Mathematics Teachers Association Malappuram (MAM)
Mathematics Test Series - III May 2022
Sequence and Series, straight lines, Conic sections and Introduction to Three Dimensional Geometry

Class : XI
Max. Score : 60

## ANSWER KEY

| | | | | |
|---|---|---|---|---|
| | Unit 1 : Answer any six, each question 3 marks. | | | |
| 1 | (a) $a_n = 3n-2$ ; $d = 3$ | 1 | 1 | |
| | (b) $a_n = \frac{n(n^2+5)}{4}$ | | | |
| | $a_1 = \frac{1(1^2+5)}{4} = \frac{6}{4} = \frac{3}{2}$ | $\frac{1}{2}$ | | |
| | $a_2 = \frac{2(2^2+5)}{4} = \frac{9}{2}$ | $\frac{1}{2}$ | | 3 |
| | $a_3 = \frac{3(3^2+5)}{4} = \frac{3\times 14}{4} = \frac{21}{2}$ | $\frac{1}{2}$ | 2 | |
| | $a_4 = \frac{4(4^2+5)}{4} = 21$ | $\frac{1}{2}$ | | |
| 2 | (a) $a_n = a+(n-1)d$ | 1 | 1 | |
| | (b) $a = -6$, $d = -\frac{11}{2}+6 = \frac{-11+12}{2} = \frac{1}{2}$ | 1 | | 3 |
| | $n = 10$ | | | |
| | $a_{10} = -6 + 9\times\frac{1}{2}$ | $\frac{1}{2}$ | 2 | |
| | $= \frac{-12+9}{2} = -\frac{3}{2}$ | $\frac{1}{2}$ | | |
| 3 | $3x - 2y - 6 = 0$ | | | |
| | (a) Slope, $m = -\frac{A}{B} = \frac{-3}{-2} = \frac{3}{2}$ | 1 | 1 | |
| | (b) $3x-2y = 6 \Rightarrow \frac{3x-2y}{6} = 1 \Rightarrow \frac{x}{2}+\frac{y}{-3} = 1$ | | | 3 |
| | $\frac{x}{a}+\frac{y}{b} = 1$ | | | |
| | $x$ intercept $= 2$ | 1 | 2 | |
| | $y$ intercept $= -3$ | 1 | | |
| 4 | (a) $-1$ | 1 | 1 | |
| | (b) Slope of line passing through $(-2,6)$ and $(4,8)$, | | | |
| | $m_1 = \frac{8-6}{4-{}^-2} = \frac{2}{6} = \frac{1}{3}$ | $\frac{1}{2}$ | | |

Slope of line passing through (8,12) and (x, 24),

$m_2 = \frac{24-12}{x-8} = \frac{12}{x-8}$ ½

Since the lines are perpendicular, $m_1 m_2 = -1$ 2 3

$\Rightarrow \frac{1}{3} \times \frac{12}{x-8} = -1$ ½

$\Rightarrow 4 = -x + 8 \Rightarrow x = 4$ ½

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5 $c = 8$ , $a = 6$ 1

$c^2 = a^2 + b^2 \Rightarrow b^2 = c^2 - a^2$ ½

$\Rightarrow b^2 = 64 - 36 = 28$ ½ 3 3

$a^2 = 36$

Eqn is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ ½

ie $\frac{x^2}{36} - \frac{y^2}{28} = 1$ ½

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6 (a) $x^2 + y^2 = r^2$ 1

(b) $x^2 + y^2 - 4x + 6y - 12 = 0$

$\Rightarrow x^2 - 4x + 2^2 + y^2 + 6y + 3^2 = 12 + 4 + 9$

$\Rightarrow (x-2)^2 + (y+3)^2 = 25$ 1

$\Rightarrow (x-2)^2 + (y - -3)^2 = 5^2$ 2

$(x-h)^2 + (y-k)^2 = r^2$

Centre $(h,k) = (2,-3)$ ½ 3

radius $r = 5$ ½

OR

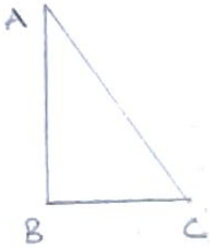
(b) $x^2 + y^2 - 4x + 6y - 12 = 0$

comparing with $x^2 + y^2 + 2gx + 2fy + c = 0$ ½

$2g = -4$, $2f = 6$, $c = -12 \Rightarrow g = -2$, $f = 3$, $c = -12$ ½ 2

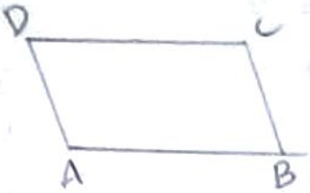
Centre $= (-g, -f) = (2, -3)$ ½

radius, $r = \sqrt{g^2 + f^2 - c} = \sqrt{4 + 9 + 12} = \sqrt{25} = 5$ ½

| | | | | |
|---|---|---|---|---|
| 7 | (a) xz plane | 1 | 1 | |
| | (b) A(−2, 3, 5), B(1, 2, 3) | | | 3 |
| | Distance $AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$ | | | |
| | $= \sqrt{(1+2)^2+(2-3)^2+(3-5)^2}$ | 1 | 2 | |
| | $= \sqrt{9+1+4} = \sqrt{14}$ | 1 | | |
| 8 | Let A(0, 7, 10), B(−1, 6, 6) and C(−4, 9, 6) | | | |
| | $AB = \sqrt{(-1-0)^2+(6-7)^2+(6-10)^2} = \sqrt{1+1+16} = \sqrt{18}$ | 1 | | |
| | $BC = \sqrt{(-4+1)^2+(9-6)^2+(6-6)^2} = \sqrt{9+9} = \sqrt{18}$ | $\frac{1}{2}$ | | |
| | $AC = \sqrt{(-4-0)^2+(9-7)^2+(6-10)^2} = \sqrt{16+4+16} = \sqrt{36}$ | $\frac{1}{2}$ | 3 | 3 |
| |  Now $AB^2 + BC^2 = 18+18 = 36$ | | | |
| | $= AC^2$ | 1 | | |
| | Pythegorus theorem is verified. | | | |
| | ∴ $\Delta ABC$ is a right angled triangle. | | | |
| | Unit II : Answer any Six, each question 4 marks | | | |
| 9 | (a) 304, 312, 320, · · · , 496 → AP | | | |
| | $a_1 = 304$, $a_n = 496$, $d = 8$ | $\frac{1}{2}$ | | |
| | $n = \frac{a_n - a_1}{d} + 1 = \frac{496-304}{8} + 1 = \frac{192}{8} + 1 = 24 + 1 = 25$ | $\frac{1}{2}$ | | |
| | $S_n = \frac{n}{2}(a_1 + a_n)$ | $\frac{1}{2}$ | 2 | |
| | $\therefore S_{25} = \frac{25}{2}(304 + 496) = \frac{25}{2} \times 800$ | | | |
| | $= 10000$ | $\frac{1}{2}$ | | 4 |
| | (b) $a_n = n(n+1) = n^2 + n$ | $\frac{1}{2}$ | | |
| | $S_n = \Sigma a_n = \Sigma n^2 + \Sigma n$ | $\frac{1}{2}$ | 2 | |
| | $= \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$ | $\frac{1}{2}$ | | |

| | | | | |
|---|---|---|---|---|
| | $\frac{n(n+1)}{2}\left[\frac{2n+1}{3}+1\right] = \frac{n(n+1)}{2}\left(\frac{2n+1+3}{3}\right)$ | | | |
| | $= \frac{n(n+1)}{2}\left(\frac{2n+4}{3}\right) = \frac{n(n+1)(n+2)}{3}$ | $\frac{1}{2}$ | | |
| 10 | (a) $\sqrt{16\times 4} = 4\times 2 = 8$; (iv) 8 | 1 | 1 | |
| | (b) Let $G_1, G_2, G_3$ be the 3 nos such that | | | |
| | $1, G_1, G_2, G_3, 256$ is a GP. | 1 | | |
| | $a_n = ar^{n-1}$ $a = 1,\ a_5 = 256$ | | | 4 |
| | i.e. $ar^4 = 256$ | $\frac{1}{2}$ | 3 | |
| | $\Rightarrow r^4 = 4^4 \Rightarrow r = 4$ | $\frac{1}{2}$ | | |
| | $G_1 = 4,\ G_2 = 16,\ G_3 = 64$ | 1 | | |
| | ∴ GP → 1, 4, 16, 64, 256 | | | |
| 11 | Let $m_1 = \frac{1}{2}$, $m_2 = m$; $\tan\theta = \left\lvert\frac{m_2 - m_1}{1 + m_1 m_2}\right\rvert$ | 1 | | |
| | $\Rightarrow \tan\frac{\pi}{4} = \left\lvert\frac{m - \frac{1}{2}}{1 + \frac{m}{2}}\right\rvert$ | $\frac{1}{2}$ | | |
| | $\Rightarrow 1 = \left\lvert\frac{2m-1}{2+m}\right\rvert \Rightarrow \frac{2m-1}{2+m} = \pm 1$ | $\frac{1}{2}$ | 4 | 4 |
| | $\frac{2m-1}{2+m} = 1 \Rightarrow 2m-1 = 2+m \Rightarrow m = 3$ | 1 | | |
| | $\frac{2m-1}{2+m} = -1 \Rightarrow 2m-1 = -2-m \Rightarrow 3m = -1 \Rightarrow m = \frac{-1}{3}$ | 1 | | |
| | slope of the line $= 3$ or $-\frac{1}{3}$ | | | |
| 12 | (a) (iii) 0 | 1 | 1 | |
| | (b) $d = \left\lvert\frac{Ax_1 + By_1 + C}{\sqrt{A^2+B^2}}\right\rvert$ | 1 | | |
| | $= \left\lvert\frac{12\times -1 - 5\times 1 + 82}{\sqrt{12^2+5^2}}\right\rvert$ | 1 | 3 | 4 |
| | $= \left\lvert\frac{-12-5+82}{\sqrt{144+25}}\right\rvert = \frac{65}{13} = 5$ | 1 | | |

|  |  |  |  |  |  |
|---|---|---|---|---|---|
| 13 | (a) | $y^2 = 8x$    $4a = 8 \Rightarrow a = 2$ | $\frac{1}{2}$ |  |  |
|  |  | $\boxed{y^2 = 4ax}$    focus $= (a, 0)$ | $\frac{1}{2}$ | 2 |  |
|  |  | $= (2, 0)$ | $\frac{1}{2}$ |  |  |
|  |  | Latus rectum $= 4a = 8$ | $\frac{1}{2}$ |  | 4 |
|  | (b) | $a = 6$ | $\frac{1}{2}$ |  |  |
|  |  | Eqn of parabola is $y^2 = 4ax$ | 1 | 2 |  |
|  |  | ie $y^2 = 24x$ | $\frac{1}{2}$ |  |  |
| 14 |  | $\frac{y^2}{9} - \frac{x^2}{27} = 1$ |  |  |  |
|  | (a) | $a^2 = 9, \quad b^2 = 27$ | $\frac{1}{2}$ |  |  |
|  |  | $c^2 = a^2 + b^2 = 36$ | 1 | 3 |  |
|  |  | $a = 3, \quad c = 6$ | $\frac{1}{2}$ |  | 4 |
|  |  | Length of Latus rectum $= \frac{2b^2}{a} = \frac{2 \times 27}{3} = 18$ | 1 |  |  |
|  | (b) | $e = \frac{c}{a} = \frac{6}{3} = 2$ | 1 | 1 |  |
| 15 | (a) | 0 | 1 | 1 |  |
|  | (b) | Let $A(-2, 4, 7)$, $B(3, -5, 8)$ |  |  |  |
|  |  | Let $k:1$ be the ratio in which the YZ plane divides AB. | $\frac{1}{2}$ |  |  |
|  |  | On YZ plane $x = 0$ | $\frac{1}{2}$ |  |  |
|  |  | ie $\frac{3k-2}{k+1} = 0$ | 1 | 3 | 4 |
|  |  | $\Rightarrow 3k - 2 = 0 \Rightarrow k = \frac{2}{3}$ | $\frac{1}{2}$ |  |  |
|  |  | ∴ required ratio is $2:3$ | $\frac{1}{2}$ |  |  |
| 16 | (a) | Let $A(1, 2, 3)$, $B(-1, -2, -1)$, $C(2, 3, 2)$, $D(4, 7, 6)$ |  |  |  |



$AB = \sqrt{(-1-1)^2+(-2-2)^2+(-1-3)^2}$

$= \sqrt{4+16+16} = \sqrt{36} = 6$ $\frac{1}{2}$

$BC = \sqrt{(2+1)^2+(3+2)^2+(2+1)^2} = \sqrt{9+25+9} = \sqrt{43}$ $\frac{1}{2}$

$CD = \sqrt{(4-2)^2+(7-3)^2+(6-2)^2} = \sqrt{4+16+16} = \sqrt{36} = 6$ 2

$AD = \sqrt{(4-1)^2+(7-2)^2+(6-3)^2} = \sqrt{9+25+9} = \sqrt{43}$ $\frac{1}{2}$

$AB = CD$, $BC = AD$

Opposite sides are equal $\frac{1}{2}$

$\therefore$ ABCD is a parallelogram

4

(b) $AC = \sqrt{(2-1)^2+(3-2)^2+(2-3)^2} = \sqrt{1+1+1}$

$= \sqrt{3}$ 1

$BD = \sqrt{(4+1)^2+(7+2)^2+(6+1)^2}$

$= \sqrt{25+81+49}$ $\frac{1}{2}$ 2

$= \sqrt{155}$

$AC \neq BD$ $\frac{1}{2}$

Since the diagonals are not equal

ABCD is not a rectangle.

Unit III : Answer any Three, each question 6 marks

17 (a) $a_m = n$, $a_n = m$

$a+(m-1)d = n$ — ①

$a+(n-1)d = m$ — ② $\frac{1}{2}$

① − ② → $(m-n)d = n-m \Rightarrow d = -1$ $\frac{1}{2}$

① → $a+(m-1)(-1) = n \Rightarrow a+1-m = n$

$\Rightarrow a = n+m-1$ 1 3

Now $p^{th}$ term $a_p = a+(p-1)d = n+m-1+(p-1)(-1)$

$\Rightarrow a_p = n+m-1+1-p$

$= m+n-p$ 1

(b) $S_n = 7 + 77 + 777 + \cdots$ upto n terms

$= 7\left[1 + 11 + 111 + \cdots\right]$ ½

$= \frac{7}{9}\left[9 + 99 + 999 + \cdots\right]$ ½ 6

$= \frac{7}{9}\left[10 - 1 + 100 - 1 + 1000 - 1 + \cdots\right]$ ½

$= \frac{7}{9}\left[10^1 + 10^2 + 10^3 + \cdots - n\right]$ ½ 3

$S_n = \frac{a(r^n - 1)}{r - 1}$

$= \frac{7}{9}\left[\frac{10(10^n - 1)}{10 - 1} - n\right]$ ½

$= \frac{7}{9}\left[\frac{10(10^n - 1)}{9} - n\right]$ ½

18 (a) Let the first 3 terms be $\frac{a}{r}, a, ar$

Given $\frac{a}{r} \times a \times ar = -1 \Rightarrow a^3 = -1 \Rightarrow a = -1$ 1

Also $\frac{a}{r} + a + ar = \frac{13}{12} \Rightarrow -\frac{1}{r} - 1 - r = \frac{13}{12}$

$\Rightarrow -12 - 12r - 12r^2 = 13r$ [Multiply by 12r]

$\Rightarrow 12r^2 + 25r + 12 = 0$ ½

$\Rightarrow r = \frac{-25 \pm \sqrt{25^2 - 4 \times 12 \times 12}}{2 \times 12} = \frac{-25 + 7}{24} = \frac{-32}{24}$ or $\frac{-18}{24}$ 3

$\Rightarrow r = -\frac{4}{3}$ or $-\frac{3}{4}$ ½

If $a = -1$ and $r = -\frac{4}{3}$, GP is

$\frac{-1}{\frac{-4}{3}}, -1, -1 \times -\frac{4}{3}$ ie $\frac{3}{4}, -1, \frac{4}{3}$ ½

If $a = -1$ and $r = \frac{-3}{4}$, GP is $\frac{4}{3}, -1, \frac{3}{4}$ ½ 6

(b) $3, \frac{3}{2}, \frac{3}{4}, \cdots$ $a = 3, r = \frac{1}{2}, S_n = \frac{3069}{512}$ ½

$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{3\left(1 - \left(\frac{1}{2}\right)^n\right)}{1 - \frac{1}{2}} = \frac{3\left(1 - \frac{1}{2^n}\right)}{\frac{1}{2}} = 6\left(1 - \frac{1}{2^n}\right)$ ½ ie $\frac{3069}{512} = 6\left(1-\frac{1}{2^n}\right) \Rightarrow \frac{3069}{512\times6} = 1-\frac{1}{2^n}$ $\frac{1}{2}$

$\Rightarrow \frac{1}{2^n} = 1 - \frac{3069}{3072} = \frac{3072-3069}{3072}$ $\frac{1}{2}$ 3

$\Rightarrow \frac{1}{2^n} = \frac{3}{3072} = \frac{1}{1024}$ $\frac{1}{2}$

$\Rightarrow 2^n = 1024 = 2^{10}$ $\frac{1}{2}$

$\Rightarrow n = 10$

19 (a) $A(2,2)$ , $B(5,3)$

Eqn of line is $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$ 1

$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$

$\Rightarrow \frac{y-2}{3-2} = \frac{x-2}{5-2}$ $\frac{1}{2}$ 2

$\Rightarrow y-2 = \frac{x-2}{3} \Rightarrow 3y-6 = x-2$ $\frac{1}{2}$

$\Rightarrow x-3y+4=0$

OR

(a) $A(2,2)$, $B(5,3)$

slope $m = \frac{y_2-y_1}{x_2-x_1} = \frac{3-2}{5-2} = \frac{1}{3}$ $\frac{1}{2}$

Eqn of line is $y-y_1 = m(x-x_1)$ $\frac{1}{2}$ 2

$\Rightarrow y-2 = \frac{1}{3}(x-2)$ $\frac{1}{2}$

$\Rightarrow 3y-6 = x-2$ $\frac{1}{2}$

$\Rightarrow x-3y+4=0$

20 (b) $\sqrt{3}x + y = 8$

$Ax + By = c$

$\sqrt{A^2+B^2} = \sqrt{3+1} = 2$

Dividing throughout by 2,

$\frac{\sqrt{3}}{2}x + \frac{1}{2}y = 4$ 1

Comparing with the normal form $x\cos w + y\sin w = p$

$\cos w = \frac{\sqrt{3}}{2}$ , $\sin w = \frac{1}{2}$ , $P = 4$ $\frac{1}{2}$ 2

$\Rightarrow w = 30^\circ (= \frac{\pi}{6})$ $\frac{1}{2}$

∴ normal form is $x \cos 30^\circ + y \sin 30^\circ = 4$

6

(c) $y - \sqrt{3}x - 5 = 0$ — ① $\sqrt{3}y - x + 6 = 0$ — ②

Slope of ① $m_1 = -\frac{A}{B} = \sqrt{3}$

slope of ② $m_2 = \frac{1}{\sqrt{3}}$ $\frac{1}{2}$

$\tan\theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$ $\frac{1}{2}$

$\Rightarrow \tan\theta = \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1+1} = \frac{3-1}{2\sqrt{3}}$ 2

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$ $\frac{1}{2}$

$\Rightarrow \theta = 30^\circ (= \frac{\pi}{6})$ $\frac{1}{2}$

or $180 - \theta = 180 - 30^\circ = 150^\circ$

20 $4x^2 + 9y^2 = 36 \Rightarrow \frac{x^2}{9} + \frac{y^2}{4} = 1$

(a) $a^2 = 9$, $b^2 = 4$, $c^2 = a^2 - b^2 = 5$

$a = 3$, $b = 2$, $c = \sqrt{5}$

foci $= (\pm c, 0) = (\pm\sqrt{5}, 0)$ 1 2

Vertices $= (\pm a, 0) = (\pm 3, 0)$ 1

(b) Length of major axis $= 2a = 6$ 1 2 6

Length of minor axis $= 2b = 4$ 1

(c) Length of Latus rectum $= \frac{2b^2}{a} = \frac{8}{3}$ 1

$e = \frac{c}{a} = \frac{\sqrt{5}}{3}$ 1 2

| | | | | |
|---|---|---|---|---|
| 21 | (a) (i) $(-4, 2, -5)$ | 1 | 1 | |
| | (b) Any point on $x$ axis is of the form $(x,0,0)$ | | | |
| | Let $A(-2,3,5)$, $B(1,2,3)$, $C(x,0,0)$ | $\frac{1}{2}$ | | |
| | $AC = BC \Rightarrow AC^2 = BC^2$ | | | |
| | $\Rightarrow (x+2)^2+(0-3)^2+(0-5)^2 = (x-1)^2+(0-2)^2+(0-3)^2$ | 1 | 2 | |
| | $\Rightarrow x^2+4x+4+9+25 = x^2-2x+1+4+9$ | | | |
| | $\Rightarrow 6x = -24 \Rightarrow x = -4$ | $\frac{1}{2}$ | | |
| | $\therefore$ The point is $(-4,0,0)$ | | | |
| | (c) Let the 3rd vertex be $C(a,b,c)$. | | | |
| | $A(3,-5,7)$, $B(-1,7,-6)$ | | | 6 |
| | Centroid is $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$ | 1 | | |
| | Given centroid $=(1,2,3)$ | | | |
| | Then $\frac{3+-1+a}{3} = 1 \Rightarrow 2+a=3 \Rightarrow a=1$ | $\frac{1}{2}$ | | |
| | $\frac{-5+7+b}{3} = 2 \Rightarrow 2+b=6 \Rightarrow b=4$ | $\frac{1}{2}$ | 3 | |
| | $\frac{7+-6+c}{3} = 3 \Rightarrow 1+c=9 \Rightarrow c=8$ | $\frac{1}{2}$ | | |
| | $\therefore$ 3rd vertex is $(1,4,8)$ | $\frac{1}{2}$ | | |