_{3\times3}$ and $C=\left[c_{ij}\right]_{3\times3}$ be three matrices , where det(A) = 2 and b_{ij} , c_{ij} are the corresponding cofactors of a_{ij} and b_{ij} respectively , then $det(2AB^TC)$ is equal to :

(a) $\sum_{r=1}^{10} {}^{10}C_r$ (b) $\sum_{r=1}^{10} {}^{11}C_r$

(c) $\sum_{r=1}^{11} {}^{10}C_{r-1}$ (d) $\sum_{r=1}^{11} {}^{11}C_{r+1}$

4. Let $A=\left[a_{ij}\right]_{10\times10}$ be a matrix for which $a_{ij}=2i^3+ij-2i^2j+\left[\frac{i+j}{4}\right]-\sin^2(i-j)$, where [.] represents the greatest integer function , then $trace(A)$ is equal to :

(a) 420 (b) 400

(c) 410 (d) 500

5. Let $'S'$ be the set of all 3×3 symmetric matrices for which all the entries are either 1 or 2 , if five of these entries are 2 and four of them are 1 , then $n(S)$ is equal to : ($n(S)$ represents the cardinal number of S)

(a) 10 (b) 12

(c) 20 (d) 18

6. If A and B are two square matrices of order $n\times n$ and $AB=B$, $BA=A$, then $A^2+B^2=2I$ holds true for the condition :

(a) $|A|=|B|=0$ (b) $|A|=|B|\neq0$

(c) $|A|\neq|B|\neq0$ (d) $|A|$ and $|B|$ are non-zero

7. Let $A=\left[a_{ij}\right]_{3\times3}$; where $a_{ij}=\begin{cases}\min\{i\ ,\ j\}\ ;\ i\neq j\\ \left[\frac{2i+j}{2}\right]\ ;\ i=j\end{cases}$

and [.] represents the greatest integer function , then $det\{adj(adj(A))\}$ is equal to :

(a) 5 (b) 25

(c) 625 (d) 125

8. Total number of matrices that can be formed using all the seven different one digit numbers such that no digit is repeated in any matrix , is given by :

(a) 7! (b) $(7)^7$

(c) 2(7!) (d) 7(7!)

9. Let $A=\left[a_{ij}\right]_{3\times3}$ and $B=\left[b_{ij}\right]_{3\times3}$ be two matrices and $\alpha,\beta,\in\{1,2,3\}$, then which one of the following is always true :

(a) $\sum_{\beta=1}^{3}a_{\alpha\beta}.b_{\beta\alpha}=\sum_{\beta=1}^{3}a_{\alpha\beta}.b_{\beta\alpha}$

(b) $\sum_{\beta=1}^{3}a_{\alpha\beta}.b_{\beta\alpha}\neq\sum_{\beta=1}^{3}a_{\alpha\beta}.b_{\beta\alpha}$

(c) $\sum_{\alpha=1}^{3}\left(\sum_{\beta=1}^{3}a_{\alpha\beta}.b_{\beta\alpha}\right)=\sum_{\alpha=1}^{3}\left(\sum_{\beta=1}^{3}a_{\alpha\beta}.b_{\beta\alpha}\right)$

(d) $\sum_{\alpha=1}^{3}\left(\sum_{\beta=1}^{3}a_{\alpha\beta}.b_{\beta\alpha}\right)\neq\sum_{\alpha=1}^{3}\left(\sum_{\beta=1}^{3}a_{\alpha\beta}.b_{\beta\alpha}\right)$

10. Let $'\omega'$ be the non-real cube root of unity , where $A=\begin{bmatrix}\omega&0&0\\0&\omega&0\\0&0&\omega\end{bmatrix}$, then A^{2010} is equal to :

(a) A (b) $-A$

(c) 0 (d) I

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11. Let $A=\begin{bmatrix}\cos(\pi/6) & \sin(\pi/6)\\ -\sin(\pi/6) & \cos(\pi/6)\end{bmatrix}$ and $B=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$; where $C=ABA^T$, then $A^TC^{2010}A$ is equal to :

(a) $\begin{bmatrix}1 & 2010\\ 0 & 1\end{bmatrix}$

(b) $\begin{bmatrix}\sqrt{3}/2 & -1\\ 2010 & 1\end{bmatrix}$

(c) $\begin{bmatrix}\sqrt{3}/2 & 2010\\ 1 & -1\end{bmatrix}$

(d) $\begin{bmatrix}1 & \sqrt{3}/2\\ 0 & 2010\end{bmatrix}$

12. Let $\alpha_k = k({}^8C_k)$ and $\beta_k=(2-k)\,{}^8C_k$, and $A_k=\begin{bmatrix}\alpha_k & 0\\ 0 & \beta_k\end{bmatrix}$. If $\sum_{k=1}^{7}A_k=\begin{bmatrix}p & 0\\ 0 & q\end{bmatrix}$; then value of $(p+q)$ is equal to :

(a) 1020 (b) 508

(c) 204 (d) 420

13. Let $A=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 1\\ 0 & -2 & 4\end{bmatrix}$ and $6A^{-1}=A^2+pA+qI$, then $(2p+q)$ is equal to :

(a) 0 (b) -1

(c) 1 (d) 2

14. If matrix $A=\begin{bmatrix}a & b & c\\ b & c & a\\ c & a & b\end{bmatrix}$, where $a, b, c \in \boldsymbol{C}$, $abc=1$ and $AA^T=I$, then $a^3+b^3+c^3$ is equal to :

(a) 0 (b) $a+b+c$

(c) 3 (d) $3+a+b+c$

15. Let $A=[a_{ij}]_{3\times3}$ represents a matrix and $(-1)^{i+j}a_{ij}+(-1)^{j+k}a_{jk}+(-1)^{k+i}a_{ki}=0$ for all i, j, k belongs to $\{1, 2, 3\}$, then $'A'$ is :

(a) symmetric matrix.

(b) singular matrix.

(c) non-singular matrix.

(d) orthogonal matrix.

Multiple choice questions with MORE than ONE correct answer : (Questions No. 16-20)

16. Let $\theta\in[0, 2\pi)$, $\phi\in\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$ and

$$A=\begin{bmatrix}\sin\phi & \sin\theta & \cos\theta\\ -\sin\theta & -\sin\phi & 1\\ \cos\theta & 1 & \sin\phi\end{bmatrix}, \text{ then}$$

(a) det (A) is independent from θ.

(b) det (A) is independent from ϕ.

(c) det $(A)\in\left[\frac{-3\sqrt{3}}{8}, \frac{-1}{8}\right]$.

(d) det $(A)\in[-1, 1]$.

17. Let $A=[a_{ij}]_{3\times3}$, where $a_{ij}=\begin{cases}\min\{i, j\}, & i\neq j\\ \left[\frac{i+2j}{10}\right]; & i=j\end{cases}$.

If a_{ij} represents the element of i^{th} row and j^{th} column in matrix $'A'$, then : ([.] represents G.I.F.).

(a) det $(A)=0$

(b) det $(A)=4$

(c) A is symmetric matrix

(d) $Tr(A)=0$

18. Let $A(\theta)=\begin{bmatrix}\sin\theta & i\cos\theta\\ i\cos\theta & \sin\theta\end{bmatrix}$, where $i^2=-1$, then

(a) $A(\theta)$ is invertible $\forall\ \theta\in R$.

(b) Inverse of $A(\theta)=A(-\theta)$.

(c) Inverse of $A(\theta)=A(\pi-\theta)$.

(d) $A(\theta)+A(\pi+\theta)=O_{2\times2}$.

19. Let $P=\begin{pmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\\ \frac{-1}{2} & \frac{\sqrt{3}}{2}\end{pmatrix}$ and $A=\begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix}$, then

(a) $(A^{-1})^n=\begin{bmatrix}1 & -n\\ 0 & 1\end{bmatrix}$

(b) Matrices A and P both are orthogonal matrix

(c) If $A^n=I+nB$, then det $(B)=0$

(d) $det\{adj\,(adj\,(2AP))\}=4$

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20. Let matrix 'A' be singular matrix , and $\theta \in [0 , \pi]$.

If $A = \begin{bmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{bmatrix}$, then

possible values of $'\theta'$ can be :

(a) $\frac{7\pi}{24}$ (b) $\frac{11\pi}{24}$

(c) $\frac{23\pi}{24}$ (d) $\frac{19\pi}{24}$

Assertion Reasoning questions :
(Questions No. 21-25)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements , Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers , only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

21. **Statement 1 :** Let $A = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$ and $A^2 = 3A - 2I$, then $A^8 = 255A - 256\,I_2$

because

Statement 2 : $A^n = \begin{bmatrix} 1 & 0 \\ 2^n - 1 & 2^n \end{bmatrix}$

22. Let A and B be two square matrices of order 3 , and $'O'$ represents the null matrix of order 3×3.

Statement 1 : If $AB = 0$, and A is non-singular matrix , then matrix B is necessarily a singular matrix

because

Statement 2 : Product of two equal order square matrices can only be zero matrix if both the matrices are not non-singular matrices.

23. Let A be a 2×2 matrix with real entries , and satisfy the condition $A^2 = I$, where $'I'$ is unit matrix of order 2.

Statement 1 : If $A \neq I$ and $A \neq -I$, then $det(A) = -1$

because

Statement 2 : If $\pm A \neq I$, then $Tr(A) \neq 0$

24. **Statement 1 :** Let $A^5 = 0$ and $A^n \neq I$ for all $n \in \{1 , 2 , 3 , 4\}$, then $(I - A)^{-1} = A^4 + A^3 + A^2 + A + I$

because

Statement 2 : $1 + x + x^2 + x^3 + x^4 = \left(\frac{1-x^5}{1-x}\right)$, where $x \neq 1$.

25. Let A and B be square matrices of order 3 , where $A = \left[a_{ij}\right]_{3\times3}$; $a_{ij} = \sin^3(i - j)$.

Statement 1 : If $n = 7!$, then $B^T A^n B$ is skew-symmetric matrix

because

Statement 2 : determinant value of skew-symmetric matrix of odd order is always zero.

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Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1-3)

Let $'S'$ be the set of all 3×3 symmetric matrices all of whose entries are either 0 or 1. If five of these entries are 1 and four of them are 0, then answer the following questions.

1. The number of matrices in $'S'$ is :

(a) 12 (b) 6

(c) 9 (d) 3

2. The number of matrices A in $'S'$ for which the system of linear equation $A\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$ has a unique solution, is :

(a) less than 4

(b) at least 4 but less than 7

(c) at least 7 but less than 10

(d) at least 10

3. The number of matrices A in $'S'$ for which the system of linear equation $A\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$ is inconsistent, is

(a) 0 (b) more than 2

(c) 2 (d) 1

Comprehension passage (2)
(Questions No. 4-6)

For a given square matrix $'A'$, if $AA^T = A^TA = I$ holds true, then matrix is termed as orthogonal matrix. If $a, b, c \in R$ and matrix $'P'$ is orthogonal, where $P=\begin{bmatrix}0 & a & a\\2b & b & -b\\c & -c & c\end{bmatrix}$, then answer the following questions :

4. If square matrices of order 2 is formed with the entries $0, a, b$ and c, then maximum number of matrices which can be formed without repetition of the entries, is equal to :

(a) 840 (b) 24 (c) 256 (d) 192

5. Let $\alpha, \beta, \gamma \in R$ and matrix $Q=\begin{bmatrix}\alpha a^2 & 0 & 0\\0 & \beta b^2 & 0\\0 & 0 & \gamma c^2\end{bmatrix}$.

If $'Q'$ is orthogonal matrix then maximum number of ordered triplets (α, β, γ) which are possible is given by :

(a) 1 (b) 8

(c) 2 (d) 6

6. If $k=\dfrac{1}{(abc)^2}$; then number of positive integral solutions for the equation $x_1.x_2.x_3=k$, is equal to :

(a) 18 (b) 20

(c) 36 (d) 72

Comprehension passage (3)
(Questions No. 7-9)

Let $A=\begin{bmatrix}1 & 0 & 0\\2 & 1 & 0\\3 & 2 & 1\end{bmatrix}$, and R_1, R_2, R_3 be the row matrices satisfying the relations, $R_1A=\begin{bmatrix}1 & 0 & 0\end{bmatrix}$, $R_2A=\begin{bmatrix}2 & 3 & 0\end{bmatrix}$ and $R_3A=\begin{bmatrix}2 & 3 & 1\end{bmatrix}$. If B is square matrix of order 3 with rows R_1, R_2, R_3, then answer the following questions.

7. The value of $det(B)$ is equal to :

(a) –3 (b) 3

(c) 0 (d) 1

8. Let $C=(2A^{100}.B^3)-(A^{99}.B^4)$, then value of $det(C)$ is equal to :

(a) 27 (b) –27

(c) 100 (d) –100

9. Sum of all the elements of matrix B^{-1} is equal to :

(a) 8 (b) 0

(c) 5 (d) 10

Questions with Integral Answer :
(Questions No. 10-13)

10. Let matrix $A=\begin{bmatrix}1 & 1 & 3\\5 & 2 & 6\\-2 & -1 & -3\end{bmatrix}$, then the least positive integer $'K'$ for which A^K becomes null matrix, is equal to ....

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11. Let $A=\left[a_{ij}\right]_{4\times4}$, where $|A| = 2$ and $B=\left[b_{ij}\right]_{4\times4}$. If b_{ij} is the cofactor of a_{ij}, and $AB^T = C$, then sum of diagonal elements of matrix $'C'$ is equal to ....

12. Let a, x, $y \in R$, where $x+y=0$, and the system of equations is given by :

$$\begin{bmatrix} 2x^2 & -2ay^2 \\ x^2+axy & xy \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} (a+1)^2 \\ 1 \end{bmatrix}$$

If the system has at least one solution, then number of possible integral value(s) of $'a'$ is/are ....

13. Let a, x, $y \in R$, and matrices A, B and C be defined as $A=\begin{bmatrix} 2^{|x|}+|x| & y \\ x^2 & -y^2 \end{bmatrix}$, $B=\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $C=\begin{bmatrix} x^2+a \\ 1 \end{bmatrix}$. If the matrix equations $AB = C$ is having only one solution, then total number of possible value(s) of $'a'$ is/are ....

Matrix Matching Questions :
(Questions No. 14-15)

14. Let $A=\begin{bmatrix} 1 & 4 & 5 \\ \alpha & 8 & 8\alpha-6 \\ 1+\alpha^2 & 8\alpha+4 & 2\alpha+21 \end{bmatrix}$, then match columns (I) and (II) for the values of α and the rank of matrix $'A'$.

Column (I)	Column (II)
(a) If $\alpha=2$, then rank of matrix A is :	(p) 1
(b) If $\alpha=-1$, then rank of matrix A is :	(q) 2
(c) If $\alpha\in R-\{2\}$, then rank of matrix A can be :	(r) 3
(d) If $\alpha=4$, then rank of matrix A is :	(s) 0

15. Match columns (I) and (II)

Column (I)	Column (II)
(a) Let $A=\left[a_{ij}\right]_{3\times3}$ and $B=\left[k^{i-j}a_{ij}\right]_{3\times3}$; if $k_1\vert A\vert+k_2\vert B\vert=0$; where $\vert A\vert \neq 0$, then (k_1+k_2) is	(p) 0
(b) Maximum value of third order determinant if each of its entries are either 1 or –1 , is	(q) 4
(c) If $\begin{vmatrix} 1 & \cos\alpha & \cos\beta \\ \cos\alpha & 1 & \cos\gamma \\ \cos\beta & \cos\gamma & 1 \end{vmatrix}=\begin{vmatrix} 0 & \cos\alpha & \cos\beta \\ \cos\alpha & 0 & \cos\beta \\ \cos\beta & \cos\gamma & 0 \end{vmatrix}$ then $\cos^2\alpha+\cos^2\beta+\cos^2\gamma$ is equal to :	(r) 1
(d) $\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix}=Ax+B$ where A and B are determinant of 3×3, then $(A + 2B)$ is equal to	(s) 2

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ANSWERS Exercise No. (1)

1. (b)	**2.** (c)	**3.** (c)	**4.** (c)	**5.** (b)
6. (d)	**7.** (c)	**8.** (c)	**9.** (c)	**10.** (d)
11. (a)	**12.** (b)	**13.** (b)	**14.** (d)	**15.** (b)
16. (a , c)	**17.** (b , c , d)	**18.** (a , c , d)	**19.** (a , c , d)	**20.** (a , b , c , d)
21. (d)	**22.** (a)	**23.** (c)	**24.** (b)	**25.** (d)

ANSWERS Exercise No. (2)

1. (a)	**2.** (b)	**3.** (b)	**4.** (d)	**5.** (b)
6. (c)	**7.** (b)	**8.** (b)	**9.** (c)	**10.** (3)
11. (8)	**12.** (3)	**13.** (1)		

14. (a) $\rightarrow$ p
(b) $\rightarrow$ q
(c) $\rightarrow$ q , r
(d) $\rightarrow$ r

15. (a) $\rightarrow$ p
(b) $\rightarrow$ q
(c) $\rightarrow$ r
(d) $\rightarrow$ p

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Chapter 8

Determinants

Exercise No. (1)

Multiple choice questions with ONE correct answer : (Questions No. 1-20)

1. If system of equations : $4x + 5y - z = 0$, $x - y - 4z = 0$ and $(K + 1)x + (2K - 1)y + (K - 4)z = 0$ have non-trivial solution , then :

(a) $K = 3$ (b) $K = 0$

(c) $K = 3$ or 0 (d) $K \in R$

2. Let (x , y , z) be points with integer co-ordinates satisfying the system of homogeneous equation :

$$3x - y - z = 0$$
$$-3x + z = 0$$
$$-3x + 2y + z = 0.$$

Then the number of such points for which $x^2 + y^2 + z^2 \le 100$ are :

(a) 6 (b) 5

(c) 10 (d) 7

3. If $\theta \in [0 , 2\pi)$ and $A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$;

then det (A) lies in the interval :

(a) $[2 , 4]$ (b) $[2 , 3]$

(c) $[1 , 4]$ (d) $(2 , 4)$

4. The existence of unique solution for the system of equations , $x + y + z = p$, $5x - y + qz = 10$ and $2x + 3y - z = 6$ depends on :

(a) $'p'$ only. (b) $'q'$ only.

(c) $'p'$ and $'q'$ both. (d) neither $'p'$ nor $'q'$.

5. Let $f(x) = \begin{vmatrix} 2 & 1 & 0 \\ -3 & 2 & -1 \\ x|x| & \tan^{-1}x & \sin\pi[x] \end{vmatrix}$, where $[.]$ represents the greatest integer function , then $\int_{-2}^{2} f(x)dx$ is :

(a) $2\cos^2 1$ (b) $\sin^2 2 + \sec 1$

(c) $1 + \cos 2 - 2\sin^2 1$ (d) $\cos 2 + 1 - 2\cos^2 1$

6. Let $f(x)$, $g(x)$ and $h(x)$ be cubic functions of x and $\phi(x) = \begin{vmatrix} f'(x) & f''(x) & f'''(x) \\ g'(x) & g''(x) & g'''(x) \\ h'(x) & h''(x) & h'''(x) \end{vmatrix}$, then

(a) $\phi''(x) = 2$.

(b) graph of $\phi(x)$ is symmetric about origin.

(c) graph of $\phi(x)$ is symmetric about y-axis.

(d) $\phi(x)$ is polynomial of degree 3.

7. Let $P_K = a^K + b^K$, where $K \in N$ & $(a + b) = 2ab = 4$, then value of $\begin{vmatrix} 3 & 1+P_1 & 1+P_2 \\ 1+P_1 & 1+P_2 & 1+P_3 \\ 1+P_2 & 1+P_3 & 1+P_4 \end{vmatrix}$ is equal to :

(a) 4 (b) 0 (c) 8 (d) 2

8. Let α , β and γ be internal angles of a triangle , then minimum value of $\begin{vmatrix} -2 & \cos\gamma & \cos\beta \\ \cos\gamma & -1 & \cos\alpha \\ \cos\beta & \cos\alpha & -1 \end{vmatrix}$ is equal to :

(a) 0 (b) 1 (c) -1 (d) 2

9. Let α , β , γ and δ be the positive real roots of the equation $x^4 - 12x^3 + px^2 + qx + 81 = 0$, where $p , q \in R^+$, then value of $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$ is equal to :

(a) $-\frac{5}{2}$ (b) $-\frac{9}{2}$

(c) $-\frac{3}{2}$ (d) none of these

10. Let set $'S'$ consists of all the determinants of order 3×3 with entries zero or one only and set $'P'$ is subset of $'S'$ consisting of all the determinants with value 1. If set $'Q'$ is subset of $'S'$ consisting of all the determinants with value -1 , then :

(a) $n(S) = n(P) + n(Q)$ (b) $n(P) = 2n(Q)$

(c) $n(P) = n(Q)$ (d) $P \cup Q = S$

11. Let $'M'$ be a 3×3 matrix, where $MM^T = I$ and $\det(M) = 1$, then :

(a) $\det(M - I) \neq 0$.
(b) $\det(M - I)$ is always zero.
(c) $\det(M + 2I) = 0$.
(d) $\det(M + I)$ is always zero.

12. Let $px^4 + qx^3 + rx^2 + sx + t = \begin{vmatrix} x-3 & x+4 & 3x \\ x^2+3x & x-1 & x+3 \\ x+1 & -2x & x-4 \end{vmatrix}$ be an identity in x, where p, q, r, s and t are constants, then $(q + s)$ is equal to :

(a) 52 (b) 51 (c) 50 (d) 102

13. Let A, B and C be the angles of a triangle, where $A, B, C \neq \dfrac{\pi}{2}$, then the value of $\begin{vmatrix} \tan A & 1 & 1 \\ 1 & \tan B & 1 \\ 1 & 1 & \tan C \end{vmatrix}$ is equal to :

(a) 0 (b) –1 (c) 2 (d) –2

14. Let $\vec{a}_r = x_r\hat{i} + y_r\hat{j} + z_r\hat{k}$, where $r = 1, 2, 3$ be three vectors and $|\vec{a}_r| = r$, $\vec{a}_1.\vec{a}_2 = \vec{a}_2.\vec{a}_3 = \vec{a}_3.\vec{a}_1 = 1$, then value of $\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$ is equal to :

(a) ±4 (b) $\pm2\sqrt{6}$
(c) $\pm6\sqrt{2}$ (d) $\pm\sqrt{6}$

15. Let $f(x) = \begin{vmatrix} 1+\alpha & 1+\alpha x & 1+\alpha x^2 \\ 1+\beta & 1+\beta x & 1+\beta x^2 \\ 1+\gamma & 1+\gamma x & 1+\gamma x^2 \end{vmatrix}$; then $f(x)$ is independent of :

(a) α and β (b) β and γ
(c) α and γ (d) α, β and γ

16. If a homogenous system of equations is represented by: $ax + by + cz = 0$, $bx + cy + az = 0$ and $cx + ay + bz = 0$, and infinite ordered triplets (x, y, z) are possible without any linear constraint, then

(a) $a + b + c \neq 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$
(b) $a + b + c = 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$
(c) $a + b + c \neq 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$
(d) $a + b + c = 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$

17. If a determinant is chosen at random from the set of all determinants of order 2×2 with elements zero or one only, then the probability that the value of determinant chosen is non-negative is equal to :

(a) $\dfrac{3}{16}$ (b) $\dfrac{5}{8}$
(c) $\dfrac{3}{8}$ (d) $\dfrac{13}{16}$

18. Let α, β, γ be non-zero real numbers, then system of equations in x, y and z, $\dfrac{x^2}{\alpha^2} + \dfrac{y^2}{\beta^2} - \dfrac{z^2}{\gamma^2} = 1$, $\dfrac{x^2}{\alpha^2} - \dfrac{y^2}{\beta^2} + \dfrac{z^2}{\gamma^2} = 1$ and $\dfrac{y^2}{\beta^2} + \dfrac{z^2}{\gamma^2} = 1 + \dfrac{x^2}{\alpha^2}$ has :

(a) no solution.
(b) unique solution.
(c) infinitely many solutions.
(d) finitely many solutions.

19. The number of values of $'K'$ for which the system of equations
$(2K + 1)x + (3K + 1)y + K + 2 = 0$ and
$(5K + 1)x + (7K + 1)y + 4K + 2 = 0$
is consistent and indeterminate is given by :

(a) 0 (b) 1
(c) 2 (d) infinite

20. If the system of equations ; $2x + y - 3 = 0$, $6x + ky - 4 = 0$ and $6x + 3y - 10 = 0$ is consistent, then

(a) $k = 1$ (b) $k = 3$
(c) $k = 1$ or 3 (d) $k \in \phi$

Multiple choice questions with MORE than ONE correct answer : (Questions No. 21-25)

21. Let a, b, c be non-zero real numbers and function $f(x)$ is given by $\begin{vmatrix} a^2+x^2 & ab & ac \\ ab & b^2+x^2 & bc \\ ac & bc & c^2+x^2 \end{vmatrix}$, then $f(x)$ is divisible by :

(a) x^4 (b) x^6
(c) $x^2 - a^2 - b^2 - c^2$ (d) $x^2 + a^2 + b^2 + c^2$

22. System of equations : $x + 3y + 2z = 6$, $x + \lambda y + 2z = 7$, $x + 3y + 2z = \mu$ has :

(a) Infinitely many solutions if $\lambda = 4$, $\mu = 6$.
(b) No solution if $\lambda = 5$, $\mu = 7$.
(c) Unique solution if $\lambda = 5$, $\mu = 7$.
(d) No solution if $\lambda = 3$, $\mu = 5$.

23. Consider the system of linear equations in x , y , z :

$$2x+7y+7z=0$$
$$(\sin 3\theta)x-y+z=0$$
$$(\cos 2\theta)x+4y+3z=0$$

If the system has non-trivial solutions , then angle 'θ' can be :

(a) $\frac{25\pi}{6}$ (b) $\frac{17\pi}{6}$

(c) 4π (d) $\frac{7\pi}{6}$

24. Let determinant $'D'$ is having all the elements as either 1 or –1. If the product of all the elements of any row or any column of $'D'$ is negative , then it is represented by $'D_N'$. If the order of $'D'$ is 3×3 , then :

(a) minimum value of D_N is –2.

(b) minimum value of D_N is –4.

(c) total number of D_N is 16.

(d) total number of D_N is 32.

25. Let $f(x)$ be real valued polynomial function , and

$$\begin{vmatrix} 1 & x & x \\ x & 2 & x \\ x & x & 3 \end{vmatrix} = x f'(x)-f(x) \text{ , then}$$

(a) $\int_1^3 f(x)dx=0$

(b) $\int_{-1}^1 f(x)dx=0$

(c) $y=f(x+2)$ is odd function

(d) $y=|f(x)|$ is symmetrical about line $x-2=0$

Assertion Reasoning questions :
(Questions No. 26-30)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements , Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers , only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

26. Statement 1 : Let $\alpha_K=\cos\frac{2K\pi}{9}+i\sin\frac{2K\pi}{9}$ for all $K\in W$, then value of determinant $\begin{vmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \alpha_4 & \alpha_5 & \alpha_6 \\ \alpha_7 & \alpha_8 & \alpha_9 \end{vmatrix}$ is zero

because

Statement 2 : $\sum_{K=1}^{9}\alpha_K=0$

27. Let $f(x)$, $g(x)$ and $h(x)$ be the polynomial functions of degree 3 , 4 and 5 respectively , where

$$\phi(x)=\begin{vmatrix} f'(\theta) & g'(\theta) & h'(\theta) \\ f'(x) & g'(x) & h'(x) \\ f''(\theta) & g''(\theta) & h''(\theta) \end{vmatrix} \text{ and } \theta\in R.$$

Statement 1 : $\phi(x)$ is divisible by $(x-\theta)^2$

because

Statement 2 : $\phi(\theta)=\phi'(\theta)=0$

28. Let $S=\{\Delta_1 , \Delta_2 , \Delta_3 , ... , \Delta_n\}$ be the set of 3×3 determinants that can be formed with the distinct non-zero real numbers $a_1 , a_2 , a_3 , ... a_9$, where repeatition of elements is not permissible , then

Statement 1 : $\sum_{i=1}^{n}\Delta_i=0$

because

Statement 2 : $n=\prod_{i=1}^{9} i$

29. Let $f(x)=\begin{vmatrix} 1 & 1 & 1 \\ \sin 2x & \sin 4x & \sin 6x \\ \cos^2 2x & \cos^2 4x & \cos^2 6x \end{vmatrix}$

Statement 1 : If $x\in\left(0 , \frac{\pi}{2}\right)$, then number of solutions of the equation $f(x)=0$ are five

because

Statement 2 : $|3\sin\pi x|-x=0$ is having five solutions if $x\in R^+$.

30. Let $'A_r'$ represents the number of positive integral solutions of $x+y+z=r$, where $r \in N-\{1, 2\}$,

and $\Delta = \begin{vmatrix} A_r & A_{r+1} & A_{r+2} \\ A_{r+1} & A_{r+2} & A_{r+3} \\ A_{r+2} & A_{r+3} & A_{r+4} \end{vmatrix}$.

Statement 1 : Value of $\Delta = 0$

because

Statement 2 : In a determinant if any two rows or any two columns are identical , then determinant value is zero.

Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1-3)

Let $y = f(x)$ be quadratic function , and

$$\begin{bmatrix} 4a^2 & 4a & 1 \\ 4b^2 & 4b & 1 \\ 4c^2 & 4c & 1 \end{bmatrix} \begin{bmatrix} f(-1) \\ f(1) \\ f(2) \end{bmatrix} = \begin{bmatrix} 3a^2+3a \\ 3b^2+3b \\ 3c^2+3c \end{bmatrix}.$$

If a , b and c are distinct real numbers , and maximum value of $f(x)$ occurs at point $'V'$, then answer the following questions.

1. Let $A = [a_{ij}]_{3\times3}$, $a_{ij} = f\left(\dfrac{i+2j}{3}\right)$ $\forall\ i = j$ and $a_{ij} = 0$ for all $i \neq j$, then det (A) is equal to :

(a) 0 (b) 2 (c) –1 (d) –3

2. Let $'A'$ is the point of intersection of $y = f(x)$ with x-axis and point $B(\alpha\ ,\ f(\alpha))$ is such that chord AB subtends a right angle at $'V'$, then area (in square units) enclosed by $f(x)$ with chord AB is :

(a) $\dfrac{250}{3}$ (b) $\dfrac{125}{3}$

(c) $\dfrac{75}{4}$ (d) $\dfrac{301}{3}$

3. Let $g(x) = \dfrac{1}{f(x)}$ $\forall\ x \in (-2\ ,\ 2)$, then total number of points of discontinuity for $y = [g(x)]$ in $\left[-\sqrt{3}\ ,\ \sqrt{3}\right]$ are given by : ([.] represents G.I.F.)

(a) 4 (b) 6 (c) 8 (d) 2

Comprehension passage (2)
(Questions No. 4-6)

Consider the matrices , $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$,

$C_1 = \begin{bmatrix} -4 \\ 1 \\ 4 \end{bmatrix}$, $C_2 = \begin{bmatrix} -3 \\ 0 \\ 4 \end{bmatrix}$ and $C_3 = \begin{bmatrix} -3 \\ 1 \\ 3 \end{bmatrix}$.

Let matrix $'B_1'$ of order 3×3 is formed with the column vectors of the matrices C_1 , C_2 and C_3 , and $B_{n+1} = adj(B_n)$, $n \in N$, then answer the following questions :

4. Matrix addition for $B_2 + B_3 + B_4 + \dots + B_{100}$ is equal to :

(a) $100\,B_1$ (b) $99\,B_1$

(c) $99\,I_3$ (d) $98\,I_3$

5. Let $M = AB_1^2 + A^2B_2^3 + A^3B_3^4 + \dots A^{100}B_{100}^{101}$, then det (M) is equal to :

(a) 100 (b) –100

(c) 0 (d) 1000

6. For a variable matrix X. the matrix equation $AX = C_2$ will have :

(a) Unique solution
(b) No solution
(c) Finitely many solutions
(d) Infinitely many solutions

Questions with Integral Answer :
(Questions No. 7-10)

7. If a , b , $c \in I^+$ and $\begin{vmatrix} 1+a^3 & a^2b & a^2c \\ ab^2 & 1+b^3 & b^2c \\ ac^2 & bc^2 & 1+c^3 \end{vmatrix} = 11$, then total number of possible triplets of $(a\ ,\ b\ ,\ c)$ is/are..........

8. Let $f(x) = \begin{vmatrix} 2\cos^2 x & \sin 2x & -\sin x \\ \sin 2x & 2\sin^2 x & \cos x \\ \sin x & -\cos x & 0 \end{vmatrix}$, and

$I = \displaystyle\int_0^{\pi/2} \{f(x) + f'(x)\}\,dx$, then the least integer just greater than $'I'$ is equal to ....

9. Let $U_n = \displaystyle\int_0^{\pi/2} \left(\dfrac{1-\cos 2nx}{1-\cos 2x}\right) dx$, then value of

$\begin{vmatrix} U_1 & U_2 & U_3 \\ U_4 & U_5 & U_6 \\ U_7 & U_8 & U_9 \end{vmatrix}$ is equal to ....

10. Consider the system of equations :

$$\lambda x + (\sin\alpha)y + (\cos\alpha)z = 0$$
$$x + (\cos\alpha)y + (\sin\alpha)z = 0$$
$$x - (\sin\alpha)y + (\cos\alpha)z = 0$$

If λ and α are real numbers , and the system of equations has non-trivial solutions , then number of integral values of λ which are possible for different values of α are ....

Matrix Matching Questions :
(Questions No. 11-12)

11. Let $f(x)$ be polynomial function having local minima at $x = \frac{5}{2}$ and $f(0) = 2f(1) = 2$. If for all $x \in R$, $f'(x) = \begin{vmatrix} 2ax & 2ax-1 & 2ax+b+1 \\ b & b+1 & -1 \\ 2ax+2b & 2ax+2b+1 & 2ax+b \end{vmatrix}$ where $'a'$ and $'b'$ are some constants , then match the following column (I) and II.

Column (I)	**Column (II)**
(a) Value of $(a+b)$	(p) 1
(b) Value of $f(5)$	(q) 0
(c) Number of solutions for $4f(x) = \lvert x-1 \rvert$	(r) –1
(d) $\lim_{x\to\infty}\left(\frac{f(x)}{x^2+1}\right)^{\frac{f(x)}{x}}$	(s) 2

12. Consider the system of equations :

$$Kx + y + z = 1$$
$$x + Ky + z = K$$
$$x + y + Kz = K^2$$

Match column (I) and (II) for the values of $'K'$ and the nature of solution for the system of equations.

Column (I)	**Column (II)**
(a) If $K = 1$, then system of equations have	(p) Unique solution.
(b) If $K \neq 1$, then system of equations may have	(q) Infinitely many solutions.
(c) If $K \in R - \{1, -2\}$, then system of equations have	(r) No solution.
(d) If $K \in \{1, -2\}$, then system of equations may have	(s) Finitely many solutions.

ANSWERS Exercise No. (1)

1. (d)	**2.** (d)	**3.** (a)	**4.** (b)	**5.** (d)
6. (c)	**7.** (c)	**8.** (c)	**9.** (d)	**10.** (c)
11. (b)	**12.** (b)	**13.** (c)	**14.** (b)	**15.** (d)
16. (d)	**17.** (d)	**18.** (d)	**19.** (c)	**20.** (d)
21. (a , d)	**22.** (a , b , d)	**23.** (a , b , c)	**24.** (b , c)	**25.** (a , c , d)
26. (b)	**27.** (a)	**28.** (b)	**29.** (b)	**30.** (d)

ANSWERS Exercise No. (2)

1. (a)	**2.** (b)	**3.** (b)	**4.** (b)	**5.** (c)
6. (b)	**7.** (3)	**8.** (4)	**9.** (0)	**10.** (3)

11. (a) $\to$ r
(b) $\to$ s
(c) $\to$ p
(d) $\to$ q

12. (a) $\to$ q
(b) $\to$ p , r
(c) $\to$ p
(d) $\to$ q , r

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Chapter 9

Logarithm

Exercise No. (1)

Multiple choice questions with ONE correct answer : (Questions No. 1-25)

1. If $log_7 2 = m$, then $log_{49} 28$, is equal to :

(a) $2(2m+1)$ (b) $\frac{2m+1}{2}$

(c) $\frac{2}{2m+1}$ (d) $m+1$

2. If $ln\left(\frac{a+b}{2}\right)=\frac{1}{2}(ln\,a+ln\,b)$, where $a, b \in R^+$ then relation between a and b is :

(a) $a=b$ (b) $a=\frac{b}{2}$

(c) $a=2b$ (d) $a=\frac{b}{3}$

3. The value of $(81)^{\frac{1}{\log_5 3}}+(27)^{\log_9 36}+(3)^{\frac{4}{\log_7 9}}$ is :

(a) 49 (b) 625

(c) 216 (d) 890

4. $7\,log\left(\frac{16}{15}\right)+5\,log\left(\frac{25}{24}\right)+3\,log\left(\frac{81}{80}\right)$ is equal to :

(a) 0 (b) 1

(c) $log\,2$ (d) $log\,3$

5. If $A=log_2\left\{log_2\left(log_4 256\right)\right\}+2log_{\sqrt{2}}2$, then A is :

(a) 2 (b) 3

(c) 5 (d) 7

6. If $x=log_a(bc)$, $y=log_b(ac)$, $z=log_c(ab)$, then which one of the following is equal to 1 ?

(a) $x+y+z$

(b) $\frac{1}{1+x}+\frac{1}{1+y}+\frac{1}{1+z}$

(c) $x\,y\,z$

(d) $(1+x)^{-2}+(1+y)^{-2}+(1+z)^{-2}$

7. If $a=log_{24}12$, $b=log_{36}24$, $c=log_{48}36$, then value of $(1+abc)$ is :

(a) $2ab$ (b) $2ac$

(c) $2bc$ (d) 0

8. If $a^x=b$, $b^y=c$, $c^z=a$, then value of (xyz) is :

(a) 0 (b) 1

(c) 2 (d) 3

9. $\sum_{r=1}^{89} log_3(\tan(r^{o}))$ is equal to :

(a) 3 (b) 1

(c) 2 (d) 0

10. $\sum_{r=1}^{n}\frac{1}{log_{2^r}a}$ is equal to :

(a) $\frac{n(n+1)}{2}log_a 2$ (b) $\frac{n(n+1)}{2}log_2 a$

(c) $\frac{(n+1)^n.n^2}{4}log_2 a$ (d) none of these

11. If $log_7\left\{log_5\sqrt{(x^2+x+5)}\right\}=0$, then x is equal to :

(a) 2 (b) 3

(c) 4 (d) –2

12. The value of $(0.05)^{log_{\sqrt{20}}(0.1+.01+.001+.....\infty)}$ is :

(a) 81 (b) $\frac{1}{81}$

(c) 20 (d) 10

13. If $log_{12}27=a$, then $log_6 16$ is :

(a) $2\left(\frac{3-a}{3+a}\right)$ (b) $3\left(\frac{3-a}{3+a}\right)$

(c) $4\left(\frac{3-a}{3+a}\right)$ (d) $2\left(\frac{4-a}{4+a}\right)$

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14. If $n = 2010!$, then $\frac{1}{log_2 n} + \frac{1}{log_3 n} + ... + \frac{1}{log_{2010} n}$ is equal to :

(a) -1 (b) 0

(c) 1 (d) 2

15. The number of solution(s) of $log_2(x+5) = 6 - x$ is/are :

(a) 2 (b) 0

(c) 3 (d) 1

16. If $log_{\cos x} \sin x \geq 2$, then the values of $\sin x$ lies in the interval :

(a) $\left[\frac{\sqrt{5}-1}{2}, 1\right]$ (b) $\left(0, \frac{\sqrt{5}-1}{2}\right]$

(c) $\left(0, \frac{1}{2}\right]$ (d) $\left[\frac{\sqrt{5}-1}{2}, \frac{\sqrt{5}+1}{4}\right]$

17. $log_{\frac{1}{\sqrt{2}}} (\sin x) > 0$, $x \in [0, 4\pi]$, then number of values of x which are integral multiples of $\frac{\pi}{4}$, is :

(a) 4 (b) 12

(c) 3 (d) 10

18. Set of real values of x satisfying the inequation $log_{0.5}(x^2 - 6x + 12) \geq -2$ is :

(a) $(-\infty, 2]$ (b) $[2, 4]$

(c) $[4, \infty)$ (d) none of these

19. Set of real x for which $2^{log_{\sqrt{2}}(x-1)} > (x+5)$ is :

(a) $(-\infty, -1) \cup (4, \infty)$ (b) $(4, \infty)$

(c) $(-1, 4)$ (d) $[1, 4) \cup (4, \infty)$

20. If $log_{0.2}\left(\frac{x+2}{x}\right) \leq 1$, then x belongs to :

(a) $\left(-\infty, -\frac{5}{2}\right] \cup (0, \infty)$

(b) $\left(\frac{5}{2}, \infty\right)$

(c) $(-\infty, -2) \cup (0, \infty)$

(d) none of these

21. If $\left(\frac{1}{2}\right)^{x^2-2x} < \frac{1}{4}$, then set of $'x'$ contains :

(a) $(-\infty, 0)$ (b) $(-\infty, 1)$

(c) $(1, \infty)$ (d) none of these

22. If $log_x\left(\frac{5x - x^2}{4}\right) \geq 0$, then exhaustive set of values of x is :

(a) $[0, 4]$ (b) $(0, 4] - \{1\}$

(c) $(0, 4)$ (d) none of these

23. The value of $\frac{log_2 24}{log_{96} 2} - \frac{log_2 192}{log_{12} 2}$ is :

(a) 3 (b) 0

(c) 2 (d) 1

24. If $log_3 2$, $log_3(2^x - 5)$ and $log_3\left(2^x - \frac{7}{2}\right)$ are in A.P., then x is equal to :

(a) 2 (b) 3

(c) 4 (d) 8

25. If $log\, x^2 - log\, 2x = 3log\, 3 - log\, 6$, then x is :

(a) 10 (b) 9

(c) 1 (d) 2

Multiple choice questions with MORE than ONE correct answer : (Questions No. 26-30)

26. If $(x)^{\frac{3}{4}(log_3 x)^2 + log_3 x - \frac{5}{4}} = \sqrt{3}$, then x has :

(a) one positive integral value.

(b) one irrational value.

(c) two positive rational values.

(d) no real value.

27. If $x = 9$ satisfy the equation $ln(x^2 + 15a^2) - ln(a-2) = ln\left(\frac{8ax}{a-2}\right)$, then

(a) value of $'a'$ is 3

(b) value of $'a'$ is $\frac{9}{5}$

(c) $x = 15$ is other solution

(d) $x = 12$ is other solution

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28. Let $p = \frac{\ln 3}{\ln 20}$, then the correct statements are :

(a) p is a rational number

(b) p is an irrational number

(c) p lies in $\left(\frac{1}{3}, \frac{1}{2}\right)$

(d) p lies in $\left(\frac{1}{4}, \frac{1}{3}\right)$

29. Let set $'S'$ contain the values of x for which the equation $|x-1|^{(\log_{10} x)^2 - \log_{10} x^2} = |x-1|^3$ is satisfied , then :

(a) total number of elements in $'S'$ are 4

(b) set $'S'$ contains only one fractional number

(c) set $'S'$ contains only one irrational number

(d) total number of elements in $'S'$ are 2

30. If set $'S'$ contains all the real values of x for which $\log_{(2x+3)} x^2 < 1$ is true , then set $'S'$ contain :

(a) $(\log_2 5 , \log_2 7)$

(b) $[\log_3 4 , \log_3 8]$

(c) $\left(-\frac{3}{2}, 1\right)$

(d) $(-1, 0)$

Questions with Integral Answer :
(Questions No. 31-35)

31. Let $x \in (1, \infty)$ and $y \in (1, 16)$, where $xy = 16$. If x and y satisfy the relation $\log_y x - \log_x y = \frac{8}{3}$, then value of $(x-y)$ is equal to ....

32. If $a \in R^+ - \{1\}$, $\alpha = \frac{6}{5}(a)^{\log_a x . \log_{10} a . \log_a 5}$ and $\beta = (3)^{\log_{10}\left(\frac{x}{10}\right)} + (9)^{\log_{100} x + \log_4 2}$, where $\alpha - \beta = 0$, then value of $\sqrt{\frac{x}{4}}$ is equal to ....

33. If $M = \sum_{r=1}^{4} \log_2\left(\sin\left(\frac{r\pi}{5}\right)\right)$, then value of $(2)^{M+4}$ is equal to ....

34. If $\frac{\ln a}{(y-z)} = \frac{\ln b}{(z-x)} = \frac{\ln c}{(x-y)}$, then value of $\left\{(a)^{y^2+yz+z^2} . (b)^{z^2+zx+x^2} . (c)^{x^2+xy+y^2}\right\}$ is equal to ....

35. Total number of integral solution(s) of the equation $x + \log_{10}(2^x + 1) = x\log_{10} 5 + \log_{10} 6$ is/are ....

ANSWERS — Exercise No. (1)

1. (b)	**2.** (a)	**3.** (d)	**4.** (c)	**5.** (c)
6. (b)	**7.** (c)	**8.** (b)	**9.** (d)	**10.** (a)
11. (c)	**12.** (a)	**13.** (c)	**14.** (c)	**15.** (d)
16. (b)	**17.** (a)	**18.** (b)	**19.** (b)	**20.** (a)
21. (d)	**22.** (d)	**23.** (a)	**24.** (b)	**25.** (b)
26. (a , b , c)	**27.** (a , c)	**28.** (b , c)	**29.** (a , b)	**30.** (a , b , d)
31. (6)	**32.** (5)	**33.** (5)	**34.** (1)	**35.** (1)

Chapter 10

Functions

Exercise No. (1)

Multiple choice questions with ONE correct answer : (Questions No. 1-40)

1. Which one of the following functions is an odd function ?

(a) $f(x)=\log_e\left(\dfrac{x^4+x^2+1}{(x^2+x+1)^2}\right)$

(b) $f(x)=\log_e\left(\dfrac{(x+1)(2-x)}{(x+1)(2+x)}\right)$

(c) $f(x)$, where $f(x)+f(y)=f(x+y).f(x-y)$ for all $x\,,y\in R$

(d) $f(x)=\dfrac{e^{-|x|}}{1+e^{-|x|}}$

2. Domain of function $f(x)=\log_{(2x-1)}(x-1)$ is :

(a) $(1\,,\infty)$ (b) $\left(\dfrac{1}{2},\infty\right)$

(c) $(0\,,\infty)$ (d) $\left(-\dfrac{1}{2},0\right)\cup(0,\infty)$

3. If [.] represents the greatest integer function , then $\displaystyle\sum_{r=0}^{99}\left[\frac{3}{4}+\frac{r}{100}\right]$ is equal to :

(a) 30 (b) 70

(c) 75 (d) 100

4. Let $f(x)=\sin ax+\cos ax$ and $g(x)=|\sin x|+|\cos x|$ have equal fundamental period , then $'a'$ is :

(a) 1 (b) 2

(c) 3 (d) 4

5. Let $f(x)+f(1-x)=2\ \ \forall\ x\in R$ and $g(x)=f(x)-1$, then $g(x)$ is symmetrical about :

(a) the line $x=\dfrac{1}{2}$ (b) the point (1 , 0)

(c) the line $x=1$ (d) the point $\left(\dfrac{1}{2}\,,\,0\right)$

6. The values of $'a'$ and $'b'$ for which equation $\left|e^{|x-b|}-a\right|=2$ has four distinct solutions , are :

(a) $a\in(3\,,\infty)\,;\,b=0$ (b) $a\in(2\,,\infty)\,;\,b\in R$

(c) $a\in(3\,,\infty)\,;\,b\in R$ (d) $a\in(2\,,\infty)\,;\,b=0$

7. Let $f:R\to R$ and $g:R\to R$ be functions defined as $f(x)=\begin{cases}0\,;\ x\in Q\\ x\,;\ x\notin Q\end{cases}$ and $g(x)=\begin{cases}x\,;\ x\in Q\\ 0\ ;\ x\notin Q\end{cases}$, then composition $f(x)-g(x)$ is :

(a) one-one onto (b) one-one into

(c) many-one onto (d) many-one into

8. If $0<\lambda<1$ and $f(x)=\left\{\dfrac{\log_\lambda|x-2|}{|x|}\right\}^{\frac{1}{2}}$, then domain of $f(x)$ is :

(a) $[1\,,2)\cup(2\,,3)$ (b) $[1\,,2)\cup(2\,,3]$

(c) $(1\,,2)\cup(2\,,5)$ (d) none of these

9. The number of solutions of equation $6\,|\cos x|-x=0$ in $[0\,,2\pi]$ are :

(a) 6 (b) 4

(c) 3 (d) 2

10. If $f:(3\,,6)\to(2\,,5)$ is a function defined as $f(x)=x-\left[\dfrac{x}{3}\right]$, where [.] represents the greatest integer function , then $f^{-1}(x)$ is given by :

(a) $x+1$ (b) $3x+2$

(c) $3x+1$ (d) none of these

11. If $x\in R^+$, then range of $f(x)=\dfrac{(1+x+x^2)(x^4+1)}{x^3}$ is :

(a) $\left[2\,,\dfrac{5}{3}\right)$ (b) $[6,\infty)$

(c) $\left[\dfrac{2}{3},\infty\right)$ (d) $\left[\dfrac{2}{3},\dfrac{5}{3}\right]$

12. If $3f(x)+2f\left(\frac{x+59}{x-1}\right)=10x+30 \;\forall\; x\in R-\{1\}$, then $f(7)$ is equal to :

(a) 7 (b) 5

(c) 4 (d) 2

13. Let $f:[-2, 2] \to R$ be an odd function defined as $f(x) = x^3 + \tan x + \left[\frac{x^2+1}{\lambda}\right]$, then λ belongs to :

(a) $(5, \infty)$ (b) $(7, \infty)$

(c) R^- (d) R^+

14. If $f(x)=\sin 3x.\cos[3x]-\cos 3x.\sin[3x]$, where [.] represents the greatest integer function , then fundamental period of $f(x)$ is :

(a) 3 (b) $\frac{1}{3}$

(c) 6 (d) $\frac{1}{6}$

15. Let $f: R \to [0, \pi/2)$ be defined as $f(x) = \tan^{-1}(x^2 + x + a)$ then set of values of 'a' for which $f(x)$ is onto , is :

(a) $\left[\frac{1}{4},\infty\right)$ (b) $[4, \infty)$

(c) $\left(\frac{1}{8},\infty\right)$ (d) none of these

16. If $f: R\to R$ be a function satisfying $f(2x+3)+f(2x+7) = 2 \;\forall\; x\in R$ then fundamental period of $f(x)$ is :

(a) 2 (b) 4

(c) 8 (d) 16

17. Interval of x satisfying the inequality $\frac{5}{2}\le|x-1|+|x-2|+|x-3|<6$ is given by :

(a) $\left(0,\frac{1}{2}\right]\cup\left[\frac{3}{2},4\right)$ (b) $(0, 1]\cup[2, 5)$

(c) $\left[-1,\frac{3}{2}\right)\cup(4, 5]$ (d) $\left(0,\frac{3}{2}\right]\cup\left[\frac{5}{2},4\right)$

18. Area enclosed by inequality $2\le|x+y|+|x-y|\le 4$ is :

(a) 12 sq. units (b) 5 sq. units

(c) 4 sq. units (d) 8 sq. units

19. Number of solutions of equation $e^{-|x|}=|1-|2-x||$ are :

(a) 3 (b) 4

(c) 5 (d) 2

20. The number of integral values of 'm' for which function $f(x)=\frac{x^3}{3}+(m-1)x^2+(m+5)x+11$ is invertible , are :

(a) 4 (b) 10

(c) 6 (d) 8

21. If $3^{\log_a x}+3x^{\log_a 3}=2$, where $a\in R^+-\{1\}$, then value of x is :

(a) $a^{\log_2 3}$ (b) $a^{-\log_2 3}$

(c) $a^{-\log_3 2}$ (d) $2^{\log_3 a}$

22. If $x^4 - 18x^2 + \lambda - 2 = 0$ is having all four real roots , then *exhaustive set for* 'λ' belongs to :

(a) [3, 67] (b) [–1, 61]

(c) [0, 75] (d) [2, 83]

23. Domain of function $f(x) = \sin^{-1}(x^2 - 5x + 5)$ is :

(a) $[1, 2]\cup[4, 5]$ (b) $[1, 2]\cup[3, 4]$

(c) $[2, 3]\cup[4, 5]$ (d) $[1, 2]\cup[3, 5]$

24. Let $f(x)=\left\{\cos^2 x+\cos^2\left(\frac{\pi}{3}+x\right)-\cos x.\cos\left(\frac{\pi}{3}+x\right)\right\}$, then $f\left(\frac{\pi}{8}\right)$ is equal to :

(a) $\frac{3}{4}$ (b) $\frac{5}{4}$

(c) $\frac{4}{5}$ (d) $\frac{2}{3}$

25. Domain of $f(x)=\sqrt{10[x]-21-[x]^2}$, where [.] is greatest integer function , is :

(a) [3, 8) (b) [3, 7]

(c) (2, 7] (d) (2, 8)

26. Let $f(x) = (\sin x + \sin 3x)\sin x$, then $\forall\ x \in R$, $f(x)$ is :

(a) positive (b) non-positive

(c) negative (d) non-negative

27. Number of solution(s) of the equation $x^2 - 4x + 5 - e^{-|x|} = 0$ is/are :

(a) 0 (b) 1

(c) 2 (d) 4

28. If $f : R \to R$ is defined by $f(x) = \dfrac{x^2+2x+3}{x^2+2x+2}$, then range of $f(x)$ is :

(a) $[1, 2]$ (b) $(1, 2]$

(c) $[1, 2)$ (d) $\left[1, \frac{3}{2}\right) \cup \left(\frac{3}{2}, 2\right]$

29. Number of integral values of x which satisfy the inequality $\dfrac{x^4(x-1)^2(x+4)^3}{(x+2)^4(6-x)^5} \geq 0$ are :

(a) infinite (b) 8

(c) 9 (d) 10

30. Let $f(x) = x^2 + (a-b)x + (1-a-b)$ cuts the x-axis at two distinct points for all values of b, where $a, b \in R$, then the interval of 'a' is :

(a) $[1, \infty)$ (b) $(1, \infty)$

(c) $(-\infty, 1)$ (d) $(-\infty, 1]$

31. If $\dfrac{(\ln x)^2 - 3\ln x + 3}{(\ln x - 1)} < 1$, then x belongs to :

(a) $(0, e)$ (b) $(1, e)$

(c) $(1, 2e)$ (d) $(0, 3e)$

32. Let $g(x) = 1 + x - [x]$ and $f(x) = sgn\,(x)$, where $[.]$ is greatest integer function, then for all $x \in R$ $f(g(x))$ is :

(a) $f(x)$ (b) $g(x)$

(c) $[g(x)]$ (d) x

33. Let $f(x) = \begin{cases} x & ; \text{ if } x \in Q \\ 1-x & ; \text{ if } x \notin Q \end{cases}$, then $f(f(f(x)))$ is :

(a) 0 (b) $f(x)$

(c) x (d) $1-x$

34. Let $f(x) = \dfrac{x}{e^x - 1} + \dfrac{x}{2} + 1$, then $f(x)$ is :

(a) even function

(b) odd function

(c) neither even nor odd function

(d) both even and odd function

35. Let $f(x) = \dfrac{x-[x]}{1+x-[x]}$, where $[.]$ is greatest integer function, then range of $f(x)$ is :

(a) $\left[0, \frac{1}{2}\right]$ (b) $[0, 1)$

(c) $\left[0, \frac{1}{2}\right)$ (d) $[0, 1]$

36. If $f(x) = \dfrac{(K)^x}{(K)^x + \sqrt{K}}$; $K > 0$, then which one of the following statements is true :

(a) $f(x) + f(1-x) = 2$ (b) $f(x) + f(1-x) = 1$

(c) $f(x) + f(1+x) = 1$ (d) $f(x) = f(1-x)$

37. Let $f(x) = |x|$ and $g(x) = [x]$, where $[.]$ represents the greatest integer function, then the inequality $g(f(x)) \leq f(g(x))$ is valid, if

(a) $x \in (-\infty, 0) \cup I$ (b) $x \in I$

(c) $x \in (-\infty, 0)$ (d) $x \in R$

38. Let $f(x) = \sin x - ax$ and $g(x) = \sin x - bx$, where $a < 0$, $b < 0$. If number of roots of $f(x) = 0$ is greater than number of roots of $g(x) = 0$, then :

(a) $a < b$ (b) $a > b$

(c) $ab = \dfrac{\pi}{6}$ (d) $a + b = 0$

39. Let $f(x) = \dfrac{x-3}{x+1}$, $x \neq -1$, then $f^{2010}(2009)$, where $f(f(f(x)))$ is represented by $f^3(x)$, is :

(a) 2010 (b) 2009

(c) 4013 (d) none of these

40. If $|f(x) + 6 - x^2| = |f(x)| + |4 - x^2| + 2$, then $f(x)$ is necessarily non-negative in :

(a) $[-2, 2]$ (b) $(-\infty, -2) \cup (2, \infty)$

(c) $\left[-\sqrt{6}, \sqrt{6}\right]$ (d) none of these

Multiple choice questions with MORE than ONE correct answer : (Questions No. 41-50)

41. Let $f : R \to R$ be a function defined as $f(x) = x^3 + k^2x^2 + 5x + 2\cos x$. If $f(x)$ is invertible function , then possible values of 'k' may lie in the interval :

(a) $(-\sqrt{2}\ ,\ \sqrt{2})$ (b) $(2\ ,\ \sqrt{5})$

(c) $(-1\ ,\ 1)$ (d) $(-e\ ,\ -2)$

42. Let $f(x)$ be real valued function and $f(x+y) = f(x)f(a-y) + f(y)f(a-x)$ for all $x,\ y \in R$. If for some real 'a' , $2f(0) - 1 = 0$, then :

(a) $f(x)$ is even function.

(b) $f(x)$ is periodic function.

(c) $f(x) = \dfrac{1}{2}\ \ \forall\ \ x \in R$.

(d) $f''(x)$ is both even and odd function.

43. Let $f(x).f\left(\dfrac{1}{x}\right) = f(x) + f\left(\dfrac{1}{x}\right)\ \ \forall\ \ x \in R - \{0\}$,

then function $f(x)$ may be :

(a) $f(x) = 1 \pm x^n$ (b) $f(x) = \dfrac{\pi}{2\tan^{-1}|x|}$

(c) $f(x) = 2$ (d) $f(x) = \dfrac{2}{1 + ln\,x^4}$

44. Let $f(x) = \begin{cases} 0 & ;\ |x| = \dfrac{1}{n} \\ \left[|x|\left[\dfrac{1}{|x|}\right]\right] & ;\ |x| \neq \dfrac{1}{n} \end{cases}$, where $n \in N$

and $[\alpha]$ represents the greatest integer just less than or equal to α , then which of the following statement(s) are true :

(a) $f(x)$ is odd function.

(b) $f(x)$ is not periodic.

(c) $sgn(f(x)) = 1\ \ \forall\ \ x \in R.$

(d) $f(x)$ is even function.

45. Let $f : R \to R$ be a function defined as $f(x) = 3 - 3x + 2|x+2| - |x-3|$, then :

(a) $f(x)$ is surjective function.

(b) number of integral solutions of the equation $f(x) - 4 = 0$ are six.

(c) number of real solutions of the equation $f(x) - 4 = 0$ are infinitely many.

(d) number of real solutions of the equation $f(x) - |4\sin \pi x| = 0$ are more than eight.

46. Let $f(x) = \begin{cases} -(x+3) & ;\ -2 \le x < -1 \\ x - 1 & ;\ -1 \le x \le 4 \end{cases}$ and

$g(x) = 1 - x\ \ \forall\ \ x \in [-1\ ,\ 2]$. If $h(x) = g(f(x))$, then :

(a) Range of $h(x)$ is $[-2\ ,\ 2]$.

(b) Domain of $h(x)$ is $[0\ ,\ 3]$.

(c) Domain of $h(x)$ is $[-2\ ,\ 3]$.

(d) Number of solutions of the equation $h(x) - 2\,sgn(x^2 + 2x + 8) = 0$ are two.

47. Let $A = \{x : [5\sin x] + [\cos x] + 6 = 0\ ,\ x \in R\}$, where $[.]$ represents the greatest integer function. If $f(x) = \sqrt{3}\sin x + \cos x\ \ \forall\ \ x \in A$, then :

(a) value of $f(x)$ is less than $\tan\left(\dfrac{2\pi}{3}\right)$.

(b) value of $f(x)$ is less than $2\cos(\pi)$.

(c) value of $f(x)$ is more than $\dfrac{4 - 3\sqrt{3}}{5}$.

(d) value of $f(x)$ is more than $\dfrac{-3 - 4\sqrt{3}}{5}$.

48. Let $n \in N$ and $[.]$ represents the greatest integer function , where $f : [0\ ,\ \pi] \to \left[\dfrac{n^2+n}{2}\ ,\ \dfrac{n^2+n+2}{2}\right]$

be defined as $f(x) = \sum_{r=1}^{n}\left[r + \sin\left(\dfrac{x}{r}\right)\right]$, then :

(a) $f(x)$ is one-one function.

(b) $f(x)$ is onto function.

(c) $f(x)$ is into function.

(d) $f(x)$ is many-one function.

49. Let $\alpha\ ,\ \beta\ ,\ \gamma$ be non-zero real numbers and $f : [0\ ,\ 3] \to [0\ ,\ 3]$ be a function defined as $f(x) = \alpha x^2 + \beta x + \gamma$. If $f(x)$ is bijective function , then :

(a) value of γ is 0. (b) value of γ is 3.

(c) α is root of $\gamma x^2 + \beta x + \alpha = 0$.

(d) one of the possible values of 'α' can be $1/\pi$.

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50. Consider the function

$f(x) = 3x^4 - 8kx^3 + 24(6-k)x^2 + 24$ for all $x \in R$. If the graph of function $f(x)$ is convex downwards , then possible values of 'k' can be :

(a) $\cos^{-1}(\cos 2)$ (b) $\cot^{-1}(\cot e)$

(c) $-2\tan 2$ (d) $-3\tan 1$

Assertion Reasoning questions :
(Questions No. 51-55)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements , Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers , only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

51. Let $f : R \to R$ and $g : R \to R$ be two bijective functions and both the functions are mirror images of one another about the line $y - 2 = 0$.

Statement 1 : If $h : R \to R$ be a function defined as $h(x) = f(x) + g(x)$, then $h(x)$ is many one onto function

because

Statement 2 : $h(2) = h(-2) = 4$.

52. **Statement 1 :** If $x \in (0, 2\pi)$, then the equation $\tan x + \sec x = 2\cos x$ is having three distinct solutions

because

Statement 2 : graph of $y = 1 + \sin x$ and $y = 2\cos^2 x$ intersect each other at three distinct points in $(0, 2\pi)$.

53. If $[x]$ represents the greatest integer function and $f(x) = \sin^{-1}\left[x^2 + \frac{1}{2}\right] + \cos^{-1}\left[x^2 - \frac{1}{2}\right]$, then

Statement 1 : Range of $f(x)$ is $\left\{\frac{\pi}{2}, \pi\right\}$

because

Statement 2 : $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$ for all $x \in [-1, 1]$.

54. Consider the function $\phi(x) = \log_2\left(\frac{\sqrt{9-x^2}}{2-x}\right)$ and $f(x) = 3\sin\phi(x) + 4\cos\phi(x)$, then

Statement 1 : Range of $f(x)$ is $[-5, 5]$

because

Statement 2 : If $\theta \in R$, then value of $(a\sin\theta + b\cos\theta)$ lies in $\left[-\sqrt{a^2+b^2}, \sqrt{a^2+b^2}\right]$.

55. Let function $f : N \to N$ be defined as $f(x) = x - (sgn(\cos 2))^x$, then

Statement 1 : $f(x)$ is bijective in nature

because

Statement 2 : $sgn(\cos x) = 1 \;\forall\; x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1-3)

Let $A = \{(x\,,\ y) : \max\{|x+y|\,,|x-y|\} \ge 10\,;\, x\,,\, y \in R\}$

and $B = \{(x\,,\ y) : \max\{|x+y|\,,|x-y|\} \le 20\,;\, x\,,\, y \in R\}$.

On the basis of given set of ordered pairs $(x\,,\, y)$ in the 2-dimensional plane , answer the following questions.

1. Area of the region which contain all the ordered pairs $(x\,,\, y)$ that belongs to the set of $A \cap B$ is equal to :

(a) 300 square units.
(b) 800 square units.
(c) 400 square units.
(d) 600 square units.

2. Let the ordered pair $(x\,,\, y)$ be termed as integral point if both x and y belong to the set of integers , then total number of integral points which belong to the set of $A \cap B$ are :

(a) 600 (b) 1000
(c) 660 (d) 860

3. Number of ordered pairs $(x\,,\ y)$ which satisfy the condition $|y| = 10\left\{\dfrac{x}{10}\right\}$ and belong to set *'A'*, where $\{\alpha\}$ represents the fractional part of α , are :

(a) 100 (b) 420
(c) finitely many (d) infinitely many.

Comprehension passage (2)
(Questions No. 4-6)

Let $f : A \to B$ be bijective function and its inverse exists , where the inverse function of $f(x)$ is given by $g : B \to A$. If the functions $y = f(x)$ and $y = g(x)$ are represented graphically by the continuous curves C_1 and C_2 respectively , then answer the following questions.

4. If the points (4 , 2) and (2 , 4) lie on the curve $'C_2'$ then minimum number(s) of solutions of the equation $f(x) - g(x) = 0$ is/are :

(a) 1 (b) 3
(c) 6 (d) 2

5. Let $f(x) = \int (\cos x - 2)dx$, where $f(0) = 0$, then which one of the following statements is true :

(a) C_1 and C_2 meet only at point (0 , 0).
(b) C_1 and C_2 meet at infinitely many points on the line $y - x = 0$.
(c) C_1 and C_2 meet at finitely many points on the line $y + 2x = 0$.
(d) All the points of intersection of C_1 and C_2 lie on the line $y + 2x = 0$.

6. Let $p \in A$ and $q \in B$, where $p - q \ne 0$. If point $(p\,,\, q)$ lies on C_1 but not on C_2 , then :

(a) C_1 and C_2 can't meet on the line $y - x = 0$.
(b) C_1 and C_2 don't meet each other.
(c) either C_1 and C_2 don't meet each other or they meet on the line $y - x = 0$.
(d) C_1 and C_2 meet on the line $y - x = 0$.

Comprehension passage (3)
(Questions No. 7-9)

Let $f : N \to N$ be a function defined by $f(x) = D_x$, where D_k represents the largest natural number which can be obtained by rearranging the digits of natural number k.
For example : $f(3217) = 7321$, $f(568) = 865$, $f(89) = 98$ .... etc.
On the basis of given definition of $f(x)$, answer the following questions.

7. Function $f(x)$ is :

(a) one-one and into.
(b) many-one and into.
(c) one-one and onto.
(d) many-one and onto.

8. If natural number $'n_0'$ divides $f(\alpha) - \alpha$ for every $\alpha \in N$, then maximum possible value of $'n_0'$ is equal to :

(a) 3 (b) 4
(c) 9 (d) 11

9. Let $f(\alpha) = 99852$, where $\alpha \in N$, then maximum number of possible distinct values of $'\alpha'$ are :

(a) more than 100. (b) less than 50.
(c) more than 55. (d) less than 30.

Comprehension passage (4)
(Questions No. 10-12)

Let $f: R \to R$ be a function defined as $f(x) = 3x^5 - 25x^3 + 60x + 5$, and

$$g(x) = \begin{cases} \max\{f(t)\,;\, -4 \le t \le x\} & ; \quad -4 \le x \le 0 \\ \min\{f(t)\,;\, 0 < t \le x\} & ; \quad 0 < x \le 2 \\ f(x) - 16 & ; \quad x > 2 \end{cases}$$

On the basis of given definitions of $f(x)$ and $g(x)$, answer the following questions.

10. Total number of location(s) at which the graph of $y = g(x)$ breaks in $[-4, \infty)$ is/are :

(a) 2 (b) 1
(c) 0 (d) 4

11. If the equation $f(x) + \lambda = 0$ is having exactly three distinct real roots, then total number of possible integral values of 'λ' are :

(a) 20 (b) 21
(c) 40 (d) 42

12. If the equation $g(x) + \mu = 0$ is having infinitely many real solutions, then number of possible integral values of 'μ' is/are :

(a) 0 (b) 1
(c) 2 (d) 3

Questions with Integral Answer :
(Questions No. 13-17)

13. Let $f(x) = \sin\left(\frac{5x}{n}\right).\cos(nx)$, where $n \in I$, and the period of $f(x)$ is 3π, then total number of possible values of 'n' is equal to ....

14. Total number of integral values of x in $\left[-\frac{3\pi}{2}, \frac{3\pi}{2}\right]$ for which the equation

$$\left|\sin^{-1}(\sin x) + x^4 - 17x^2 + 16\right| = \left|x^4 - 17x^2 + 16\right| + \left|\sin^{-1}(\sin x)\right|$$

is satisfied, are ....

15. Let $n \in N$, and $f: N \to N$ be a function defined by $f(n) = \sum_{r=1}^{n} (r)!$. If $P(n)$ and $Q(n)$ are polynomials in n such that $f(n+2) = P(n)\,f(n+1) + Q(n)\,f(n)$ for all $n \in N$, then value of $P(10) + Q(6)$ is equal to ....

16. Let the equation $(P+1)(x^4 + x^2 + 1) - (P-1)(x^2 + x + 1)^2 = 0$ is having two distinct and real roots and $f(x) = \frac{1-x}{1+x}$, where $f(f(x)) + f\left(f\left(\frac{1}{x}\right)\right) = \alpha P$, then value of '$\alpha$' is ......

17. Let $f(x)$ and $g(x)$ be even and odd functions respectively, where $x^2 f(x) - 2f\left(\frac{1}{x}\right) = g(x)$, then value of $f(4)$ is equal to ....

Matrix Matching Questions :
(Questions No. 18-20)

18. Let $f(x) = \frac{|3-x| + |x+1|}{|1-x| + |x+3|}$ $\forall\ x \in R$, and $[x]$ represents the greatest integer function of x, then match the conditions/expressions in column (I) with statement(s) in column (II).

Column (I)	Column (II)
(a) If $x \in (-\infty, -3)$, then $f(x)$ satisfies	(p) $0 \le [f(x)] \le 2$
(b) If $x \in [-1, 1]$, then $f(x)$ satisfies	(q) $[f(x)] \ge 0$
(c) If $x \in [-4, -2]$, then $f(x)$ satisfies	(r) $[f(x)] = 0$
(d) If $x \in [2, \infty)$, then $f(x)$ satisfies	(s) $[f(x)] \ge 1$

19. Match the functions in column (I) with their corresponding range in column (II).

Column (I)	**Column (II)**
(a) $f(x)=\cos(\sin x)+\sin(\cos x)$ for all $x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	(p) $[\cos 1, 1]$
(b) $f(x)=\cos(\cos(\sin x))$ for all $x\in[0,\pi]$	(q) $[\cos 1, \cos(\cos 1)]$
(c) $f(x)=\cos(\cos x)$ all $x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	(r) $[\cos(\cos 1), \cos 1]$
(d) $f(x)=\cos(\sin\sqrt{2x-x^2})$ for all $x\in\left[0,\frac{3\pi}{8}\right]$	(s) $[\cos 1, 1+\sin 1]$
	(t) $[\sin 1, 1+\cos 1]$

20. Match the following columns (I) and (II)

Column (I)	**Column (II)**
(a) Domain of $f(x)=\cos^{-1}\left(\frac{\tan^2 2x+\cot^2 2x}{2}\right)$ contain(s)	(p) $\frac{3\pi}{4}$
(b) Domain of $f(x)=\left\{log_3 \sin^2(2x)\right\}^{1/2}$ contain(s)	(q) $\frac{\pi}{12}$
(c) Range of $f(x)=\tan^{-1}\left(\frac{x^2-x+1}{x^2+x+1}\right)$ contain(s)	(r) $\frac{\pi}{8}$
(d) If $[\alpha]$ represents the greatest integer function of α,	(s) $\frac{3\pi}{8}$
and $f(x)=\sqrt{[\cos^{-1}x]-[\sin^{-1}x]}$, then domain of $f(x)$ contain(s)	(t) $\frac{\pi}{4}$

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ANSWERS Exercise No. (1)

1. (a)	**2.** (a)	**3.** (c)	**4.** (d)	**5.** (d)
6. (c)	**7.** (a)	**8.** (b)	**9.** (c)	**10.** (a)
11. (b)	**12.** (c)	**13.** (a)	**14.** (b)	**15.** (d)
16. (c)	**17.** (d)	**18.** (a)	**19.** (b)	**20.** (c)
21. (c)	**22.** (d)	**23.** (b)	**24.** (a)	**25.** (a)
26. (d)	**27.** (a)	**28.** (b)	**29.** (c)	**30.** (b)
31. (a)	**32.** (c)	**33.** (b)	**34.** (a)	**35.** (c)
36. (b)	**37.** (d)	**38.** (b)	**39.** (b)	**40.** (a)
41. (a , c)	**42.** (a , b , c , d)	**43.** (a , b , c , d)	**44.** (a , d)	**45.** (a ,b, c , d)
46. (a , d)	**47.** (a , d)	**48.** (c , d)	**49.** (b , c , d)	**50.** (a , b , d)
51. (d)	**52.** (d)	**53.** (d)	**54.** (a)	**55.** (b)

ANSWERS Exercise No. (2)

1. (d)	**2.** (c)	**3.** (d)	**4.** (b)	**5.** (d)
6. (c)	**7.** (b)	**8.** (c)	**9.** (c)	**10.** (c)
11. (d)	**12.** (c)	**13.** (8)	**14.** (7)	**15.** (5)
16. (1)	**17.** (0)	**18.** (a) $\rightarrow$ p , q , s (b) $\rightarrow$ p , q , s (c) $\rightarrow$ p , q , s (d) $\rightarrow$ p , q , r	**19.** (a) $\rightarrow$ s (b) $\rightarrow$ q (c) $\rightarrow$ p (d) $\rightarrow$ p	**20.** (a) $\rightarrow$ r (b) $\rightarrow$ p , t (c) $\rightarrow$ r , s , t (d) $\rightarrow$ q , r , t

Chapter 11

Limits

Exercise No. (1)

Multiple choice questions with ONE correct answer : (Questions No. 1-15)

1. $\lim_{x\to 0}\dfrac{\int_0^{x^2} t^2.e^{-t^2}dt}{1-\cos(x^3)}$ is equal to :

(a) $-\dfrac{3}{2}$ (b) $\dfrac{2}{3}$

(c) $\dfrac{4}{3}$ (d) $\dfrac{1}{3}$

2. $\lim_{n\to\infty}\left(\sin\dfrac{\pi}{2n}.\sin\dfrac{2\pi}{2n}......\sin\dfrac{(n-1)\pi}{n}\right)^{1/n}$ is equal to :

(a) $\dfrac{1}{4}$ (b) $e^{4/\pi}$

(c) $e^{2/\pi}$ (d) $e^{\pi/8}$

3. Let $f(x)$ be differentiable and $f(1)=2$ and $f'(1)=4$,

then $\lim_{x\to 1}\left(\dfrac{f(x)}{f(1)}\right)^{\frac{1}{x-1}}$ is equal to :

(a) 1 (b) e^2

(c) 0 (d) e^{-1}

4. $\lim_{n\to\infty}\dfrac{(1+2^5+3^5+4^5+........+n^5)}{n^8}$ is :

(a) 0 (b) $\dfrac{1}{5}$

(c) $\dfrac{1}{6}$ (d) $\dfrac{1}{4}$

5. If $f(x)$ is differentiable and $f(0)=0$, such that $2f(x+y)+f(x-y)+3y^2=3f(x)-2xy$, then $\lim_{x\to 1}\dfrac{f(x)-1}{x-1}$ is equal to :

(a) –3 (b) 0

(c) –2 (d) 1

6. $\lim_{x\to 0}\left(2\sin^2\dfrac{x}{2}\right)^{ln(\cos x)}$ is :

(a) 1 (b) 2

(c) $\dfrac{1}{2}$ (d) $\dfrac{1}{e}$

7. $\lim_{x\to 0}(\cos x)^{\cot(x^2)}$ is :

(a) 1 (b) $\sqrt{e}$

(c) e^2 (d) $\dfrac{1}{\sqrt{e}}$

8. The value of $\lim_{x\to\infty}\left(x+\dfrac{1}{x}\right)e^{1/x}-x$ is equal to :

(a) 1 (b) ∞

(c) 0 (d) none of these

9. Let $f(x)$ be real function and $g(x)$ is bounded function for all $x\in R^+$, then $\lim_{n\to\infty}\dfrac{f(x).e^{nx}+g(x)}{1+e^{nx}}$ is :

(a) $f(x)$ (b) $g(x)$

(c) 0 (d) 1

10. If the graph of function $y=f(x)$ is having a unique tangent of finite slope at location $(a, 0)$, then $\lim_{x\to a}\dfrac{\log_e(1+6f(x))}{3f(x)}$ is equal to :

(a) 0 (b) 1

(c) 2 (d) 1/3

11. Let $\lim_{x\to 0}\dfrac{x(1+a\cos x)-b\sin x}{x^3}=1$, then $(a+b)$ is :

(a) –3 (b) –2

(c) – 4 (d) –1

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12. $\lim_{n\to\infty}\frac{n}{3}\left\{\left(\frac{3}{n}+\frac{9}{n^2}\right)^2+\left(\frac{3}{n}+\frac{18}{n^2}\right)^2+\left(\frac{3}{n}+\frac{27}{n^2}\right)^2 ....+\left(\frac{3}{n}+\frac{9}{n}\right)^2\right\}$

is equal to :

(a) 62 (b) 63

(c) 64 (d) none of these

13. If normal to curve $y=f(x)$ at $x=0$ is $3x-y+3=0$,

then $\lim_{x\to 0}\left\{\frac{x^2}{f(x^2)-5f(4x^2)+4f(7x^2)}\right\}$ is :

(a) $\frac{1}{2}$ (b) $-\frac{1}{3}$

(c) $\frac{1}{3}$ (d) $-\frac{1}{2}$

14. Let $f:[-1,1]\to R$ and $f(0)=0, f'(0)=\lim_{n\to\infty} n\, f\left(\frac{1}{n}\right)$,

where $0<\left|\lim_{n\to\infty}\cos^{-1}\left(\frac{1}{n}\right)\right|<\frac{\pi}{2}$, then value of

$\lim_{n\to\infty}\left\{\frac{2}{\pi}(n+1)\cos^{-1}\left(\frac{1}{n}\right)-n\right\}$ is equal to :

(a) $\frac{2}{\pi}$ (b) 0

(c) $1-\frac{2}{\pi}$ (d) $1+\frac{2}{\pi}$

15. $\lim_{x\to 0}\frac{\sin(\pi(1-\sin^2 x))}{\tan^2 x}$ is equal to :

(a) π (b) $-\pi$

(c) $\frac{\pi}{2}$ (d) 1

Multiple choice questions with MORE than ONE correct answer : (Questions No. 16-20)

16. Let $m, n\in I^+$ and $f(x)=\frac{(x-1)^{2m}}{log_e\left(\cos^n(x-1)\right)}$ for all $x\in(0,2)$. If $g(x)=e^{-|x-1|}\ \forall\ x\in R$ and $\lim_{x\to 1^+} f(x)=g'(1^+)$, then :

(a) $m+2n=5$ (b) $2m+n=4$

(c) $m-n=1$ (d) $2m-n=0$

17. In which of the following case(s), the limit doesn't exist ?

(a) $\lim_{x\to 0}\frac{x}{\sqrt{\sec^2 x-1}}$ (b) $\lim_{x\to 0}(\sin^3 x)^{\tan x}$

(c) $\lim_{x\to\infty}\left(\frac{3x^2+1}{4x^2+x}\right)^{\frac{x^2+1}{2x}}$ (d) $\lim_{x\to 0}(ln\, x^2)^{2x}$

18. Let $f(x)$ be differentiable function for all $x\in R^+$ and $f(1)=1$. If $\lim_{\alpha\to x}\frac{\alpha^2 f(x)-x^2 f(\alpha)}{\alpha-x}=1$ for every $x>0$, then :

(a) $f(2)=\frac{17}{6}$

(b) $f(x)$ has local minima at $x=\frac{(2)^{1/3}}{2}$

(c) $f(x)$ is strictly increasing for all $x\geq 2$

(d) $f''(x)>0\ \forall\ x\in R^+$

19. Let $f(x)=\lim_{k\to 0}\left(\frac{2x}{\pi}\cot^{-1}\left(\frac{x}{k^2}\right)\right)$, then

(a) $f(x)$ is increasing function for all $x\in R$.

(b) $f(x)$ is differentiable for all $x\in R-\{0\}$.

(c) $\int_{-1}^{\infty}[f(x)]dx=0$, where [.] represents greatest integer function.

(d) $f(|x|)$ is odd function.

20. If $\lim_{x\to 1}(1+\alpha x+\beta x^2)^{\frac{\gamma}{x-1}}=\lim_{x\to\infty}\left(\frac{x^2+4x}{x+x^2}\right)^{\frac{x^2}{x+1}}$, then :

(a) $\alpha+\beta=1$ (b) $\alpha+\beta=0$

(c) $\beta\gamma=4$ (d) $\beta\gamma=3$

Assertion Reasoning questions : (Questions No. 21-25)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements, Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers, only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

21. Statement 1 : Let $L = \lim\limits_{x \to -\infty}\left(\sqrt{4x^2 + 7x} + 2x\right)$, then limiting value $'L'$ approaches to positive infinity

because

Statement 2 : The form of indeterminacy in $'L'$ is $\infty - \infty$ form.

22. Statement 1 : Let $a_1 = \sqrt{3}$ and $a_{n+1} = \dfrac{a_n}{1 + \sqrt{1 + a_n^2}}$, for all $n \in N$, then $\lim\limits_{n \to \infty} 2^n(a_n)$ is equal to $\dfrac{2\pi}{3}$

because

Statement 2 : Sequence $\{a_n\}$ for all $n \in N$ is converging in nature.

23. Let $S_n = \sum\limits_{r=0}^{n} \dfrac{r.2^r}{(r+2)!}$, then

Statement 1 : $\lim\limits_{n \to \infty} S_n = 1$

because

Statement 2 : $\lim\limits_{n \to \infty} \dfrac{2^{n+1}}{(n+2)!} = 0$

24. Statement 1 :

Let $L = \lim\limits_{n \to \infty}\left(\dfrac{1}{1+n^2} + \dfrac{2}{2+n^2} + \ldots\ldots + \dfrac{n}{n+n^2}\right)$, then value of limit $'L'$ is equal to $\dfrac{1}{2}$

because

Statement 2 : $\lim\limits_{n \to \infty} \dfrac{1}{n}\sum\limits_{r=1}^{n} f\left(\dfrac{r}{n}\right) = \int\limits_0^1 f(x)dx$

25. Statement 1 : Let $L = \lim\limits_{n \to \infty}\left\{(\sin 1)^n + (\cos 1)^n\right\}^{\frac{1}{n}}$, then value of $\sin^{-1}(L) = 1$

because

Statement 2 : $\lim\limits_{x \to 0} \dfrac{\sin\left(\cos^2 \dfrac{x}{2}\right).\sin\left(x^2\right)}{\tan^2 x} = \sin 1$

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Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1-3)

Let $f(x)$ and $g(x)$ be continuous functions for all $x \in R$ and $f(0) = g(0) = 0$. If $\lim\limits_{x\to 0} \dfrac{g(x).\sin^2 x}{f(1-\cos x)} = \alpha$ and $\lim\limits_{x\to 0} \dfrac{f(x)}{x^2} = \beta$, then answer the following questions.

1. $\lim\limits_{x\to\infty} x^2.g\left(\dfrac{1}{x}\right)$ is equal to :

(a) $\dfrac{\alpha}{2\beta}$ (b) $2\alpha\beta^2$

(c) $\dfrac{\alpha\beta}{4}$ (d) $\dfrac{4\alpha}{\beta}$

2. $\lim\limits_{x\to 0} \dfrac{g(\cos 2x - 1)}{x^4}$ is equal to :

(a) $\alpha\beta$ (b) $-\alpha\beta$

(c) $\dfrac{\beta}{\alpha}$ (d) $-\dfrac{\alpha}{\beta}$

3. $\lim\limits_{x\to 0} \left\{ \dfrac{f(x)g(x)}{\left(\sin(2x^2)\right)^2} \right\}$ is equal to :

(a) $\dfrac{\alpha^2\beta}{16}$ (b) $\dfrac{\alpha^2\beta}{4}$

(c) $\dfrac{\alpha\beta^2}{16}$ (d) $\dfrac{\alpha\beta^2}{4}$

Comprehension passage (2)
(Questions No. 4-6)

Let points $'A'$ and $'B'$ lies on the circle $C_1 : x^2 + y^2 - 1 = 0$, where $\angle AOB = \theta$, $'O'$ being the origin. If tangents drawn at $'A'$ and $'B'$ to $'C_1'$ meet at $'P'$, and the tangent to $'C_1'$ drawn at the mid-point of arc $\overset{\frown}{AB}$ meet the lines PA and PB at $'C'$ and $'D'$ respectively , then answer the following questions.

4. If area of triangles PAB and PCD are $'A_1'$ and $'A_2'$ respectively , then $\lim\limits_{\theta\to 0^+} \left(\dfrac{A_1}{A_2}\right)$ is equal to :

(a) 2 (b) 4

(c) 1 (d) 6

5. If area of triangle PAB is $'A_1'$ and area enclosed by arc $\overset{\frown}{AB}$ with the chord AB is $'A_3'$, then $\lim\limits_{\theta\to 0^+} \left(\dfrac{A_1}{A_3}\right)$ is equal to :

(a) 3/2 (b) 5/2

(c) 2 (d) 1

6. If area of triangle PCD is $'A_2'$ and area enclosed by arc $\overset{\frown}{AB}$ with the chord AB is $'A_3'$, then $\lim\limits_{\theta\to 0^+} \left(\dfrac{A_2}{A_3}\right)$ is equal to :

(a) $\dfrac{3}{8}$ (b) $\dfrac{5}{4}$

(c) 1 (d) $\dfrac{1}{6}$

Questions with Integral Answer :
(Questions No. 7-10)

7. Let $f(x) = x\sin(\sin x) - \sin^2 x$ and $L = \lim\limits_{x\to 0} \dfrac{f(x)}{x^n}$. If limiting value $'L'$ is non-zero and finite , then value of $'n'$ must be equal to ....

8. Let $L = \lim\limits_{x\to 0} \left\{ \dfrac{\sin^3 x}{a\,x e^x - b\ln(1+x) + c\,x e^{-x}} \right\}$. If the value of L is 3/2 , then $(2b + a - c)$ is equal to ....

9. Let $S_n = \left(\sum\limits_{r=1}^{n} r\right) + 2\left(\sum\limits_{r=1}^{n-1} r\right) + 3\left(\sum\limits_{r=1}^{n-2} r\right) + \ldots + n$ and $\lim\limits_{n\to\infty} \dfrac{n^4}{S_n}$ is equal to $'L'$, then value of $\dfrac{L}{3}$ is equal to ....

10. Let $p(x)$ be a polynomial of degree 4 having the points of extremum at $x = 1$ and $x = 2$, where $\lim\limits_{x\to 0} \left(1 + \dfrac{p(x)}{x^2}\right) = 2.$ The value of $p(2)$ is ....

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Matrix Matching Questions :
(Questions No. 11-12)

11. Let $[x]$ represents the greatest integer which is just less than or equal to x , then match the following columns (I) and (II) .

Column (I)	**Column (II)**
(a) $\lim_{x\to 0}\left(\left[\frac{\sin x}{x}\right]+\left[\frac{\tan x}{x}\right]\right)$	(p) 2
(b) $\lim_{x\to 0}\left(\left[\frac{2x}{\sin x}\right]+\left[\frac{3\sin x}{x}\right]\right)$	(q) 0
(c) $\lim_{x\to 0}\left([x^2+2]+[x^3+3]\right)$	(r) 1
(d) $\lim_{x\to 0^+}\left(\frac{x}{2}\left[\frac{4}{x}\right]\right)$	(s) 4
	(t) limit doesn't exist

12. Let $L=\lim_{x\to\infty}\left(\sqrt{x^4+ax^3+3x^2+bx+2}-\sqrt{x^4+2x^3-cx^2+3x-d}\right)$, then match the columns (I) and (II).

Column (I)	**Column (II)**
(a) If $L=4$, then value of $(c-a)$ is	(p) 1
(b) If $L=2$, then value of $'c'$ is	(q) 2
(c) If $L=6$, $b\in R^+$, then value of $(a+b)$ can be	(r) 3
	(s) 4
(d) If $L=3$, $d\in R^-$, then value of $(c+d)$ can be	(t) 0

ANSWERS Exercise No. (1)

1. (b)	**2.** (a)	**3.** (b)	**4.** (a)	**5.** (c)
6. (a)	**7.** (d)	**8.** (a)	**9.** (a)	**10.** (c)
11. (c)	**12.** (d)	**13.** (b)	**14.** (c)	**15.** (a)
16. (a , b , d)	**17.** (a , b)	**18.** (a , b , c , d)	**19.** (b , d)	**20.** (b , d)
21. (d)	**22.** (b)	**23.** (a)	**24.** (b)	**25.** (b)

ANSWERS Exercise No. (2)

1. (c)	**2.** (a)	**3.** (c)	**4.** (b)	**5.** (a)
6. (a)	**7.** (6)	**8.** (6)	**9.** (8)	**10.** (0)

11. (a) $\rightarrow$ r
(b) $\rightarrow$ s
(c) $\rightarrow$ t
(d) $\rightarrow$ p

12. (a) $\rightarrow$ r
(b) $\rightarrow$ p
(c) $\rightarrow$ r , s
(d) $\rightarrow$ p , q , t

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Chapter 12

Continuity and Differentiability

Exercise No. (1)

Multiple choice questions with ONE correct answer : (Questions No. 1-20)

1. Let $f(x) = \min\{2, x^2-4x+5, x^3+2\}$, then total number of points of non-differentiability is/are :

(a) 4 (b) 2

(c) 3 (d) 1

2. Total number of locations of non-differentiability for the function $f(x) = |x| + |\cos x| + \tan\left(\frac{\pi}{4}+x\right)$ in the interval $x \in (-1, 2)$ is/are :

(a) 3 (b) 1

(c) 2 (d) 4

3. If function $f : R \to R$ satisfy the condition

$$\frac{f(2x+2y)-f(2x-2y)}{f(2x+2y)+f(2x-2y)} = \frac{\cos x \sin y}{\sin x \cos y} \text{ and}$$

$f'(0) = \frac{1}{2}$, then :

(a) $f''(x) - f(x) = 0$ (b) $4f''(x) + f'(x) = 0$

(c) $4f''(x) + f(x) = 0$ (d) $4f'(x) + f''(x) = 0$

4. The number of points of non-differentiability of $f(x) = \max\{\sin x, \cos x, 0\}$ in $(0, 2n\pi)$, where $n \in N$, are given by :

(a) $4n$ (b) $2n$

(c) $6n$ (d) $3n$

5. Let $f(x) = \left||e^x-1|-1\right|$ then $f(x)$ is non-differentiable for x belongs to :

(a) $\{0, 2\}$ (b) $\{0, 1\}$

(c) $\{1, \ln 2\}$ (d) $\{0, \ln 2\}$

6. Let $f(x) = 3x^{10} - 7x^8 + 5x^6 - 21x^3 + 3x^2 - 7$, then $\lim\limits_{h\to 0}\frac{f(1-h)-f(1)}{h^3+3h}$, is equal to :

(a) $\frac{22}{3}$ (b) $\frac{53}{3}$

(c) $-\frac{53}{3}$ (d) $\frac{25}{3}$

7. Let $f(x) = x^3 + x$ and $g(x) = \begin{cases} f(|x|) & ; x \ge 0 \\ f(-|x|) & ; x < 0 \end{cases}$, then :

(a) $g(x)$ is continuous $\forall\ x \in R$

(b) $g(x)$ is continuous $\forall\ x \in R^-$

(c) $g(x)$ is discontinuous $\forall\ x \in R^-$

(d) $g(x)$ is continuous $\forall\ x \in R^+$

8. Let $f(x) = \begin{cases} x^2+3x+a & ; x \le 1 \\ bx+2 & ; x > 1 \end{cases}$ be a differentiable function for all $x \in R$, then $(a+3b)$ is :

(a) 20 (b) 18

(c) 15 (d) 25

9. If $f(x) = \left|x^2 + a|x| + b\right|$ has exactly three points of non-differentiability, then

(a) $b \in R, a < 0$ (b) $a > 0, b = 0$

(c) $b = 0, a \in R$ (d) $a < 0, b = 0$

10. If $f(x) = [2x^3 - 5]$, [.] is greatest integer function, then total number of points in $(1, 2)$ where $f(x)$ is not continuous is/are :

(a) 10 (b) 12

(c) 13 (d) 15

11. Let $f(x) = \begin{cases} (\cos x - \sin x)^{\text{cosec}\, x} & ; -\frac{\pi}{2} < x < 0 \\ a & ; x = 0 \\ \dfrac{e^{1/x}+e^{2/x}+e^{3/x}}{ae^{2/x}+be^{3/x}} & ; 0 < x < \frac{\pi}{2} \end{cases}$ be continuous at location $x = 0$, then value of $(a+b)$ is :

(a) $e - \frac{1}{e}$ (b) $e + \frac{1}{e}$ (c) $e + \frac{2}{e}$ (d) $2e - \frac{1}{e}$

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12. If $f:[-2a,2a]\to R$ is an odd-function such that left hand derivative at $x=a$ is zero and $f(x)=f(2a-x)$ for all $x\in(a,2a)$ then left hand derivative at $x=-a$ is :

(a) 1 (b) –1
(c) 0 (d) Data insufficient

13. If $\int_0^x t\,f(t)\,dt=\sin x-x\cos x-\dfrac{x^2}{2}$ for all $x\in R-\{0\}$,

then $f\left(\dfrac{\pi}{6}\right)$ is equal to :

(a) 0 (b) $\dfrac{1}{2}$
(c) $-\dfrac{1}{2}$ (d) $-\dfrac{1}{4}$

14. Let $f(x)=[\sin x]+[\sin 2x]\ \ \forall\ x\in(0\,,10)$, [.] is the greatest integer function , then $f(x)$ is discontinuous at :

(a) 8 points (b) 9 points
(c) 10 points (d) 11 points

15. If $f(x)=\begin{cases}\dfrac{x}{2x^2+|x|} & ;\ x\neq 0\\ 1 & ;\ x=0\end{cases}$, then $f(x)$ is :

(a) differentiable at $x=0$
(b) discontinuous at $x=0$
(c) continuous but not differentiable at $x=0$
(d) $f\,'(0^+)=-1$

16. Let function $y=f(x)$ be defined parametrically as $x=3t-|t|$; $y=2t^2+t|t|$ for all $t\in R$, then :

(a) $f(x)$ is continuous but non-differentiable at $x=0$.
(b) $f(x)$ is discontinuous at $x=0$.
(c) $f(x)$ is differentiable at $x=0$.
(d) $f\,'(0^+)=2$.

17. Let $f(x)=[x]^2-[x^2]$, where [.] represents the greatest integer function , then $f(x)$ is discontinuous at :

(a) $x\in I$ (b) $x\in I-\{0\}$
(c) $x\in I-\{0\,,1\}$ (d) $x\in I-\{1\}$

18. Let $f(x)=\sum_{r=0}^{n}a_r x^r$ and if $|f(x)|\le|e^{x-1}-1|$ for all $x\in[0\,,\infty)$, then value of $|na_n+(n-1)a_{n-1}+\ldots..+2a_2+a_1|$ is :

(a) less than one
(b) greater than one
(c) not less than one
(d) not greater than one

19. Let $f(x)=\begin{cases}\dfrac{x}{1+e^{1/x}} ; x\neq 0\\ 0\quad ; x=0\end{cases}$; then :

(a) $f(x)$ is discontinuous at $x=0$
(b) $f\,'(0^+)=1$
(c) $f\,'(0^-)=1$
(d) $f\,'(0^+)=f\,'(0^-)=1$

20. Let $f(x)$ be differentiable function with property $f(x+y)=f(x)+f(y)+xy$ and $\lim\limits_{h\to 0}\dfrac{1}{h}f(h)=3$, then $f(x)$ is :

(a) linear function (b) $3x+x^2$
(c) $3x+\dfrac{x^2}{2}$ (d) x^3+3x

Multiple choice questions with MORE than ONE correct answer : (Questions No. 21-25)

21. Let $f(x)$ be defined in $[-2\,,2]$ by

$$f(x)=\begin{cases}\max\left\{\sqrt{4-x^2}\,,\sqrt{1+x^2}\right\} ; -2\le x\le 0\\ \min\left\{\sqrt{4-x^2}\,,\sqrt{1+x^2}\right\} ;\ 0<x\le 2\end{cases}$$, then

(a) $f(x)$ is continuous at $x=\pm\sqrt{\dfrac{3}{2}}$ but non-differentiable
(b) $f(x)$ is discontinuous at $x=\pm\sqrt{\dfrac{3}{2}}\,,0$
(c) $f(x)$ is non-differentiable at $x=0$
(d) $f(x)$ is differentiable $\forall\ x\in(-2\,,2)$

22. Let $f:R\to R$ be defined by functional relationship $f\left(\dfrac{x+y}{3}\right)=\dfrac{2+f(x)+f(y)}{3}$ and $f\,'(0)=2$, then which of the following statements are correct ?

(a) $y=|f(x)|$ is continuous and non-differentiable at $x=-1$.
(b) $y=\sin(f(x))$ is differentiable for all real x.
(c) $\int_{-1}^{1}[f(x)]dx=2$, where [.] represents the greatest integer function.
(d) $\int_{1}^{2}f([x])dx=4$.

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23. Let $f(x)=|\sin^{-1}(\sin x)|\ \forall\ x\in R$, then

(a) $f(x)$ is non-differentiable at $x=\frac{n\pi}{2}$; $n\in I$.

(b) Number of solutions of the equation $\frac{2}{\pi}f(x)-log_{3\pi}x=0$ are five.

(c) $\int_0^{\pi}[f(x)]dx=\pi-2$, where [.] represents the greatest integer function.

(d) $y=sgn(f(x))$ is continuous $\forall\ x\in R$.

24. Let [.] denotes the greatest integer function , and $f(x)=\frac{\sin\frac{\pi}{4}[x]}{[x]}$, then $f(x)$ is :

(a) continuous at x = 2. (b) discontinuous at x = 2.

(c) continuous at x = 3/2. (d) discontinuous at x = 3/2.

25. If $|c|\le\frac{1}{2}$ and $f(x)$ is differentiable function at $x = 0$

where $f(x)=\begin{cases} b\sin^{-1}\left(\frac{x+c}{2}\right) & ; & -\frac{1}{2}<x<0 \\ 1/2 & ; & x=0 \\ \frac{e^{ax/2}-1}{x} & ; & 0<x<\frac{1}{2}\end{cases}$, then

(a) $a = 2$
(b) $64b^2 + c^2 = 4$
(c) $a = 1$
(d) $16b^2 + c^2 = 64$

Assertion Reasoning questions :
(Questions No. 26-30)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements , Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers , only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

26. Statement 1 : Let $f(x)$ be discontinuous at $x=\pi$ and $\lim\limits_{x\to a}g(x)=\pi$, then $\lim\limits_{x\to a}f(g(x))$ can't be equal to $f\left(\lim\limits_{x\to a}g(x)\right)$.

because

Statement (2) : If $f(x)$ is continuous at $x=\pi$ and $\lim\limits_{x\to a}g(x)=\pi$, then $\lim\limits_{x\to a}f(g(x))=f\left(\lim\limits_{x\to a}g(x)\right)$.

27. Let $g(x)=[x^2-3x+4]\ \forall\ x\in R$, where [.] is greatest integer function , and $f(x)=\frac{\sin(\pi g(x))}{1+[x]^2}$ for all $x\in R$.

Statement 1 : $f(x)$ is discontinuous at infinitely many point locations

because

Statement 2 : $g(x)$ is discontinuous at infinitely many point locations.

28. Statement 1 : $f(x) = sgn(x)$, then $y = |f(x)|$ is not continuous at $x = 0$

because

Statement 2 : If $y = g(x)$ is discontinuous at location $x = a$, then $y = |g(x)|$ is also discontinuous at $x = a$.

29. Let $f:R\to R$ be defined as

$$f(x)=\begin{cases}\sqrt{x-2} & ; & x\ge 2 \\ -\sqrt{2-x} & ; & x<2\end{cases}$$

Statement 1 : $f(x)$ is non-differentiable at $x=2$

because

Statement 2 : $f(x)$ is not having a unique tangent at x = 2.

30. Let $f(x)=\begin{cases}\max\{g(t)\,;\,0\le t\le x\} & ; & 0\le x\le 4 \\ x^2-8x+17 & ; & x>4\end{cases}$ &

$g(x) = \sin x$ for all $x\in[0\ ,\ \infty)$.

Statement 1 : $f(x)$ is differentiable for all $x\in[0\ ,\ \infty)$

because

Statement 2 : $f(x)$ is continuous for all $x\in[0\ ,\ \infty)$.

Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1-3)

Let $f : R \to R$ and $g : R \to R$ be the functions which are defined as $f(x) = max\left\{2x(1-x)\ ,\ x^2\ ,\ (x-1)^2\right\}$ and $g(x) = 2 - |1-2x|$. On the basis of defined functions answer the following questions.

1. Total number of locations at which the function $h(x) = min\ \{\ f(x)\ , g\ (x)\}$ is non-differentiable is/are :

(a) 1 (b) 2
(c) 4 (d) 6

2. If α and β are the real roots of equation $f(x) - g(x) = 0$, then value of $(\alpha + \beta)$ is equal to :

(a) 2 (b) 1
(c) 0 (d) 3

3. If the equation $min\ \{\ f\ (x)\ ,\ g(x)\} - \lambda = 0$ is having exactly four distinct real roots, then value of λ should not be :

(a) $\frac{4}{5}$ (b) $\frac{1}{2}$
(c) $\frac{3}{4}$ (d) $\frac{4}{9}$

Comprehension passage (2)
(Questions No. 4-6)

Let $\phi(x) = mid\{\ f\ (x)\ ,\ g\ (x)\ ,\ h\ (x)\}$ represents the function which is second in order when the values of three functions $(viz: f(x), g\ (x), h\ (x))$ are arranged in ascending or descending order at any given location of x. If $\phi(x) = mid\{\ x\ , x(4-x)^2\ , 4x\ \}$, then answer the following questions.

4. Exhaustive set of values of x at which the function $y = \phi(x)$ is non-differentiable , is given by :

(a) $\{0\ ,2\ ,3\ ,5\}$ (b) $\{2\ ,3\ ,4\ ,6\}$
(c) $\{3\ ,4\ ,5\ ,6\}$ (d) $\{2\ ,3\ ,5\ ,6\}$

5. Value of $\left\{\phi'(3^+) - \phi'(2^+) + \phi'(6^+)\right\}$ is equal to :

(a) 10 (b) 8
(c) 9 (d) 6

6. Value of $\int_3^5 \phi(x)dx$ is equal to :

(a) 10 (b) 8
(c) 12 (d) 6

Questions with Integral Answer :
(Questions No. 7-10)

7. Let $f(x) = \begin{cases} \cot^{-1}(x) & ; \quad |x| \geq 1 \\ \dfrac{|x|-1}{2} + \dfrac{\pi}{4} & ; \quad |x| < 1 \end{cases}$, then total number of locations which domain of $f'(x)$ doesn't contain is/are ....

8. Let $f(x) = \begin{cases} x+a & ; \quad 0 \leq x < 2 \\ b-x & ; \quad x \geq 2 \end{cases}$ and

$g(x) = \begin{cases} 1+\tan x & ; \quad 0 \leq x < \pi/4 \\ 3-\cot x & ; \quad \pi/4 \leq x < \pi \end{cases}$.

If $f\left(g(x)\right)$ is continuous at the location $x = \frac{\pi}{4}$, then value of $2(b-a)$ is equal to ....

9. Consider $f(x) = \begin{cases} a\sin x + b\cos x & ; \quad x \leq 0 \\ \left(\dfrac{x+e^x}{2x+1}\right)^{1/x} & ; \quad x > 0 \end{cases}$, if $f(x)$ is continuous for all $x \in R$ and $f'(1) = f\left(-\frac{\pi}{2}\right)$, where $[x]$ represents the greatest integer less than or equal to x, then value of $[b] + [a]$ is equal to ....

10. Let $f : R^+ \to R^+$ be a differentiable function satisfying $f(xy) = \frac{f(x)}{y} + \frac{f(y)}{x}$ $\forall$ $x\ ,\ y \in R^+$ also $f(1) = 0$, $f'(1) = 1$. If M be the greatest value of $f(x)$ then the value of $[M+3]$. , (where [.] denotes the greatest integer function) , is equal to ....

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Matrix Matching Questions :
(Questions No. 11-12)

11. Match the functions in columns (I) with their cosrespending properties in column (II).

Column (I)	**Column (II)**
(a) $f(x) = min\{x^3, x^2\}$	(p) continuous in $(-2, 2)$.
(b) $f(x) = min\{\|x\|, \|x-1\|, \|x+1\|\}$	(q) differentiable in $(-2, 2)$.
(c) $f(x) = \|2x+4\| - 2\|x-2\|$	(r) not differentiable at least at one point in $(-2, 2)$.
(d) $f(x) = \|\sin x\| + \|\cos x\|$	(s) increasing in $(-2, 2)$.

12. Let $f: R \to R$ be continuous quadratic function such that $f(x) - 2f\left(\frac{x}{2}\right) + f\left(\frac{x}{4}\right) = x^2$. If $f(0) = 0$, then match the following columns (I) and (II).

Column (I)	**Column (II)**
(a) Value of $f'\left(\frac{9}{8}\right)$ is equal to	(p) 0
(b) Total number of points of non-differentiability for $y = \big\|1 - \|f(x) - 2\|\big\|$ is/are	(q) 2
(c) If $g(x) = min\{f(t) ; 0 \le t \le x\}$, where $x \in [0, 4]$, then value of $g'(3)$ is	(r) 4
(d) Number of locations at which $y = \|f(x)\|$ is non-differentiable is/are	(s) 6

ANSWERS Exercise No. (1)

1. (b)	**2.** (a)	**3.** (c)	**4.** (d)	**5.** (d)
6. (b)	**7.** (a)	**8.** (b)	**9.** (d)	**10.** (c)
11. (b)	**12.** (c)	**13.** (c)	**14.** (b)	**15.** (b)
16. (c)	**17.** (d)	**18.** (d)	**19.** (c)	**20.** (c)
21. (a , c)	**22.** (a , b , d)	**23.** (a , b , c)	**24.** (b , c)	**25.** (b , c)
26. (d)	**27.** (d)	**28.** (c)	**29.** (c)	**30.** (b)

ANSWERS Exercise No. (2)

1. (c)	**2.** (b)	**3.** (b)	**4.** (d)	**5.** (c)
6. (b)	**7.** (3)	**8.** (8)	**9.** (0)	**10.** (3)

11. (a) $\to$ p , r , s
(b) $\to$ p , r
(c) $\to$ p , q , s
(d) $\to$ p , r

12. (a) $\to$ r
(b) $\to$ s
(c) $\to$ p
(d) $\to$ p

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Chapter 13

Differentiation

Exercise No. (1)

Multiple choice questions with ONE correct answer : (Questions No. 1-15)

1. Let $y = e^{2x}$, then $\left(\frac{d^2y}{dx^2}\right)\left(\frac{d^2x}{dy^2}\right)$ is equal to :

(a) 1 (b) e^{-2x}
(c) $2e^{-2x}$ (d) $-2e^{-2x}$

2. Let $g(x)$ is reflection of $f(x)$ about the line mirror $y = x$ and $f'(x) = \frac{x^{12}}{1+x^2}$, if $g(3) = a$, then $g'(3)$ is :

(a) $\frac{1+a^2}{a}$ (b) $\frac{a^{12}}{1+a^2}$

(c) $\frac{1+a^2}{a^{12}}$ (d) $\frac{a}{1+a^2}$

3. Let $x = \tan\left(\frac{1}{a}\log_e y\right)$ and $(1+x^2)\frac{d^2y}{dx^2} = (\lambda - 2x)\frac{dy}{dx}$ then 'λ' is equal to :

(a) $2a$ (b) $3a$
(c) a (d) $-4a$

4. If $y = f(x)$ and $y\cos x + x\cos y = \pi$ for all $x \in R$, then $f''(0)$ is :

(a) π (b) $-\pi$
(c) 0 (d) 2π

5. If $y = f\left(\frac{2x-1}{1+x^2}\right)$ and $f'(x) = \sin^2 x$, then $\left.\frac{dy}{dx}\right|_{x=0}$ is :

(a) $\sin^2(1)$ (b) $-2\sin^2(1)$
(c) $1 - \cos 2$ (d) $1 + \cos(1)$

6. Second derivative of $a\sin^3 t$ w.r.t. $a\cos^3 t$ at $t = \frac{\pi}{4}$ is :

(a) $\frac{4\sqrt{2}}{3a}$ (b) 2

(c) $\frac{1}{2a}$ (d) $\frac{3\sqrt{2}}{4a}$

7. If $\int_0^y \cos t^2\, dt = \int_0^{x^2} \frac{\sin t}{t}\, dt$, then $\frac{dy}{dx}$ is equal to :

(a) $\frac{2\sin^2 x}{x\cos y^2}$ (b) $\frac{2\sin x^2}{x^2\cos y}$

(c) $\frac{\sin x^2}{x\cos y^2}$ (d) $\frac{2\sin x^2}{x\cos y^2}$

8. Let $y = \tan^{-1}\left(\frac{ln(e/x^2)}{ln(ex^2)}\right) + \tan^{-1}\left(\frac{3+2\,ln\,x}{1-6\,ln\,x}\right)$; then $\frac{d^2y}{dx^2}$ is :

(a) 0 (b) 1
(c) 2 (d) -1

9. Let $f(x)$ be a polynomial function, then second derivative of $f(e^x)$ is :

(a) $e^{2x}f'(x) + e^x f''(e^x)$ (b) $e^x f''(x) + f''(e^x)$

(c) $f''(e^x) + e^x f'(e^x)$ (d) $e^x f'(e^x) + e^{2x} f''(e^x)$

10. If $xe^{xy} = y + \sin^2 x$, then $\left.\frac{dy}{dx}\right|_{x=0}$ is :

(a) -1 (b) 2
(c) 1 (d) 0

11. Let $f(x)$ be differentiable and $\int_0^{t^2} x\, f(x)\, dx = \frac{2}{5}t^5$, then $f\left(\frac{4}{25}\right)$ is :

(a) $\frac{2}{5}$ (b) $\frac{5}{2}$

(c) $-\frac{5}{2}$ (d) 1

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12. For an invertible function $y = f(x)$, value of

$$\frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{\frac{d^2y}{dx^2}} + \frac{\left\{1+\left(\frac{dx}{dy}\right)^2\right\}^{3/2}}{\frac{d^2x}{dy^2}} \text{ is :}$$

(a) 1 (b) 0

(c) –1 (d) $\sqrt{2}$

13. Let (α, β), where $\alpha, \beta \neq 0$, satisfy the equation $ax^2 + 2hxy + by^2 = 0$, then $\left.\frac{dy}{dx}\right|_{(\alpha,\beta)}$ is equal to :

(a) 1 (b) $\frac{\alpha}{\beta}$

(c) $\frac{\beta}{\alpha}$ (d) 0

14. Let $F(x) = \frac{1}{x^2}\int_4^x \left\{4z^2 - 2F'(z)\right\} dz$, then value of $F'(4)$ is equal to :

(a) $\frac{64}{9}$ (b) $\frac{32}{9}$

(c) $\frac{64}{3}$ (d) $\frac{32}{3}$

15. If $f(x) = (1+x)^n$, then the value of $f(0) + f'(0) + \frac{f''(0)}{2!} + ... + \frac{f^n(0)}{n!}$ is

(a) n (b) 2^n

(c) 2^{n-1} (d) 0

Multiple choice questions with MORE than ONE correct answer : (Questions No. 16-20)

16. Let $n \in N$ and $f(x)$ is twice differentiable positive function on $(0, \infty)$ such that $x f(x) - f(x+1) = 0$. If $f(x) = e^{g(x)}$, then :

(a) $g''(x+1) + g''(x) = \frac{1}{x^2}$

(b) $g''(x) - g''(x-1) = -\frac{1}{(x-1)^2}$

(c) $g''\left(n+\frac{1}{2}\right) - g''\left(n-\frac{1}{2}\right) = -\frac{4}{(1-2n)^2}$

(d) $g''\left(\frac{1}{2}\right) - g''\left(n+\frac{1}{2}\right) = 4\sum_{r=1}^{n}\frac{1}{(2r+1)^2}$

17. Let $f : R \to R$ be strictly increasing function for all $x \in R$ and $f''(x) - 2f'(x) + f(x) = 2e^x$, then which of the following may be correct :

(a) $|f(x)| = f(x) \;\; \forall \;\; x \in R$

(b) $f(5) = -8$

(c) $f(3) = 8$

(d) $|f(x)| = -f(x) \;\; \forall \;\; x \in R$

18. Let $p, q \in R$, and $f(x) = (x^2 - 6x + p)(x^2 - 8x + q)$. If exactly one real value of 'α' exists for which $f(\alpha) = f'(\alpha) = 0$ and $f''(\alpha) \neq 0$, then which of the following ordered pairs (p, q) are applicable :

(a) (9 , 16) (b) (9 , 15)

(c) (5 , 15) (d) (8 , 12)

19. Let $f(x) = \sin^{-1}(\sin x)$ and $g(x) = \cos^{-1}(\cos x)$ for all $x \in R$, then which of the following statements are correct :

(a) $f'(7) = g'(7) = 1$ (b) $f'(2) + g'(2) = 0$

(c) $f'(-4) = g'(-4) = -1$ (d) $f'(e) = g'(2e) = -1$

20. Let $f(x) = \cos^2(x+1) - \cos x \,.\, \cos(x+2)$ for all $x \in R$, then :

(a) $f'(x) = 0 \;\; \forall \;\; x \in R$

(b) $f''(x) \neq 0 \;\; \forall \;\; x \in R$

(c) $f'(x) \neq 0$ for some real values of x

(d) $f(x)$ is non-decreasing $\forall \;\; x \in R$

Assertion Reasoning questions :
(Questions No. 21-25)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements , Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers , only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

21. Let $f^n(x)$ denotes the n^{th} derivative of $f(x)$ and $f(x)=(x^2-1)^k$, where $k \in N$.

Statement 1 : If the equation $f^n(x)=0$ is having 10 distinct real roots for exactly one value of $'n'$, then $'k'$ equals to 9

because

Statement 2 : A polynomial function of $'m'$ degree, where $m \in N$, vanishes after m^{th} derivative.

22. Statement 1 : Let $f(x)=\sin x - x\cos x$, then $f'(x)=x\sin x$. Both the functions $f(x)$ and $f'(x)$ are non-periodic

because

Statement 2 : The derivative of non-periodic differentiable function is non-periodic in nature.

23. Statement 1 : Let $f(x)=\cos^{-1}(4x^3-3x)$, then $f'\left(\frac{1}{4}\right)=\frac{4}{5}\sqrt{15}$

because

Statement 2 : $\cos^{-1}(4x^3-3x)=-3\cos^{-1}x$ for all $x \in \left(-\frac{1}{2},\frac{1}{2}\right)$

24. Let $f_n(x) = exp\left(f_{n-1}(x)\right) \;\; \forall \;\; n \in N$ and $f_0(x)-x=0$, then

Statement 1 : $\frac{d}{dx}\left(f_n(x)\right)=\prod_{i=1}^{n} f_i(x)$

because

Statement 2 : $\prod_{i=1}^{n} f_i(x)=\exp\left(\sum_{i=1}^{n} f_{i-1}(x)\right)$

25. Statement 1 : Let $y=t^2$ and $x=t+1 \;\; \forall \;\; t \in R$, then $\frac{d^3y}{dx^3}=0$ at $t=0$

because

Statement 2 : $\frac{dy}{dx}=\frac{d^2y}{dx^2}=0$ at $t=0$

Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1-3)

Let $f(x)$ be a cubic polynomial function for which $x^3 + f'(1)x^2 + f''(3)x - f(x) = 0$ holds true for all $x \in R$, then answer the following questions which are based on $f(x)$.

1. With reference to $f(x)$, the incorrect statement is :

(a) $f(0) + f(2) = -12$ (b) $f(0) + f(3) \neq -26$

(c) $f(1) + f(3) \neq -26$ (d) $f(1) + f(2) = -14$

2. Let $[x]$ represents the greatest integer which is just less than equal to x, and α, β, γ are the roots of $f(x) = 0$, where $\alpha < \beta < \gamma$, then value of $[\alpha] + 2[\beta] + 3[\gamma]$ is equal to :

(a) 18 (b) 15

(c) 20 (d) 12

3. If $g(x) = |f(x)|$, then total number of critical points for $y = g(x)$ are :

(a) 4 (b) 5

(c) 3 (d) 2

Comprehension passage (2)
(Questions No. 4-6)

Let $f(x) = x^4 - 8x^3 + 22x^2 - 24x \quad \forall \; x \in R$ and function $g(x)$ is defined as :

$$g(x) = \begin{cases} \min\{f(t) : 0 \le t \le x\} \;;\; 0 \le x < 2 \\ \max\{f(t) : 2 < t \le x\} \;;\; 2 \le x \le 5 \end{cases}$$

On the basis of given definition of $f(x)$ and $g(x)$ answer the following questions :

4. Function $g(x)$ in $(0, 5)$ is non-differentiable at :

(a) one point location. (b) two point locations.

(c) three point locations. (d) infinite point locations.

5. Value of $g'\left(\frac{3}{2}\right)$ is equal to :

(a) -2

(b) 1

(c) 0

(d) -4

6. In which one of the following intervals, $f(x) = g(x)$ holds true :

(a) $\left(\frac{1}{e}, e\right)$

(b) $[\cos 1, 2]$

(c) $[\sin 3, \sin 1]$

(d) $(1, \pi)$

Questions with Integral Answer :
(Questions No. 7-10)

7. If $x = \sec\theta - \cos\theta$, $y = (\sec\theta)^n - (\cos\theta)^n$, where $n \in N$, and $\left(\frac{dy}{dx}\right)^2 = n^2\left(\frac{y^2 + \alpha}{x^2 + \beta}\right)$, then value of $(\alpha - \beta)$ is equal to ....

8. Let $f(x) = -\frac{x^3}{3} + (\sin 6)x^2 - (\sin 4)(\sin 8)x$, and $f'(\sin 8) = K(\sin^2 1)(\sin 8)(\sin 6)$, then value of $'K'$ is equal to ....

9. Let $f(x) = \int_0^\infty \frac{e^{-xt}}{1+t^2}\,dt$, then value of $f''\left(\frac{1}{4}\right) + f\left(\frac{1}{4}\right)$ is equal to ....

10. Let the function $f(x)$ be defined as $f(x) = x^3 + e^{x/2}$ and $g(x) = f^{-1}(x)$, then the value of $g'(1)$ is equal to ....

Matrix Matching Questions :
(Questions No. 11-12)

11. Let α , $\beta \in R$, where $\alpha \neq \beta$, and $f(x) = x^3 - 9x^2 + 24x + k \equiv (x-\alpha)^2(x-\beta)$ then match the following columns.

Column (I)	**Column (II)**
(a) Absolute value of the difference of the two possible values for $'k'$ is	(p) 0
(b) If $\alpha < \beta$, then $'\alpha'$ is	(q) 2
(c) If $\alpha > \beta$, then $'\alpha'$ is	(r) 4
(d) If $\alpha > \beta$, then $'\beta'$ is	(s) 1

12. Match the following columns for the function and their derivatives.

Column (I)	**Column (II)**
(a) If $f(x) = 2\tan^{-1}x$, then $f'(x)$ is :	(p) $\frac{2}{1+x^2}$; $\lvert x \rvert < 1$
	(q) $\frac{-2}{1+x^2}$; $x < 0$
(b) If $f(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, then $f'(x)$ is :	(r) $\frac{2}{1+x^2}$; $\lvert x \rvert \neq 1$
(c) If $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, then $f'(x)$ is :	(s) $\frac{-2}{1+x^2}$; $\lvert x \rvert > 1$
(d) If $f(x) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, then $f'(x)$ is :	(t) $\frac{2}{1+x^2}$; $x \in R$

ANSWERS Exercise No. (1)

1. (d)	**2.** (c)	**3.** (c)	**4.** (a)	**5.** (c)
6. (a)	**7.** (d)	**8.** (a)	**9.** (d)	**10.** (c)
11. (a)	**12.** (b)	**13.** (c)	**14.** (b)	**15.** (b)
16. (b , c)	**17.** (a , c)	**18.** (c , d)	**19.** (a , b , d)	**20.** (a , d)
21. (b)	**22.** (c)	**23.** (c)	**24.** (b)	**25.** (c)

ANSWERS Exercise No. (2)

1. (c)	**2.** (a)	**3.** (b)	**4.** (b)	**5.** (c)
6. (c)	**7.** (0)	**8.** (4)	**9.** (4)	**10.** (2)

11. (a) $\to$ r
(b) $\to$ q
(c) $\to$ r
(d) $\to$ s

12. (a) $\to$ t
(b) $\to$ p , r
(c) $\to$ p , s
(d) $\to$ q

Chapter 14

Tangent and Normal

Exercise No. (1)

Multiple choice questions with ONE correct answer : (Questions No. 1-10)

1. Let 'P' be a point on the curve $y=\dfrac{x}{1+x^2}$ and tangent drawn at P to the curve has greatest slope in magnitude , then point 'P' is

(a) $\left(\sqrt{3}, \dfrac{\sqrt{3}}{4}\right)$ (b) $(0, 0)$

(c) $\left(-\sqrt{3}, -\dfrac{\sqrt{3}}{4}\right)$ (d) $\left(1, \dfrac{1}{2}\right)$

2. The equation of common tangent to the curves $y = 6-x-x^2$ and $xy = x+3$ is :

(a) $3x-y=8$ (b) $3x+y=10$

(c) $2x+y=4$ (d) $3x+y=7$

3. If $\alpha>0$, then set of values of α for which $\alpha e^x - x = 0$ has real roots is :

(a) $\left(0, \dfrac{1}{e}\right]$ (b) $\left(\dfrac{1}{e}, 1\right]$

(c) $\left[\dfrac{1}{e}, \infty\right)$ (d) $[0, 1]$

4. If $|f(x_1)-f(x)_2| \le (x_1-x_2)^2 \;\; \forall \;\; x_1, x_2 \in R$, then equation of tangent to the curve $y = f(x)$ at point $(2, 8)$ is :

(a) $x-8=0$ (b) $y-2=0$

(c) $y-8=0$ (d) $x-2=0$

5. Any normal to the curve $x = a(\cos\theta + \theta\sin\theta)$; $y = a(\sin\theta - \theta\cos\theta)$ at any point 'θ' is such that :

(a) it passes through $(0, 0)$.

(b) it makes constant angle with x-axis.

(c) it is at a constant distance from $(0, 0)$.

(d) none of these.

6. Angle of intersection between the curves given by $x^3-3xy^2+2=0$ and $y^3-3x^2y-2=0$ is :

(a) $\dfrac{\pi}{6}$ (b) $\dfrac{\pi}{2}$ (c) $\dfrac{\pi}{3}$ (d) $\dfrac{\pi}{4}$

7. Equation of normal to curve $y=(1+x)^y+\sin^{-1}(\sin^2 x)$ at $x=0$ is :

(a) $x+y+1=0$

(b) $x-y+1=0$

(c) $x+y-1=0$

(d) $x+y=0$

8. Let at point 'P' on the curve $y^3+3x^2=12y$, the tangent is vertical , then 'P' may be :

(a) $(0, 0)$ (b) $\left(\pm\dfrac{4}{\sqrt{3}}, -2\right)$

(c) $\left(\pm\sqrt{\dfrac{11}{3}}, 1\right)$ (d) $\left(\pm\dfrac{4}{\sqrt{3}}, 2\right)$

9. Acute angle of intersection between the curves $y=|1-x^2|$ and $y=|x^2-3|$ is given by :

(a) $\tan^{-1}\left(\dfrac{4\sqrt{3}}{7}\right)$ (b) $\sin^{-1}\left(\dfrac{3\sqrt{2}}{7}\right)$

(c) $\cos^{-1}\left(\dfrac{7}{9}\right)$ (d) $\cos^{-1}\left(\dfrac{7}{9\sqrt{2}}\right)$

10. If the tangent and normal to the curve $y=e^{-x}$ at point $P(0, 1)$ intersects the x-axis at 'T' and 'N' respectively , then area (in sq. units) of equilateral triangle which is circumscribed by the incircle of ΔPTN is :

(a) $\dfrac{3\sqrt{3}}{2}(\sqrt{2}+1)^2$ (b) $\dfrac{3\sqrt{3}}{4}(\sqrt{2}-1)^2$

(c) $\dfrac{\sqrt{3}}{4}(\sqrt{2}+1)^2$ (d) $\dfrac{\sqrt{3}}{4}(\sqrt{2}-1)^2$

Multiple choice questions with MORE than ONE correct answer : (Questions No. 11-15)

11. Let $x+2y-k=0$ be the tangent to the curve $y=\cos(x+y)$, $-2\pi \le x \le 2\pi$, then possible values of 'k' can be :

(a) $\pi/2$ (b) $-\pi/2$

(c) $3\pi/2$ (d) $-3\pi/2$

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12. If a function is having horizontal tangent at origin then it holds the H-property , functions having H-property are :

(a) $y=\begin{cases} x\sin\frac{1}{x} & ; x\neq 0 \\ 0 & ; x=0 \end{cases}$ (b) $y=\begin{cases} x^2\sin\frac{1}{x} & ; x\neq 0 \\ 0 & ; x=0 \end{cases}$

(c) $y=x\,|x|$ (d) $y=\min\{x^2 , |x|\}$

13. Let a curve in parametric form be represented by $x=3t^2$, $y=2t^3$ for all $t\in R$, then which of the following lines are tangent to curve at one point and normal at another point of curve ?

(a) $\sqrt{2}x+y-2\sqrt{2}=0$ (b) $\frac{x}{2}-\frac{\sqrt{2}}{4}y-1=0$

(c) $\frac{x}{2}+\frac{\sqrt{2}}{2}y-\sqrt{2}=0$ (d) $x-\sqrt{2}y+\sqrt{2}=0$

14. Let $f:R\to R$ and $g:R^+\to[0\,,\infty)$ be the functions which are given by $f(x)=kx$ and $g(x)=|\log_e x|$. If the equation $f(x)-g(x)=0$ is having three distinct real roots , then possible values of $'k'$ can be :

(a) $\frac{1}{e^2}$ (b) $\frac{1}{\sqrt{e}}$

(c) $\frac{1}{e^3}$ (d) $\frac{1}{2+\sqrt{\pi}}$

15. Functions which are having vertical tangent at point $x=1$ are :

(a) $f(x)=sgn(x-1)$

(b) $f(x)=\sqrt[3]{x-1}$

(c) $f(x)=(x-1)^{2/3}$

(d) $f(x)=\begin{cases} \sqrt{x-1} & ; x\geq 1 \\ \sqrt{1-x} & ; x<1 \end{cases}$

Assertion Reasoning questions :
(Questions No. 16-20)

Following questions are assertion and reasoning type questions. Each of these questions contains two statements , Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions has four alternative answers , only one of them is the correct answer. Select the correct answer from the given options :

(a) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1.

(b) Both Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

16. Consider the curves $C_1 : y^2=2x$ and $C_2 : y=e^{-|x|}$.

Statement 1 : Curves $'C_1'$ and $'C_2'$ form an orthogonal pair of curves

because

Statement 2 : Curves $'C_1'$ and $'C_2'$ intersect each other at only one point location

17. Let $a\in(0\,,\sqrt{2})$ and $b\in R^+$, where

$$D=(a-b)^2+\left(\sqrt{2-a^2}-\frac{9}{b}\right)^2$$

Statement 1 : For given conditions on $'a'$ and $'b'$, the minimum value of $'D'$ is 8

because

Statement 2 : The minimum distance between the curves $xy=9$ and $x^2+y^2=2$ is equal to $2\sqrt{2}$ units.

18. **Statement 1 :** Let $y=f(x)$ be polynomial function , and tangent at point $A(a\,,f(a))$ is normal to the curve of $y=f(x)$ at point $B(b\,,f(b))$, then at least one point $(c\,,f(c))$ exists for which $f'(c)=0$, where $c\in(a\,,b)$

because

Statement 2 : Product of the slopes of tangents to the curve $y=f(x)$ at $'A'$ and $'B'$ is equal to –1 if tangents are not parallel to the axes.

19. Consider the curves $C_1 : y = x^2+x+1$ and $C_2 : y=x^2-5x+6$.

Statement 1 : Equation of common tangent to the curves C_1 and C_2 is given by $9y+3x-4=0$

because

Statement 2 : Acute angle of intersection of the curves C_1 and C_2 is $\tan^{-1}\left(\frac{54}{71}\right)$.

20. **Statement 1 :** Length of subtangent at point $P\,(2\,,2)$ for the curve $x^2y^3=32$ is equal to 3 units

because

Statement 2 : Length of subtangent at any point $(\alpha\,,\beta)$ for the curve $x^2y^3=32$ is equal to $\left|\frac{3\beta}{\alpha}\right|$.

Exercise No. (2)

Comprehension based Multiple choice questions with ONE correct answer :

Comprehension passage (1)
(Questions No. 1-3)

Consider the curve $C_1: 5x^5 - 10x^3 + x + 2y + 6 = 0$. If the normal $'N'$ to curve C_1 at point $P(0, -3)$ meets the curve again at two points Q and R, then answer the following questions.

1. Minimum area (in square units) of the circle passing through the points Q and R is equal to :

(a) 5π (b) 4π
(c) 8π (d) 2π

2. With reference to line of normal $'N'$, which of the following statement is correct ?

(a) line $'N'$ is tangential to curve C_1 at point Q only.
(b) line $'N'$ is tangential to curve C_1 at point R only.
(c) line $'N'$ is tangential to curve C_1 at both the points Q and R..
(d) line $'N'$ is not tangential to curve C_1 at either of the point Q and R.

3. Let the length of subtangents at the points Q and R for the curve C_1 be l_1 and l_2 respectively, where $OQ > OR$, $'O'$ being the origin, then $\dfrac{l_1}{l_2}$ is equal to :

(a) 4 (b) 1
(c) 2 (d) 5

Comprehension passage (2)
(Questions No. 4-6)

Let $f: R \to R$ be defined as $f(x) = ax^3 + bx^2 + cx + 27$, where the curve of $y = f(x)$ touches the x-axis at point $P(-3, 0)$ and meets the y-axis at point Q. If $f'(0) = 9$, then answer the following questions.

4. If $f(\alpha) = f(\beta) = 0$ and $\alpha \neq \beta$, then value of $[\alpha]+[\beta]$ is equal to : ([.] represents the greatest integer function)

(a) 0 (b) 2
(c) 5 (d) 10

5. Area (in square units) of the triangle formed by normal at $(\alpha, 0)$, where $\alpha \neq -3$, with the co-ordinate axes is equal to :

(a) $\dfrac{1}{2}$

(b) $\dfrac{1}{8}$

(c) 1

(d) $\dfrac{1}{4}$

6. Let $g(x) = f(x) + \lambda$, where $(g'(x))^2 + g''(x).g(x) = 0$ is having exactly four distinct real roots, then exhaustive set of values of $'\lambda'$ belong to :

(a) $(-27, 8)$
(b) $(-24, 4)$
(c) $(-32, 0)$
(d) $(-20, 32)$

Questions with Integral Answer :
(Questions No. 7-10)

7. Let tangent at $'t_1'$ point to the curve $C: y = 8t^3 - 1$, $x = 4t^2 + 3$ is normal at another point $'t_2'$ to the curve $'C'$, then value of $729(t_1)^6$ is equal to ....

8. Let any point $'P'$ lies on the curve $y^2(3-x) = (x-1)^3$, where the distance of $'P'$ from the origin is $'r_1'$ and the distance of tangent at $'P'$ from the origin is $'r_2'$. If point P is $(2, 1)$, then value of $\left|\dfrac{(r_1^2+15)r_2^2}{r_1^2-1}\right|$ is equal to ....

9. Let l_1 and l_2 be the intercepts made on the x-axis and y-axis respectively by tangent at any point of the curve $x = a\cos^3\theta$; $y = b\sin^3\theta$, then the value of $\left\{\dfrac{l_1^2}{a^2} + \dfrac{l_2^2}{b^2}\right\}$ is ....

10. Let chord PQ of the curve $y + \lambda^2 x^2 - 5\lambda x + 4 = 0$ be tangential to curve $y(1-x) = 1$ at the point $R(2, -1)$, if $PR = RQ$, then the least possible value of 4λ is equal to ....

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Matrix Matching Questions :
(Questions No. 11-12)

11. Match the following columns (I) and (II)

Column (I)	Column (II)
(a) If the angle between the curves $yx^2 = 1$ and $y = e^{2-2\|x\|}$ at point (1 , 1) is θ , then value of $\cos\theta$ is	(p) 3
(b) If the acute angle of intersection of the curves $x^2 = 4ay$ and $y = \dfrac{8a^3}{x^2+4a^2}$, $a \in R^+$, is $\tan^{-1}(\lambda)$, then $'\lambda'$ is equal to	(q) 2
	(r) 1
(c) The length of subtangent at any point on the curve $y = ae^{x/3}$ is equal to	(s) 5/4
(d) If the slope of tangent , if exists , varies at every point of the curve $y = \max\{e^x, 1+e^{-x}, k\}$, then *'k'* can be	(t) 1/2

12. Match the following columns (I) and (II)

Column (I)	Column (II)
(a) If the non-vertical common tangent of the curves $xy = -1$ and $y^2 = 8x$ is line *'L'* , then area (in square units) of the triangle formed by line *'L'* with the co-ordinate axes is	(p) 1
(b) If the curves $y = 1 - \cos x$, $-\pi < x < \pi$ and $y = \dfrac{\sqrt{3}}{2}\|x\| + \lambda$ touch each other , then the number of possible values of $'\lambda'$ is/are	(q) 1/2
(c) The area (in square units) of triangle formed by normal at the point (1 , 0) to the curve $x = e^{\sin y}$ with coordinate axes is :	(r) 4
	(s) 2
(d) If the inequation $3 - x^2 > \|x - \lambda\|$ has at least one negative solution , then the possible values of $'\lambda'$ can be	(t) -4

ANSWERS — Exercise No. (1)

1. (b)	**2.** (d)	**3.** (a)	**4.** (c)	**5.** (c)
6. (b)	**7.** (c)	**8.** (d)	**9.** (c)	**10.** (b)
11. (a , d)	**12.** (a , b , c , d)	**13.** (a , b)	**14.** (a , c , d)	**15.** (a , b)
16. (b)	**17.** (a)	**18.** (a)	**19.** (d)	**20.** (c)

ANSWERS — Exercise No. (2)

1. (a)	**2.** (c)	**3.** (d)	**4.** (a)	**5.** (b)
6. (c)	**7.** (8)	**8.** (9)	**9.** (1)	**10.** (1)

11. (a) $\rightarrow$ r
(b) $\rightarrow$ p
(c) $\rightarrow$ p
(d) $\rightarrow$ r , s , t

12. (a) $\rightarrow$ s
(b) $\rightarrow$ s
(c) $\rightarrow$ q
(d) $\rightarrow$ p , q , s

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