Assignments in Mathematics Class X (Term II)

13. SURFACE AREAS AND VOLUMES

IMPORTANT TERMS, DEFINITIONS AND RESULTS

- **Cuboid :**

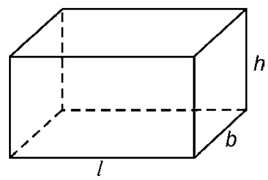
  (*a*) Lateral surface area $= 2h(l + b)$

  (*b*) Surface area $= 2(lb + bh + lh)$

  (*c*) Volume $= lbh$

  (*d*) Length of diagonal $= \sqrt{l^2 + b^2 + h^2}$

  where l, b, h are length, breadth and thickness of the cuboid.



- **Cube :**

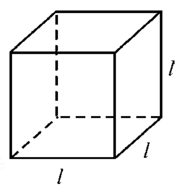
  (*a*) Lateral surface area $= 4l^2$

  (*b*) Surface area $= 6l^2$

  (*c*) Volume $= l^3$

  (*d*) Length of diagonal $= \sqrt{3}l.$

  where, l is the edge of the cube.



- **Cylinder :** r = radius, h = height

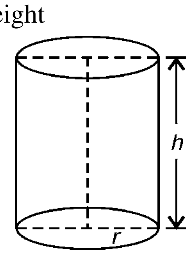
  (*a*) Area of curved surface $= 2\pi rh$

  (*b*) Total surface area $=2\pi r^2 + 2\pi rh = 2\pi r(r + h)$

  (*c*) Volume $= \pi r^2 h$

  (*d*) Curved surface area of hollow cylinder $= 2\pi h(\text{R} + r)$

  e)(Total surface area of hollow cylinder $= 2\pi h\ (\text{R} + r) + 2\pi(\text{R}^2 - r^2)$

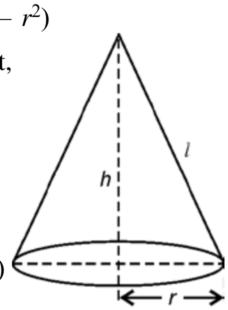


- **Cone :** r = radius, h = height, l = slant height.

  (*a*) Curved surface area $= \pi rl = \pi r\sqrt{h^2 + r^2}$

  (*b*) Total surface area $= \pi r^2 + \pi rl = \pi r\ (r + l)$

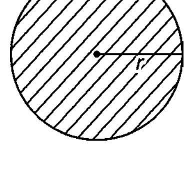
  (*c*) Volume $= \frac{1}{3}\pi r^2 h$



- **Sphere :** r = radius

  (*a*) Surface area $= 4\pi r^2$

  (*b*) Volume $= \frac{4}{3}\pi r^3$

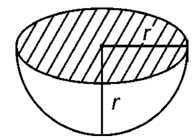


- **Hemisphere (solid) :** r = radius

  (*a*) Curved surface area $= 2\pi r^2$

  (*b*) Total surface area $= 3\pi r^2$

  (*c*) Volume $= \frac{2}{3}\pi r^3$



- **Spherical Shell :**

  Outer radius = R,

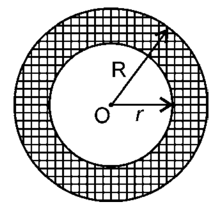
  Inner radius = r

  (*a*) Surface area (outer) $= 4\pi\text{R}^2$

  (*b*) Surface area (inner)$= 4\pi r^2$

  (*c*) Volume of the material $= \frac{4}{3}\pi(\text{R}^3 - r^3)$

  $= \frac{4}{3}(\text{R} - r)(\text{R}^2 + \text{R}r + r^2)$



- When a cone is cut by a plane parallel to the base of the cone, then the portion between the plane and the base is called the frustum of the cone.

  (*a*) Slant height of the frustum,

  $l = \sqrt{h^2 + (\text{R} - r)^2}$

  (*b*) Volume of the frustum of the cone

  $= \frac{\pi h}{3}\left[\text{R}^2 + r^2 + \text{R}r\right]$

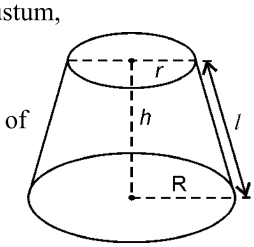
  (*c*) Lateral surface area of the frustum of the cone $= \pi l(\text{R} + r)$

  (*d*) Total surface area of the frustum of the cone

  = (area of the base) + (area of the top) + (lateral surface area)

  $= [\pi\text{R}^2 + \pi r^2 + \pi l(\text{R} + r)]$

  $= \pi[\text{R}^2 + r^2 + l\ (\text{R} + r)].$



SUMMATIVE ASSESSMENT

MULTIPLE CHOICE QUESTIONS [1 Mark]

A. Important Questions

1. Diagonal of a cube of edge a is :

(a) $\sqrt{3} \times a$ units (b) $\frac{\sqrt{3}}{2} \times a$ units

(c) $\frac{2}{\sqrt{3}} a$ units (d) none of these

2. If each edge of a cube is increased by 50%, the percentage increase in the surface area is :

(a) 25% (b) 50% (c) 75% (d) 125%

3. A *surahi* is the combination of :

(a) a sphere and a cylinder
(b) a hemisphere and a cylinder
(c) two hemispheres
(d) a cylinder and a cone

4. A funnel is combination of :

(a) a cone and a cylinder
(b) frustum of a cone and a cylinder
(c) a hemisphere and a cylinder
(d) a hemisphere and a cone

5. The shape of a bucket is usually in the form of:

(a) a cone (b) frustum of a cone

(c) a cylinder (d) a sphere

6. A flask used in the laboratory is the combination of :

(a) a cylinder and a cone
(b) a sphere and a cone
(c) a sphere and a cylinder
(d) frustum of a cone and a sphere

7. Volume of material of a hollow right circular cylinder of height h and external and internal radii R and r respectively is :

(a) $\pi h(R^2 - r^2)$ (b) $\pi^2 rh(R^2 - r^2)$

(c) $\pi r^2 h(R^2 + r^2)$ (d) $\pi Rh(R^2 - r^2)$

8. The ratio of the volumes of two spheres is 8 : 27. The ratio between their surface areas is :

(a) 2 : 3 (b) 4 : 27 (c) 8 : 9 (d) 4 : 9

9. On increasing each of the radius of the base and the height of a cone by 20%, its volume will be increased by :

(a) 25% (b) 40% (c) 50% (d) 72.8%

10. The curved surface area of a cylinder is 264 m^2 and its volume is 924 m^3. The height of the pillar is :

(a) 3 m (b) 4 m (c) 6 m (d) 8 m

11. A cylindrical vessel of radius 5 cm and height 6 cm is full of water. A marble of radius 2.1 cm is put into the vessel. The volume of water which flows out of the vessel is :

(a) 471.4 cm^3 (b) 19.4 cm^3

(c) 55.4 cm^3 (d) 38.8 cm^3

12. If two solid hemispheres of same base radius r are joined together along their bases, then curved surface area of the new solid is :

(a) $4\pi r^2$ (b) $6\pi r^2$ (c) $3\pi r^2$ (d) $8\pi r^2$

13. A right circular cylinder of radius r cm and height h ($h > 2r$) just encloses a sphere of diameter :

(a) r cm (b) $2r$ cm

(c) h cm (d) $2h$ cm

14. Volumes of two spheres are in the ratio 27 : 64. The ratio of their surface areas is :

(a) 3 : 4 (b) 4 : 3

(c) 9 : 16 (d) 16 : 9

15. In a right circular cone, the cross section made by a plane parallel to the base is a :

(a) circle (b) frustum of a cone

(c) sphere (d) hemisphere

16. The total surface area of a hemisphere of radius 7 cm is :

(a) 447π cm^2 (b) 239π cm^2

(c) 147π cm^2 (d) 174π cm^2

17. The height of a conical tent is 14 m and its floor area is 346.5 m^2. The length of 1.1 m wide canvas required to built the tent is :

(a) 490 m (b) 525 m

(c) 665 m (d) 860 m

18. The ratio of the total surface area to the lateral surface area of a cylinder with base diameter 160 cm and height 20 cm is :

(a) 1 : 2 (b) 2 : 1 (c) 3 : 1 (d) 5 : 1

19. The radius of the base of a cone is 5 cm and its height is 12 cm. Its curved surface area is :

(a) 30π cm^2 (b) 65π cm^2

(c) 80π cm^2 (d) none of these

20. The volume of the largest right circular cone that can be cut out from a cube of edge 4.2 cm is :

(a) 9.7 cm^3 (b) 58.2 cm^3

(c) 77.6 cm^3 (d) 19.4 cm^3

21. A solid piece of iron in the form of a cuboid of dimensions 49 cm × 33 cm × 24 cm is moulded to form a solid sphere. The radius of the sphere is :

(a) 21 cm (b) 23 cm

(c) 25 cm (d) 19 cm

22. A cubical icecream brick of edge 22 cm is to be distributed among some children by filling the ice cream cones of radius 2 cm and height 7 cm up to its brim. How many children will get the icecream cones?

(a) 163 (b) 263 (c) 363 (d) 463

23. The radii of two circular ends of a bucket are 12 cm and 22 cm. If the height of the bucket is 35 cm, the capacity of the bucket is :

(a) 32.7 litres (b) 33.7 litres

(c) 34.7 litres (d) 31.7 litres

24. The height of a cylinder is 14 cm and its curved surface area is 264 cm^2. The volume of the cylinder is:

(a) 296 cm^3 (b) 396 cm^3

(c) 369 cm^3 (d) 503 cm^3

25. The ratio between the radius of the base and the height of the cylinder is 2 : 3. If its volume is 1617 cm^3, the total surface area of the cylinder is:

(a) 208 cm^2 (b) 77 cm^2

(c) 707 cm^2 (d) 770 cm^2

26. The diameters of the ends of a frustum of a cone of height h are 2 R and 2r. The volume of the frustum of the cone is :

(a) $\frac{\pi h}{3}\left[R^2 + r^2 + Rr\right]$ (b) $\frac{\pi h}{3}\left[R^2 + r^2 - Rr\right]$

(c) $\frac{\pi h}{3}\left[R^2 - r^2 + Rr\right]$ (d) $\frac{\pi h}{3}\left[R^2 - r^2 - Rr\right]$

27. 12 solid spheres of the same size are made by melting a solid metallic cylinder of base radius 1 cm and height 16 cm. The radius of each sphere is :

(a) 2 cm (b) 3 cm (c) 4 cm (d) 5 cm

28. The curved surface area of the bucket whose top and bottom radii are 28 cm and 7 cm and slant height is 45 cm, is :

(a) 4950 cm^2 (b) 4951 cm^2

(c) 5921 cm^2 (d) 9450 cm^2

29. A cone of base diameter 8 cm is formed by melting a metallic spherical shell of internal and external radii 2 cm and 4 cm, respectively. The height of the cone is :

(a) 12 cm (b) 14 cm (c) 15 cm (d) 16 cm

30. The surface area of a sphere is 154 cm^2. The volume of the sphere is :

(a) $179\frac{2}{3}$ cm^3 (b) $359\frac{1}{2}$ cm^3

(c) $1215\frac{2}{3}$ cm^3 (d) $1374\frac{1}{3}$ cm^3

B. Questions From CBSE Examination Papers

1. If a cone is cut into two parts by a horizontal plane passing through the mid-points of its axis, the ratio of the volumes of the upper part and the cone is : **[2011 (T-II)]**

(a) 1 : 2 (b) 1 : 4 (c) 1 : 6 (d) 1 : 8

2. The curved surface area of a right circular cone of height 15 cm and base diameter 16 cm is : **[2011 (T-II)]**

(a) 60 π cm^2 (b) 68 π cm^3

(c) 120 π cm^2 (d) 136 π cm^2

3. A cone, a hemisphere and a cylinder stand on equal bases and have the same height. The ratio of their volumes is : **[2011 (T-II)]**

(a) 3 : 2 : 1 (b) 1 : 3 : 2

(c) 2 : 3 : 1 (d) 1 : 2 : 3

4. A child reshapes a cone made up of China clay of height 24 cm and radius of base 6 cm into a sphere. The radius of the sphere is : **[2011 (T-II)]**

(a) 24 cm (b) 12 cnm (c) 6 cm (d) 48 cm

5. A frustum of a right circular cone of height 16 cm with radii of its circular ends as 8 cm and 20 cm has its slant height equal to : **[2011 (T-II)]**

(a) 18 cm (b) 16 cm (c) 20 cm (d) 24 cm

6. A solid piece of iron in the form of a cuboid of dimensions 49 cm × 33 cm × 24 cm is moulded to form a solid sphere. The radius of the sphere is : **[2011 (T-II)]**

(a) 25 cm (b) 21 cm (c) 19 cm (d) 23 cm

7. The volume of a sphere (in cu. cm) is equal to its surface area (in sq. cm). The diameter of the sphere (in cm) is : **[2011 (T-II)]**

(a) 3 (b) 6 (c) 2 (d) 4

8. The volume of the largest right circular cone that can be cut out from a cube of edge 4.2 cm is : **[2011 (T-II)]**

(a) 9.7 cm^3 (b) 77.6 cm^3

(c) 58.2 cm^3 (d) 19.4 cm^3

9. A solid sphere of radius x cm is melted and cast into a shape of a solid cone of radius x cm. Then the height of the cone is : **[2011 (T-II)]**

(a) $3x$ cm (b) x cm (c) $4x$ cm (d) $2x$ cm

10. The radii of bases of a cylinder and a cone are in the ratio 3 : 4 and their heights are in the ratio 2 : 3. The ratio between the volume of cylinder to that of cone is : **[2011 (T-II)]**

(a) 8 : 9 (c) 9 : 8 (c) 5 : 7 (d) 7 : 5

11. A soild sphere of diameter 6 cm is melted and drawn into a wire of radius 4 mm. The length of wire is : **[2011 (T-II)]**

(a) 900 cm (b) 90 cm (c) 900 m (d) 225 cm

12. The number of cubes of side 2 cm which can be cut from a cube of side 6 cm is : **[2011 (T-II)]**

(a) 56 (b) 54 (c) 28 (d) 27

13. A shuttle cock used for playing badminton has the shape of the combination of : **[2011 (T-II)]**

(a) a cylinder and a sphere

(b) a sphere and a cone

(c) a cylinder and a hemisphere

(d) a hemisphere and frustum cone

14. A metallic cube of edge 1 cm is drawn into a wire of diameter 4 mm, then the length of the wire is : **[2011 (T-II)]**

(a) 100/ π cm (b) 100 π cm

(c) 25/ π cm (s) 10000 cm

15. The radii of the base of a cylinder and a cone of the same height are in the ratio 3 : 4. The ratio of their volumes is : **[2011 (T-II)]**

(a) 9 : 8 (b) 9 : 4 (c) 3 : 1 (d) 27 : 16

16. A garden roller has a circumference of 4 m. The no. of revolutions it makes in moving 40 metres are : **[2011 (T-II)]**

(a) 12 (b) 16 (c) 8 (d) 10

17. The ratio of volume of a cone and a cylinder of equal diameter and equal height is : **[2011 (T-II)]**

(a) 3 : 1 (b) 1 : 3 (c) 1 : 2 (d) 2 : 1

18. If a solid right circular cone of height 24 cm and base radius 6 cm is melted and recast in the shape of a sphere, then the radius of the sphere is : **[2011 (T-II)]**

(a) 6 cm (b) 4 cm (c) 8 cm (d) 12 cm

19. If the radius of base of a cylinder is doubled and the height remains unchanged, its curved surface area becomes : **[2011 (T-II)]**

(a) double (b) three times

(c) half (d) no change

20. The total surface area of a solid hemisphere of radius 7 cm is : **[2011 (T-II)]**

(a) 447π cm^2 (b) 239π cm^2

(c) 147π cm^2 (d) 174π cm^2

21. A solid sphere of radius r is melted and recast into the shape of a solid cone of height r, then the radius of the base of the cone is : **[2011 (T-II)]**

(a) r (b) $2r$ (c) r^2 (D) $\frac{r}{2}$

22. The volume of a largest sphere that can be cut from cylindrical log of wood of base radius 1 m and height 4 m is : **[2011 (T-II)]**

(a) $\frac{8}{3}\pi m^3$ (b) $\frac{10}{3}\pi m^3$ (c) $\frac{16}{3}\pi m^3$ (d) $\frac{4}{3}\pi m^3$

23. Total surface area of a cube is 216 cm^2, it's volume is : **[2011 (T-II)]**

(a) 216 cm^3 (b) 144 cm^3

(c) 196 cm^3 (d) 212 cm^3

SHORT ANSWER TYPE QUESTIONS [2 Marks]

A. Important Questions

1. Find the radius of the sphere whose surface area is 154 cm^2.

2. The dimensions of a box are 12 cm × 4 cm × 3 cm. Find the length of the longest rod which can be placed in this box.

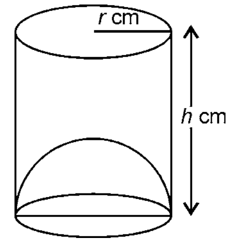
3. How many cubes of sides 3 cm can be cut from a cuboid measuring 18 cm × 12 cm × 9 cm?

4. Find the height of the cone whose volume is 1570 cm^3 and area of base is 314 cm^2.

5. The circumference of the edge of a hemispherical bowl is 132 cm. Find the capacity of the bowl.

6. If the lateral surface area of a cylinder is 94.2 cm^2, and its height is 5 cm, then find the radius of its base.

7. Two cubes each of edge 4 cm are joined face to face. Find the surface area of the resulting cuboid.
8. The diameter of a garden roller is 1.4 m and it is 2 m long. How much area will it cover in 5 revolutions?
9. The largest possible sphere is carved out from a solid cube of side 7 cm. Find the volume of the sphere.
10. A right circular cone is 3.6 cm high and has base radius 1.6 cm. It is melted and recast into a right circular cone with radius of its base as 1.2 cm. Find its height.
11. Find the ratio of the volumes of a cylinder, a cone and a sphere, if each has the same diameter and same height.
12. A cone and a sphere have equal radii and equal volume. What is the ratio of the diameter of the sphere to the height of the cone? **[HOTS]**
13. What is the ratio of the volume of a cube to that of a sphere which will fit exactly inside the cube?
14. A solid cylinder of radius r and height h is placed over other cylinder of same height and radius. Find the total surface area of the shape so formed. **[HOTS]**
15. A solid cone of base radius r and height h is placed over a solid cylinder having same base radius and height as that of the cone. Find the curved surface area and the total surface area of the shape thus formed.
16. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have ? Find the surface area of the solid.
17. A solid cube is cut into two cuboids of equal volumes. Find the ratio of the total surface area of the given cube and that of one of the cuboids.
18. Find the capacity of a cylindrical vessel with a hemispherical portion raised upward at the bottom as shown in the figure. **[HOTS]**



19. A spherical iron ball of radius r is melted to make 8 new idential balls. Find the radius of each new ball.
20. The circumference of the base of a 9 m high wooden solid cone is 44 m. Find the volume of the cone.
21. Radius of a cone is 4 cm. Find the height of the cone so that its volume may be equal to that of a sphere of radius 2 cm.
22. The height of a cone is 5 cm. Find the height of another cone whose volume is sixteen times its volume and radius equal to its diameter.
23. If the radii of the ends of a bucket 24 m high are 5 cm and 15 cm, find the surface area of the bucket.
24. The sum of length, breadth and height of a cuboid is 19 cm and its diagonal is $5\sqrt{5}$ cm. What is its surface area? **[HOTS]**
25. A cylindrical vessel of radius 4 cm contains water. A solid sphere of radius 3 cm is covered into the water untill it is completely immersed. Find the rise in the water level in the vessel.
26. A hemisphere of iron of radius 12 cm is melted and cast into a right circular cone of height 54 cm. Find the radius of the base of the cone.

B. Questions From CBSE Examination Papers

1. The radius and slant height of a right circular cone are in the ratio of 7 : 13 and its curved surface area is 286 cm^2. Find its radius. $\left(\text{use } \pi = \frac{22}{7}\right)$ **[2011 (T-II)]**
2. A toy is in the form of a cone mounted on a hemisphere of common base radius 7 cm. The total height of the toy is 31 cm. Find the total surface area of the toy. **[2011 (T-II)]**
3. A solid cone of radius 4 cm and vertical height 3 cm has to be painted from outside except the base. Find the surface area to be painted. **[2011 (T-II)]**
4. Three cubes of volume 64 cm^3 each are joined end to end to form a solid. Find the surface area of the cuboid so formed. **[2011 (T-II)]**
5. The slant height of the frustum of a cone is 5 cm. If the difference between the radii of its two circular ends is 4 cm, find the height of the frustum. **[2011 (T-II)]**
6. The volume of a right circular cylinder of height 7 cm is 567 π cm^3. Find its curved surface area. $\left(\text{Take } \pi = \frac{22}{7}\right)$. **[2011 (T-II)]**
7. Find the number of solid cylindrical structures of

radius 7 cm and height 10 cm which can be made from a solid cylinder of radius 7 m and height 10 m. **[2011 (T-II)]**

8. A solid cuboidal slab of iron of dimensions 66 cm × 20 cm × 27 cm is used to cast an iron pipe. If the outer diameter of the pipe is 10 cm and thickness is 1 cm, then calculate the length of the pipe. **[2011 (T-II)]**

9. The circumference of the circular end of a hemispherical bowl is 132 cm. Find the capacity of the bowl. **[2011 (T-II)]**

10. A solid cylinder of radius r and height h is placed over other cylinder of the same height and radius. Find the total surface area of the shape so formed. **[2011 (T-II)]**

11. How many coins 1.75 cm in diameter and 2 mm thick must be melted to form a cuboid of dimensions 11 cm × 10 cm × 7 cm? $\left(\text{Take } \pi = \frac{22}{7}\right)$. **[2011 (T-II)]**

12. Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/h. How much area will it irrigate in 30 minutes, if 8 m of standing water is needed? **[2011 (T-II)]**

13. The total surface area of a right circular cone is 90 π cm^2. If the radius of base of the cone is 5 cm, find the height of the cone. **[2011 (T-II)]**

14. The radius of the base and the height of a right circular cylinder are in the ratio of 2 : 3 and its volume is 1617 cu.cm. Find the curved surface area of the cylinder. $\left(\text{use } \pi = \frac{22}{7}\right)$. **[2011 (T-II)]**

15. A solid is hemispherical at the bottom and conical above. If the curved surface area of the two parts are equal, then find the ratio of the radius and height of the conical part. **[2011 (T-II)]**

16. A spherical solid ball of diameter 21 cm is melted and recast into cubes, each of side 1 cm. Find the number of cubes thus formed. $\left(\text{use } \pi = \frac{22}{7}\right)$ **[2011 (T-II)]**

17. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel. $\left(\text{Take } \pi = \frac{22}{7}\right)$. **[2011 (T-II)]**

18. The surface area of a sphere is 616 cm^2. Find its radius. **[2008]**

19. A cylinder and a cone are of same base radius and of same height. Find the ratio of the volume of cylinder to that of the cone. **[2009]**

20. The slant height of the frustum of a cone is 5 cm. If the difference between the radii of its two circular ends is 4 cm, write the height of the frustum. **[2010]**

SHORT ANSWER TYPE QUESTIONS [3 Marks]

A. Important Questions

1. From a solid cube of side 7 cm, a conical cavity of height 7 cm and radius 3 cm is hollowed out. Find the volume of the remaining solid.

2. Two cones with same base radius 8 cm and height 15 cm are joined together along their bases. Find the surface area of the shape so formed.

3. A spherical bowl of internal diameter 36 cm contains a liquid. This liquid is to be filled in cylindrical bottles of radius 3 cm and height 6 cm. How many bottles are required to empty the bowl? **[HOTS]**

4. A cone of maximum size is carved out from a cube of edge 14 cm. Find the surface area of the cone and of the remaining solid left out after the cone carved out.

5. An iron pipe 20 cm long has exterior diameter equal to 25 cm. If the thickness of the pipe is 1 cm, find the whole surface area of the pipe.

6. A canal 3 m wide and 1m 20 cm deep. The water in the canal is flowing with speed of 20 km/hr. How much area will it irrigate in 20 minutes if 8 cm of standing water is desired?

7. The interior of a building is in the form of a cylinder of diameter 4.3 m and height 3.8 m surmounted by cone whose vertical angle is 90°. Find the surface area of the interior of the building. **[HOTS]**

8. A cone of radius 4 cm is divided into two parts by drawing a plane through the mid-point of its axis and parallel to its base. Compare the volumes of the two parts.

9. Find the number of metallic circular disc with 1.5 cm base diameter and of height 0.2 cm to be metled to form a right circular cylinder of height 10 cm and diameter 4.5 cm. **[HOTS]**

10. If the radius of base of a right circular cylinder is halved, keeping the height same, what is the ratio of the volume of the reduced cylinder to that of the original?

11. How many metres of cloth 1 m 10 cm wide will be required to make a conical circus tent whose height is 12 m and the radius of whose base is 10 m? Also, find the cost of cloth at Rs. 7 per m.

12. Three cubes of iron whose edges are in the ratio 3 : 4 : 5 are melted and converted into a single cube whose diagonal is $12\sqrt{3}$ cm. Find the edges of the three cubes.

13. A metallic toy in the form of a cone of radius 11 cm and height 62 cm mounted on a hemisphere of the same radius is melted and recast into a solid cube. Find the surface area of the cube thus formed. **[HOTS]**

14. A bucket is in the form of a frustum of a cone and holds 28.490 litres of water. The radii of the top and bottom are 28 cm and 21 cm respectively. Find the height of the bucket.

15. A wall 24 m long, 0.4 m thick and 6 m high is constructed with the bricks each of dimensions 25 cm × 16 cm × 10 cm. If the mortar occupies $\frac{1}{10}$th of the volume of the wall, then find the number of bricks used in constructing the wall. **[HOTS]**

B. Questions From CBSE Examination Papers

1. The circumference of the base of a 9 m high wooden solid cone is 44 m. Find the volume of the cone. **[2011 (T-II)]**

2. A solid metallic sphere of diameter 21 cm is melted and recast into a number of smaller cones each of diameter 7 cm and height 3 cm. Find the number of cones so formed. **[2011 (T-II)]**

3. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder of same radius. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel. **[2011 (T-II)]**

4. A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform. **[2011 (T-II)]**

5. A well with 7 m inside diameter is dug 22 m deep, earth taken out of it has been spread all round it to a width of 10.5 m to form an embankment. Find the height of the embankment so formed. **[2011 (T-II)]**

6. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the reamining solid to the nearest cm^2. $\left(\text{use } \pi = \frac{22}{7}\right)$ **[2011 (T-II)]**

7. Right circular cylinder having diameter 12 cm and height 15 cm is full of ice-cream. This ice-cream is to be filled in cones of height 12 cm and diameter 6 cm having a hemispherical shape on the top. Find the number of such cones which can be filled with ice-cream. **[2011 (T-II)]**

8. The rain water from a roof 22 m × 20 m drains into a conical vessel having the diameter of base as 2 m and height 3.5 m. If the vessel is just full, find the rainfall in mm. **[2011 (T-II)]**

9. An open container made up of a metal sheet is in the form of a frustum of a cone of height 8 cm with radii of its lower and upper ends as 4 cm and 10 cm respectively. Find the cost of the metal used, if it costs Rs 5 per 100 cm^2 (Use π = 3.14). **[2011 (T-II)]**

10. Solid shperes of diameter 6 cm each are dropped into a cylindrical beaker containing some water and are fully submerged. The water in the beaker rises by 40 cm. Find the number of soild spheres dropped into the beaker if the diameter of the beaker is 18 cm. **[2011 (T-II)]**

11. Water is flowing at the rate of 5 km/hour through a pipe of diameter 14 cm into a rectangular tank, which is 50 m long and 44 m wide. Determine the time in which the level of water in the tank will rise by 7 cm. **[2011 (T-II)]**

12. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid. **[2011 (T-II)]**

13. An ice cream cone consisting of a cone is surmounted by a hemisphere. The common radius of hemisphere and cone is 3.5 cm and the total height of ice-cream cone is 12.5 cm. Calcualte the volume of ice-cream in the cone. **[2011 (T-II)]**

14. A semi-circular sheet of paper of diameter 28 cm is bent into an open conical cup. Find the depth and capacity of the cup. **[2011 (T-II)]**

15. 50 circular plates, each of radius 7 cm and thickness 0.5 cm, are placed one above another to form a solid right circular cylinder. Find the total surface

area and the volume of the cylinder so formed. **[2011 (T-II)]**

16. A rectangular sheet of paper 44 cm × 18 cm is rolled along its length (44 cm) and a cylinder is formed. Find the volume of the cylinder. **[2011 (T-II)]**

17. A toy is in the form of a cone mounted on a hemisphere of same radius 3.5 cm and total height of the toy is 15.5 cm, find the total surface area and the volume of the toy. **[2011 (T-II)]**

18. A solid sphere of diameter 14 cm is cut into two halves by a plane passing through the centre. Find the combined surface area of the two hemispheres so formed. **[2011 (T-II)]**

19. The internal and external radii of a hollow spherical shell are 3 cm and 5 cm respectively. If it is melted to form a solid cylinder of height $10\frac{2}{3}$ cm, find the diameter of the cylinder. **[2011 (T-II)]**

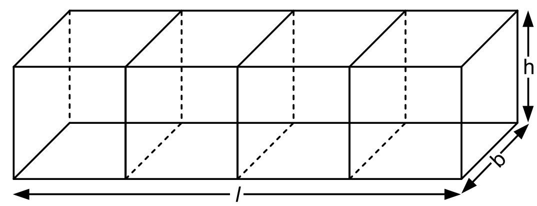
20. A cylindrical copper rod of diameter 1 cm and length 8 cm is drawn into a cylindrical wire of length 18 m and of uniform thickness. Find the thickness of the wire. **[2011 (T-II)]**

21. A farmer connects a pipe of internal diameer 20 cm from a canal into a cylindrical tank in his field which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/hr, in how much time will the tank be filled? **[2011 (T-II)]**

22. A sphere of radius 6 cm is dropped into a cylindrical vessel party filled with water. The radius of the vessel is 8 cm. If the sphere is submerged compleely, then find the increase in level of the water. **[2011 (T-II)]**

23. If the radius of the base of a right circular cylinder is halved, keeping the height same, find the ratio of the volume of the reduced cylinder to that of the original cylinder. **[2011 (T-II)]**

24. Four cubes of volume 125 cm^3 each are joined end to end, in a row. Find the surface area and volume of the resutling cuboid (see fig.) **[2011 (T-II)]**



25. A cylindrical pipe has inner diameter of 4 cm and water flows through it at the rate of 20 m per minute. How long would it take to fill a conical tank, with diameter of base as 80 cm and depth 72 cm? **[2011 (T-II)]**

26. A hemispherical bowl of internal diameter 36 cm. is full of liquid. This liquid is to be filled in cylindrical bottles of radius 3 cm and height 6 cm. How many such bottles are required to empty the bowl? **[2011 (T-II)]**

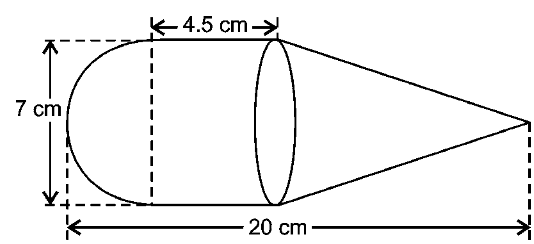
27. Solid spheres of diameter 6 cm are dropped into a cylindrical beaker containing some water and are fully submerged. If the diameter of the beaker is 18 cm and the water rises by 40 cm, find the number of solid spheres dropped in the water. **[2004]**

LONG ANSWER TYPE QUESTIONS [4 Marks]

A. Important Questions

1. A building is in the form of a cylinder surmounted by hemispherical dome. The base diameter of the dome is equal of $\frac{2}{3}$ of the total height of the building. Find the height of the building, if it contains $67\frac{1}{21}$ m^3 of air.

2. The given figure shows a solid consisting of a cylinder with a cone at one end and a hemisphere at the other end. The total length of the solid is 20 cm and the common diameter is 7 cm. If the cylindrical portion has height 4.5 cm, find the total surface area of the solid. **[HOTS]**



3. How many cubic centimetres of iron is required to construct an open box whose external dimensions are 36 cm, 25 cm and 16.5 cm, provided the thickness of the iron is 1.5 cm. If one cubic centimetre of iron weighs 7.5 gm, find the weight of the box.

4. Water flows through a cylindrical pipe whose inner radius is 1 cm, at the rate of 80 cm/sec in an empty cylindrical tank the radius of whose base is 40 cm. What is the rise of water level in tank in half an hour?

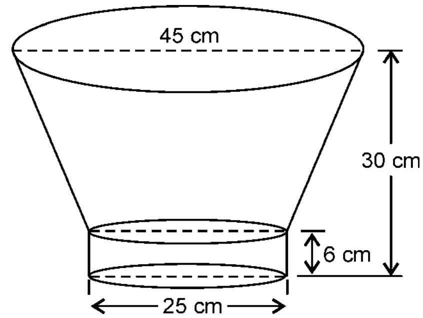
5. A cylindrical vessel is filled with sand and this vessel is emptied on the ground and a conical heap of sand is formed. If the base radius and height of the vessel are 18 cm and 32 cm respectively and the height of the conical heap is 24 cm, find the radius and the stant height of the heap

6. The radius of the base of a right circular cone is r. It is cut by a plane parallel to the base at a height h from the base. The distance of the boundary of the upper surface from the centre of the base of the frustum is $\sqrt{h^2 + \frac{r^2}{9}}$. Show that the volume of the frustum is $\frac{13}{27\pi r^2 h}$ **[HOTS]**

7. Water is flowing at the rate of 10 m/min through a cylindrical pipe 5 mm in diameter into a conical vessel whose base diameter is 40 cm and depth 24 cm. In what time will the vessel be filled ?

8. A solid right circular cone of height 120 cm and radius 60 cm is placed in a right circular cylinder full of water of height 180 cm such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is equal to the radius of the cone.

9. The height of a cone is 16 cm. A small cone is cut off at the top by a plane parallel to the base. If its volume be $\frac{1}{8}$ of the volume of the given cone, at what height above the base is the section made?



10. An open metallic bucket is in the shape of frustum of a cone mounted on a hollow cylindrical base made of metallic sheet. If the diameters of the two ends of the bucket are 45 cm and 25 cm, the total vertical height of the bucket is 30 cm, and that of the cylindrical portion is 6 cm, find the area of the metallic sheet used to make the bucket. Also, find the volume of water it can hold.

11. The dimensions of a cuboidal box are 16 cm × 8 cm × 8 cm. 16 glass spheres each of radius 2 cm are packed into this box and then the box is filled with a liquid. Find the volume of the liquid filled in the box.

12. A hemispherical tank full of water is emptied by a pipe at the rate of $3\frac{4}{7}$ litres per second. How much time will it take to empty half the tank, if it is 3 m in diameter?

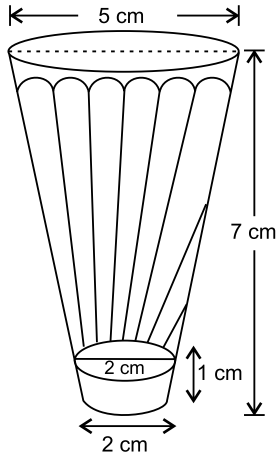
13. The barrel of a fountain pen, cylindrical in shape is 7 cm long and 5 mm in diameter. A full barrel of ink in the pen is used up on writing 3300 words on an average. How many words can be written in a bottle of ink containing one fifth of the litre? **[HOTS]**

14. A conical vessel of radius 6 cm and height 8 cm is completely filled with water. A sphere is lowered in the water and its size is such that when it touches the sides, it is just emerged. Whose fraction of water overflows?

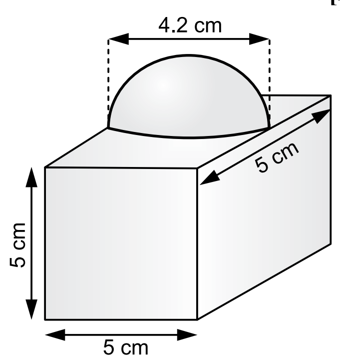
B. Questions From CBSE Examination Papers

1. A building is in the form of a right circular cylinder surmounted by a hemispherical dome both having the same base radii. The base diameter of the dome is equal to $\frac{2}{3}$ of the total height of the building. Find the height of the building, if it contains $67\frac{1}{21}\text{m}^3$ of air. **[2011 (T-II)]**

2. A shuttle cock used for playing Badminton has the shape of a frustum of a cone mounted on a hemisphere (see figure). The diameters of the ends of the frustum are 5 cm and 2 cm, the height of the entire shuttle cock is 7 cm. Find the external surface area. $\left(\text{use } \pi = \frac{22}{7}\right)$ **[2011 (T-II)]**



3. Decorative block shown in the figure is made of two solids, a cube and a hemisphere. The base of the block is a cube with edge 5 cm and hemisphere fixed on the top has a diameter 4.2 cm. Find the total surface area of the block $\left(\text{Take } \pi = \frac{22}{7}\right)$. **[2011 (T-II)]**



4. If the radii of the ends of a bucket 45 cm high are 28 cm and 7 cm. Find its capacity and surface area. **[2011 (T-II)]**

5. A bucket made up of a metal sheet is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of the bucket if the cost of metal sheet used is Rs 15 per 100 cm^2. (use $\pi = 3.14$). **[2011 (T-II)]**

6. From a solid cylinder whose height is 8 cm and radius 6 cm, a conical cavity of height 8 cm and of base radius 6 cm, is hollowed out. Find the volume of the remaining solid. (Take $\pi = 3.1416$). Also find the slant height of the cone.

[2011 (T-II)]

7. A container open from the top, made up of a metal sheet is in the form of a frustum of a cone of height 8 cm with radii of its lower and upper ends as 4 cm and 10 cm respectively. Find the cost of oil which can completely fill the container at the rate of Rs 50 per litre. Also, find the cost of metal used, if it costs Rs 5 per 100 cm^2 (Use $\pi = 3.14$). **[2011 (T-II)]**

8. Water is flowing at the rate of 3 km/h through a circular pipe of 20 cm internal diameter into a cylindrical custern of diameter 10 m and depth 2 m. In how much time will the cistern be filled? $\left(\text{use } \pi = \frac{22}{7}\right)$ **[2011 (T-II)]**

9. A solid toy is in the form of a hemisphere surmounted by a right circular cone of the same base radius. The height of the cone is 2 cm and diameter of the base is 4 cm. Determine the volume of the toy. If a right circular cylinder circumscribes the toy, find the difference of the volumes of the cylinder and the toy. [Take $\pi = 3.14$].

[2011 (T-II)]

10. A tent is of the shape of a right circular cylinder upto a height of 3 metres and conical above it. The total height of the tent is 13.5 metres above the ground. Calculate the cost of painting the inner side of the tent at the rate of Rs 2 per square metre, if the radius of the base is 14 metres. **[2011 (T-II)]**

11. A solid is in the form of a right circular cylinder with hemispherical ends. The total height of the solid is 19 cm and the diameter of the cylinder and the hemispheres is 7 cm. Find the volume and total surface area of the solid. **[2011 (T-II)]**

12. A solid is composed of a cylinder with hemispherical ends. If the whole height of the solid is 100 cm and the diameter of cylindrical part and the hemispherical ends is 28 cm, find the cost of polishing the surface of the solid at the rate of 5 paise per sq. cm. $\left(\text{use } \pi = \frac{22}{7}\right)$ **[2011 (T-II)]**

13. A milk container is made of a metal sheet in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of milk which the container can hold when fully filled at Rs 20 per litre. **[2011 (T-II)]**

14. A sphere and cube have same surface. Show that the ratio of the volume of sphere to that of the cube is $\sqrt{6} : \sqrt{\pi}$. **[2011 (T-II)]**

15. Find the mass of a 3.5 m long lead pipe, if the external diameter of pipe is 2.4 cm, thickness of the metal is 2 mm and 1 cm^3 of lead weighs 11.4 kg. **[2011 (T-II)]**

16. The difference between the outer and inner curved surface areas of a hollow right circular cylinder 14 cm long, is 88 cm^2. If the volume of metal used in making the cylinder is 176 cm^3, find the outer and inner diameter of the cylinder.

[2011 (T-II)]

17. A bucket is in the form of a frustum of a cone of height 30 cm with radii of its lower and upper ends as 10 cm and 20 cm respectively. Find the capacity and surface area of the bucket. Also find the cost of milk which can completely fill the container at the rate of Rs 25 per litre. (use π = 3.14). **[2011 (T-II)]**

18. A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 such gulab jamuns, each shaped like a cylinder with two hemispherical ends with total length 5 cm and diameter 2.8 cm. **[2011 (T-II)]**

19. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has 8 gm mass. **[2011 (T-II)]**

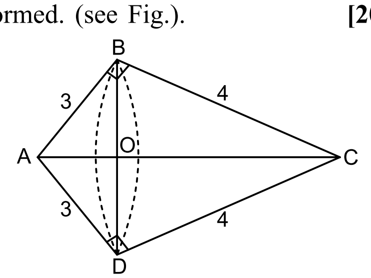
20. The perimeters of the ends of a frustum of a right circular cone are 44 cm and 33 cm. If the height of frustum is 16 cm, find its volume and total surface area. **[2011 (T-II)]**

21. A milk container is made of metal sheet in the shape of frustum of cone whose volume is $10459\frac{3}{7}\,cm^3$. The radii of its lower and upper circular ends are 8 cm and 20 cm. Find the cost of metal used in making the container at the rate of Rs 1.40 per square cm. **[2011 (T-II)]**

22. A solid wooden toy is in the form of a cone mounted on a hemisphere. If the radii of hemisphere and base of cone are 4.2 cm each and the total height of toy is 10.2 cm, find the volume of wood used in the toy. Also, find the total surface area of toy. **[2011 (T-II)]**

23. A circus tent is in the form of right circular cylinder and right circular cone above it. The diameter and height of cylindrical part of tent at 126 m and 5 m respectively. The total height of tent is 21 m. Find the total cost of tent if the canvas used costs Rs 12 per m^2. **[2011 (T-II)]**

24. A right triangle, whose sides other than hypotenuse, are 3 cm and 4 cm is made to revolve about its hypotenuse. Find the volume of the double cone so formed. (see Fig.). **[2011 (T-II)]**

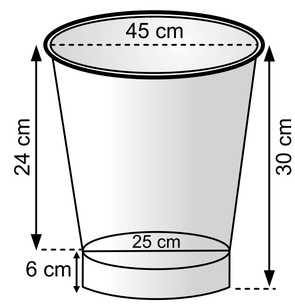


25. Wax cylinder of diameter 21 cm and height 21 cm is chipped off and shaped to form a cone of maximum volume. The chipped off wax is recast into a solid sphere. Find the diameter of the sphere. **[2011 (T-II)]**

26. The height of a cone is 30 cm. A small cone is cut off at the top by a plane parallel to the base. If its volume be $\frac{1}{27}$th volume of the cone, at what height above the base is the section made? **[2011 (T-II)]**

27. A container shaped like a circular cylinder having diameter 12 cm and height 15 cm is full of ice-cream. The ice-cream is filled into cones of height 12 cm and diameter 6 cm each having a hemispherical shape on the top. Find the number of such cones which can be filled with ice-cream. **[2011 (T-II)]**

28. A metallic bucket is in the shape of a frustum of a cone. If the diameter of two circular ends of the bucket are 45 cm and 25 cm, respectively and the total vertical height is 24 cm, find the area of the metallic sheet used to make the bucket. Also find the volume of water it can hold. **[2011 (T-II)]**



29. A spherical copper shell, of external diameter 18 cm, is melted and recast into a solid cone of base radius 14 cm and height $4\frac{3}{7}$ cm. Find the inner diameter of the shell. **[2011 (T-II)]**

30. A bucket is in the form of a frustum of a cone with a capacity of 12308.8 cm^3. The radii of the top and bottom circular ends of the bucket are 20 cm and 12 cm respectively. Find the height of the bucket and also the area of metal sheet used in making it. (Use π = 3.14). **[2011 (T-II)]**

31. Water is flowing at the rate of 15 km per hour through a pipe of diameter 14 cm into a rectangular tank which is 50 m long and 44 m wide. Find the time in which the level of water in the tank will rise by 21 cm. **[2011 (T-II)]**

32. A drinking glass open at the top is in the shape of a frustum of a cone of height 24 cm. The diameters of its top and bottom circular ends are 18 cm and 4 cm respectively. Find the capacity and total surface area of the glass. **[2011 (T-II)]**

33. A solid in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and height of the cone is equal to its radius. Find the volume and surface area of the solid.

[2011 (T-II)]

34. An iron pillar has some part in the form of a right circular cylinder and the remaining in the form of a right circular cone. The radius of the base of each of the cone and the cylinder is 8 cm. The cylindrical part is 240 cm high and conical part is 36 cm high. Find the weight of the pillar if 1 cu cm of iron weighs 7.5 grams. **[2011 (T-II)]**

35. The internal radii of the ends of a bucket, full of milk and of internal height 16 cm, are 14 cm and 7 cm. If this milk is poured into a hemispherical vessel, the vessel is completely filled. Find the internal diameter of the hemispherical vessel.

[2011 (T-II)]

36. A right triangle, whose sides are 6 cm and 8 cm (other than hypothenuse) is made to revolve about its hypotenuse. Find the volume and surface area of the double cone so formed. **[2011 (T-II)]**

37. A vessel in the form of a hemispherical bowl is full of water. Its water is emptied into a cylinder. The internal radii of bowl and the cylinder are $10\frac{1}{2}$ cm and 7 cm respectively. Find the height of water in the cylinder. **[2011 (T-II)]**

38. A hollow cone is cut by a plane parallel to the base and the upper portion is removed. If the curved surface of the remainder is $\frac{8}{9}$th of the curved surface of the whole cone, find the ratio of the line segments into which the cone's altitude is divided by the plane. **[2004C]**

39. A sphere, of diameter 12 cm, is dropped in a right circular cylindrical vessel, partly filled with water. If the sphere is completely submerged in water, the water level in the cylindrical vessel rises by $3\frac{5}{9}$ cm. Find the diameter of the cylindrical vessel.

[2007]

40. Water flows out through a circular pipe whose internal radius is 1 cm, at the rate of 80 cm/second into an empty cylindrical tank, the radius of whose base is 40 cm. By how much will the level of water rise in the tank in half an hour.

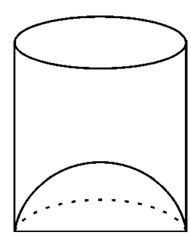
[2007]

41. A tent consists of a frustum of a cone, surmounted by a cone. If the diameter of the upper and lower circular ends of the frustum be 14 m and 26 m respectively, the height of the frustum be 8 m and the slant height of the surmounted conical portion be 12 m, find the area of canvas required to make the tent. (Assume that the radii of the upper circular end of the frustum and the base of surmounted conical portion are equal.) **[2008]**

42. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder. If the height of the cylinder is 20 cm and radius of the base is 3.5 cm, find the total surface area of the article. **[2008]**

43. A juice seller serves his customers using a glass as shown in figure. The inner diameter of the cylindrical glass is 5 cm, but the bottom of the glass has a hemispherical portion raised which reduces the capacity of the glass. If the height of the glass is 10 cm, find the apparent capacity of the glass and its actual capacity. (Use $\pi = 3.14$)

[2009]



FORMATIVE ASSESSMENT

Activity 1

Objective : To compare the curved surface areas and the total surface areas of two right circular cylinders which are formed from rectangular sheets of paper having same dimensions.

Materials Required : Rectangular sheets of paper, a pair of scissors, sellotape, geometry box, etc.

Procedure :

1. Take two rectangular sheets ABCD each of dimensions 22 cm × 11cm.

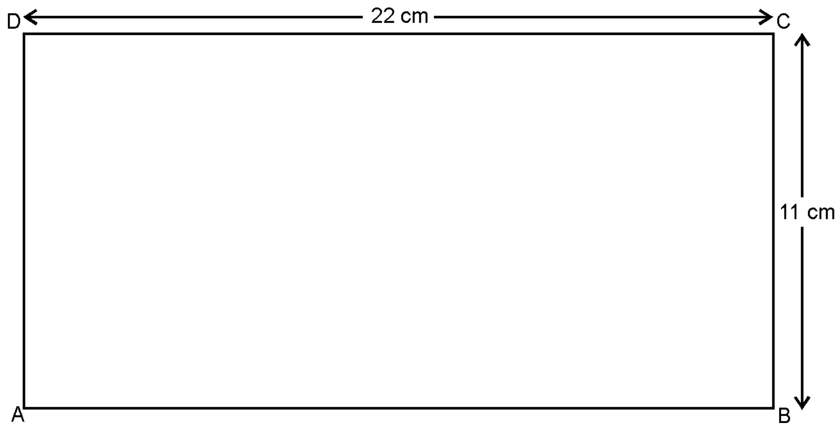


Figure 1

2. Fold one of the sheets ABCD along AB such that AD and BC coincide. Use sellotape to get a cylinder as shown below.

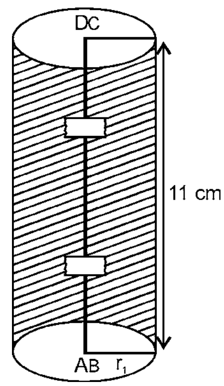


Figure 2

3. Put the cylinder on a white sheet of paper and draw the outline of its base. Using a pair of scissors cut out the circular region. Make one more such circular cut out.

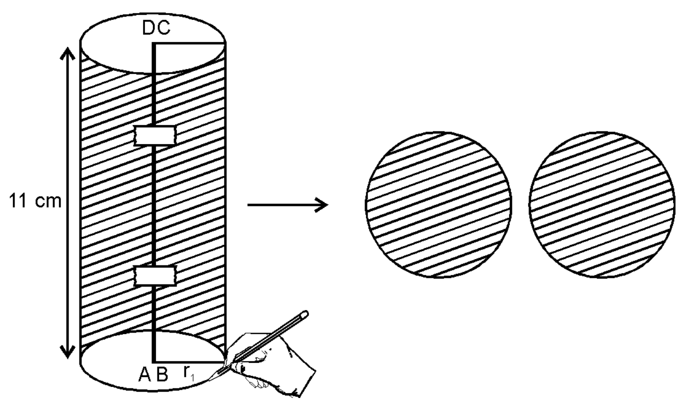


Figure 3

4. Using sellotape, fix these circular cut outs on the top and bottom of the hollow cylinder as shown in the figure.

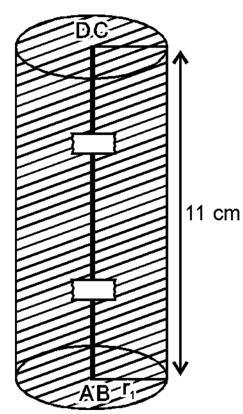


Figure 4

5. Take another sheet ABCD and fold it along AD such that AB and DC coincide. Use sellotape to get a cylinder as shown.

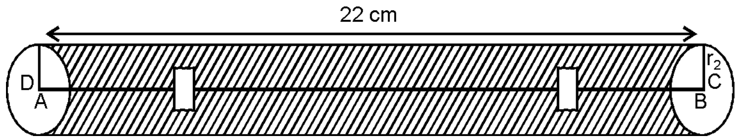


Figure 5

6. Repeat steps 3 and 4 for the cylinder obtained above.

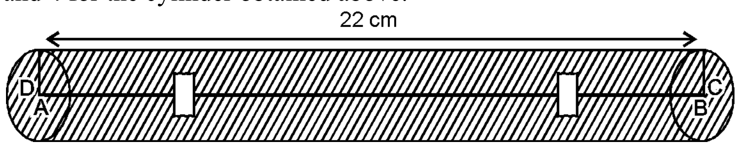


Figure 6

Observations :

1. Let r_1 be the base radius of the cylinder in figure 2.
   Then, circumference of its base = $2\pi\ r_1 = 22$

   $\Rightarrow r_1 = \dfrac{22 \times 7}{2 \times 22}$ cm = 3.5 cm.

   Also, height, h_1 of the cylinder = 11 cm.

2. So, curved surface area of the cylinder in figure 2 = $2\pi\ r_1 h_1 = 2 \times \dfrac{22}{7} \times 3.5 \times 11\ \text{cm}^2$
   $= 242\ \text{cm}^2$.

3. Total surface area of the cylinder in figure 4 = curved surface area + $2\pi\ r_1^2$
   $= (242 + 2 \times \dfrac{22}{7} \times 3.5 \times 3.5)\ \text{cm}^2 = 319\ \text{cm}^2$

4. Let r_2 be the base radius of the cylinder in figure 5.
   Then circumference of its base = $2\pi\ r_2 = 11$

   $\Rightarrow r_2 = \dfrac{11 \times 7}{2 \times 22}$ cm = 1.75 cm.

   Also, height, h_2 of the cylinder = 22 cm.

5. So, curved surface area of the cylinder in figure 5 = $2\pi\ r_2 h_2$
   $= 2 \times \dfrac{22}{7} \times 1.75 \times 22\ \text{cm}^2 = 242\ \text{cm}^2$.

6. Total surface area of the cylinder in figure 6 = curved surface area + $2\pi\ r_2^2$
   $= (242 + 2 \times \dfrac{22}{7} \times 1.75 \times 1.75)\ \text{cm}^2 = 261.25\ \text{cm}^2$.

7. From 2 and 5, we see that the curved surface areas of both the cylinders are same.
8. From 3 and 6, we see that the total surface areas of the two cylinders are different.

Conclusion :

1. The curved surface areas of two cylinders formed by folding the rectangular sheets having same dimensions are same.
2. The total surface areas of two cylinders formed by folding the rectangular sheets having same dimensions are not same.

Activity 2

Objective : To compare the volumes of two right circular cylinders which are formed from rectangular sheets of paper having same dimensions.

Materials Required : Rectangular sheets of paper, a pair of scissors, sellotape, geometry box, etc.

Procedure :

1. Take two rectangular sheets ABCD of each dimensions 22 cm × 11 cm.

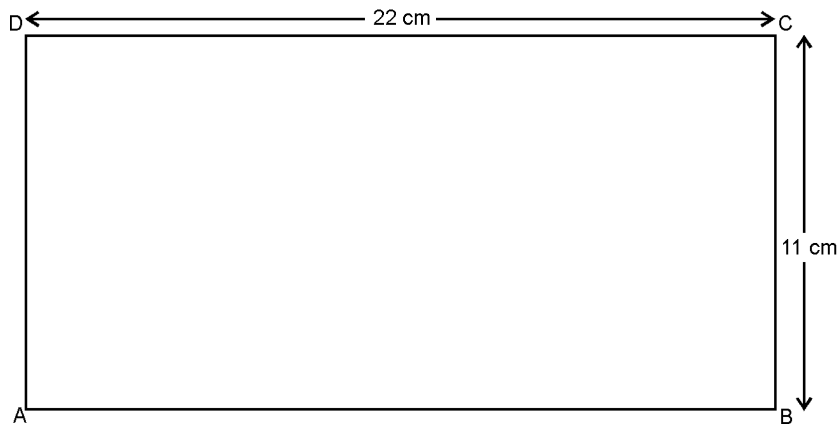


Figure 7

2. Fold one of the sheets ABCD along AB such that AD and BC coincide. Use sellotape to get a cylinder as shown below.

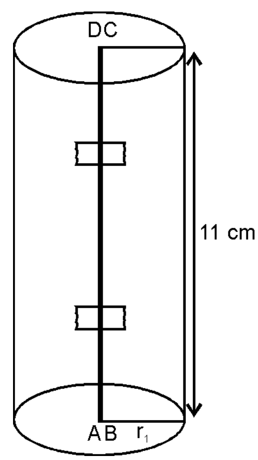


Figure 8

3. Take another sheet ABCD and fold it along AD such that AB and DC coincide. Use sellotape to get a cylinder as shown.

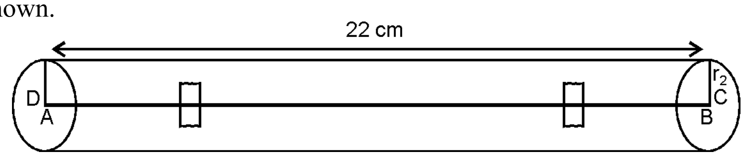


Figure 9

Observations :

1. Let r_1 be the radius of the base of the cylinder in figure 8.

   Then, circumference of its base = $2\pi r_1 = 22$

   $\Rightarrow r_1 = \dfrac{22\times7}{2\times22}$ cm = 3.5 cm.

   Also, height h_1 of the cylinder = 11 cm.

2. Volume of this cylinder = $\pi\, r_1^2 h_1 = \dfrac{22}{7} \times 3.5 \times 3.5 \times 11$ cm^3 = 443.50 cm^3

3. Let r_2 be the radius of the base of the cylinder in figure 9.

Then circumference of its base = $2\pi r_2 = 11$

$\Rightarrow r_2 = \dfrac{11 \times 7}{2 \times 22}$ cm = 1.75 cm.

Also, height, h_2 of the cylinder = 22 cm.

4. Volume of this cylinder = $\pi\, r_2^2 h_2 = \dfrac{22}{7} \times 1.75 \times 1.75 \times 22 \text{ cm}^3 = 211.75 \text{ cm}^3$.

5. From 2 and 4 above, we see that the two volumes are different.

Conclusion : The volumes of two cylinders formed by folding the rectangular sheets having same dimensions are different.

Activity 3

Objective : To make a cone of given slant height l and base circumference.

Materials Required : White sheets of paper, a pair of scissors, colour pencils, sellotape, geometry box, etc.

Procedure : Let us make a cone of slant height, l = 7 cm and base circumference = 11 cm.

1. On a white sheet of paper, draw a circle of radius 7 cm and centre O.

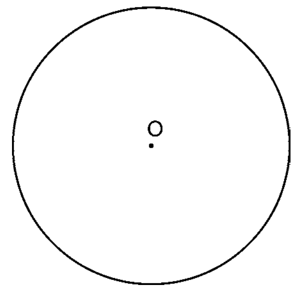


Figure 10

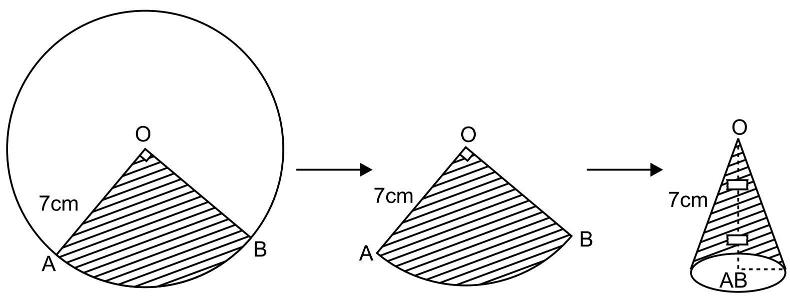


Figure 11

2. Mark a sector OAB such that $\angle$AOB = 90°. Cut the sector OAB and bring the radii OA and OB together. Use sellotape to get a cone as shown above.

Observations :

1. The radius of the circle in figure 11 becomes slant height of the cone.

or, l = 7 cm

2. $\angle$AOB = 90°

$\therefore$ So, length of the arc AB = $\dfrac{\pi r \theta°}{180°}$, where θ = 90°

$$= \frac{\frac{22}{7} \times 7 \times 90°}{180”} \text{ cm} = 11\text{cm}$$

Hence, base circumference of the cone = length of the arc AB = 11 cm

Conclusion : From the above activity, we see that a cone of given slant height and base circumference can be made from a sector of a circle.

Activity 4

Objective : To give a suggestive demonstration of the formula for the lateral surface area of a cone.

Materials Required : White sheets of paper, a pair of scissors, colour pencils, geometry box, sellotape, gluestick etc.

Procedure :

1. On a white sheet of paper, draw a circle of any convenient radius and with centre O. Mark a sector OAB on the circle and cut it out. From the previous activity, we know that this sector can be folded to make a cone.

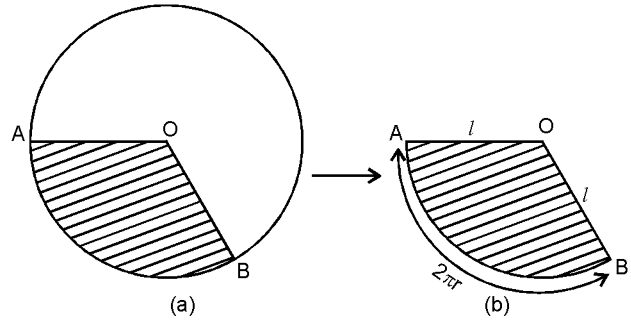


Figure 12

2. Fold the sector such that OA and OB coincide. Again fold it as shown. Press it to make creases.

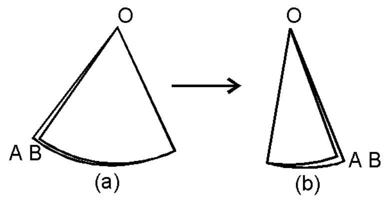


Figure 13

3. Unfold the cut out and draw lines along the creases. Now, cut the sectors along the creases to get four smaller sectors as shown below.

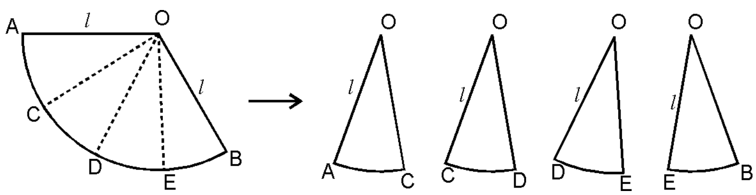


Figure 14

4. Paste the four smaller sectors on a white sheet of paper to get an approximate parallelogram as shown.

Observations :

1. From figure, slant height of the cone = l and base circumference of the cone = $2\pi r$, where r is the base radius of the cone.
2. Figure 15 is an approximate parallelogram whose base is half of the base circumference of the cone and height is approximately the slant height of the cone.

   i.e, base of the parallelogram = $\frac{1}{2} \times 2\pi r = \pi r$
   Height of the parallelogram = l

   $\therefore$ Area of the parallelogram = base × height = $\pi r \times l = \pi rl$

   Thus, curved surface area of the cone = area of the parallelogram = πrl

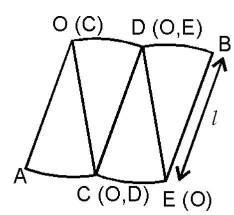


Figure 15

Conclusion : The curved (lateral) surface area of a cone of base radius r and slant height l is equal to πrl.

Activity 5

Objective : To give a suggestive demonstration of the formula for the volume of a right circular cone.

Materials Required : Cones and cylinders of same base radius and height, sand etc.

Procedure :

1. Take a set of cone and cylinder having same base radius (r) and same height (h).

2. Fill the cone with sand.
3. Pour the sand from the cone into the cylinder.
4. Repeat step 3 untill the cylinder gets completely filled with sand.
5. Repeat steps 2 to 4 for another set of cone and cylinder having same base radius and same height.

Observations :

1. We see that after filling the cone with sand and pouring the sand into the cylinder, it needs three pourings to fill the cylinder completely.

   So, volume of the cylinder = 3 times the volume of the cone

   or, volume of the cone = $\frac{1}{3}$ volume of the cylinder.
2. Volume of the cylinder = $\pi r^2 h$
3. So, volume of the cone = $\frac{1}{3}\pi r^2 h$

Conclusion : From the above activity, it is verified that the volume of a cone of base radius r and height h is given by $\frac{1}{3}\pi r^2 h$.

Activity 6

Objective : To give a suggestive demonstration of the formula for the volume of a sphere in terms of its radius.

Materials Required : A hollow sphere and two cylinders whose base diameter and height are equal to the diameter of the sphere, sand, etc.

Procedure :

1. Take a hollow sphere and two cylinders whose base diameter and height are equal to the diameter of the sphere.
2. Fill the sphere with sand and pour it into one of the cylinders.
3. Fill the sphere with sand second time and pour it into the same cylinder as in step 2, till it gets completely filled.
4. Pour the remaining sand into the other cylinder.
5. Again fill the sphere with sand and pour it into the semi-filled cylinder to fill the cylinder with sand completely.

Observations :

1. We see that the total sand poured in three pourings completely filled the two cylinders.
2. So, three times the volume of the sphere = two times the volume of the cylinder

   $= 2\pi r^2 h$

   $= 2\pi r^2 (2r)$ $\quad$ [$\because$ $h = 2\,r$ for each cylinder]

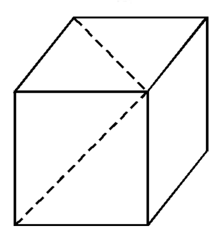
   $\Rightarrow$ 3 × volume of the sphere = $4\pi r^3$

   $\Rightarrow$ Volume of the sphere = $\frac{4}{3}\pi r^3$

Conclusion : From the above activity, it is verified that the volume of a sphere of radius r is given by $\frac{4}{3}\pi r^3$.

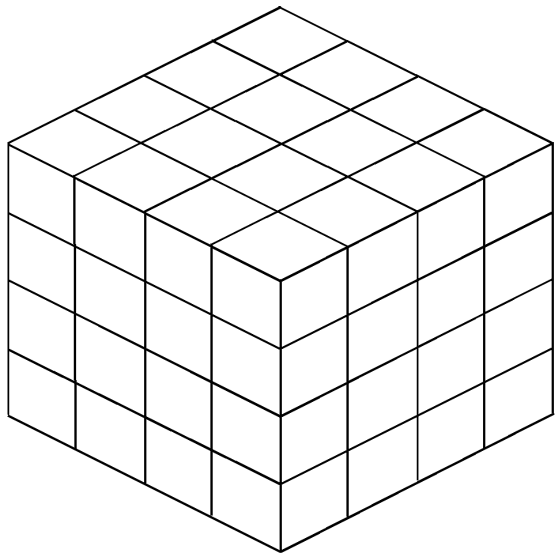
Angles in a Cube

Find the angle between the two dotted lines drawn on the surface of a cube.



Cutting Cubes

A cube of side 4 cm painted red, yellow and green on opposite faces. It was cut into cubical blocks each of the side 1 cm.



1. How many cubes are obtained?
2. How many cubes are there whose only one face is painted?
3. How many cubes are there whose no face is painted?
4. How many cubes are there whose two faces are painted?
5. How many cubes are there whose three faces are painted?
6. How many cubes are there whose two faces are painted with the same colour?
7. How many cubes are there whose three faces are painted all with different colours.
8. How many cubes are there whose all faces are painted?
9. At most how many painted faces a cube can have?

Exercise 13.1

Question 1:

2 cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboids.

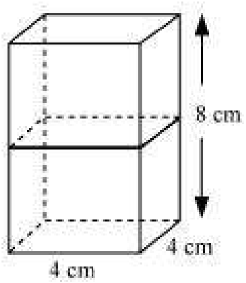
Answer:

Given that,

Volume of cubes = 64 cm^3

$(\text{Edge})^3 = 64$

Edge = 4 cm



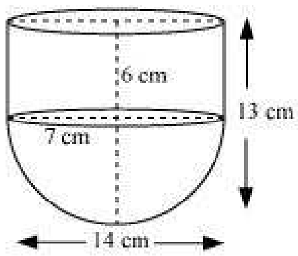
If cubes are joined end to end, the dimensions of the resulting cuboid will be 4 cm, 4 cm, 8 cm.

$$\begin{aligned}\therefore \text{Surface area of cuboids} &= 2(lb + bh + lh)\\ &= 2(4\times4 + 4\times8 + 4\times8)\\ &= 2(16+32+32)\\ &= 2(16+64)\\ &= 2\times80 = 160\ \text{cm}^2\end{aligned}$$

Question 2:

A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Answer:



It can be observed that radius (r) of the cylindrical part and the hemispherical part is the same (i.e., 7 cm).

Height of hemispherical part = Radius = 7 cm

Height of cylindrical part (h) = 13 −7 = 6 cm

Inner surface area of the vessel = CSA of cylindrical part + CSA of hemispherical part

$= 2\pi rh + 2\pi r^2$

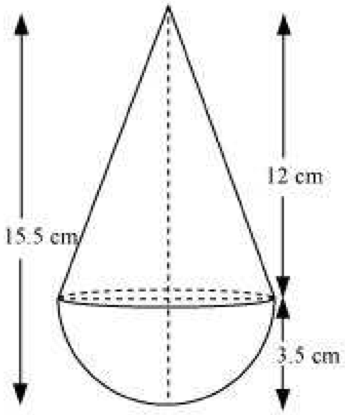
$$\text{Inner surface area of vessel} = 2\times\frac{22}{7}\times 7\times 6 + 2\times\frac{22}{7}\times 7\times 7$$
$$= 44(6+7) = 44\times 13$$
$$= 572\ \text{cm}^2$$

Question 3:

A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

$\left[\text{Use } \pi = \frac{22}{7}\right]$

Answer:



It can be observed that the radius of the conical part and the hemispherical part is same (i.e., 3.5 cm).

Height of hemispherical part = Radius (r) = 3.5 = $\frac{7}{2}$ cm

Height of conical part (h) = 15.5 −3.5 = 12 cm

Slant height (l) of conical part $= \sqrt{r^2 + h^2}$

$$= \sqrt{\left(\frac{7}{2}\right)^2 + (12)^2} = \sqrt{\frac{49}{4} + 144} = \sqrt{\frac{49 + 576}{4}}$$

$$= \sqrt{\frac{625}{4}} = \frac{25}{2}$$

Total surface area of toy = CSA of conical part + CSA of hemispherical part

$$= \pi r l + 2\pi r^2$$

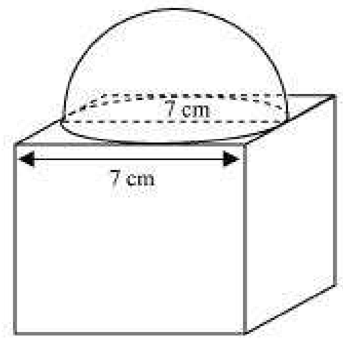
$$= \frac{22}{7} \times \frac{7}{2} \times \frac{25}{2} + 2 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

$$= 137.5 + 77 = 214.5 \text{cm}^2$$

Question 4:

A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Answer:



From the figure, it can be observed that the greatest diameter possible for such hemisphere is equal to the cube's edge, i.e., 7cm.

Radius (r) of hemispherical part = $\frac{7}{2}$ = 3.5cm

Total surface area of solid = Surface area of cubical part + CSA of hemispherical part − Area of base of hemispherical part

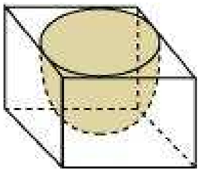
= 6 (Edge)^2 $+2\pi r^2 - \pi r^2$ = 6 (Edge)^2 + πr^2

$$\text{Total surface area of solid} = 6(7)^2 + \frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}$$
$$= 294 + 38.5 = 332.5\,\text{cm}^2$$

Question 5:

A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter *l* of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Answer:



Diameter of hemisphere = Edge of cube = l

Radius of hemisphere = $\frac{l}{2}$

Total surface area of solid = Surface area of cubical part + CSA of hemispherical part − Area of base of hemispherical part

= 6 (Edge)^2 $+2\pi r^2 - \pi r^2$ = 6 (Edge)^2 + πr^2

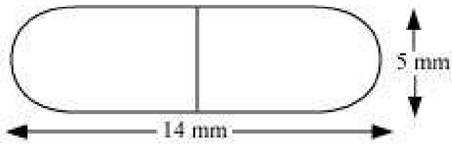
$$\text{Total surface area of solid} = 6l^2 + \pi \times \left(\frac{l}{2}\right)^2$$

$$= 6l^2 + \frac{\pi l^2}{4}$$

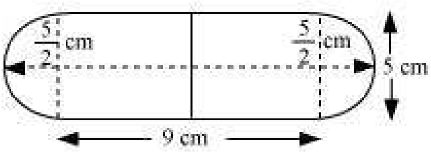
$$= \frac{1}{4}(24 + \pi)l^2 \text{ unit}^2$$

Question 6:

A medicine capsule is in the shape of cylinder with two hemispheres stuck to each of its ends (see the given figure). The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer:



It can be observed that

Radius (r) of cylindrical part = Radius (r) of hemispherical part

$$= \frac{\text{Diameter of the capsule}}{2} = \frac{5}{2}$$

Length of cylindrical part (h) = Length of the entire capsule − 2 × r

= 14 − 5 = 9 cm

Surface area of capsule = 2×CSA of hemispherical part + CSA of cylindrical part

$= 2 \times 2\pi r^2 + 2\pi rh$

$= 4\pi\left(\frac{5}{2}\right)^2 + 2\pi\left(\frac{5}{2}\right)(9)$

$= 25\pi + 45\pi$

$= 70\pi \text{ mm}^2$

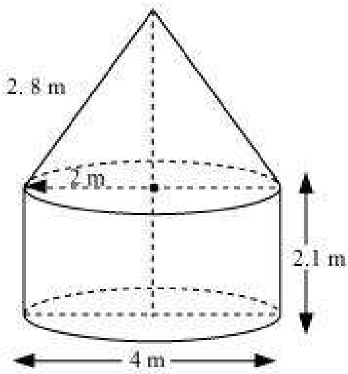
$= 70 \times \frac{22}{7}$

$= 220 \text{ mm}^2$

Question 7:

A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of Rs 500 per m^2. (Note that the base of the tent will not be covered with canvas.) $\left[\text{Use } \pi = \frac{22}{7}\right]$

Answer:



Given that,

Height (h) of the cylindrical part = 2.1 m

Diameter of the cylindrical part = 4 m

Radius of the cylindrical part = 2 m

Slant height (l) of conical part = 2.8 m

Area of canvas used = CSA of conical part + CSA of cylindrical part

$= \pi r l + 2\pi r h$

$= \pi \times 2 \times 2.8 + 2\pi \times 2 \times 2.1$

$= 2\pi[2.8 + 2 \times 2.1] = 2\pi[2.8 + 4.2] = 2 \times \frac{22}{7} \times 7$

$= 44 \text{ m}^2$

Cost of 1 m^2 canvas = Rs 500

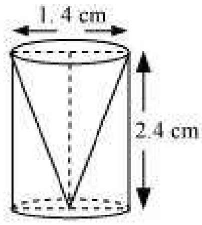
Cost of 44 m^2 canvas = 44 × 500 = 22000

Therefore, it will cost Rs 22000 for making such a tent.

Question 8:

From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Answer:



Given that,

Height (h) of the conical part = Height (h) of the cylindrical part = 2.4 cm

Diameter of the cylindrical part = 1.4 cm

Therefore, radius (r) of the cylindrical part = 0.7 cm

$$\text{Slant height } (l) \text{ of conical part} = \sqrt{r^2 + h^2}$$
$$= \sqrt{(0.7)^2 + (2.4)^2} = \sqrt{0.49 + 5.76}$$
$$= \sqrt{6.25} = 2.5$$

Total surface area of the remaining solid will be

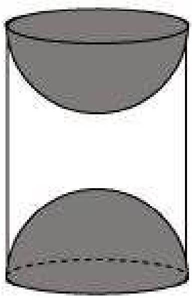
= CSA of cylindrical part + CSA of conical part + Area of cylindrical base

$$= 2\pi rh + \pi rl + \pi r^2$$
$$= 2\times\frac{22}{7}\times 0.7\times 2.4 + \frac{22}{7}\times 0.7\times 2.5 + \frac{22}{7}\times 0.7\times 0.7$$
$$= 4.4\times 2.4 + 2.2\times 2.5 + 2.2\times 0.7$$
$$= 10.56 + 5.50 + 1.54 = 17.60 \text{ cm}^2$$

The total surface area of the remaining solid to the nearest cm^2 is 18 cm^2

Question 9:

A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in given figure. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the total surface area of the article. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer:

Given that,

Radius (r) of cylindrical part = Radius (r) of hemispherical part = 3.5 cm

Height of cylindrical part (h) = 10 cm

Surface area of article = CSA of cylindrical part + 2 × CSA of hemispherical part

$= 2\pi rh + 2 \times 2\pi r^2$

$= 2\pi \times 3.5 \times 10 + 2 \times 2\pi \times 3.5 \times 3.5$

$= 70\pi + 49\pi$

$= 119\pi$

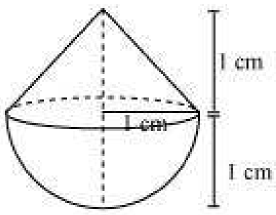
$= 17 \times 22 = 374 \text{ cm}^2$

Exercise 13.2

Question 1:

A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π.

Answer:



Given that,

Height (h) of conical part = Radius(r) of conical part = 1 cm

Radius(r) of hemispherical part = Radius of conical part (r) = 1 cm

Volume of solid = Volume of conical part + Volume of hemispherical part

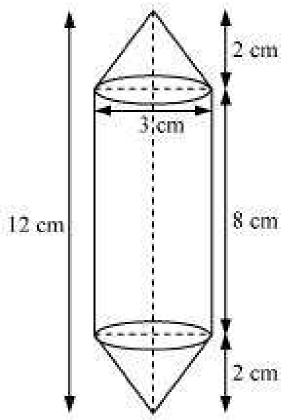
$$=\frac{1}{3}\pi r^2 h+\frac{2}{3}\pi r^3$$

$$=\frac{1}{3}\pi(1)^2(1)+\frac{2\pi}{3}\pi(1)=\frac{2\pi}{3}+\frac{\ }{3}=\pi\ \text{cm}^3$$

Question 2:

Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminum sheet. The diameter of the model is 3 cm and its length is 12 cm. if each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same.) $\left[\text{Use } \pi=\frac{22}{7}\right]$

Answer:



From the figure, it can be observed that

Height (h_1) of each conical part = 2 cm

Height (h_2) of cylindrical part = 12 − 2 × Height of conical part

= 12 − 2 ×2 = 8 cm

Radius (r) of cylindrical part = Radius of conical part = $\frac{3}{2}$ cm

Volume of air present in the model = Volume of cylinder + 2 × Volume of cones

$$= \pi r^2 h_2 + 2 \times \frac{1}{3}\pi r^2 h_1$$

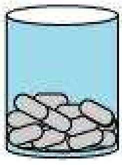
$$= \pi\left(\frac{3}{2}\right)^2 (8) + 2 \times \frac{1}{3}\pi\left(\frac{3}{2}\right)^2 (2) \quad = \pi \times \frac{9}{4} \times 8 + \frac{2}{3}\pi \times \frac{9}{4} \times 2$$

$$= 18\pi + 3\pi = 21\pi = 66 \text{ cm}^2$$

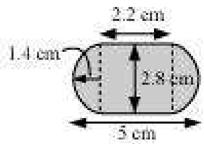
Question 3:

A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm (see the given figure).

$\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer:



It can be observed that

Radius (r) of cylindrical part = Radius (r) of hemispherical part = $\frac{2.8}{2} = 1.4\,\text{cm}$

Length of each hemispherical part = Radius of hemispherical part = 1.4 cm

Length (h) of cylindrical part = 5 − 2 × Length of hemispherical part

= 5 − 2 × 1.4 = 2.2 cm

Volume of one gulab jamun = Vol. of cylindrical part + 2 × Vol. of hemispherical part

$$= \pi r^2 h + 2 \times \frac{2}{3}\pi r^3 = \pi r^2 h + \frac{4}{3}\pi r^3$$

$$= \pi \times (1.4)^2 \times 2.2 + \frac{4}{3}\pi (1.4)^3$$

$$= \frac{22}{7} \times 1.4 \times 1.4 \times 2.2 + \frac{4}{3} \times \frac{22}{7} \times 1.4 \times 1.4 \times 1.4$$

$$= 13.552 + 11.498 = 25.05\ \text{cm}^3$$

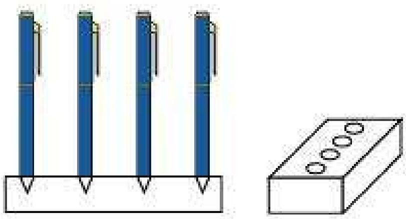
Volume of 45 gulab jamuns = 45×25.05 = 1,127.25 cm^3

Volume of sugar syrup = 30% of volume

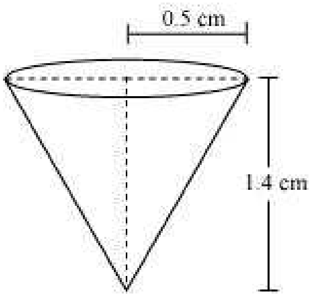
$$= \frac{30}{100} \times 1,127.25$$
$$= 338.17 \text{ cm}^3$$
$$\simeq 338 \text{ cm}^3$$

Question 4:

A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboids are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand (see the following figure). $\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer:



Depth (h) of each conical depression = 1.4 cm

Radius (r) of each conical depression = 0.5 cm

Volume of wood = Volume of cuboid − 4 × Volume of cones

$$= lbh - 4 \times \frac{1}{3}\pi r^2 h$$

$$= 15 \times 10 \times 3.5 - 4 \times \frac{1}{3} \times \frac{22}{7} \times \left(\frac{1}{2}\right)^2 \times 1.4$$

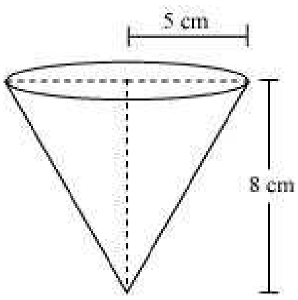
$$= 525 - 1.47$$

$$= 523.53 \text{ cm}^3$$

Question 5:

A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.

Answer:



Height (h) of conical vessel = 8 cm

Radius (r_1) of conical vessel = 5 cm

Radius (r_2) of lead shots = 0.5 cm

Let n number of lead shots were dropped in the vessel.

Volume of water spilled = Volume of dropped lead shots

$$\frac{1}{4}\times \text{Volume of cone} = n\times\frac{4}{3}\pi r_2^3$$

$$\frac{1}{4}\times\frac{1}{3}\pi r_1^2 h = n\times\frac{4}{3}\pi r_2^3$$

$$r_1^2 h = n\times 16 r_2^3$$

$$5^2\times 8 = n\times 16\times(0.5)^3$$

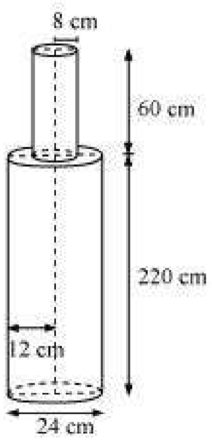
$$n = \frac{25\times 8}{16\times\left(\frac{1}{2}\right)^3} = 100$$

Hence, the number of lead shots dropped in the vessel is 100.

Question 6:

A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8 g mass. [Use π = 3.14]

Answer:



From the figure, it can be observed that

Height (h_1) of larger cylinder = 220 cm

Radius (r_1) of larger cylinder = $\frac{24}{2}$ = 12 cm

Height (h_2) of smaller cylinder = 60 cm

Radius (r_2) of smaller cylinder = 8 cm

$$\text{Total volume of pole} = \text{Volume of larger cylinder} + \text{Volume of smaller cylinder}$$
$$= \pi r_1^2 h_1 + \pi r_2^2 h_2$$
$$= \pi (12)^2 \times 220 + \pi (8)^2 \times 60$$
$$= \pi [144 \times 220 + 64 \times 60]$$
$$= 35520 \times 3.14 = 1{,}11{,}532.8 \text{ cm}^3$$

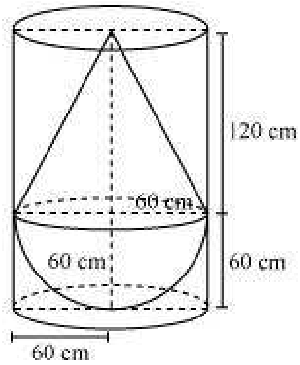
Mass of 1 cm^3 iron = 8 g

Mass of 111532.8 cm^3 iron = 111532.8 × 8 = 892262.4 g = 892.262 kg

Question 7:

A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Answer:



Radius (r) of hemispherical part = Radius (r) of conical part = 60 cm

Height (h_2) of conical part of solid = 120 cm

Height (h_1) of cylinder = 180 cm

Radius (r) of cylinder = 60 cm

Volume of water left = Volume of cylinder − Volume of solid

$$= \text{Volume of cylinder } - (\text{Volume of cone} + \text{Volume of hemisphere})$$

$$= \pi r^2 h_1 - \left(\frac{1}{3}\pi r^2 h_2 + \frac{2}{3}\pi r^3\right)$$

$$= \pi (60)^2 (180) - \left(\frac{1}{3}\pi (60)^2 \times 120 + \frac{2}{3}\pi (60)^3\right)$$

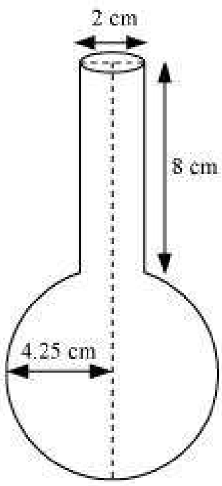
$$= \pi (60)^2 \left[(180) - (40 + 40)\right]$$

$$= \pi (3,600)(100) = 3,60,000\pi \text{ cm}^3 = 1131428.57 \text{ cm}^3 = 1.131 \text{ m}^3$$

Question 8:

A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter o the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm^3. Check whether she is correct, taking the above as the inside measurements, and π = 3.14.

Answer:



Height (h) of cylindrical part = 8 cm

Radius (r_2) of cylindrical part = $\frac{2}{2}=1$ cm

Radius (r_1) spherical part = $\frac{8.5}{2}=4.25\text{ cm}$

Volume of vessel = Volume of sphere + Volume of cylinder

$$=\frac{4}{3}\pi r_1^3+\pi r_2^2 h$$

$$=\frac{4}{3}\pi\left(\frac{8.5}{2}\right)^3+\pi(1)^2(8)$$

$$=\frac{4}{3}\times 3.14\times 76.765625+8\times 3.14$$

$$=321.392+25.12$$

$$=346.512$$

$$=346.51\text{ cm}^3$$

Hence, she is wrong.

Exercise 13.3

Question 1:

A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.

Answer:

Radius (r_1) of hemisphere = 4.2 cm

Radius (r_2) of cylinder = 6 cm

Let the height of the cylinder be h.

The object formed by recasting the hemisphere will be the same in volume.

Volume of sphere = Volume of cylinder

$$\frac{4}{3}\pi r_1^3 = \pi r_2^2 h$$

$$\frac{4}{3}\pi(4.2)^3 = \pi(6)^2 h$$

$$\frac{4}{3}\times\frac{4.2\times4.2\times4.2}{36} = h$$

$$h = (1.4)^3 = 2.74 \text{ cm}$$

Hence, the height of the cylinder so formed will be 2.74 cm.

Question 2:

Metallic spheres of radii 6 cm, 8 cm, and 10 cm, respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.

Answer:

Radius (r_1) of 1st sphere = 6 cm

Radius (r_2) of 2nd sphere = 8 cm

Radius (r_3) of 3rd sphere = 10 cm

Let the radius of the resulting sphere be r.

The object formed by recasting these spheres will be same in volume as the sum of the volumes of these spheres.

Volume of 3 spheres = Volume of resulting sphere

$$\frac{4}{3}\pi\left[r_1^3 + r_2^3 + r_3^3\right] = \frac{4}{3}\pi r^3$$

$$\frac{4}{3}\pi\left[6^3 + 8^3 + 10^3\right] = \frac{4}{3}\pi r^3$$

$$r^3 = 216 + 512 + 1000 = 1728$$

$$r = 12 \text{ cm}$$

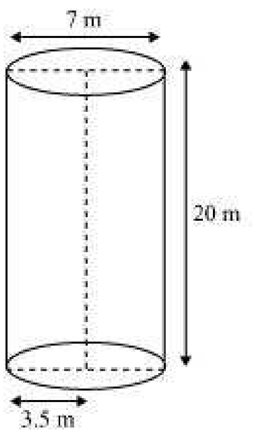
Therefore, the radius of the sphere so formed will be 12 cm.

Question 3:

A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform.

$$\left[\text{Use } \pi = \frac{22}{7}\right]$$

Answer:



The shape of the well will be cylindrical.

Depth (h) of well = 20 m

Radius (r) of circular end of well = $\frac{7}{2}$ m

Area of platform = Length × Breadth = 22 × 14 m^2

Let height of the platform = *H*

Volume of soil dug from the well will be equal to the volume of soil scattered on the platform.

Volume of soil from well = Volume of soil used to make such platform

$\pi \times r^2 \times h = \text{Area of platform} \times \text{Height of platform}$

$\pi \times \left(\frac{7}{2}\right)^2 \times 20 = 22 \times 14 \times H$

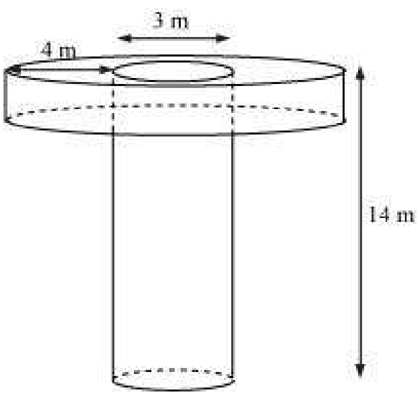
$\therefore H = \frac{22}{7} \times \frac{49}{4} \times \frac{20}{22 \times 14} = \frac{5}{2} \text{ m} = 2.5 \text{ m}$

Therefore, the height of such platform will be 2.5 m.

Question 4:

A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4 m to form an embankment. Find the height of the embankment.

Answer:



The shape of the well will be cylindrical.

Depth (h_1) of well = 14 m

Radius (r_1) of the circular end of well = $\frac{3}{2}$ m

Width of embankment = 4 m

From the figure, it can be observed that our embankment will be in a cylindrical shape having outer radius (r_2) as $4+\frac{3}{2}=\frac{11}{2}$ m and inner radius (r_1) as $\frac{3}{2}$ m.

Let the height of embankment be h_2.

Volume of soil dug from well = Volume of earth used to form embankment

$$\pi\times r_1^2\times h_1=\pi\times\left(r_2^2-r_1^2\right)\times h_2$$

$$\pi\times\left(\frac{3}{2}\right)^2\times 14=\pi\times\left[\left(\frac{11}{2}\right)^2-\left(\frac{3}{2}\right)^2\right]\times h$$

$$\frac{9}{4}\times 14=\frac{112}{4}\times h$$

$$h=\frac{9}{8}=1.125\text{ m}$$

Therefore, the height of the embankment will be 1.125 m.

Question 5:

A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream.

Answer:

Height (h_1) of cylindrical container = 15 cm

Radius (r_1) of circular end of container = $\frac{12}{2}=6\text{ cm}$

Radius (r_2) of circular end of ice-cream cone = $\frac{6}{2}=3\text{ cm}$

Height (h_2) of conical part of ice-cream cone = 12 cm

Let n ice-cream cones be filled with ice-cream of the container.

Volume of ice-cream in cylinder = n × (Volume of 1 ice-cream cone + Volume of hemispherical shape on the top)

$$\pi r_1^2 h_1 = n\left(\frac{1}{3}\pi r_2^2 h_2 + \frac{2}{3}\pi r_2^3\right)$$
$$n = \frac{6^2 \times 15}{\frac{1}{3}\times 9\times 12 + \frac{2}{3}\times(3)^3}$$
$$n = \frac{36\times 15\times 3}{108+54}$$
$$n = 10$$

Therefore, 10 ice-cream cones can be filled with the ice-cream in the container.

Question 6:

How many silver coins, 1.75 cm in diameter and of thickness 2 mm, must be melted to form a cuboid of dimensions $5.5 \text{ cm}\times 10 \text{ cm}\times 3.5 \text{ cm}$? $\left[\text{Use } \pi = \frac{22}{7}\right]$

Answer:

Coins are cylindrical in shape.

Height (h_1) of cylindrical coins = 2 mm = 0.2 cm

Radius (r) of circular end of coins = $\frac{1.75}{2} = 0.875 \text{ cm}$

Let n coins be melted to form the required cuboids.

Volume of n coins = Volume of cuboids

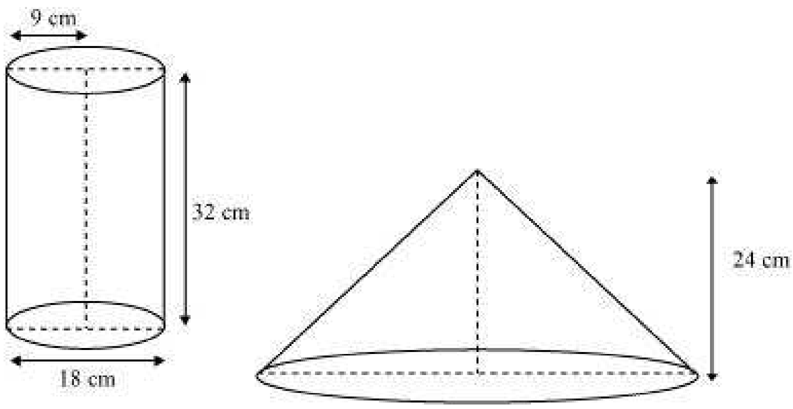
$$n\times\pi\times r^2\times h_1 = l\times b\times h$$
$$n\times\pi\times(0.875)^2\times 0.2 = 5.5\times 10\times 3.5$$
$$n = \frac{5.5\times 10\times 3.5\times 7}{(0.875)^2\times 0.2\times 22} = 400$$

Therefore, the number of coins melted to form such a cuboid is 400.

Question 7:

A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm. Find the radius and slant height of the heap.

Answer:



Height (h_1) of cylindrical bucket = 32 cm

Radius (r_1) of circular end of bucket = 18 cm

Height (h_2) of conical heap = 24 cm

Let the radius of the circular end of conical heap be r_2.

The volume of sand in the cylindrical bucket will be equal to the volume of sand in the conical heap.

Volume of sand in the cylindrical bucket = Volume of sand in conical heap

$$\pi \times r_1^2 \times h_1 = \frac{1}{3}\pi \times r_2^2 \times h_2$$

$$\pi \times 18^2 \times 32 = \frac{1}{3}\pi \times r_2^2 \times 24$$

$$\pi \times 18^2 \times 32 = \frac{1}{3}\pi \times r_2^2 \times 24$$

$$r_2^2 = \frac{3 \times 18^2 \times 32}{24} = 18^2 \times 4$$

$r_2 = 18 \times 2 = 36$ cm

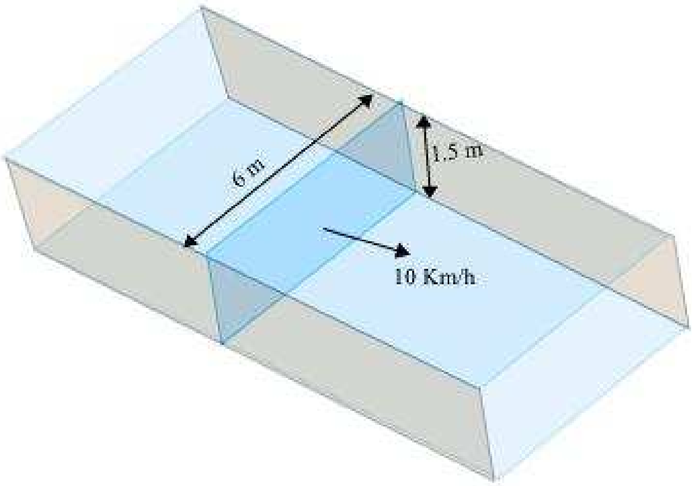
Slant height = $\sqrt{36^2 + 24^2} = \sqrt{12^2 \times (3^2 + 2^2)} = 12\sqrt{13}$ cm

Therefore, the radius and slant height of the conical heap are 36 cm and $12\sqrt{13}$ cm respectively

Question 8:

Water in canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/h. how much area will it irrigate in 30 minutes, if 8 cm of standing water is needed?

Answer:



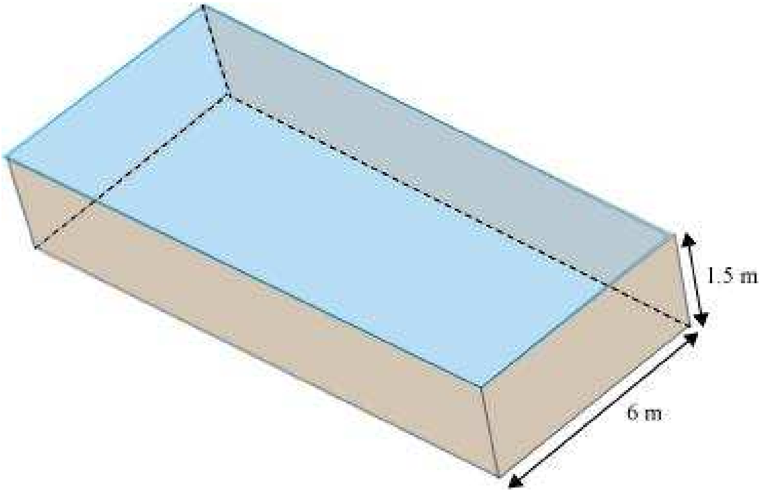
Consider an area of cross-section of canal as ABCD.

Area of cross-section = 6 × 1.5 = 9 m^2

Speed of water = 10 km/h = $\frac{10000}{60}$ metre/min

Volume of water that flows in 1 minute from canal = $9 \times \frac{10000}{60}$ =1500 m^3

Volume of water that flows in 30 minutes from canal = 30 × 1500 = 45000 m^3



Let the irrigated area be A. Volume of water irrigating the required area will be equal to the volume of water that flowed in 30 minutes from the canal.

Vol. of water flowing in 30 minutes from canal = Vol. of water irrigating the reqd. area

$$45000 = \frac{A \times 8}{100}$$

A = 562500 m^2

Therefore, area irrigated in 30 minutes is 562500 m^2.

Question 9:

A farmer connects a pipe of internal diameter 20 cm form a canal into a cylindrical tank in her field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/h, in how much time will the tank be filled?

Answer:

Consider an area of cross-section of pipe as shown in the figure.

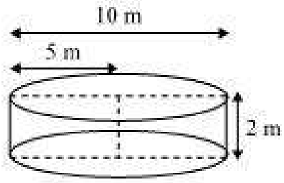
Radius (r_1) of circular end of pipe = $\frac{20}{200} = 0.1\text{ m}$

Area of cross-section = $\pi \times r_1^2 = \pi \times (0.1)^2 = 0.01\pi \text{ m}^2$

Speed of water = 3 km/h = $\frac{3000}{60} = 50 \text{ metre/min}$

Volume of water that flows in 1 minute from pipe = $50 \times 0.01\pi = 0.5\pi \text{ m}^3$

Volume of water that flows in t minutes from pipe = $t \times 0.5\pi \text{ m}^3$



Radius (r_2) of circular end of cylindrical tank = $\frac{10}{2} = 5$ m

Depth (h_2) of cylindrical tank = 2 m

Let the tank be filled completely in t minutes.

Volume of water filled in tank in t minutes is equal to the volume of water flowed in t minutes from the pipe.

Volume of water that flows in t minutes from pipe = Volume of water in tank

$t \times 0.5\pi = \pi \times (r_2)^2 \times h_2$

$t \times 0.5 = 5^2 \times 2$

$t = 100$

Therefore, the cylindrical tank will be filled in 100 minutes.

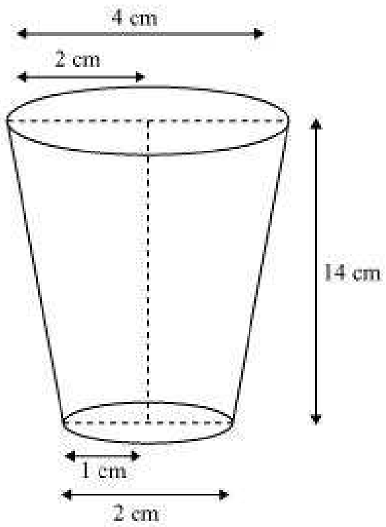
Exercise 13.4

Question 1:

A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm. Find the capacity of the glass.

$\left[\text{Use } \pi = \frac{22}{7}\right]$

Answer:



Radius (r_1) of upper base of glass $= \frac{4}{2} = 2$ cm

Radius (r_2) of lower base of glass $= \frac{2}{2} = 1$ cm

Capacity of glass = Volume of frustum of cone

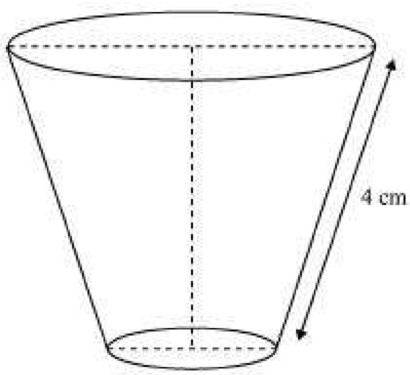
$$= \frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$$
$$= \frac{1}{3}\pi h\left[(2)^2 + (1)^2 + (2)(1)\right]$$
$$= \frac{1}{3} \times \frac{22}{7} \times 14\left[4+1+2\right]$$
$$= \frac{308}{3} = 102\frac{2}{3}\text{cm}^3$$

Therefore, the capacity of the glass is $102\frac{2}{3}\text{cm}^3$.

Question 2:

The slant height of a frustum of a cone is 4 cm and the perimeters (circumference) of its circular ends are 18 cm and 6 cm. find the curved surface area of the frustum.

Answer:



Perimeter of upper circular end of frustum = 18

$2\pi r_1 = 18$

$$r_1 = \frac{9}{\pi}$$

Perimeter of lower end of frustum = 6 cm

$2\pi r_2 = 6$

$r_2 = \frac{3}{\pi}$

Slant height (l) of frustum = 4

CSA of frustum = π (r_1 + r_2) l

$= \pi\left(\frac{9}{\pi}+\frac{3}{\pi}\right)4$

$= 12 \times 4$

$= 48\,\text{cm}^2$

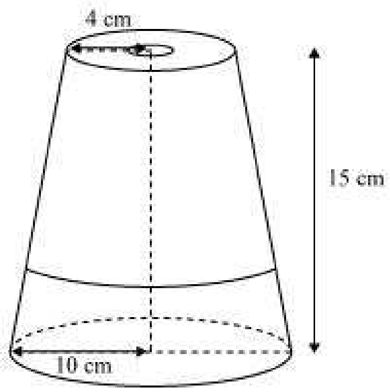
Therefore, the curved surface area of the frustum is 48 cm^2.

Question 3:

A *fez*, the cap used by the Turks, is shaped like the frustum of a cone (see the figure given below). If its radius on the open side is 10 cm, radius at the upper base is 4 cm and its slant height is 15 cm, find the area of material use for making it.

$\left[\text{Use } \pi = \frac{22}{7}\right]$

Answer:



Radius (r_2) at upper circular end = 4 cm

Radius (r_1) at lower circular end = 10 cm

Slant height (l) of frustum = 15 cm

Area of material used for making the fez = CSA of frustum + Area of upper circular end

$= \pi(r_1 + r_2)l + \pi r_2^2$

$= \pi(10 + 4)15 + \pi(4)^2$

$= \pi(14)15 + 16\pi$

$= 210\pi + 16\pi = \frac{226 \times 22}{7}$

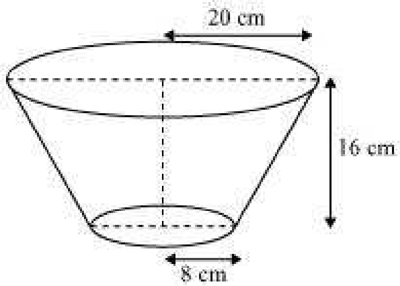
$= 710\frac{2}{7}\text{cm}^2$

Therefore, the area of material used for making it is $710\frac{2}{7}\text{cm}^2$.

Question 4:

A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of the milk which can completely fill the container, at the rate of Rs.20 per litre. Also find the cost of metal sheet used to make the container, if it costs Rs.8 per 100 cm^2. [Take π = 3.14]

Answer:



Radius (r_1) of upper end of container = 20 cm

Radius (r_2) of lower end of container = 8 cm

Height (h) of container = 16 cm

Slant height (l) of frustum = $\sqrt{(r_1 - r_2)^2 + h^2}$

$= \sqrt{(20-8)^2 + (16)^2}$

$= \sqrt{(12)^2 + (16)^2} = \sqrt{144 + 256}$

$= 20 \text{ cm}$

Capacity of container = Volume of frustum

$= \frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$

$= \frac{1}{3} \times 3.14 \times 16 \times \left[(20)^2 + (8)^2 + (20)(8)\right]$

$= \frac{1}{3} \times 3.14 \times 16(400 + 64 + 160)$

$= \frac{1}{3} \times 3.14 \times 16 \times 624$

$= 10449.92 \text{cm}^3$

$= 10.45 \text{ litres.}$

Cost of 1 litre milk = Rs 20

Cost of 10.45 litre milk = 10.45 × 20

= Rs 209

Area of metal sheet used to make the container

$= \pi\left(r_1+r_2\right)l+\pi r_2^2$

= π (20 + 8) 20 + π $(8)^2$

= 560 π + 64 π = 624 π cm^2

Cost of 100 cm^2 metal sheet = Rs 8

$$\text{Cost of } 624\ \pi\ \text{cm}^2 \text{ metal sheet} = \frac{624\times3.14\times8}{100}$$
$$=156.75$$

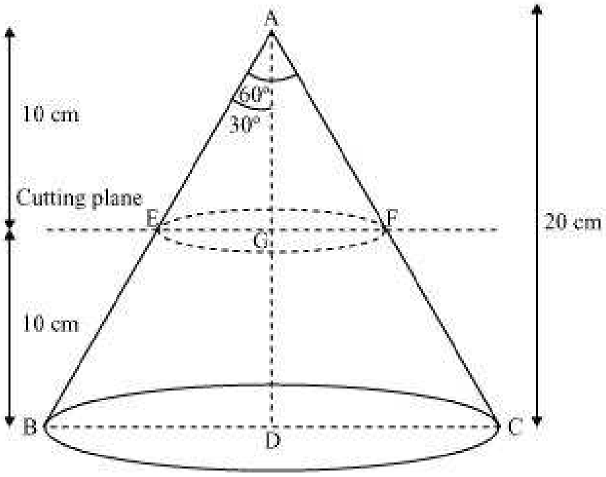
Therefore, the cost of the milk which can completely fill the container is Rs 209 and the cost of metal sheet used to make the container is Rs 156.75.

Question 5:

A metallic right circular cone 20 cm high and whose vertical angle is 60° is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained is drawn into a wire of diameter $\frac{1}{16}$ cm, find the length of the wire.

$\left[\text{Use } \pi=\frac{22}{7}\right]$

Answer:



In ΔAEG,

$$\frac{EG}{AG} = \tan 30°$$

$$EG = \frac{10}{\sqrt{3}} \text{cm} = \frac{10\sqrt{3}}{3}$$

In ΔABD,

$$\frac{BD}{AD} = \tan 30°$$

$$BD = \frac{20}{\sqrt{3}} = \frac{20\sqrt{3}}{3} \text{cm}$$

Radius (r_1) of upper end of frustum = $\frac{10\sqrt{3}}{3}$ cm

Radius (r_2) of lower end of container = $\frac{20\sqrt{3}}{3}$ cm

Height (h) of container = 10 cm

Volume of frustum $= \frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$

$$=\frac{1}{3}\times\pi\times10\left[\left(\frac{10\sqrt{3}}{3}\right)^2+\left(\frac{20\sqrt{3}}{3}\right)^2+\frac{(10\sqrt{3})(20\sqrt{3})}{3\times3}\right]$$

$$=\frac{10}{3}\pi\left[\frac{100}{3}+\frac{400}{3}+\frac{200}{3}\right]$$

$$=\frac{10}{3}\times\frac{22}{7}\times\frac{700}{3}=\frac{22000}{9}\text{cm}^3$$

Radius (r) of wire = $\frac{1}{16}\times\frac{1}{2}=\frac{1}{32}\text{cm}$

Let the length of wire be l.

Volume of wire = Area of cross-section × Length

= (πr^2) (l)

$$=\pi\left(\frac{1}{32}\right)^2\times l$$

Volume of frustum = Volume of wire

$$\frac{22000}{9}=\frac{22}{7}\times\left(\frac{1}{32}\right)^2\times l$$

$$\frac{7000}{9}\times1024=l$$

$$l=796444.44\ \text{cm}$$

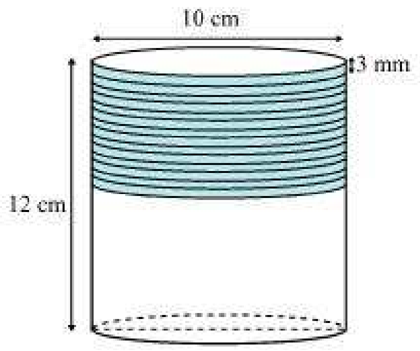
$$=7964.44\ \text{metres}$$

Exercise 13.5

Question 1:

A copper wire, 3 mm in diameter, is wound about a cylinder whose length is 12 cm, and diameter 10 cm, so as to cover the curved surface of the cylinder. Find the length and mass of the wire, assuming the density of copper to be 8.88 g per cm^3.

Answer:



It can be observed that 1 round of wire will cover 3 mm height of cylinder.

$$\text{Number of rounds} = \frac{\text{Height of cylinder}}{\text{Diameter of wire}}$$

$$= \frac{12}{0.3} = 40 \text{ rounds}$$

Length of wire required in 1 round = Circumference of base of cylinder

$= 2\pi r = 2\pi \times 5 = 10\pi$

Length of wire in 40 rounds = $40 \times 10\pi$

$$= \frac{400 \times 22}{7} = \frac{8800}{7}$$

= 1257.14 cm = 12.57 m

Radius of wire $= \frac{0.3}{2} = 0.15 \text{ cm}$

Volume of wire = Area of cross-section of wire × Length of wire

$= \pi(0.15)^2 \times 1257.14$

$= 88.898 \text{ cm}^3$

Mass = Volume × Density

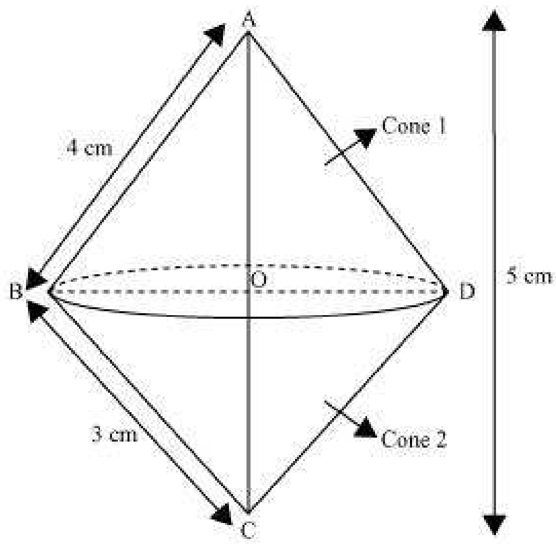
= 88.898 × 8.88

= 789.41 gm

Question 2:

A right triangle whose sides are 3 cm and 4 cm (other than hypotenuse) is made to revolve about its hypotenuse. Find the volume and surface area of the double cone so formed. (Choose value of π as found appropriate.)

Answer:



The double cone so formed by revolving this right-angled triangle ABC about its hypotenuse is shown in the figure.

Hypotenuse $AC = \sqrt{3^2 + 4^2}$

$= \sqrt{25}$ = 5 cm

Area of ΔABC $= \frac{1}{2} \times AB \times AC$

$$\frac{1}{2}\times AC\times OB=\frac{1}{2}\times 4\times 3$$

$$\frac{1}{2}\times 5\times OB=6$$

$$OB=\frac{12}{5}=2.4\text{ cm}$$

Volume of double cone = Volume of cone 1 + Volume of cone 2

$$=\frac{1}{3}\pi r^2 h_1+\frac{1}{3}\pi r^2 h_2$$

$$=\frac{1}{3}\pi r^2\left(h_1+h_2\right)=\frac{1}{3}\pi r^2\left(OA+OC\right)$$

$$=\frac{1}{3}\times 3.14\times\left(2.4\right)^2\left(5\right)$$

= 30.14 cm^3

Surface area of double cone = Surface area of cone 1 + Surface area of cone 2

= $\pi r l_1 + \pi r l_2$

$$=\pi r\left[4+3\right]=3.14\times 2.4\times 7$$

= 52.75 cm^2

Question 3:

A cistern, internally measuring 150 cm × 120 cm × 110 cm, has 129600 cm^3 of water in it. Porous bricks are placed in the water until the cistern is full to the brim. Each brick absorbs one-seventeenth of its own volume of water. How many bricks can be put in without overflowing the water, each brick being 22.5 cm × 7.5 cm × 6.5 cm?

Answer:

Volume of cistern = 150 × 120 × 110

= 1980000 cm^3

Volume to be filled in cistern = 1980000 − 129600

= 1850400 cm^3

Let n numbers of porous bricks were placed in the cistern.

Volume of n bricks = $n \times 22.5 \times 7.5 \times 6.5$

$= 1096.875n$

As each brick absorbs one-seventeenth of its volume, therefore, volume absorbed by these bricks $= \frac{n}{17}(1096.875)$

$$1850400 + \frac{n}{17}(1096.875) = (1096.875)n$$

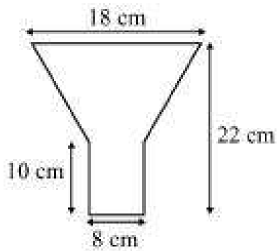
$$1850400 = \frac{16n}{17}(1096.875)$$

$n = 1792.41$

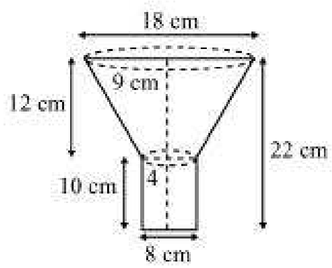
Therefore, 1792 bricks were placed in the cistern.

Question 5:

An oil funnel made of tin sheet consists of a 10 cm long cylindrical portion attached to a frustum of a cone. If the total height is 22 cm, diameter of the cylindrical portion is 8 cm and the diameter of the top of the funnel is 18 cm, find the area of the tin sheet required to make the funnel (see the given figure).



Answer:



Radius (r_1) of upper circular end of frustum part $= \frac{18}{2} = 9$ cm

Radius (r_2) of lower circular end of frustum part = Radius of circular end of cylindrical part

$= \frac{8}{2} = 4$ cm

Height (h_1) of frustum part = 22 − 10 = 12 cm

Height (h_2) of cylindrical part = 10 cm

Slant height (l) of frustum part $= \sqrt{(r_1 - r_2)^2 + h^2} = \sqrt{(9-4)^2 + (12)^2} = 13$ cm

Area of tin sheet required = CSA of frustum part + CSA of cylindrical part

$= \pi(r_1 + r_2)l + 2\pi r_2 h_2$

$= \frac{22}{7} \times (9+4) \times 13 + 2 \times \frac{22}{7} \times 4 \times 10$

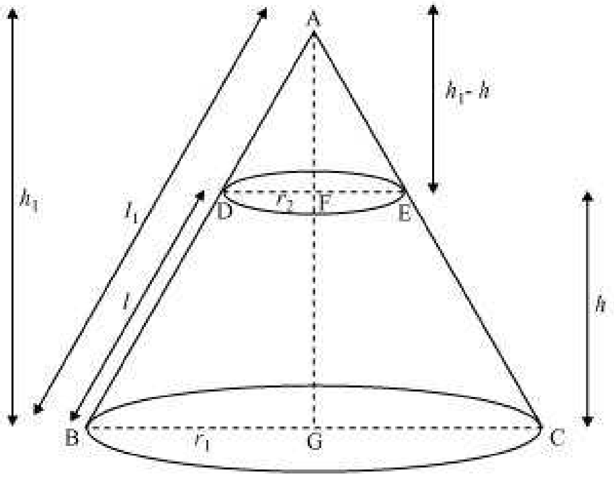
$= \frac{22}{7}[169 + 80] = \frac{22 \times 249}{7}$

$= 782\frac{4}{7}$ cm^2

Question 6:

Derive the formula for the curved surface area and total surface area of the frustum of cone.

Answer:



Let ABC be a cone. A frustum DECB is cut by a plane parallel to its base. Let r_1 and r_2 be the radii of the ends of the frustum of the cone and h be the height of the frustum of the cone.

In ΔABG and ΔADF, DF||BG

∴ ΔABG ~ ΔADF

$$\frac{DF}{BG} = \frac{AF}{AG} = \frac{AD}{AB}$$

$$\frac{r_2}{r_1} = \frac{h_1 - h}{h_1} = \frac{l_1 - l}{l_1}$$

$$\frac{r_2}{r_1} = 1 - \frac{h}{h_1} = 1 - \frac{l}{l_1}$$

$$1 - \frac{l}{l_1} = \frac{r_2}{r_1}$$

$$\frac{l}{l_1} = 1 - \frac{r_2}{r_1} = \frac{r_1 - r_2}{r_1}$$

$$\frac{l_1}{l} = \frac{r_1}{r_1 - r_2}$$

$$l_1 = \frac{r_1 l}{r_1 - r_2}$$

CSA of frustum DECB = CSA of cone ABC – CSA cone ADE

$$= \pi r_1 l_1 - \pi r_2 \left(l_1 - l\right)$$

$$= \pi r_1 \left(\frac{l r_1}{r_1 - r_2}\right) - \pi r_2 \left[\frac{r_1 l}{r_1 - r_2} - l\right]$$

$$= \frac{\pi r_1^2 l}{r_1 - r_2} - \pi r_2 \left(\frac{r_1 l - r_1 l + r_2 l}{r_1 - r_2}\right)$$

$$= \frac{\pi r_1^2 l}{r_1 - r_2} - \frac{\pi r_2^2 l}{r_1 - r_2}$$

$$= \pi l \left[\frac{r_1^2 - r_2^2}{r_1 - r_2}\right]$$

CSA of frustum = $\pi\left(r_1 + r_2\right) l$

Total surface area of frustum = CSA of frustum + Area of upper circular end + Area of lower circular end

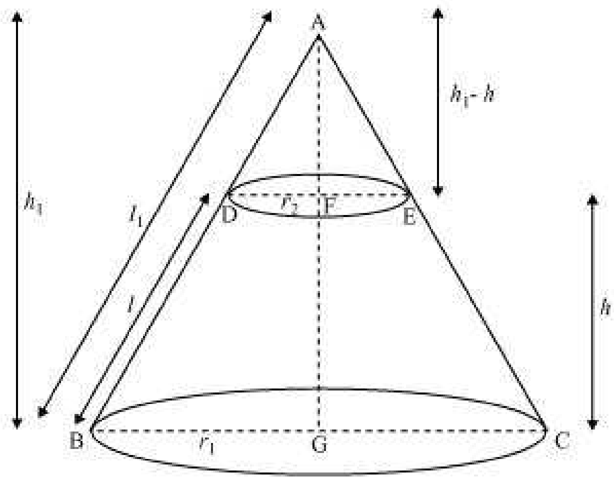
$$= \pi\left(r_1 + r_2\right) l + \pi r_2^2 + \pi r_1^2$$

$$= \pi\left[\left(r_1 + r_2\right) l + r_1^2 + r_2^2\right]$$

Question 7:

Derive the formula for the volume of the frustum of a cone.

Answer:



Let ABC be a cone. A frustum DECB is cut by a plane parallel to its base.

Let r_1 and r_2 be the radii of the ends of the frustum of the cone and h be the height of the frustum of the cone.

In ΔABG and ΔADF, DF||BG

∴ ΔABG ~ ΔADF

$$\frac{DF}{BG} = \frac{AF}{AG} = \frac{AD}{AB}$$

$$\frac{r_2}{r_1} = \frac{h_1 - h}{h_1} = \frac{l_1 - l}{l_1}$$

$$\frac{r_2}{r_1} = 1 - \frac{h}{h_1} = 1 - \frac{l}{l_1}$$

$$1-\frac{h}{h_1}=\frac{r_2}{r_1}$$

$$\frac{h}{h_1}=1-\frac{r_2}{r_1}=\frac{r_1-r_2}{r_1}$$

$$\frac{h_1}{h}=\frac{r_1}{r_1-r_2}$$

$$h_1=\frac{r_1h}{r_1-r_2}$$

Volume of frustum of cone = Volume of cone ABC – Volume of cone ADE

$$=\frac{1}{3}\pi r_1^2h_1-\frac{1}{3}\pi r_2^2\left(h_1-h\right)$$

$$=\frac{\pi}{3}\left[r_1^2h_1-r_2^2\left(h_1-h\right)\right]$$

$$=\frac{\pi}{3}\left[r_1^2\left(\frac{hr_1}{r_1-r_2}\right)-r_2^2\left(\frac{hr_1}{r_1-r_2}-h\right)\right]$$

$$=\frac{\pi}{3}\left[\left(\frac{hr_1^3}{r_1-r_2}\right)-r_2^2\left(\frac{hr_1-hr_1+hr_2}{r_1-r_2}\right)\right]$$

$$=\frac{\pi}{3}\left[\frac{hr_1^3}{r_1-r_2}-\frac{hr_2^3}{r_1-r_2}\right]$$

$$=\frac{\pi}{3}h\left[\frac{r_1^3-r_2^3}{r_1-r_2}\right]$$

$$=\frac{\pi}{3}h\left[\frac{\left(r_1-r_2\right)\left(r_1^2+r_2^2+r_1r_2\right)}{r_1-r_2}\right]$$

$$=\frac{1}{3}\pi h\left[r_1^2+r_2^2+r_1r_2\right]$$