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## INTERBELL MATH

VERSION - 22

## INTERBELL STUDENT SUPPORT MATERIAL SSLC Mathematics - 2022




## Workshop participants



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## Preface

Mathematics is one of the most support needed subjects for students writing SSLC exams. The main reason is, that children do not understand in the right sense, right meaning and application of ideas. It is hoped that this will reduce some of the learning gaps in the online period of teaching by providing realtime support. INTERBELL MATH version 22 prepared by the Interbell team members of the district under the guidance of Palakkad Diet will give enough strength for that ..

Congratulations

## 1 ARITHMETIC SEQUENCES

## CONCEPTS

> A sequence got by starting with any number and adding a fixed number repeatedly is called an arithmetic sequence. That fixed number is called common difference

- Find the common difference of the sequence $10,14,18, \ldots$
answer : 14-10 = 18-14 = 4

1. write the sequence with first term 5 and common difference 3
2. Find the tenth term of the sequence $a+1, a+2, a+3, \ldots$
(1)
3. Write the common difference of the arithmetic sequence $13,23,33, \ldots$
4. Write the arithmetic sequence with second term 6 and common difference 4

## CONCEPTS

5. The difference between any two terms of an arithmetic sequence is the product of the difference of positions and the common difference.
6. The algebraic form of any arithmetic sequence is of the form $x_{n}=d n+x_{1}-d$
$x$ The algebraic form of an arithmetic sequence is $7 n+3$ what is its common difference? answer: 7
$x$ The first term of an arithmetic sequence is 3 , its common difference is 5 , write its algebraic form. What is its $10^{\text {th }}$ term ?
(4)

$$
\begin{aligned}
\mathrm{x}_{\mathrm{n}} & =\mathrm{dn}+\mathrm{x}_{1}-\mathrm{d} \\
& =5 \mathrm{n}+(3-5) \\
& =5 n-2 \\
10^{\text {th }} \text { term } & =5 \times 10-2 \\
& =50-2=48
\end{aligned}
$$

If we add 9 times common difference to the first term, we will get ${ }^{10 \text { th }}$ term

$$
3+9 \times 5=3+45=48
$$

5. Find the first term of an arithmetic sequence with algebraic form $2 n+3$

6 . Find the common difference of an arithmetic sequence with algebraic form $5 n+1$
7. Write the algebraic form of the sequence $4,8,12,16, \ldots$
8. Find the first and $20^{\text {th }}$ term of the sequence $3 n+2$
9. What is the common difference of the sequence $15,20,25, \ldots$ ? Find the difference between $10^{\text {th }}$ and $20^{\text {th }}$ terms
10. What is the common difference of the sequence $10,20,30, \ldots$ Which is the first three digit number in this sequence?
11. What is the remainder when the terms of the sequence $5 n+2$ divided by 5? Is 102 a term in this sequence?
12. Find the $25^{\text {th }}$ term of the arithmetic sequence with $5^{\text {th }}$ term 17 and common difference 5
13. Write the sequence of numbers between 100 and 200 which leaves remainder 3 on division by 5 . Mention its last term
14. Find the $30^{\text {th }}$ term of the sequence with algebraic form $3 n+1$. Also find the sum of its first 30 terms. Is 302 a term in this sequence ?
15. The sum of first and tenth term of a sequence with 10 terms is 50 . What is the sum of second and ninth terms? If its fifth term is 24 ,then what is its sixth term? Find the first term and common difference
16. If in a sequence the $10^{\text {th }}$ term is 40 and $18^{\text {th }}$ term is 88 then what is its common difference ? Is 168 a term in this sequence?

Write its algebraic form.
17. What is the remainder when the terms of the sequence $100,109,118, \ldots$ are divided by 9 ? write the three digit number sequence of multiples of 9 . What is the position of 999 in this sequence?

## CONCEPTS

> In an arithmetic sequence, if the sums of positions of two pairs of terms are equal, then the sums of the pairs of the terms are also equal.
> If the number of terms n is even, then there will be $\frac{n}{2}$ such pairs. Then Sum $=$ number of pairs $x$ sum of a pair
> If n is odd, then Sum $=$ middle term x number of terms
> Sum to n terms of an arithmetic sequence, $S_{n}=\frac{n}{2}$ (first term + last term)
$>$ Algebraic form of an arithmetic sequence is $s_{n}=\frac{d}{2} n^{2}+\left(f-\frac{d}{2}\right) n$

- The sum of first $n$ terms of an arithmetic sequence is $2 n^{2}+3 n$, what is its first term?
$(2,3,4,5)$ answer : 5

18. The sum of first $n$ terms of the sequence is $n^{2}$, What is its first term?
19. The sum of first $n$ terms of the sequence is $2 n^{2}+3 n$, what is its common difference? (1)
20. What is the remainder when the terms of the sequence $99,108,117, \ldots$ are divided by 9 ?
21. Find the common difference of the sequence $15,13,11,9, \ldots$
22. The first term of an arithmetic sequence is 8 , its $21^{\text {st }}$ term is 108 . Find the sum of first 21 terms.
23. The sum of first 9 terms of an arithmetic sequence is 45 , write the sequence.
24. What is the second term of the sequence with sum of first n-terms is $3 n^{2}+4 n$
25. Find the $20^{\text {th }}$ term of the sequence $10,20,30 \ldots$ What is the sum of its first 20 terms? Also find the sum first 20 terms of $9,19,29, \ldots$
26. Find the sum of first 20 terms of the arithmetic sequence $19,39,59, \ldots$
27. The sum of first 9 terms of a sequence is 45 , sum of first 18 terms is 171 . what is the sum of $10^{\text {th }}$ term to $18^{\text {th }}$ term? Find its $5^{\text {th }}$ and $14^{\text {th }}$ term. Find the sum of $5^{\text {th }}$ to $14^{\text {th }}$ terms?
28. The sum of first 20 term of an arithmetic sequence is 80 more than the sum of first 19 terms. What is its $20^{\text {th }}$ term? What is its common difference if the $10^{\text {th }}$ term is 20 ? Will the difference between any two terms be 2022?
29. The first term of a sequence is 10 and is common difference is 6 , what is its $11^{\text {th }}$ term? Find the sum of first 21 terms. Write an arithmetic sequence having sum first 21 terms is 1491
30. Find the sum of numbers between 100 and 300 which leaves remainder 2 when divided by 3 .
31. The sum of first 4 terms of an arithmetic sequence and the sum of its first 9 terms are equal and is equal to 72 . What is its $5^{\text {th }}$ term? Find the sum of its first 5 terms. Write the sequence.
32. The sum of $1^{\text {st }}$ and $21^{\text {st }}$ terms of an arithmetic sequence is 140 . What is its $11^{\text {th }}$ term? If the first term is 10 , what is its common difference? Write the sequence. Find the sum of its first 11 terms. Also find the sum of first 11 terms of $20,25,30, \ldots$
33. The sum of first $n$-terms of an arithmetic sequence is $n^{2}+2 n$, find the first term and common difference. Write the sequence. Find the sum of its first 20 terms.
34. Prove that 1 added to the sum of first $n$-terms of an arithmetic sequence
$3,5,7, \ldots$ gives a perfect square.
35. In an arithmetic sequence if the $5^{\text {th }}$ term is 38 and the $8^{\text {th }}$ term is 62 , what is its common difference? Will the difference between any two terms be 100? Write the sequence. Find the algebraic form of sum of first n-terms
36. The algebraic form of an arithmetic sequence is $2 n+3$, find its first term and common difference. What is the sum of its first n-terms? What about sum of its first 10 terms? Prove that 4 added to the sum of its first $n$-terms gives a perfect square
37. a) If the $n^{\text {th }}$ term of a sequence is $2 n-1$, what is its $5^{\text {th }}$ term?Find the sum of first five terms. Also find the sum of first $n$ terms
b) Find the sum of first n-terms of $\frac{1}{n}, \frac{3}{n}, \frac{5}{n}, \ldots$
c) Find the sum of first 2022 terms of $\frac{1}{2022}, \frac{3}{2022}, \frac{5}{2022}, \ldots$
38. Consider the following pattern

1
234
$\begin{array}{lllll}5 & 6 & 7 & 8 & 9\end{array}$
a)Write the next two lines
b) Find the sequence of last numbers in this pattern
c) What is the last number in the $9^{\text {th }}$ line ?
d) What are the first and last numbers in the $10^{\text {th }}$ line?
39. The sum of first 31 terms of an arithmetic sequence is 620 . What is its $16^{\text {th }}$ term? Find the sum of $15^{\text {th }}$ and $17^{\text {th }}$ terms. What about the sum of $1^{\text {st }}$ and $31^{\text {st }}$ terms? Write the sequence.
40. Find the sum of the following
a) $1+2+3+\ldots+10$
b) $2+4+6+\ldots+20$
c) $3+6+9+\ldots+30$
d) $6+12+18+\ldots+60$
41. The $n^{\text {th }}$ term of an arithmetic sequence is $3 n-2$, What are its first term and common difference? Write the sequence. Find the sum of first 11 terms. Write algebraic form of sum to n-terms.
42. Find the sum of numbers between 200 and 500 which leaves remainder 4 on division by 7 . What is the first 4-digit number which leaves remainder 4 when divided by 7 ?
43. If the $n^{\text {th }}$ term of an arithmetic sequence is $5 n-2$, find the sum of first $n$ terms. How many numbers are there in this sequence less than 500?What is the position of 998 in this sequence? Can the difference between any two terms be 56 ?
44. a) Find the sum of numbers between 100 and 500 which leaves remainder 3 on division by 6 .
b) Find the sum of numbers between 100 and 500 which leaves remainder 3 on division by 5 .
c) Write the algebraic form of above mentioned sequences
45. a) The sum of $7^{\text {th }}$ and $17^{\text {th }}$ term of an arithmetic sequence is 94 and the sum of $4^{\text {th }}$ and $14^{\text {th }}$ term is 70 . Find the first term and common difference.
b) in an arithmetic sequence with common difference 4, the sum of first 8 terms is equal
to the sum of next 7 terms. What is the difference between first and ninth terms? Find the difference between seventh and fifteenth terms? Find the eighth term

## ANSWERS

1. $5,8,11,14, \ldots$
2. $a+10$
3. 10
4. $2,6,10,14, \ldots$
5. $2 \times 1+3=5$
6. 5
7. $x_{n}=d n+f-d=4 n+4-4=4 n$
8. $x_{1}=3 \times 1+2=5, x_{20}=3 \times 20+2=62$
9. $d=5, x_{20}-x_{10}=(20-10) \times 5=50$
10. $\quad d=10, \quad$ first 3 digit number $=100$
11. remainder $=2 . \quad 102$ is term in this sequence
12. $x_{25}=x_{5}+20 d=17+20 x 5=117$
13. 103,108,111,...,198
14. $x_{30}=3 \times 30+1=91, x_{1}=4, S_{30}=\frac{30}{2}(4+91)=1425,302$ is not a term
15. $x_{2}+x_{9}=50 x_{5}+x_{6}=50 \quad x_{6}=50-24=26 \quad d=26-24=2 \quad x_{1}=x_{5}-4 d=24-$ $8=16$
16. $\frac{x_{18}-x_{10}}{18-10}=\mathrm{d} \quad \mathrm{d}=\frac{88-40}{8}=6$ 168 is not a term in this sequence

$$
x_{1}=x_{10}-9 d=40-54=-14
$$

$$
x_{n}=6 n+(-14-6)=6 n-20
$$

17. remainder $=1$

$$
\text { sequence: } 108,117,126, \ldots \quad x_{n}=
$$

$9 n+99$
$9 \mathrm{n}+99=999 \quad \mathrm{n}=100 \quad 999$ is $100^{\text {th }}$ term
18. $\quad 1^{2}=1$
19. $2 \times 2=4$
20. 0
21. -2
22. $\mathrm{S}_{21}=\frac{21}{2}(8+108)=21 \times 58=1218$
23. middle term $=\frac{45}{9}=5 \quad$ sequence : $1,2,3,4,5,6,7,8,9$
24. $x_{1}=3+4=7, \quad x_{1}+x_{2}=3 \times 2^{2}+4 \times 2=20 \quad x_{2}=20-7=13$
25. $\mathrm{x}_{20}=10+19 \times 10=200 \quad \mathrm{~S}_{20}=\frac{20}{2}(10+200)=2100$

$$
S_{20}=2100-20=2080
$$

26. 

$$
S_{20}=2100+2080=4180
$$

27. Sum of $10^{\text {th }}$ to $18^{\text {th }}$ terms $=171-45=126$

$$
x_{5}=\frac{45}{9}=5 \quad x_{14}=\frac{126}{9}=14
$$

Sum of $5^{\text {th }}$ to $14^{\text {th }}$ terms $=\frac{10}{2} \quad(5+14)=95$
28.

$$
\mathrm{X}_{20}=\mathrm{S}_{20}-\mathrm{S}_{19}=80
$$

$\mathrm{d}=\frac{x_{20}-x_{10}}{20-10}=6$
2022 will be a term difference
29.

$$
x_{11}=10+10 \times 6=70
$$

$$
x_{21}=10+20 \times 6=130
$$

$$
\mathrm{S}_{21}=\frac{21}{2}
$$

$$
(10+130)=
$$

1470
sequence with sum 1470 is $10,16,22, \ldots$
sequence with sum 1491 is 11,17,33,...
30. $101,104,107, \ldots .299 \quad$ number of terms $=\frac{299-101}{3}+1=67$ $S_{67}=13400$
31. $x_{5}=\frac{72}{9}=8$
$\mathrm{S}_{5}=\mathrm{S}_{4}+\mathrm{X}_{5}=72+8=80$
$x_{3}=\frac{80}{5}=16$
$x_{5}=x_{3}+2 d \quad 8=16+2 d \quad d=-4 \quad$ sequence $; 24,20,16,12, \ldots$
32.
$\mathrm{x}_{11}=\frac{140}{2}=70 \quad \mathrm{~d}=\frac{x_{11}-x_{1}}{11-1}=\frac{60}{10}=6 \quad 10,16,22, \ldots$
$S_{11}=\frac{11}{2}(10+70)=440 \quad \mathrm{X}_{11}=20+10 \times 5=70, \quad S_{11}=\frac{11}{2}(20+70)=$ 495
33.
$x_{1}=S_{1}=1^{2}+2 \times 1=3$ $x_{2}=S_{2}-S_{1}=8-3=5$
sequence;
$3,5,7, \ldots$
$S_{20}=\frac{20}{2}(3+41)=440$
34. $S_{n}=n^{2}+2 n \quad S_{n}+1=n^{2}+2 n+1=(n+1)^{2} \quad$ which is a perfect square
35. $\mathrm{d}=\frac{x_{8}-x_{5}}{8-5}=\frac{24}{3}=8 \quad 100$ cannot be a term difference

$$
x_{1}=x_{5}-4 d=38-32=6 \quad \text { sequence }: 6,14,22, \ldots
$$

$$
s_{n}=\frac{d}{2} n^{2}+\left(f-\frac{d}{2}\right) n=4 \mathrm{n}^{2}+2 \mathrm{n}
$$

36. 

$$
x_{1}=2 \times 1+3=5, d=2
$$

$$
\mathrm{S}_{\mathrm{n}}=\frac{n}{2}(5+2 \mathrm{n}+3)=\mathrm{n}(4+\mathrm{n})=\mathrm{n}^{2}+4 \mathrm{n}
$$

$\mathrm{S}_{10}=10^{2}+4 \times 10=140$
$S_{n}+4=n^{2}+4 n+4=(n+2)^{2}$ is a perfect square
37.

$$
x_{5}=9
$$

$S_{5}=25$
$S_{n}=n^{2}$
c) $\mathrm{S}_{\mathrm{n}}=\frac{n^{2}}{n}=\mathrm{n}$
d)
$\mathrm{S}_{2022}=2022$
38.
a) 10111213141516,171819202122232425
b)
$1,4,9,16,25, \ldots$
c) $9^{2}=81$
d) first term $=82$, last term $=100$
39. $x_{16}=\frac{620}{31}=20$

$$
x_{15}+x_{17}=2 x_{16}=40 \quad x_{1}+x_{31}=40
$$

$x_{15}=18, x_{17}=22 . \quad x_{1}=x_{15}-14 d=18-4 \times 2=-10 \quad$ sequence $:-10,-8,-6, \ldots$
40.
a) $\frac{10 \times 11}{2}=55$
b) $10 \times 11=110$
c) $110+55=165$
d)
$55+110+165=330$
41.

$$
\mathrm{d}=3 \mathrm{x}_{6}=3 \times 6-2=16 \quad \mathrm{x}_{1}=1, \mathrm{x}_{11}=31 \quad \mathrm{~S}_{11}=176 \quad s_{n}=\frac{3 n^{2}-n}{2}
$$ $1,4,7, \ldots$

42

$$
\begin{aligned}
& \text { 42. } \begin{array}{c}
\mathrm{X}_{1}=207 \\
\text { 43. } \quad \mathrm{x}=494 \quad \text { number of terms }=42 \\
s_{n}=\frac{d}{2} n^{2}+\left(f-\frac{d}{2}\right) n \\
\mathrm{x}_{1}=5-2=3 \\
s_{n}=\frac{n+5 n^{2}}{2}
\end{array} s_{n}=\frac{5}{2} n^{2}+\left(3-\frac{5}{2}\right) n
\end{aligned}
$$

$$
S_{42}=14721
$$

first term $=3$, last term $=498$
number of terms $=100$

$$
5 n-2=998 \quad 5 n=1000 \quad n=200 \quad 998 \text { is the } 200^{\text {th }} \text { term }
$$

term difference cannot be 56
44. a) first term $=105$, last term $=495, d=6$
number of terms $=\frac{495-105}{6}+1=65+1=66$
sum $=\frac{66}{2}(105+495)=33 \times 600=19800$
b) first term $=103, \quad$ last term $=498, d=5$
number of terms $=\frac{498-103}{5}+1=79+1=80$
sum $=\frac{80}{2}(103+498)=40 \times 601=24040$
C) for first sequence $x_{n}=6 n+3$
for the second sequence $x_{n}=5 n+3$
45.

$$
\begin{array}{lrr}
\text { a) } \quad x_{1}+6 d+x_{1}+16 d=94 & 2 x_{1}+22 d=94 \\
x_{1}+3 d+x_{1}+13 d=70 & 2 x_{1}+16 d=70 \\
2 x_{1}+16 x_{4}=70 & x_{1}=3
\end{array} \quad 6 d=24, d=4
$$

b) $\quad \mathrm{X}_{9}-\mathrm{x}_{1}=(9-1) \times 4=32 \quad \mathrm{x}_{15}-\mathrm{x}_{7}=(15-7) \times 4=32$
sum of first 8 terms $=$ sum of next 7 terms

$$
\begin{gathered}
x_{1}+x_{2}+x_{3}+x_{4}+x_{5}+x_{6}+x_{7}+x_{8}=x_{9}+x_{10}+x_{11}+x_{12}+x_{13}+x_{14}+x_{15} \\
x_{8}=\left(x_{9}-x_{1}\right)+\left(x_{10}-x_{2}\right)+\left(x_{11}-x_{3}\right)+\left(x_{12}-x_{4}\right)+\left(x_{13}-x_{5}\right)+\left(x_{14}-x_{6}\right)+\left(x_{15}-x_{7}\right) \\
x_{8}=7 \times 32=224
\end{gathered}
$$

## 2 CIRCLES

## Concepts

"Angle in a semicircle is right.
If we join the ends of a diameter of a cicle to a point inside the circle the angle formed will be more than $90^{\circ}$.....if it is outside....less than $90^{\circ}{ }^{\circ}$


1) $P Q$ is the diameter of the circle and $R$ is a point on the circle.
i) $\angle \mathrm{PRQ}=\ldots \ldots \ldots$
ii) $S$ is out side the circle, $T$ is inside the circle. Write your assumption about the measures of $\angle P S Q, \angle P T Q$
iii) In quadrilateral $\mathrm{ABCD}, \angle \mathrm{A}=60^{\circ}, \angle \mathrm{B}=90^{\circ}, \angle \mathrm{C}=100^{\circ}, \angle$ $D=110^{\circ}$ If we draw a circle with $A C$ as diameter where will be the positions of $B, D$ out side the circle, inside or on the circle?
(2)


Content

The angle made by any arc of a circle on the alternate arc is half the angle made at the centre.

All angles made by an arc on the alternate arc are equal, and a pair of angles on an arc and its alternate are supplementary.

If all the four vertices of a quadrilateral are on a circle, then its opposite angles are supplementary.
2) Find the value of $x$ in each figure given below.

3) $O$ is the circum centre of triangle $A B C$.
i) If $\angle C=40^{\circ}$ then $\angle A O B=$ $\qquad$
ii) If $\angle A=60^{\circ}$ then $\angle B O C=$
iii) Two angles of a triangle whose circum radius 2.5 are $30^{\circ}$, $80^{\circ}$. Draw the triangle and measure the smallest side. (4)

"If all the four vertices of a quadrilateral are on a circle, it is known as cyclic quadrilateral, then its opposite angles are supplementary."

5) In the figure $\angle \mathrm{APB}=50^{\circ}$,
a) Find $\angle \mathrm{AQB}$ ?
b) Find $\angle \mathrm{AOB}$ ?
c) Find $\angle \mathrm{OAB}$ ?

6) All the vertices of PQRS are on the circle.
i) What is the name of the quadrilateral ?
ii) If $\angle \mathrm{POR}=140^{\circ}$ then $\angle \mathrm{PSR}=$ $\qquad$
iii) $\angle \mathrm{PQR}=$ $\qquad$
iv ) If $\angle \mathrm{P}=100^{\circ}$ then $\angle \mathrm{R}=$


## Chords.

"If two chords of a circle intersect within the circle,then the products of the parts of the two chords are equal."
"The product of the parts into which a diameter of a circle is cut by a perpendicular chord, is equal to the square of half the chord."
$P A \times P B=P C \times P D$

$\mathrm{PA} \times \mathrm{PB}=\mathrm{PC}^{2}$

7) Chords $\mathrm{AB}, \mathrm{CD}$ intersects at P outside the circle. $\mathrm{PB}=3 \mathrm{~cm}, \mathrm{AB}=13 \mathrm{~cm}, \mathrm{PD}=4 \mathrm{~cm}$, Find CD .....?(1)
$(24,12,6,8)$

8) In the figure $\mathrm{PA}=\mathrm{PC}$ ๔ேறஸ゙.
i) $\mathrm{PA} \times \mathrm{PB}=$ $\qquad$
ii) Prove that $A B=C D$
iii) Whether PB, PD are equi distant from the centr of the circle...? Why ?

9) Two chords $A B, C D$ intersects at $P$.
i) Write the relation between $\mathrm{PA}, \mathrm{PB}, \mathrm{PC}, \mathrm{PD}$ (1)
ii) $\mathrm{PA}=8 \mathrm{~cm}, \mathrm{~PB}=3 \mathrm{~cm}, \mathrm{PC}=6 \mathrm{~cm}$ then find the length of PD?
iii) If we draw a rectangle with $\mathrm{PA}, \mathrm{PB}$ as sides and another rectangle with $\mathrm{PC}, \mathrm{PD}$ as sides what will be the relation between the areas of that two rectangles ?
(2)

10) AB is the diameter of the circle.
$C D$ is perpendicular to AB .
i ) $\mathrm{PD}=4$ then find PC . Find the length of the chord?
ii) $\mathrm{PA}=8 \mathrm{~cm}$ then find PB ?
iii) What is the area of the rectangle with PA , PB as sides ? What is the area of the square with PC as side ? What is the relation between these two areas ?
iv ) Draw a rectangle with sides $5 \mathrm{~cm}, 3 \mathrm{~cm}$ then draw a square having same area.
(4)


## Circles:- Answers

1) i) $\angle P R Q=90^{\circ}$
ii) $\angle \mathrm{PSQ} \angle 90^{\circ}, \angle \mathrm{PTQ}>90^{\circ}$
iii) $B$ is on the cicle,$\angle B=90^{\circ}$
$D$ is inside the circle, $\angle D>90^{\circ}$
2) $90^{\circ}, 53^{\circ}, 42^{0}, 67^{\circ}$
3) i) $\angle \mathrm{AOB}=80^{\circ}$
ii) $\angle B O C=120^{\circ}$
iii) Draw a circle of radius 2.5 cm . Then mark $60^{\circ}, 160^{\circ}$ at the centre of the circle then join the three points to form a triangle.
4) draw the figure.
5)a) $\angle A Q B=180^{\circ}-\angle A P B$

$$
=180^{\circ}-50^{\circ}=130^{\circ} \text { (Angles in the }
$$

opposite segments )
b) $\angle \mathrm{AOB}=2 \times \angle \mathrm{APB}=2 \times 50^{\circ}=100^{\circ}$
(Angle in the centre )
c) $O A=O B$ (radius of the same circle )
$\therefore \angle O A B=\angle O B A$
$=\frac{180^{\circ}-100^{\circ}}{2}=40^{\circ}$
6) i ) Cyclic quadrilateral
ii) $\angle P S R=70^{\circ}$
iii) $\angle P Q R=110^{\circ}$
iv) $80^{\circ}$
7) 8 cm
8)8) i) $P A x P B=P C \times P D$
ii) $\mathrm{PA}=\mathrm{PC}$ ๘ைைாிைைை
$P B=P D$

$P B-P A=P D-P C$
$A B=C D$
iii) At same distance.

Equal chords are at equal distance from the centre.
9) i) $P A \times P B=P C \times P D$
ii) $P D=24 / 6=4$
iii) Areas are equal.
10) i ) $\mathrm{PC}=4 \mathrm{~cm}, \mathrm{CD}=8 \mathrm{~cm}$
ii) $\mathrm{PB}=2 \mathrm{~cm}$
iii) Area of the rectangle $=8 \times 2=16 \mathrm{~cm}^{2}$

Area of the square $=16 \mathrm{~cm}^{2}$, Areas are equal.
iv) Draw the rectangle first. Then extends its breadth into the right-hand side then draw the perpendicular bisector of the total length and draw a semicircle with half of that total length. Draw a perpendicular from where the first side of the rectangle ends to the semicircle. Then take this length as one side of the square and complete that square.


## 3 MATHEMATICS OF CHANCE

## Probability as a number

If there are 5 black pearls and 7 white pearls in a box, what is the probability of getting one white pearl out of it? So the probability of getting a white pearl is to calculate how much of the white pearl is in total. Then the answer is 7/12
1)If 8 black balls and 6 white balls are taken out of a box, what is the probability that it will turn black?
2)A child is asked to say a two-digit number. What is the probability that the two digits are equal ?
3)Asking someone to say a two digit number What is the probability that it is an perfect square?
4) There are 4 red balls and 6 green balls in a bag. 3 red balls and 7 green balls in another bag.
a) If one participant from the first bag, what is the probability that it will turn red?
b) What if you take it out of the second bag?
c) If the balls in both the bags are put in one bag and a ball is taken out of it, what is the probability that it will turn red?
5) 10 A has 22 boys and 18 girls. If a child is selected to participate in a math fair
a) What is the probability that the child will be a boy?
b) What is the probability that the child will be a girl?
6) The numbers from 1 to 30 are written on different paper slips and put in a box. If a slip is chosen without looking,
a) What is the probability of becoming an even number?
b) What is the probability of becoming a prime number?
7) There are 6 black balls, 5 red balls and 9 yellow balls in a bowl. If you take a ball without looking at the bowl, then
a) What is the probability of getting a black ball?
b) What is the probability of getting the yellow ball?
8) The total number of children in a class is 40 . If a child is selected from this, the probability of it being a boy is $3 / 8$. If so
a) What is the number of boys in the class?
b) What is the number of girls?
c) How many boys should be added to get the probability of getting boys become $1 / 2$
9) Someone is asked to say a two digit number
a) What is the probability of being a multiple of ten?
b) What is the probability that the two digits of the said number are the same number?
c) What is the probability that the said number is an perfect square ?
10) A box contains a total of 60 pearls, including black and white pearls. If a pearl is taken from it, the chance of it becoming white is $2 / 5$. If so
a) What is the number of white pearls?
b) What is the number of black pearls?
c) How many white pearls should be added to get the probability become $1 / 2$
11) There are 7 white pearls and 13 black pearls in a box.
a) If you take a pearl without looking at the box, it will turn white How likely is it?
b) Put 2 more black pearls in the box and take one pearl what is the probability it to be black?
12) What is the probability of getting 5 Sundays in December in the calendar year?

## Geometrical probability

Most of the problems with using geometric images mean that it is possible to estimate the area of the shaded part over the total area of the image. In many pictures the total area and the shaded area can be easily divided into same geometric shapes and the answer can be easily seen.

For example, if a dot is made in the image below, what is the probability that it will be on the shaded area?


Look at the way this image is divided into identical geometric shapes

13) What is the probability that a dot in the image below will occur on the shaded area?


## Possibility when coming in pairs

Some questions may need to be considered in pairs, when considering the total number. The probability of seeing how many of the pairs we need, out of the total number of pairs in such problems means that, for example, in one box there are three slips of 1, 2,3 , and in another box there are two slips of 1,2 and if we take one slip from each box, what is the probability that the sum will be 4 ?

In this, if you take one slip from each box, you will get a pair of slips. First calculate how many such pairs there are. It is possible to calculate how many pairs of sum is 4 we need in it.
14) A bag contains 5 black balls, 7 white balls and 8 red balls. Another bag contains 6 black balls, 4 white balls and 5 red balls. If a child picks up a ball from each of the two bags without looking at the bag
a) What is the probability that both are white balls?
b) What is the probability of getting the same colour?
c) What is the probability of getting different colour?
d) What is the probability of getting at least one white ball?
15) Put 5 pieces of paper $1,2,3,4,5$ in one box and 4 pieces of paper $2,3,6,8$ in another box. Each piece of paper is taken out of each box without looking at the box. Then on the pieces of paper taken.
a) What is the probability that the two are even numbers?
b) What is the probability that both are even numbers?
c) What is the probability that the first number is an even number?
d) What is the probability that at least one is an even number?
16) There are 60 children in a class. 30 of them are boys. Another class has 50 children, 25 of whom are boys. If only one child is selected from each class
a) What is the probability that it is both girls?
b) What is the probability that there will be at least one girl in it?

Answers

1) probability $=8 / 14=4 / 7$

2 )probability $=9 / 90=1 / 10$
3) probability $=6 / 90=1 / 15$
4. a) $4 / 10$
b) $3 / 10$
c) $7 / 20$
5. a) $22 / 40$
b) $18 / 40$

6 a) $15 / 30$
b) $10 / 30$

7 a) $6 / 20$
b) $9 / 20$

8 a) $40 \times 3 / 8=15$
b) $40-15=25$
c) $\frac{15+x}{40+x}=\frac{1}{2}$
$\mathrm{x}=10$
9 a) $9 / 90$
b) $9 / 90$
c) $6 / 90$

10 a) $2 / 5 \times 60=24$
b) $60-24=36$
c) 12

11 a) $7 / 20$
b) $5 / 22$

12 There are 31 days in December 4 full weeks and 3 days

The probable three days are

| 1) | Sunday | Monday | Tuesday |
| :--- | :--- | :--- | :--- |
| 2) | Monday | Tuesday | Wednesday |
| 3 3) | Tuesday | Wednesday | Thursday |
| 4) | Wednesday | Thursday | Friday |
| 5) | Thursday | Friday | Saturday |
| 6$)$ | Friday | Saturday | Sunday |
| 7 7) | Saturday | Sunday | Monday |
| Total possibilities = 7 |  |  |  |
| having Sunday = 3 |  |  |  |
| required probability $=3 / 7$ |  |  |  |

## 4 SECOND DEGREE EQUATION

Main points to remember
*Square completion
*Quadratic equation

$$
a x^{2}+b x+c=0
$$



## 4.1 - (one mark each)

1. If square of a number is 225 , which is the number?
2. Area of a rectangle is $40 \mathrm{~cm}^{2}$. If length is " x ", what is the breadth?

3 . Which number is to be added to $x^{2}+10 x$, to get a perfect square?
Answers

1. $x^{2}=225, x=15$
2. length + breadth $=40 / 2=20$

$$
l=x, b=20-x
$$

3. $(10 / 2)^{2}=5^{2}=25$

## 4.2-(2 marks each)

1. When 4 times a number is subtracted from its square, got 221 . Which is the number?
2. The product of a number and 6 less than it is 135 . which is the number?
3. Product of two consecutive terms of an arithmetic sequence with common difference 6 is 160
a) Find the numbers
b) Find their sum
4. The product of 2 consecutive multiples of 8 is 384 .
a) Form second degree equation
b) Find the numbers
5. a) Which number is to be added to $x^{2}+12 x$ to get a perfect square?
b) $x^{2}+12 x=64, x=$ ?

## Answers

1. $x^{2}-4 x=221$

$$
\begin{aligned}
& x^{2}-4 x+4=221+4 \\
& (x-2)^{2}=225=15^{2} \\
& x-2=15 \\
& x=17
\end{aligned}
$$

2. $x(x-6)=135$

$$
x^{2}-6 x=135
$$

$$
x^{2}-6 x+9=135+9
$$

$$
(x-3)^{2}=144=12^{2}
$$

$$
x-3=12, x=15
$$

3. a) $x, x+6$

$$
x(x+6)=160
$$

$$
x^{2}+6 x=160
$$

$$
x^{2}+6 x+9=160+9
$$

$(x+3)^{2}=169=13^{2}$
$x+3=13, x=10$
Numbers $=10,16$
b) Sum $=10+16=26$
4. a) $x, x+8$

$$
x(x+8)=384
$$

$$
x^{2}+8 x=384
$$

b) $x^{2}+8 x+16=384+16=400$ $(x+4)^{2}=20^{2}$
$x+4=20, x=16$
5. a) $(12 / 2)^{2}=6^{2}=36$
b) $x^{2}+12 x=64$
$x^{2}+12 x+36=64+36=100$
$(x+6)^{2}=10^{2}, x+6=10, x=4$

## 4.3-(4 marks each)

1. Product of 2 consecutive even numbers is 528
a) If one is $x$, which is the other number?
b) Form second degree equation
c) Find the numbers
2. Perimeter of a rectangle is 44 cm and area is 120 sq.cm
a) length +breadth = ?
b) If length is $x$, what is breadth?
c) Find length and breadth
3. Fill in the blanks
a) $x^{2}+4 x=140$
b) $x^{2}+4 x+----=140+-----$
c) $(x+----)^{2}=-----$
d) $x+2=$
e) $x=$
4. What will be the length and breadth of a rectangle whose perimeter is 36 cm , and area is 32 sq.cm?
5. Rectangular shaped play ground has perimeter 60 m .
a) if breadth is $x$, what is length?
b) If the area is 176 sq . c.m, find length and breadth

## Answers

1. a) $x, x+2$
b) $x(x+2)=528$
$x^{2}+2 x=528$
$x^{2}+2 x+1=528+1=529$
$(x+1)^{2}=23^{2}$
$x+1=23, x=22$
c) 22,24
2. a) length + breadth $=22$
b) $l=x, b=22-x$
c) $x(22-x)=120$
$22 x-x^{2}=120$
$x^{2}-22 x=-120$
$x^{2}-22 x+121=-120+121=1$
$(x-11)^{2}=1, x-11=1, x=12$
$\mathrm{l}=12, \mathrm{~b}=10$
3. a) $x^{2}+4 x=140$
b) $x^{2}+4 x+4=140+4$
c) $(x+2)^{2}=12^{2}$
d) $x+2= \pm 12$
e) $x=12-2, x=-12-2$
$x=10,-14$
4. 2 (length + breadth $)=36$
$\mathrm{l}=\mathrm{x}, \mathrm{b}=18-\mathrm{x}$
$x(18-x)=32$
$18 \mathrm{x}-\mathrm{x}^{2}=32$
$x^{2}-18 x=-32$
$\mathrm{x}^{2}-18 \mathrm{x}+81=-32+81=49$
$(x-9)^{2}=7^{2}$
$x-9=7, x=16$
length and breadth- 16,2
5. a) $30-x$
b) $x(30-x)=176$
$30 x-x^{2}=176$
$x^{2}-30 x=-176$
$\mathrm{x}^{2}-30 \mathrm{x}+225=-176+225=49$
$(x-15)^{2}=7^{2}$
$\mathrm{x}=22$
if breadth is 22 cm , length $=8 \mathrm{~cm}$

## 4.4-(6 marks each)

1. There is a play ground in the shape of a square. A garden with 4 m width is set in one side. The area of play ground excluding this is 437 sq.m.
a) What is the shape of play ground now?
b) What is its length?
c) What is the length of sides of square play ground?
d) What is the area of entire play ground?
2. The biggest side of a rectangle is 8 cm more than its smallest side. Its area is $180 \mathrm{~cm}^{2}$.
a) smallest side $x$, then biggest side?
b) form second degree equation
c) Find length and breadth of the rectangle
3. If $x$ is a counting number
a) What should be added to $x^{2}+16 x$ to get a perfect square?
b) $x^{2}+a x+16$ is a perfect square, what is the value of $a$ ?
c) $a x^{2}+b x+c$ is a perfect square. Then prove that, $a^{2}=4 b$
d) $x^{2}+8 x+k$ is a perfect square, then what will be the value of $k$ ?

## Answers

1. a)Rectangle
b) $x-4$
c) $x(x-4)=437$
$x^{2}-4 x=437$
$\mathrm{x}^{2}-4 \mathrm{x}+4=437+4=441$

$$
(x-2)^{2}=21^{2}, \quad x-2=21, \quad x=23
$$

d) $23^{2}=529$ sq. c.m
2.a) $x, x+8$
b) $x(x+8)=180$
$x^{2}+8 x=180$
$x^{2}+8 x+16=180+16=196$
$(x+4)^{2}=14^{2}, \quad x+4=14, x=10$
c) 10,18
3. a) $x^{2}+16 x+64$
b) $a=8$
c) $(\mathrm{a} / 2)^{2}=\mathrm{b}, \mathrm{a}^{2} / 4=\mathrm{b}, \mathrm{a}^{2}=4 \mathrm{~b}$
d) $k=16$

## 5 Trigonometry

If a perpendicular is drawn from one vertex of an equilateral triangle to the opposite side, it becomes two identical right triangles. Its angles are $30^{\circ}, 60^{\circ}$ and $90^{\circ}$ each, of which the length of the opposite side of the $90^{\circ}$ angle is twice the length of the opposite side of the $30^{\circ}$ angle. The length of the opposite side of the $60^{\circ}$ angle is equal to $\sqrt{3}$ times of the length of the opposite side of the $30^{\circ}$ angle.


If we draw the diagonal of a square, it becomes two isosceles right triangles. Then its angles will be $45^{\circ}, 45^{\circ}$ and $90^{\circ}$ each. The opposite sides of the $45^{\circ}$ angles will be of equal length. The length of the opposite side of the $90^{\circ}$ angle is $\sqrt{2}$ times of their equal sides.


1

1. In right triangle ABC , let $\angle \mathrm{C}=30^{\circ} \quad \mathrm{AC}=6 \mathrm{~cm}$ then,
(i) What is the length of $A B$ ?
(ii) What is the length of BC?
(2)

2. In right triangle $A B C$, let $A B=3 \sqrt{3} \mathrm{~cm}$
(i) What is the length of AC?
(ii) What is the length of BC ?
(iii) What is the perimeter of the triangle?

(4)
3. In right triangle $A B C$, let $A C=10 \mathrm{~cm}$
(I) What is the length of AB ?
(ii) What is the length of BC ?
(iii) Find the area of triangle ABC.
(4)

4. In right triangle $P Q R$, let $P Q=3 \mathrm{~cm}$
(i) What is the length of PR ?
(ii) What is the perimeter of square PRST?
(iii) Find the area of square PRST?
(4)

5) If two people standing on either side of a 10 meter high tower viewed the top edge at an angle of elevation of $30^{\circ}$ and $60^{\circ}$ respectively.
(1) Draw an approximate figure showing these ideas.
(2) What is the distance between those people?
6) The angle of the triangle in the figure is $30^{\circ}$ and the length of the opposite side is 8 cm .
(a) What centre angle does that side form at its circumcircle ?
(b) If $r$ is the radius of circumcircle, what is the relation between the radius of circumcircle and that side ?
(c) Find the diameter of circumcircle.
(d) Find the circumference of the circle?
7) In the triangle $A B C$, ' $h$ ' is the perpendicular distance from the vertex $B$ to the side AC . If $\mathrm{AC}=14 \mathrm{~cm}$, then
(a) Find the length of $h$.
(b) What is the area of $\Delta \mathrm{ABC}$.
$\left[\sin \left(40^{\circ}\right)=0.6428\right]$

$\qquad$
$\qquad$

## 6 CO ORDINATES

## 9 GEOMETRY AND ALGEBRA



See the picture and try to learn the ideas
Olrigin is $\mathrm{O}(0,0)$.
The $y$-co-ordinates of the points on the line parallel to the x axis are equal.
eg: $(3,2),(6,2),(9,2)$
The $x$-co-ordinates of the points on the line parallel to the $y$ axis are equal eg: $(3,4),(3,2),(3,-2)$.

The $y$ co-ordinates of points on the $x$ axis is 0 .
eg: $(1,0),(2,0),(4,0),(-1,0)$
The $\times$ co-ordinates of points on the $y$ axis is 0
eg: $(0,1),(0,2)(0,4),(0,-1)$

In rectangle $A B C D$,
a) Since the side $A D$ is parallel to the $y$ axis, the $x \operatorname{co}$ ordinates of $A$ and $D$ are 3 $A(3,2), D(3,4)$.
b) Since the side $B C$ is parallel to the $y$ axis, the $x$ co ordinates of $B$ and $C$ are 6 $B(6,2), C(6,4)$.
c) Since the side $A B$ is parallel to the $x$ axis, the $y$ co ordinates of $A$ and $B$ are 2 $A(3,2), B(6,2)$.
d) Since the side CD is parallel to the $x$ axis, the $y$ co ordinates of $C$ and $D$ are 4 . $C(6,4), D(3,4)$.
1)Classify the points given below as,
a) Points on $x$ axis
b) Points on y axis
c) Points on the line parallel to the x axis
d) Points on the line parallel to the y axis
e) Non axis points
f) Origin

$$
\begin{equation*}
(0,5),(2,3),(-2,0),(7,3),(2,-4),(-1,3),(0,1),(4,0),(0,0) \tag{6}
\end{equation*}
$$

- The $y$-co-ordinates of the points on the line parallel to the $x$ axis are equal . $\left(x_{1}, y\right)\left(x_{2}, y\right)$
- The $x$ co ordinates of the points on the line parallel to the $y$ axis are equal . $\left(x, y_{1}\right)\left(x, y_{2}\right)$

2) Write the co-ordinates of the corners of the square $A B C D$ ?



## See the picture and try to learn the ideas

$B$ is 3 right and 6 above $A$
Slope of $A B=\frac{6}{3}$
Length of $A B=\sqrt{3^{2}+6^{2}}$
Midpoint $A B=\left(\frac{2+5}{2}, \frac{3+9}{2}\right)$
$C(4,7)$ divides the line $A B$ in the ratio $2: 1$
3) Without drawing co ordinate axes, Mark each pair of points below with right-left, top-bottom position correct.
$A(3,2), B(4,-3), C(-1,4), D(-4,-5)$
The distance between the points $(0,0)(x, y)$ is $\sqrt{x^{2}+y^{2}}$
4) Find the distance between origin and the point $(3,4)$ ?

The midpoint of the line joining the points $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)$ and $\left(\mathrm{x}_{2}, \mathrm{y}_{2}\right)$ is $\left(\frac{x_{1}+x_{2}}{2}, \frac{y_{1}+y_{2}}{2}\right)$
5) Find the co ordinates of the midpoint of the line joining the points $(2,3)$ and $(6,7)$
6)
a)Mark the points $\mathrm{A}(1,4), \mathrm{B}(4,10)$
b)Find the co ordinates of the midpoint of the line $A B$
c) What are the co ordinates of the point which divides the line $A B$ in the ratio $2: 1$ ?

If a pair of opposite vertices of a parallelogram are $\left(x_{1}, y_{1}\right)$ and $\left(x_{2}, y_{2}\right)$ and the other pair of opposite vertices are $\left(x_{3}, y_{3}\right),\left(x_{4}, y_{4}\right)$ then $x_{1}+x_{2}=$ $\mathbf{x}_{3}+\mathbf{x}_{4}, \mathbf{y}_{1}+\mathbf{y}_{2}=\mathbf{y}_{3}+\mathbf{y}_{4}$.
7) What are the co ordinates of the fourth (

In any line not parallel to either axis , the change in y is proportional to the change in $x$.
8) Find the value of $k$, If the points $(5,4), B(8,7) C(k, 16)$ are on the same line?

The $y$ co ordinates of the points on the line parallel to the $x$ axis are equal. (x1, y) (x2, y)

The $x$ co ordinates of the points on the line parallel to the $y$ axis are equal. ( $x, y 1$ ) ( $x, y 2$ )

The distance between the points with co ordinates $\left(x_{1}, y\right)\left(x_{2}, y\right)$ is $\left|x_{1}-x_{2}\right|$ The distance between the points with co ordinates $\left(x, y_{1}\right)\left(x, y_{2}\right)$ is $\left|y_{1}-y_{2}\right|$ Area of a rectangle $=$ Length $x$ Breadth
9) The rectangle below has sides parallel to the axes
(a) Find the co ordinates of the points $B, D$
(b) Find the length and breadth of the rectangle ?
(c) What is the area of $A B C D$ ?


- For any two points on the line joining $\left(x_{1}, y_{1}\right)$ and $\left(x_{2}, y_{2}\right)$ Slope $=\frac{y 2-y 1}{x 2-x 1}$
- In any line not parallel to either axis, the change in $y$ is proportional to the change in $x$.

10) If $A(3,5)$ and $B(7,9)$ are the two points on a line then
a) What is the slope of the line $A B$ ?
b) Write the co ordinates of any two points on this line?

In a point ( $x, y$ )

- The $x$ coordinate denotes the distance of the point from the $y$ axis
- The $y$ coordinate denotes the distance of the point from the $x$ axis
- The distance between $(0,0)$ and $(x, y)$ is $\sqrt{x^{2}+y^{2}}$
- Area of a parallelogram $=b \times h$

11) In rhombus $O A B C$, $O$ is the origin and $A$ is $(5,0)$, then
(a)What is the perpendicular distance from the point C to OA?
(b)What is the length of OC?
(c) What is the area of the rhombus OABC ?

$(2+1+1)$

- Area of a triangle $=\frac{1}{2} \mathrm{~b} \times \mathrm{h}$
- The distance between the points $\left.\left(x_{1} 0\right)\right)\left(x_{2}, 0\right)$ on the $x$ axis is $\left|x_{1}-x_{2}\right|$
- The distance between the points $\left.\left(0, y_{1}\right)\right)\left(0, y_{2}\right)$ on $y$ axis is $\left|y_{1}-y_{2}\right|$
- The $x$ coordinate of a point on $y$ axis is 0

12) In the figure $O$ is the origin and area of the triangle is $12 \mathrm{~cm}^{2}$.
a) Write the length of $A B$ ?
b) Write the length of $O C$ ?
c) write the co ordinates of the point $C$ ?
$(1+2+1)$


In ( $x, y$ ) The $x$ co ordinate denotes, the distance of the point from the $y$ axis The $y$ co ordinate denotes, the distance of the point from the $x$ axis

The angles of an isosceles right triangle are $45^{\circ}, 45^{\circ}, 90^{\circ}$

The distance between $(0,0)$ and $(x, y)$ is $\sqrt{x^{2}+y^{2}}$
Area of a triangle is $\frac{1}{2} \mathbf{b} \times \mathbf{h}$
In any line not parallel to either axis, the change in $y$ is proportional to the change in $x$.

The equation of a line joining the points $\left(x_{1}, y_{1}\right)\left(x_{2}, y_{2}\right)$
is $\frac{y-y_{1}}{x-x_{1}}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$
The general form of a line is $a x+b y+c=0$

13) See the figure,
(a) What are the co ordinates of $A$ ?
(b) What is $\angle A O B$ ?
(c) What is the length of OA ?
(d) What is the area of the triangle $A O B$ ?
(e) Write the co ordinates of any two points on the line OA?
(f) Write the equation of the line?

- If ( $x, y$ ) is a point on a circle with origin as its centre, then $r=\sqrt{ }\left(x^{2}+y^{2}\right)$
- If we draw a circle with origin as its centre and radius $r$,
- it cuts the $x$ axis at ( $\mathbf{r}, 0$ ) and ( $-\mathbf{r}, 0$ )
- If we draw a circle with origin as its centre and radius $r$,
- it cuts the y axis at ( $0, r$ ) and ( $0,-r$ ).
- If ( $x, y$ ) is a point on a circle with origin as its centre then
- ( $-\mathbf{x}, \mathrm{y}$ ), ( $-\mathrm{x},-\mathrm{y}$ ), ( $\mathrm{x},-\mathrm{y}$ ) are also points on it.
- The equation of the circle with origin as its centre is $\mathbf{x}^{\mathbf{2}}+\mathrm{y}^{\mathbf{2}}=\mathbf{r}^{2}$

14) A circle is drawn with origin as its centre.
(a)What is the radius of the circle?
(b)Find the co ordinates of the points, where the circle cuts the axes?
(c)Find 8 other points on the circle also write the equation of the circle?

$(2+2+4)$

- Area of a triangle $=\frac{1}{2} \mathbf{b} \times \mathbf{h}$
- The $y$ - co-ordinates of the points on the line parallel to the $x$ axis are equal . ( $\mathrm{x} 1, \mathrm{y}$ ) $(\mathrm{x} 2, \mathrm{y})$
- The areas of the triangles between two parallel lines with the same base are equal

15) 

a) Mark the points $A(-3,1) \quad B(5,1) \quad C(1,5)$ by drawing the axes and join them to form a triangle .
b) What is the area of the triangle ?
c) Draw a line through the opposite vertex of the triangle parallel to the base and mark two points on it? Draw another triangle of equal area?

- Tangent at point on a circle is perpendicular to the radius through that point
- The ratio of the sides of a triangle with angles $45^{\circ} 45^{\circ} 90^{\circ}$ is $1: 1: \sqrt{ } 2$
- The y co ordinates of the points on $x$ axis is 0 : $\left.\left(x_{1}, 0\right)\right)\left(x_{2}, 0\right)$
- The $x$ co ordinates of the points on $y$ axis is 0 : $\left(0, y_{1}\right)\left(0, y_{2}\right)$
- The length of the line joining the points $\left(x_{1}, y_{1}\right)\left(x_{2}, y_{2}\right)$ is $\sqrt{ }\left[\left(\mathbf{x}_{1}-\mathrm{x}_{2}\right)^{2}+\left(\mathrm{y}_{1}-\mathrm{y}_{2}\right)^{2}\right]$

16) a) Draw the co ordinate axes and draw a circle of radius 4 cm with centre as its origin.
b) Draw a radius OP at an angle of $45^{\circ}$ with the x -axis and draw a tangent through P .
c) If the tangent cuts the axes at the points $A, B$,then write the co ordinates of the points and also find the length of $A B$ ?

## ANSWERS

1)a) $(-2,0)(4,0)(0,0)$
12)
b) $(0,5)(0,1)(0,0)$
a) 6
c) $(2,3)(7,3)$
b) 4
d) $(2,3)(2,-4)$
e) $(2,3)(7,3)(2,-4)(-1,-3)$
f) $(0,0)$
2) $A(-2,-2) B(2,-2) C(2,2) D(-2,2)$
3)
$\mathrm{A}(3,2): 3$ right- 2 up
$\mathrm{B}(4,-3): 4$ right 3 down
C(-1,4):1 left- 4 down
D(-4,-5):4 left- 5 down
4) 5
5) $(4,5)$
6)
a) $\left(2 \frac{1}{2}, 7\right)$
b) $(3,8)$
7) $(13,6)$
8) $\mathrm{k}=17$
9)
a) $B(6,5) D(2,7)$
b) length $=4$, breadth $=2$
c) Area=8 cm ${ }^{2}$
10)
a) Slope $=4 / 4=1$
b) $(8,10)(9,11)$
11)
a) 4
b) 5
c) $\mathrm{b} \times \mathrm{h}=4 \times 5=20$
c) $(0,4)$
13)
a) $(3,3)$
b) $45^{\circ}$
c) $3 \sqrt{ } 2$
d) $9 / 2$
e) $(4,4)(5,5)$
f) $y=x$
14)
a)Darw circle through $(3,4)$ with centre as origin
b) 5

Point on x axis are $(5,0)(-5,0)$
points on $y$ axis are $(0,5)(0,-5)$
c) $x^{2}+y^{2}=25$

SET 1: $(3,4)(3-4)(-3,4)(-3,-4)$
SET 2:(4,3)(4,-3)(-4,3)(-4,-3)
15)a) pricture
b)
$\mathrm{AB}=8, \mathrm{~h}=4$
Area of the triangle $=16 \mathrm{~cm}^{2}$
c) $\mathrm{P}(1,5) \mathrm{Q}(6,5)$
$\Delta \mathrm{ABP} / \triangle \mathrm{ABQ}$ are the traingles of equal area to the area of $\Delta \mathrm{ABC}$
16)
a) Draw a circle with centre $(0,0)$ and radius 4 cm .
b) Draw the tangent at $P$
c) $(4 \sqrt{ } 2,0)(0,4 \sqrt{ } 2)$

A,B can be in other 3 ways as given below.
$(0,4 \sqrt{ } 2)(-4 \sqrt{ } 2,0)$
$(-4 \sqrt{ } 2,0)(0,-4 \sqrt{ } 2)$
$(0,-4 \sqrt{ } 2)(4 \sqrt{ } 2,0)$

## 7 TANGENTS

## "The tangent at a point on a circle is perpendicular to the diameter through that point."



1) $O$ is the centre of the circle with radius $4 \mathrm{~cm} . \mathrm{PA}, \mathrm{PB}$ are the tangents from $P$.
If $\angle A P B=90^{\circ}$ then find the lengths of $P A, P B$

2) Draw a circle of radius 2 cm then mark a point $P$ on the circle then draw a tangent through $P$.
3) $O$ is the circumcentre of triangle $A B C$.
i) If $\angle \mathrm{C}=40^{\circ}, \angle \mathrm{POQ}=\ldots . . .$. .
ii) If $\angle \mathrm{POR}=120^{\circ}, \angle \mathrm{A}=$.
iii) Draw a circle of radius 3 cm then draw a triangle having two angles $50^{\circ}, 60^{\circ}$ whose sides touches the circle.
(4)

"In a circle,the angle which a chord makes with the tangent at one end on any side is equal to the angle which it makes on the part of the circle on the other side. "


4 ) In the figure O is the centre of the circle, In triangle $\mathrm{ABC} \angle \mathrm{C}=70^{\circ}$. $\mathrm{PA}, \mathrm{PB}$ are the tangents to the circle. Then
i) $\angle \mathrm{AOB}=\ldots \ldots$.
ii) In triangle $\mathrm{AOB}, \angle \mathrm{BAO}=\angle \mathrm{ABO}=$
iii) $\angle A B P=\angle B A P=\ldots .$.
iv) What is the relation betwen $\angle C$ and the angles made by chord AB with tangents $\mathrm{PA}, \mathrm{PB}$.

"We can draw two equal tangents from a point which is outside the circle ."
In a circle, the angles between the radii through two
points and the angle between the tangents at these points are supplementary."

$\angle A O B+\angle A P B=180^{\circ}$
5) $\mathrm{PA}, \mathrm{PB}$ are the tangents to the circle with centre $O$. OA is the radius of the circle.
i) $\angle \mathrm{OAP}=\angle \mathrm{OBP}=\ldots \ldots$

What is the relation between the lengths of $\mathrm{PA}, \mathrm{PB}$ ?
ii) $\angle \mathrm{OAP}+\angle \mathrm{OBP}=\ldots$....
$\angle A O B+\angle A P B=\ldots .$.
iii) Draw a circle of radius 4 cm , then mark a point 9 cm away from the centre then draw two tangents from that point to the circle and measure the lengths.

iv) What is the shortest distance from the centre of a circleto point from where we can draw two tangents?

## Chord and tangent.



## $P A \times P B=P C^{2}$

6) The tangent through $C$ and the chord $A B$ extended intersects at $P$
i) Write the relation between PA, PB, and PC (1)
ii) $\mathrm{PB}=9 \mathrm{~cm}, \mathrm{PC}=6 \mathrm{~cm}$ then $\mathrm{PA}=$. $\qquad$
(1)
iii) What is the area of the rectangle whose sides are PA, PB? What is the area of the
 square whose side is PC ?
(4)

## Construction of incircle.

7) ME, MF are two tangents meet at $M, O$ is the centre of the circles touches these two lines.
i) What is the measures of $\angle \mathrm{MEO}, \angle \mathrm{MFO}$ ?
ii) If $\angle O M E=x^{\circ}$ then $\angle O M F=$
iii) Draw a triangle with sides $6 \mathrm{~cm}, 7 \mathrm{~cm}, 8 \mathrm{~cm}$ then draw a circle which touches the sides of this triangle.

(5)

Perimeter, Area and Inradius of a triangle If half of the perimeter of a triangle is $s$ and area $A$, inradius $r$ o. then $r=A / s$

$r=A / s$
8) The inradius of triangle with sides $5 \mathrm{~cm}, 6 \mathrm{~cm}, 7 \mathrm{~cm}$ is 2.5 cm .
i) Find the perimeter of the triangle?
ii) Find the area of the triangle?
iii) If the perpendicular sides of a triangle are $6 \mathrm{~cm}, 8 \mathrm{~cm}$ then find its inradius.

9) Three sides of a triangle are $6 \mathrm{~cm}, 7 \mathrm{~cm}, 9 \mathrm{~cm}$. Find the lengths of $A P, B R, C Q$.
(6)


## TANGENTS: Answers

1) 4 cm
2) Draw the figure

3) i) $\angle \mathrm{POQ}=140^{\circ}$
ii) $\angle A=60^{\circ}$
iii) Draw a circle of radius 3 cm , Draw the radius, Then measure $130^{\circ}, 120^{\circ}, 110^{\circ}$ at the centre and mark it then draw the radii which cuts the circle in the above measures. Then draw the tangents through that three points which intersects at three different points to complete a triangle.
4) i) $\angle \mathrm{AOB}=140^{\circ}$
ii) $\angle A B O=20^{\circ}$
iii) $\angle \mathrm{ABP}=\angle \mathrm{BAP}=70^{\circ}$
iv) equal
5) i) $\angle \mathrm{OAP}=\angle \mathrm{OBP}=90^{\circ}$ $P A=P B$
ii) $\angle \mathrm{OAP}+\angle \mathrm{OBP}=180^{\circ}$
$\angle A O B+\angle A P B=180^{\circ}$
iii)Draw a circle of radius 4 cm with centre $O$. Mark a point ' $P$ ', 9 cm away from the centre. Draw the perpendicular bisector of OP then draw a circle with radius half of which intersects the first circle at $A, B$. Then join $P A, P B$ and measue the lengths.
iv) Distance from centre has to be more than the radius.
6) i) $\mathrm{PA} \times P B=P C^{2}$
ii) $P A=4$
iii) $A B=5$
iv) Area of the rectangle $=36 \mathrm{~cm}^{2}$

Area of the square $=36 \mathrm{~cm}^{2}$
Areas are equal
7) i) $\angle \mathrm{MEO}=\angle \mathrm{MFO}=90^{\circ}$
ii) Equal right triangles.
iii) $\angle O M F=x^{\circ}$
iv) Draw the figure and measure the radius.

Draw the triangle first. Then draw the bisectors of two angles , then take the point which the two bisectors intersect as the centre of the incircle. Then draw the circle which touches the sides of the triangle. Then measure the radius.

8) i) Perimeter $=18 \mathrm{~cm}$
ii) Area $A=r \times s=2.5 \times 9=22.5 \mathrm{~cm}^{2}$
iii) Third side $=10 \mathrm{~cm}$

Area $=24 \mathrm{~cm}^{2}$
Perimeter $=24 \mathrm{~cm}$
radius $r=A / s=24 / 12=2 \mathrm{~cm}$
9) Permeter $=9+7+6=22$
$s=22 / 2=11$
$\mathrm{AP}=\mathrm{s}-\mathrm{a}=11-6=5$
$B R=s-b=11-7=4$
$C Q=s-c=11-9=2$

## 8 SOLIDS <br> CONE, SQUARE PYRAMID


$r: l=x: 360$
$h^{2}+r^{2}=l^{2}$
Lateral area of a cone = Area of the sector used to make that cone
Volume of a cone is one third of the product of its base area and height

1) Base radius is 12 cm , slant height 25 cm , Find the lateral area of the cone?
(lateral area $=\pi \times 12 \times 25$ )
2) Base diameter is 60 cm , height 40 cm , Find the total surface area of the cone?
$\left(r=30 \mathrm{~cm}, \mathrm{I}=\sqrt{30^{2}+40^{2}}=50\right.$ Total area $\left.=\pi 30 \times 50+\pi \times 30^{2}\right)$
3) Having radius 12 cm and center angle $120^{\circ}$ is bent into form a cone.
a) What is the slant height of the cone ?
b) Find the base radius of the cone.
c) Find the lateral area of the cone.
(a) 12 cm
b) $\frac{r}{12}=\frac{120}{360} \quad r=4 \mathrm{~cm}$
c) Lateral area $=\Pi \times 4 \times 12$ )
4) Base radius of the cone is 12 cm , height is 6 cm
a) Find the volume of the cone
b) Ratio of the base radii of two cones is $2: 3$, ratio of their heights is $3: 2$. Find the ratio of their volumes.
[a) Volume $=\frac{1}{3} \times \pi \times 12^{2} \times 6$
b) $2^{2} \times 3: 3^{2} \times 2$ ]
5) A cone having slant height 17 cm , and base radius 8 cm is melted and recast into small cones having height 3 cm and base radius 2 cm .
a) Find the height of the large cone.
b) Find the volume of the large cone
c) How many small cones can be made?
( a) Height $=\sqrt{17^{2}-8^{2}}=15$
b) Volume $=\frac{1}{3} 8^{2} \times 15$
c) $\frac{8^{2} \times 15}{2^{2} \times 3}$ )
6) A square pyramid is made from a square sheet of paper as shown in the figure. Base edge of the square pyramid is 10 cm . Lateral edge is 13 cm .
a) Find the slant height of the cone ?
b) What is the length of one side of the paper used?
c) Find the total surface area of the square pyramid ?
( $2+1+1$ )
a) slant height $=\sqrt{13^{2}+5^{2}}=12$
b) $13+10+13=36 \mathrm{~cm}$
c) Total area $=2 \times 10 \times 12+10^{2}$

## SPHERE

7) Radius of a sphere is 10 cm .
a) Find the surface area of the sphere.
b) Find the volume of the sphere.
c) What is the area of the hemisphere having same radius ?
d) Find the ratio of the areas of the sphere and hemisphere having same radius .
( a) Area of the sphere $=4 \Pi \times 10^{2}$
b) Volume $=\frac{4}{3} \Pi \times 10^{3}$
c) Area of the hemisphere $=3 \Pi \times 10^{2}$
d) $4: 3$
8) a) One side of a cube is 24 cm , Find the height and base radius of the square pyramid which can be carved from the cube.
b) Find the volume and area of the square pyramid.
c) What is the radius of the sphere that can be carved from a cube of side 24 cm having maximum volume ?
( a) height $=24 \mathrm{~cm}$, base edge $=24 \mathrm{~cm}$.
b) Volume $=\frac{1}{3} 24^{2} \times 24$

Slant height $=\sqrt{24^{2}+12^{2}}=\sqrt{720}=12 \sqrt{5}$
Area $=2 \times 24 \times 12 \sqrt{5}+24^{2}$
c) radius $=12 \mathrm{~cm}$
9) A hemisphere is attached to one end of a cone having radius 9 cm . Total height of the shape is 21 cm .
a) Find the height of the cone
b) Find the total volume of the shape?
(a) $21-9=12 \mathrm{~cm}$
b) Total volume of the shape $=\frac{1}{3} \Pi \times 9^{2} \times 12+\frac{2}{3} \Pi 9^{3}$

## 10 POLYNOMIALS

## Concepts

- If $p(x)=(x-a)(x-b)(x-c)$, then the factors of $p(x)$ are $x-a, x-b, x-c$.
- The solution of $(x-a)(x-b)(x-c)=0$ are , $a, b, c$
- In a polynomial $p(x)$, if $p(a)=0$, then $x-a$ is a factor of it.
- In a polynomial $p(x)$, if $p(-a)=0$, then $x+a$ is a factor of it.
- If $x-a$ is a factor of $p(x)$, then $p(a)=0$.
- If $x+a$ is a factor of $p(x)$, then $p(-a)=0$.
- If $x=a$ is a solution of $p(x)=0$, then $x-a$ is a factor of it.
- If $x=-a$ is a solution of $p(x)=0$, then $x+a$ is a factor of it.
- For any polynomial $p(x)$, and a number a, $p(x)-p(a)$ is a multiple of $x-a$



## 1 Mark questions

1. If $p(x)=x(x-3)$, find $p(4)$.
2. If $p(x)=x^{2}-7 x-8$, find $p(-1)$.
3. If $p(x)=(x+1)(x-2)$, find $p(2)$.
4. If $\mathrm{p}(\mathrm{x})$ a polynomial and $\mathrm{p}(-3)=0$, then Write a factor of $\mathrm{p}(\mathrm{x})$.
5. If $\mathrm{x}+2$ is a factor of $\mathrm{q}(\mathrm{x})$, find $\mathrm{q}(-2)$.
6. If $x^{2}+8 x-105=(x+15)(x+a)$, find the value of $a$.
7. sum of two numbers is 17 , their product is 70 , Guess the numbers.
8. In a polynomial $\mathrm{p}(\mathrm{x})$, and for a number a, write one factor of $\mathrm{p}(\mathrm{x})-\mathrm{p}(\mathrm{a})$.
9. If $p(x)$ is a polynomial , then $p(x)-p(3)$ is a multiple of $\qquad$
10. If $x^{2}+18 x-100=(x+a)(x+b)$, find $a+b$
11. $p(x)$ is a polynomial and $p(-3)=0, p(2)=0, p(-1)=0$. Which of the following is not a factor $\{x+3, x+2, x-2, x+1\}$
12. If $\mathrm{x}-3, \mathrm{x}+4, \mathrm{x}-7$ are factors of $\mathrm{p}(\mathrm{x})$, Which of the following is a wrong statement? $\{P(3)=0, p(-4)=0, p(-3)=0, p(7)=0\}$

## $\underline{2}$ Mark questions

13. Given $p(x)=4 x^{2}+x-3$
a) Find $p(-1)$
b) Is $x+1$ a factor of $p(x)$ ?
14. Given $p(x)=2 x^{2}-5 x-3$
a) Find $p(3)$.
b) Is $x-3$ a factor of $p(x)$ ?

## 3 Mark questions

15. If $p(x)=2 x^{2}+k x+3$.
a) Find $p$ (3).
b) If $x-3$ is a factor of $p(x)$, Find the value of $k$.
16. $x-2$ is a factor of $x^{3}-7 x+k$.
a) Find $p(2)$.
b) Find the value of $k$.
17. $P(x)$ is a polynomial such that $p(3)=0, p(-2)=0$.
a) Write two factors of $p(x)$.
b) Write $p(x)$.
18. $P(x)$ is a polynomial such that $p(\sqrt{ } 2)=0, p(-\sqrt{ } 2)=0$.
a) Write two factors of $p(x)$.
b) Write $\mathrm{p}(\mathrm{x})$.

## 4 Mark questions

19. $p(x)=x^{3}+a x^{2}-10 x+b$, Also $x+1, x-4$ are its factors.
a) Find $p(-1), p(4)$
b) Find the values of $a, b$.
20. a) Find the solution of the equation $x^{2}+6 x+8=0$ using formula.
b) Hence write $x^{2}+6 x+8$ as a product of two first degree polynomials.
21. a) Find the solution of the equation $x^{2}+5 x-24=0$.
b) Hence write $x^{2}+5 x-24$ as a product of two first degree polynomials.

## 6 Mark questions

22. If $p(x)=x^{2}-7 x+5$
a) Find $p(2)$.
b) Find $p(x)-p(2)$.
c) Write one factor of $p(x)-p(2)$.
d) Find the other factor also.
23. a) If $x^{2}+8 x+15=(x+a)(x+b)$. write the value of $a+b, a b$.
b) Hence write $x^{2}+8 x+15$ as a product of two first degree polynomials.
c) Hence solve the equation $x^{2}+8 x+15=0$.
24. a) Write $x^{2}+3 x-40$ as a product of two first degree polynomials.
b) Hence solve the equation $x^{2}+3 x-40=0$.

## Answers

1. -4
2. 0
3. 0
4. $x+3$
5. 0
6. 7
7. 10,7
8. $\mathrm{x}-\mathrm{a}$
9. $\mathrm{x}-3$
10. 18
11. $\mathrm{x}+2$
12. $p(-3)=0$
13. a) 11 , b) No
14. a) 0 , b) Yes
15. a) $3 k+21$, b) -7
16. a) k-6 , b) 6
17. a) $x-3, x+2, \quad$ b) $p(x)=(x-3)(x+2)=x^{2}-x-6$
18. a) $x-\sqrt{2}, x+\sqrt{2}$
b) $p(x)=x^{2}-2$
19. a) $p(-1)=a+b+9, p(4)=16 a+b+24$
b) $a=-1, b=-8$
20. a) $x=-2, x=-4, \quad$ b) $(x+2)(x+4)$
21. a) $x=-8, x=+3$,
b) $(x+8)(x-3)$
22. a) 4 , b) $x^{2}-7 x+10$, c) $x-2$ d) $x-5$
23. a) 8,15 , b) $(x+5)(x+3)$, c) $-5,-3$
24. a) $(x+8)(x-5)$, b) $x=-8,5$

## 11 STATISTICS

1. The marks obtained by five children are given below. 26, 21, 36, 32, 45
a) Find Arithmetic mean of these marks.
b) Find the Median marks of the children.
2. The marks obtained by six children are given below.
$26,21,37,33,45,48$
a) Find Arithmetic mean of these marks.
b) Find the Median marks of the children.
3. The table below shows the weight of children in a class.

| Weight (in kg) | Number of <br> children |
| :---: | :---: |
| 40 | 5 |
| 44 | 6 |
| 48 | 9 |
| 52 | 12 |
| 56 | 8 |
| 60 | 5 |
| 64 | 4 |

a) If children are placed in ascending order based on weight what is the position of child whose weight is the median?
b) Find the median weight.

| Weight (in kg) | Number of <br> children | Number of <br> children |
| :---: | :---: | :---: |
| 40 | 5 | Up to $40-5$ |
| 44 | 6 | Up to $44-11$ |
| 48 | 9 | Up to 48-20 |
| 52 | 12 | Up to 52-32 |
| 56 | 8 | Up to 56-40 |
| 60 | 5 | Up to 60 - 45 |
| 64 | 4 | Up to 64-49 |
| Total | 49 |  |

a) The median weight will be the weight of $25^{\text {th }}$ child.
b) The weight of the $21^{\text {st }}$ child to the $32^{\text {nd }}$ child is 52 according to the table. Then the weight of the $25^{\text {th }}$ child is also 52 , then the median weight is 52 kg .
4. The table below shows the 45 workers sorted according to their daily wage.

| Daily Wage (Rs) | Number of workers |
| :--- | :---: |
| $400-500$ | 8 |
| $500-600$ | 11 |
| $600-700$ | 10 |
| $700-800$ | 7 |
| $800-900$ | 9 |
| Total | 45 |

a) Workers are arranged in ascending order based on their wages, what is the position of worker whose wage is considered to be the median?
b) What is the wage of the $20^{\text {th }}$ worker?
c) Calculate the median wage.

| Daily Wage (Rs) | Number of <br> workers | Number of workers |
| :--- | :---: | :--- |
| $400-500$ | 8 | Up to $500-8$ |
| $500-600$ | 11 | Up to $600-19$ |
| $600-700$ | 10 | Up to $700-29$ |
| $700-800$ | 7 | Up to $800-36$ |
| $800-900$ | 9 | Up to $900-45$ |
| Total | 45 |  |

a) The median wage will be the wage of $23^{\text {rd }}$ person.
b) According to the table, the wage of the $23^{\text {rd }}$ person will be between 600 and 700 rupees. If 10 people from $20^{\text {th }}$ to $29^{\text {th }}$ are arranged in ascending order at equal intervals between 600 and 700, each will be at a difference of $\frac{700-600}{10}=10$ rupees. Then the $20^{\text {th }}$ person's wage will be between 600 and 610 that is 605 .

c) Median wage $=\frac{630+640}{2}=635$

If we use the method of arithmetic sequence
$20^{\text {th }}$ term $=605$
$23^{\text {rd }}$ term $=605+3 \times 10=635$

