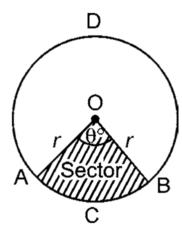
Assignments in Mathematics Class X (Term II)

12. AREAS RELATED TO CIRCLES

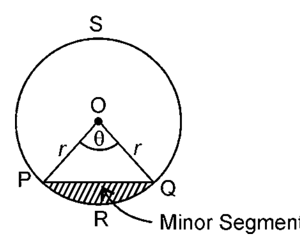
IMPORTANT TERMS, DEFINITIONS AND RESULTS

- Perimeter (circumference) of a circle with diameter d ($d = 2r$, where r is the radius) is given by $C = \pi d = 2\pi r$
- Perimeter of semicircle with radius r $= 2r + \pi r = r(\pi + 2)$
- Area of a circle with radius r is given by $A = \pi r^2$.
- Area of a semicircle of radius $r = \dfrac{\pi r^2}{2}$.
- Area of a ring whose outer and inner radii are R and r respectively $= \pi(R^2 - r^2) = \pi(R + r)(R - r)$
- If two circles touch internally, then the distance between their centres is equal to the difference of their radii.
- If two circles touch externally, then the distance between their centres is equal the sum of their radii.
- The distance moved by a rotating wheel in one revolution is equal to the circumference of the wheel.
- The number of revolutions completed by a rotating wheel in one minute
$$= \frac{\text{Distance moved in one minute}}{\text{Circumference of the wheel}}$$
- Length of an arc which subtends an angle of $\theta°$ at the centre $= \dfrac{2\pi r\theta°}{360°} = \dfrac{\pi r\theta°}{180°}$.
- Sector of a circle is a region enclosed by an arc of a circle and its two bounding radii.



  (i) Area of sector OACBO $= \dfrac{\pi r^2\theta°}{360°}$.

  (ii) Perimeter of sector OACBO $= 2r + \dfrac{2\pi r\theta°}{360°}$.
- **Minor sector :** A sector of a circle is called a **minor sector** if the **minor arc** of the circle is a part of its boundary. In the above figure **minor sector** is OACB.
- **Major sector :** A sector of a circle is called a **major sector,** if the **major arc** of the circle is a part of its boundary. In the above figure, OADB is the major sector.
- The sum of the arcs of major and minor sectors of a circle is equal to the circumference of the circle.
- The sum of the areas of major and minor sectors of a circle is equal to the area of the circle.
- The area of a sector is given by
$$A = \frac{1}{2}lr, \text{ where } l = \left(\frac{\theta r}{180°} \times \pi\right)$$
- Angle described by minute hand in 60 minutes $= 360°$.
$\therefore$ angle described by minute hand in one minute
$$= \left(\frac{360}{60}\right)^{\circ} = 6°.$$
Thus, the minute hand rotates through an angle of 6° in one minute.
- Angle described by hour hand in 12 hours = 360°.
$\therefore$ angle described by hour hand in 1 hour
$$= \left(\frac{360}{12}\right)^{\circ} = 30°.$$
Angle described by hour hand in one minute
$$= \left(\frac{30}{60}\right)^{\circ} = \frac{1}{2}^{\circ}$$
Thus, hour hand rotates through $\dfrac{1}{2}^{\circ}$ in 1 minute.
- A segment of a circle is the region bounded by an arc and a chord, including the arc and the chord.
- **Minor segment :** If the boundary of a segment is a minor arc of a circle, then the corresponding segment is called a minor segment. In the figure, segment PQR (the area which is shaded) is a minor segment.



- **Major segment :** A segment corresponding a major arc of a circle is known as the major segment. In the figure above, segment PQSP is a major segment.
- Area of minor segment PRQS
$$= \frac{\pi r^2\theta°}{360} - \frac{1}{2}r^2 \sin\theta$$
- Area of major segment PSQ $= \pi r^2$ – area of minor segment PRQ.

SUMMATIVE ASSESSMENT

MULTIPLE CHOICE QUESTIONS [1 Mark]

A. Important Questions

1. The area of a sector is given by :

(a) $\frac{1}{2}lr$, where $l = \left(\frac{\theta r}{180°} \times \pi\right)$

(b) $2lr$, where $l = \left(\frac{\theta r}{180°} \times \pi\right)$

(c) $\frac{1}{2}\frac{l}{r}$, where $l = \left(\frac{\theta r}{180°} \times \pi\right)$

(d) none of these

2. Hour hand rotates through in one minute :

(a) $\frac{1}{2}^{\circ}$ (b) $2°$ (c) $22°$ (d) $\frac{22}{7}^{\circ}$

3. A region in the circle, bounded by an arc and a chord, including the arc and the chord is :

(a) sector (b) segment

(c) minor arc (d) major arc

4. Perimeter of a quadrant of a circle of radius r is equal to :

(a) $\pi r + 2r$ (b) $2r + \frac{\pi r}{2}$

(c) $2\pi r + 2r$ (d) $r + \frac{\pi r}{2}$

5. Area of a quadrant of a circle of radius r is given by :

(a) $\frac{\pi r^2}{2}$ (b) $r^2 + \pi r^2$

(c) $\frac{\pi r^2}{4}$ (d) $2r + \frac{\pi r^2}{4}$

6. If a circle of radius 7 cm is divided into 10 equal parts, then the area of each sector is :

(a) 14.5 cm^2 (b) 15.4 cm^2

(c) 15.6 cm^2 (d) 16.5 cm^2

7. If the diameter of the wheel of a cycle is 7cm, then its area is :

(a) 77 cm^2 (b) 22 cm^2

(c) $\frac{77}{2}$ cm^2 (d) 770 cm^2

8. If the radius of a circle is doubled, then area of the circle becomes :

(a) double (b) triple (c) four times (d) same

9. The ratio of the area and circumference of a circle of radius 4 cm is :

(a) 4 : 1 (b) 2 : 1 (c) 1 : 2 (d) 8 : 1

10. The diameter of a circle of area 154 cm^2 is :

(a) 7 cm (b) 14 cm

(c) 21 cm (d) $7\sqrt{2}$ cm

11. If 100 π cm^2 is the area of a circle, then its circumference is :

(a) 50π cm (b) 20π cm

(c) 10π cm (d) 25π cm

12. The area of a circle of circumference $\sqrt{12}$ is :

(a) 3π (b) $\frac{3}{p}$ (c) 2π (d) $\frac{2}{p}$

13. If the area of a sector of a circle of radius 6 cm is 9π cm, then the angle subtended at the centre of the circle is :

(a) 30° (b) 60° (c) 90° (d) 120°

14. If a copper wire of length 88 cm is bent in the form of a circle, then the radius of the circle is :

(a) 7 cm (b) 14 cm

(c) $\frac{77}{2}$ cm (d) none of these

15. The area of the circle that can be inscribed in a square of side 6 cm is :

(a) 36π cm^2 (b) 18π cm^2

(c) 12π cm^2 (d) 9π cm^2

16. The area of the square that can be inscribed in a circle of radius 8 cm is :

(a) 256 cm^2 (b) 128 cm^2

(c) $64\sqrt{2}$ cm^2 (d) 64 cm^2

17. The radius of a circle whose circumference is equal to the sum of the circumferences of the two circles of diameters 36 cm and 20 cm is :

(a) 56 cm (b) 42 cm

(c) 28 cm (d) 16 cm

18. The diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 24 cm and 7 cm is :

(a) 31 cm (b) 25 cm

(c) 62 cm (d) 50 cm

19. If the perimeter and the area of a circle are numerically equal, then the radius of the circle is :

(a) 2 units (b) π units (c) 4 units (d) 7 units

20. A steel wire, when bent in the form of a square, enclosed an area of 121 cm^2. The same wire is bent in the form of a circle. The area of the circle is :

(a) 154 cm^2 (b) 145 cm^2
(c) 451 cm^2 (d) 541 cm^2

21. A race track is in the form of a ring whose inner and outer circumferences are 437 m and 503 m respectively. The width of the track is :

(a) 10.5 m (b) 20.5 m

(c) 21 m (d) 30 m

22. Two circles touch internally. The sum of their areas is 116π cm^2 and the distance between their centres is 6 cm. The radii of the circles are :

(a) 4 cm and 9 cm (b) 4 cm and 10 cm

(c) 5 cm and 10 cm (d) 4 cm and 20 cm

23. If the sum of the areas of two circles with radii R_1 and R_2 is equal to the area of a circle of radius R, then :

(a) $R_1 + R_2 = R$ (b) $R_1^2 + R_2^2 = R^2$

(c) $R_1 + R_2 < R$ (d) $R_1^2 + R_2^2 < R^2$

24. If the sum of the circumferences of two circles with radii R_1 and R_2 is equal to the circumference of a circle of radius R, then :

(a) $R_1 + R_2 = R$

(b) $R_1 + R_2 > R$

(c) $R_1 + R_2 < R$

(d) nothing definite can be said about the relation among R_1, R_2 and R.

25. If the circumference of a circle and the perimeter of a square are equal, then :

(a) Area of the circle = Area of the square

(b) Area of the circle > Area of the square

(c) Area of the circle < Area of the square

(d) Nothing definite can be said about the relation between the areas of the circle and square.

26. Area of the largest triangle that can be inscribed in a semi-cricle of radius *r* units is :

(a) r^2 sq. units (b) $\frac{1}{2}r^2$ sq. units

(c) $2r^2$ sq. units (d) $\sqrt{2}\,r^2$ sq. units

27. If the perimeter of a circle is equal to that of a square, then the ratio of their areas is :

(a) 22 : 7 (b) 14 : 11 (c) 7 : 22 (d) 11 : 14

28. It is proposed to build a single circular park equal in area to the sum of areas of two circular parks of diameters 16 m and 12 m in the locality. The radius of the new park would be :

(a) 10 m (b) 15 m (c) 20 m (d) 24 m

29. The length of the minute hand of a clock is 6 cm. The area swept by the minute hand in 10 minutes is:

(a) 12π cm^2 (b) 36π cm^2

(c) 9π cm^2 (d) 6π cm^2

30. If we decrease the radius of a circle by 20%, then its circumference will be reduced by :

(a) 40% (b) 10% (c) 20% (d) 50%

31. If we increase the radius of a circle by 40%, then its area will be increased by :

(a) 80% (b) 90% (c) 96% (d) 40%

32. The perimeter of the sector of a circle whose central angle is 45° and radius 7 cm is :

(a) 39 cm (b) 19.5 cm

(c) 35 cm (d) 17.5 cm

33. If a quadrant is cut off from the circle of circumference 44 cm, then area of the remaining portion is :

(a) 120 cm^2 (b) 115.5 cm^2

(c) 125.5 cm^2 (d) none of these

34. The minute hand of a clock is of length 5 cm. If it moves from 9 to 12, then the angle swept by it is :

(a) 60° (b) 90° (c) 120° (d) 150°

35. The length of a wire in the form of an equilateral triangle is 44 cm. If it is rebent into the form of a circle, then area of the circle is :

(a) 484 cm^2 (b) 176 cm^2

(c) 154 cm^2 (d) 144 cm^2

36. Assume that an umbrella is a flat circle of radius 40 cm. If the umbrella has 8 ribs, then the area of a rib is :

(a) 160 π cm^2 (b) 180 π cm^2

(c) 200 π cm^2 (d) 240 π cm^2

B. Questions From CBSE Examination Papers

1. Area of a quadrant of a circle whose circumference is 22 cm is : $\left(\pi = \frac{22}{7}\right)$ **[2011 (T-II)]**

(a) 3.5 cm^2 (b) 3.5 cm
(c) 9.625 cm^2 (d) 17.25 cm^2

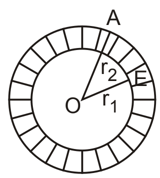
2. If the diameter of a semicircular protractor is 14 cm, then the perimeter of the protractor is : **[2011 (T-II)]**

(a) 26 cm (b) 14 cm
(c) 28 cm (d) 36 cm

3. The perimeter of a quadrant of a circle of radius $\frac{7}{2}$ cm is : **[2011 (T-II)]**

(a) 3.5 cm (b) 5.5 cm
(c) 7.5 cm (d) 12.5 cm

4. In the figure, area of shaded region is : **[2011 (T-II)]**



(a) $\pi(r_1 + r_2)$ (b) $\pi(r_1^2 + r_2^2)$
(c) $\pi(r_1 - r_2)$ (d) $\pi(r_2^2 - r_1^2)$

5. If a wire is bent into the shape of a square, then the area enclosed by the square is 81 cm^2. When the same wire is bent into a semi-circular shape, then the area enclosed by the semi circle will be : **[2011 (T-II)]**

(a) 22 cm^2 (b) 44 cm^2
(c) 77 cm^2 (d) 154 cm^2

6. The circumference of a circle is 100 cm. The side of a square inscribed in the circle is : **[2011 (T-II)]**

(a) $50\sqrt{2}$ cm (b) $\frac{100}{p}$ cm
(c) $\frac{50\sqrt{2}}{p}$ cm (d) $\frac{100\sqrt{2}}{p}$ cm

7. If the perimeter of a semicircular protractor is 36 cm, then its diameter is : **[2011 (T-II)]**

(a) 10 cm (b) 12 cm (c) 14 cm (d) 15 cm

8. The diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 40 cm and 9 cm is : **[2011 (T-II)]**

(a) 41 cm (b) 49 cm (c) 82 cm (d) 62 cm

9. The area of a sector of central angle $x°$ of a circle with radius $4r$ is : **[2011 (T-II)]**

(a) $\frac{4\pi x}{360°}$ (b) $\frac{2\pi x r^2}{45°}$
(c) $\frac{\pi r^2 x}{360°}$ (d) $\frac{2\pi r x}{360°}$

10. The area of a circle whose circumferenc is 44 cm is : **[2011 (T-II)]**

(a) 152 cm^2 (b) 153 cm^2
(c) 154 cm^2 (d) 150 cm^2

11. If the circumference of a circle of radius r and the perimeter of a square of side 'a' are equal, then the ratio of area of the circle to that of the square is : **[2011 (T-II)]**

(a) 4 : π (b) π : 4 (c) $p^2 : 16$ (d) $\pi^2 : 4$

12. The number of rounds that a wheel of diameter $\frac{7}{11}$ m will make in going 4 km is : **[2011 (T-II)]**

(a) 1500 (b) 1700 (c) 2000 (d) 2500

13. Two parallel lines touch the circle at points A and B respectively. If area of the circle is 25 π cm^2, then AB is equal to : **[2011 (T-II)]**

(a) 5 cm (b) 8 cm (c) 10 cm (d) 25 cm

14. The minute hand of a clock is 21 cm long. The distance moved by the tip of the minute hand in 1 hour is : **[2011 (T-II)]**

(a) 21 π cm (b) 42 π cm
(c) 10.5 π cm (d) 7 π cm

15. The perimeter of a quadrant of a circle of radius r is : **[2011 (T-II)]**

(a) $\frac{\pi r}{2}$ (b) $2\pi r$
(c) $\frac{r}{2}[\pi + 4]$ (d) $2\pi r + \frac{r}{2}$

16. The area of the circle that can be inscribed in a square of side 6 cm is : **[2011 (T-II)]**

(a) 36π cm^2 (b) 18π cm^2
(c) 12π cm^2 (d) 9π cm^2

17. A wire is in the shape of a circle of radius 21 cm. It is bent to form a square. The side of the square is $\left[\pi = \frac{22}{7}\right]$: **[2011 (T-II)]**

(a) 22 cm (b) 33 cm (c) 44 cm (d) 66 cm

18. The outer and inner diameters of a circular ring are 34 cm and 32 cm respectively. The area of the ring is : **[2011 (T-II)]**

(a) 66 π (b) 60 π (c) 33 π (d) 29 π

19. If the circumference of a circle increases from 2π to 4π, then its area is : **[2011 (T-II)]**

(a) halved (b) doubled

(c) tripled (d) four times

20. The angle through which the minute hand of the clock moves from 8 to 8 : 35 is : **[2011 (T-II)]**

(a) 210° (b) 90°

(c) 60° (d) 45°

21. If the area and circumference of a circle are numerically equal, then the diameter of the circle is : **[2011 (T-II)]**

(a) 3 units (b) 5 units

(c) 4 units (d) 2 units

SHORT ANSWER TYPE QUESTIONS [2 Marks]

A. Important Questions

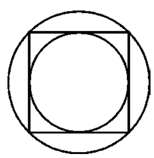
1. Will it be true to say that the perimeter of a square circumscribing a circle of radius a cm is $8a$ cm? Give reasons for your answer.

2. Is the following statement true? Give reasons for your answer.

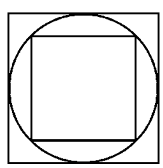
Area of a segment of a circle
= area of the corresponding sector – area of the corresponding triangle

3. Is the area of the circle inscribed in a square of side a cm, πa^2 cm^2? Give reasons for your answer.

4. In the figure, a circle is inscribed in a square of side 5 cm and another circle is circumscribing the square. Is it true to say that area of the outer circle is two times the area of the inner circle? Give reasons for your answer.



5. In the figure, a square is inscribed in a circle of diameter d and another square is circumscribing the circle. Is the area of the outer square four times the area of the inner square? Give reasons for your answer.



6. Is it true to say that area of a segment of a circle is less than the area of its corresponding sector ? Why?

7. In covering a distance s metres, a circular wheel of radius r metres makes $\frac{s}{2\pi r}$ revolutions. Is this statement true ? Why ?

8. The numerical value of the area of a circle is greater than the numerical value of its circumference. Is this statement true ? Why ?

9. If the length of an arc of a circle r is equal to that of an arc of a circle of radius $2r$, then the angle of the corresponding sector of the first circle is double the angle of the corresponding sector of the other circle. Is this statement false ? Why ?

10. The areas of two sectors of two different circles with equal corresponding arc lengths are equal. Is this statement true ? Why ?

11. The areas of two sectors of two different circles are equal. Is it necessary that their corresponding arc lengths are equal ? Why ?

12. Is the area of the largest circle that can be drawn inside a rectangle of length a cm and breadth b cm ($a > b$) is πb^2 cm^2 ? Why ?

13. Circumferences of two circles are equal. Is it necessary that their areas be equal ? Why ?

14. Areas of two circles are equal. Is it necessary that their circumferences are equal ? Why ?

15. Is it true to say that area of a square inscribed in a circle of diameter p cm is p^2 cm^2 ? Why ?

16. A bucket is raised from a well by means of a rope which is wound round a wheel of diameter 77 cm. Given that the bucket ascends in 1 minute 28 seconds with a uniform speed of 1.1 m/s. Calculate the number of complete revolutions the wheel makes in raising the bucket.

17. Diameters of three concentric circles are in the ratio 1 : 2 : 3. The sum of the circumferences of these circles is 264 cm. Find the area enclosed between second and third circles.

18. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding (i) minor segment (ii) major sector. [Use π = 3.14].

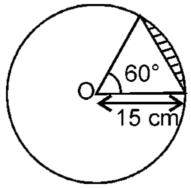
19. Find the area of both the segments of a circle of radius 15 cm, one of which makes an angle of 60° at the centre of the circle.

20. A chord 10 cm long is drawn in a circle whose radius is $5\sqrt{2}$ cm. Find the area of major segment.

21. A chord of a circle of radius 28 cm subtends an angle 45° at the centre of the circle. Find the area of the minor segment.

22. The perimeter of a sector of a circle with central angle 90° is 25 cm. Find the area of the minor segment of the circle.

23. Find the area of the shaded portions of the following figure with given measurements :

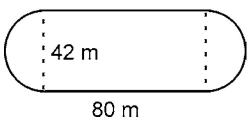


24. The diameter of a circular pond is 17.5 m. It is surrounded by a path of width 3.5 m. Find the area of the path.

25. A sector is cut from a circle of radius 21 cm. The angle of the sector is 150°. Find the area of the sector.

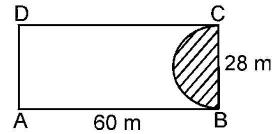
26. The diameter of the wheel of a bus is 140 cm. How many revolutions per minute must the wheel make in order to keep a speed of 66 km/hr ?

27. A playground is in the form of a rectangle having semicircles on the shorter sides. Find its area when the length of the rectangular portion is 80 m and the breadth is 42 m.



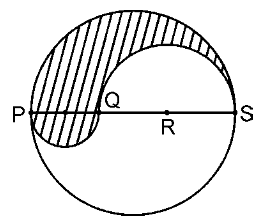
28. A bicycle wheel makes 500 revolutions in moving 22 km. Find the diameter of the wheel. $\left(\text{Take } \pi = \frac{22}{7}\right)$

29. A plot is in the form of a rectangle ABCD having semicircle on BC as shown in the figure. The semicircle portion is grassy while the remaining plot is without grass. Find the area of the plot without grass where AB = 60 m and BC = 28 m.



30. Find the area of a right angled triangle if the radius of its circumcircle is 2.5 cm and the long altitude drawn to the hypotenuse is 2 cm long.

31. PQRS is a diameter of a circle of radius 6 cm. The lengths PQ, QR and RS are equal. Semi-circles are drawn on PQ and QS as diameters. Find the perimeter of the shaded region.



32. OABC is a rhombus whose three vertices A, B and C lie on a circle with centre O. If the radius of the circle is 10 cm, find the area of the rhombus.

B. Questions From CBSE Examination Papers

1. The difference between circumference and diameter of a circle is 135 cm. Find the radius of the circle. $\left[\text{Take } \pi = \frac{22}{7}\right]$ **[2011 (T-II)]**

2. The length of the minute hand of a clock is 7 cm. Find the area swept by the minute hand from 6.00 pm to 6.10 pm. **[2011 (T-II)]**

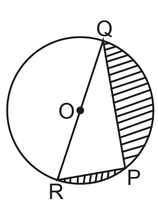
3. The sum of circumferences of two circles is 132 cm. If the radius of one circle is 14 cm, find the radius of the second circle. **[2011 (T-II)]**

4. What will be the increase in area of a circle, if its radius is increased by 40%? **[2011 (T-II)]**

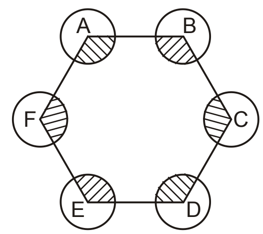
5. The radius of the wheels of a bus is 70 cm. How many revolutions per minute must a wheel make in order to move at a speed of 66 km/h? **[2011 (T-II)]**

6. What will be the ratio of perimeters of a square and a circle if their areas are equal? **[2011 (T-II)]**

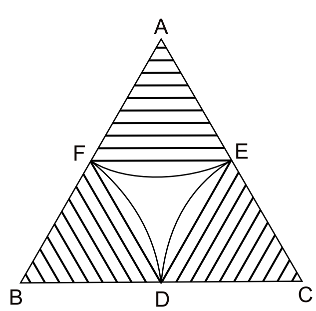
7. Find the area of the shaded region, if PQ = 24 cm, PR = 7 cm and O is the centre of the circle. **[2011 (T-II)]**



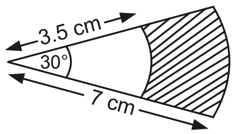
8. In the figure, ABCDEF is any regular hexagon. With different vertices A, B, C, D, E and F as the centres of circles with same radius r are drawn. Find area the of the shaded portion. **[2011 (T-II)]**



9. In the figure, arcs are drawn by taking vertices A, B and C of an equilateral triangle of side 10 cm, to intersect the sides BC, CA and AB at their respective mid-points D, E and F. Find the area of the shaded region (Use $\pi = 3.14$) **[2011 (T-II)]**



10. In the figure, sectors of two concentric circles of radii 7 cm and 3.5 cm are given. Find the area of shaded region. $\left(\text{use}\,\pi = \frac{22}{7}\right)$ **[2011 (T-II)]**



11. The perimeter of a sector of a circle of radius 5.6 cm is 27.2 cm. Find the area of the sector. **[2011 (T-II)]**

12. A horse is tied to a peg at one corner of a square shaped grass field of side 25 m by means of a 14 m long rope. Find the area of the part of the field in which the horse can graze $\left[\text{Take}\,\pi = \frac{22}{7}\right]$ **[2011 (T-II)]**

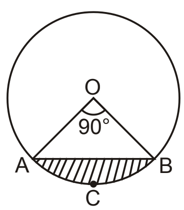
13. Area of a sector of a circle of radius 36 cm is 54π cm^2. Find the length of corresponding arc of sector. **[2011 (T-II)]**

14. The minute hand of a clock is $\sqrt{21}$ cm long. Find the area swept by the minute hand on the face of the clock from 7.00 am to 7.05 am. **[2011 (T-II)]**

15. The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at the speed of 66 km/hour ? **[2011 (T-II)]**

16. Find the area of the quadrant of that circle whose circumference is 22 cm $\left(\text{use}\,\pi = \frac{22}{7}\right)$ **[2011 (T-II)]**

17. In the figure, the chord AB of a circle of radius 10 cm subtends an angle of 90° at the centre O. Find the area of the segment ACBA. (Take $\pi = 3.14$) **[2011 (T-II)]**

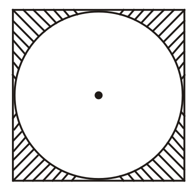


18. A wheel has diameter 84 cm. Find how many complete revolutions must it make to cover 792 m. **[2011 (T-II)]**

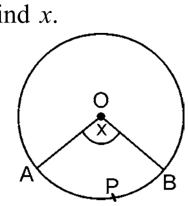
19. How many times will the wheel of a car rotate in a journey of 2002 m, if the radius of the wheel is 49 cm? **[2011 (T-II)]**

20. A chord of a circle of radius 12 cm subtends an angle of 60° at the centre. Find the area of the corresponding segment of the circle. [use $\pi = 3.14$ and $\sqrt{3} = 1.73$] **[2011 (T-II)]**

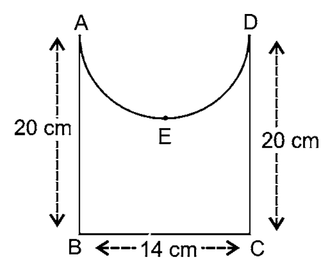
21. In the figure, a circle of radius 7 cm is inscribed in a square. Find the area of the shaded portion. $\left[\text{use}\,\pi = \frac{22}{7}\right]$ **[2011 (T-II)]**



22. In the given figure, O is the centre of a circle. The area of sector OAPB is $\frac{5}{18}$ of the area of the circle. Find x. **[2008]**



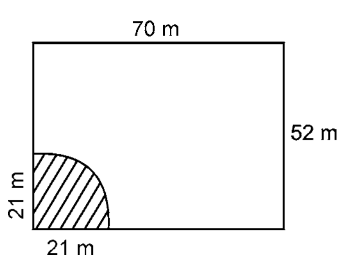
23. Find the perimeter of the given figure, where $\widehat{AED}$ is a semi-circle and ABCD is a rectangle. **[2008]**



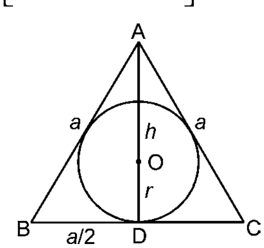
SHORT ANSWER TYPE QUESTIONS [3 Marks]

A. Important Questions

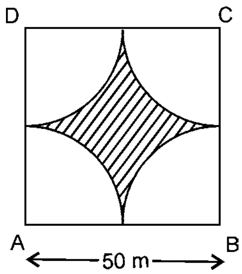
1. A horse is placed for grazing inside a rectangular field 70 m by 52 m. It is tethered to one corner by a rope 21 m long. On how much area can it graze? How much area is left ungrazed?



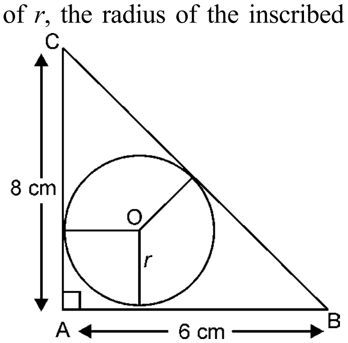
2. The area of a circle inscribed in an equilateral triangle is 154 cm^2. Find the perimeter of the triangle. $\left[\text{Take}\,\sqrt{3} = 1.73\right]$



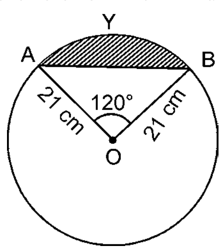
3. Four cows are tethered at the four corners of a square field of side 50 m such that each can graze the maximum unshaded area. What area will be left ungrazed? (Take π = 3.14)



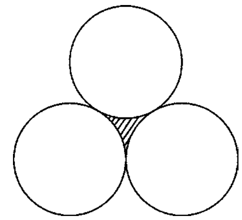
4. In the figure, ΔABC is right angled at A, with AB = 6 cm and AC = 8 cm. A circle with centre O has been inscribed inside the triangle. Find the value of r, the radius of the inscribed circle.



5. Find the area of the segment AYB in the figure, if the radius of the circle is 21 cm and $\angle$AOB = 120°.

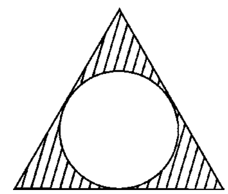


6. Three equal circles each of radius 6 cm touch one another as shown. Find the area enclosed between them.



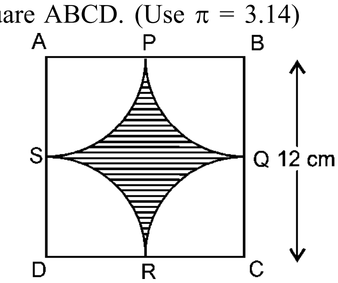
7. In an equilateral triangle of side 12 cm, a circle is inscribed touching its sides.

Find the area of the portion of the triangle not included in the circle.



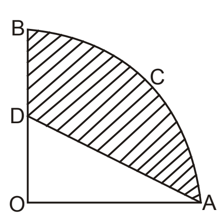
8. Prove that the area of a circular path of uniform width k surrounding a circular region of radius r is $\pi k(k + 2r)$

9. Find the area of the shaded region in the given figure, where arcs drawn with centres A, B, C and D intersect in pairs at mid points P, Q, R and S of the sides AB, BC, CD and DA, respectively of a square ABCD. (Use π = 3.14)

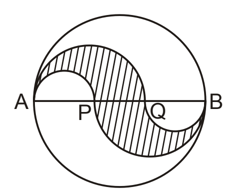


B. Questions From CBSE Examination Papers

1. In the figure, OACBO represents a quadrant of a circle of radius 7 cm with centre at O. If OD = 5 cm, find the area of the shaded region. **[2011 (T-II)]**

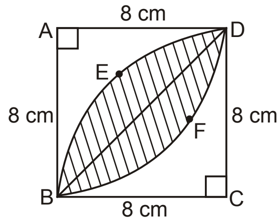


2. In the figure, diameter AB is 12 cm long. AB is trisected at points P and Q. Find the area of the shaded region. **[2011 (T-II)]**



3. A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding minor segment of the circle. (Use π = 3.14) **[2011 (T-II)]**

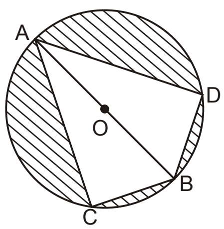
4. In the figure, ABCD is a square of side 8 cm. CBED and ADFB are quadrants of circle. Find the area of the shaded region. (Use π = 3.14) **[2011 (T-II)]**



5. The length of a rope by which a cow is tethered is increased from 16 m to 23 m. How much additional area can the cow graze now? $\left(\text{use } \pi = \frac{22}{7}\right)$ **[2011 (T-II)]**

6. The radii of two circles are 4 cm and 3 cm. Find the radius of the circle whose area is equal to the sum of the areas of the two circles. Also find the circumference of this circle. **[2011 (T-II)]**

7. Find the area of the shaded region in the figure, if BC = BD = 8 cm, AC = AD = 15 cm and O is the centre of the circle. (Take π = 3.14) **[2011 (T-II)]**

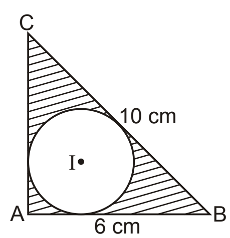


8. In a circle of radius 12 cm, an arc subtends an angle of 60° at the centre. Find

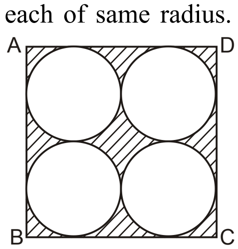
(i) Area of sector formed by the arc

(ii) Area of the segment formed by the corresponding chord. **[2011 (T-II)]**

9. In the figure, ABC is a triangle right angled at A. Find the area of the shaded region if AB = 6 cm, BC = 10 cm and *I* is the centre of incircle of ΔABC. **[2011 (T-II)]**



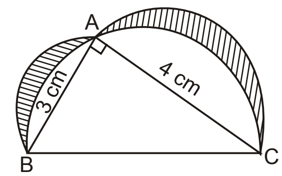
10. Find the area of the shaded region in the figure, where ABCD is a square of side 14 cm and four circles are each of same radius. **[2011 (T-II)]**



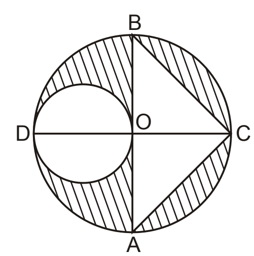
11. A square of side 4 cm is inscribed in a circle. Find the area enclosed between the circle and the square. $\left[\pi = \frac{22}{7}\right]$ **[2011 (T-II)]**

12. A piece of wire that has been bent in the form of a semicircle including the bounding diameter is straightened and then bent in the form the of a square. The diameter of the semicircle is 14 cm. Which has a larger area, the semi-circle or the square? Also, find the difference between them. **[2011 (T-II)]**

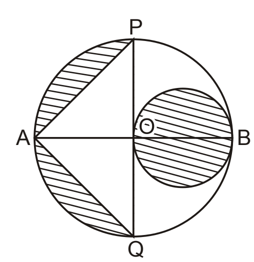
13. In the figure, ABC is a triangle right angled at A. Semicircles are drawn on AB, AC and BC as diameters. Find the area of the shaded region. **[2011 (T-II)]**



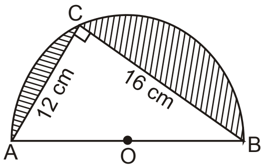
14. In the figure, AB and CD, the two diameters of a circle with centre O are perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of shaded region. **[2011 (T-II)]**



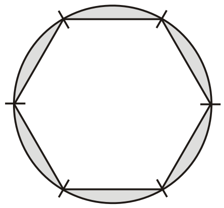
15. In the figure, AB and PQ are perpendicular diameters of the circle whose centre is O and radius OA = 7 cm. Find the area of shaded region. $\left(\text{use } \pi = \frac{22}{7}\right)$ **[2011 (T-II)]**



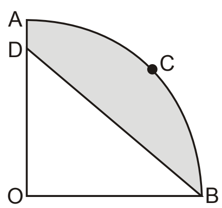
16. In the figure, O is the centre of a semi-circular arc and AOB is a straight line. Find the area of the shaded region. **[2011 (T-II)]**



17. A round table cover has six equal designs as shown in figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of Rs 0.35 per cm^2. **[2011 (T-II)]**



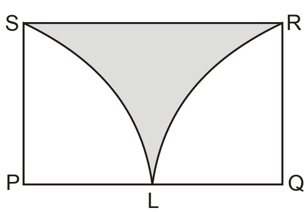
18. In the given figure, OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the : **[2011 (T-II)]**



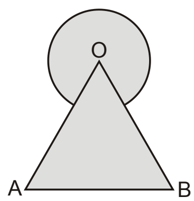
(i) quadrant OACB

(ii) shaded region

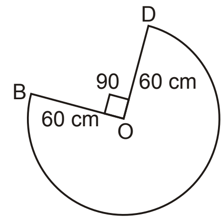
19. PQRS is a rectangle in which length is two times the breadth and L is mid point of PQ. With P and Q as centres, draw two quadrants as shown in figure. Find the ratio of the area of rectangle PQRS to the area of shaded portion.**[2011 (T-II)]**



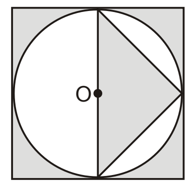
20. In the figure, find the area of the shaded region, where a circular arc of radius 6 cm is drawn with a vertex O of an equilateral tringle OAB of side 12 cm as centre. **[2011 (T-II)]**



21. In the figure, the shape of the top of a table in a restaurant is that of a sector of a circle with centre O and angle BOD = 90°. If OB = OD = 60 cm, find the perimeter of the table top. $\left(\text{use } \pi = \frac{22}{7}\right)$ **[2011 (T-II)]**

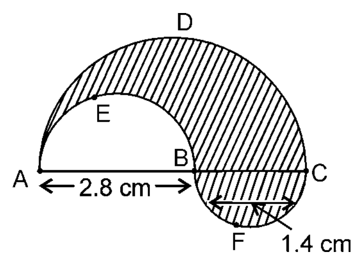


22. In the figure, a circle of radius 7 cm is inscribed in a square. Find the area of the shaded region. **[2011 (T-II)]**



23. A race track is in the form of a ring whose inner circumference is 352 m and outer circumference is 396 m. Find the width of the track $\left[\text{use } \pi = \frac{22}{7}\right]$. **[2011 (T-II)]**

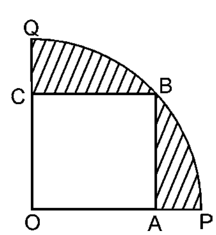
24. In the figure, find the perimeter of shaded region where ADC, AEB and BFC are semicircles on diameters AC, AB and BC respectively. **[2008]**



25. Find the area of the segment of a circle of radius 14 cm, if the length of the corresponding arc APB is 22 cm. $\left[\text{Use } \pi = \frac{22}{7}\right]$ **[2008 C]**

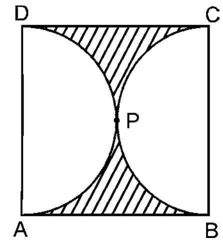
26. A square OABC is inscribed in a quadrant OPBQ of a circle as shown in figure. If OA = 14 cm, find the area of the shaded region. $\left[\text{Use } \pi = \frac{22}{7}\right]$

[2008 C]



27. The area of an equilateral triangle is $49\sqrt{3}\ \text{cm}^2$. Taking each angular point as centre, circles are drawn with radius equal to half the length of the side of the triangle. Find the area of triangle not included in the circles. $\left[\text{Take } \sqrt{3} = 1.73\right]$ **[2009]**

28. In the figure, ABCD is a square of side 14 cm and APD and BPC are semicircles. Find the area of shaded region $\left[\pi = \frac{22}{7}\right]$. **[2009]**



LONG ANSWER TYPE QUESTIONS [4 Marks]

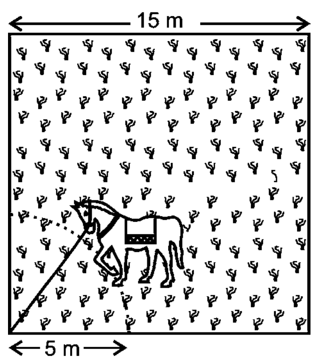
A. Important Questions

1. Find the difference between the area of a regular hexagonal plot each of whose side is 72 m and the area of the circular swimming tank inscribed in it. $\left(\text{Take } \pi = \frac{22}{7}\right)$.

2. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope. Find

(i) the area of that part of the field in which the horse can graze.

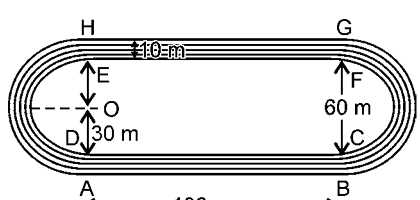
(ii) the increase in the grazing area if the rope were 10 m long instead of 5 m. (Use π = 3.14)



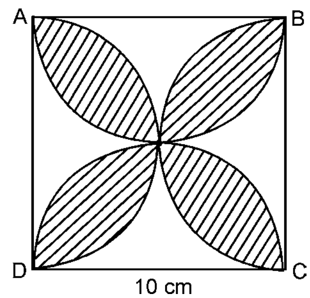
3. The given figure depicts a racing track whose left and right ends are semicircular. The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide, find:

(i) the distance around the track along its inner edge.

(ii) the area of the track.



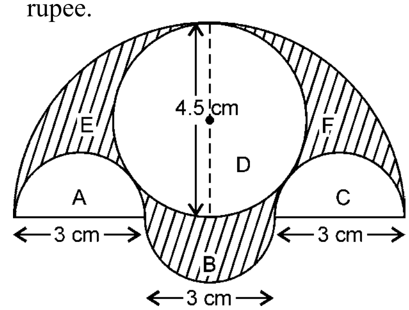
4. Find the area of the shaded design in the fig. given, where ABCD is a square of side 10 cm and semicircles are drawn with each side of the squares as diameter. (Use π = 3.14)



5. In the figure, there are three semi-circles, A, B and C having diameters 3 cm each, and another semicircle E having a circle D with diameter 4.5 cm are shown. Calculate :

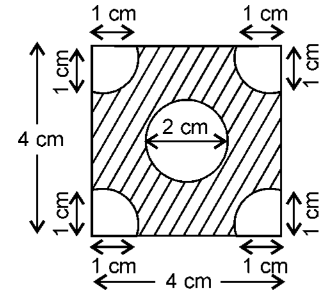
(i) the area of the shaded region.

(ii) the cost of painting the shaded region at the rate of 25 paise per cm^2, to the nearest rupee.

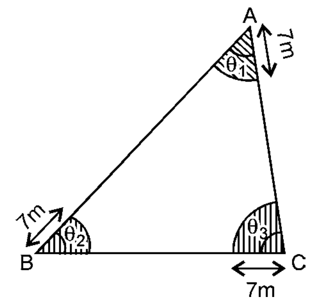


6. In an equilateral triangle of side 24 cm, a circle is inscribed touching its side. Find the area of the remaining portion of the triangle. (Take $\sqrt{3}$ = 1.732)

7. Find the area of the shaded portion shown in the figure. The four corners are quadrants and at the centre there is a circle. $\left[\pi = \frac{22}{7}\right]$

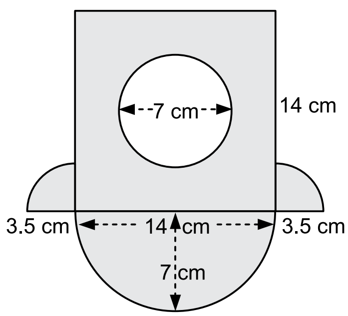


8. Three horses are tethered with 7 m long ropes at the three corners of a triangular field having sides 20 m, 34 m and 42 m. Find the area of the plot which can be grazed by the horses. Also, find the area of the plot which remains ungrazed.

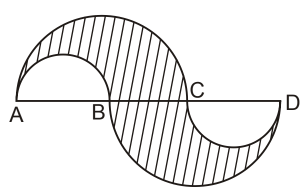


B. Questions From CBSE Examination Papers

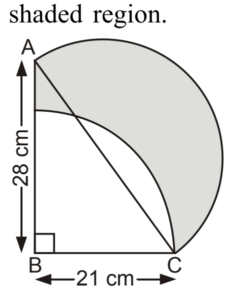
1. In the figure, find the area of the shaded region $\left(\text{Take } \pi = \dfrac{22}{7}\right)$. **[2011 (T-II)]**



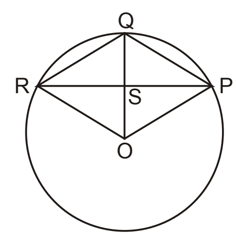
2. In the figure, AC = BD = 7 cm and AB = CD = 1.75 cm. Semicircles are drawn as shown in the figure. Find the area of the shaded region. $\left(\text{Take } \pi = \dfrac{22}{7}\right)$. **[2011 (T-II)]**



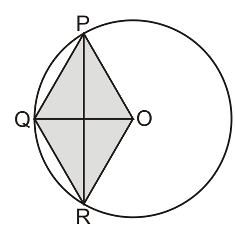
3. In the figure, ABC is a right-angled triangle, $\angle B = 90^\circ$, AB = 28 cm and BC = 21 cm. With AC as diameter, a semi-circle is drawn and with BC as radius a quarter circle is drawn. Find the area of the shaded region. **[2011 (T-II)]**



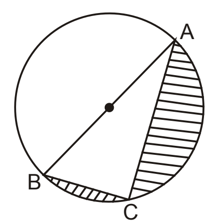
4. In the figure, OPQR is a rhombus, three of whose vertices lie on the circle with centre O. If the area of the rhombus is $32\sqrt{3}$ cm^2, find the radius of the circle. **[2011 (T-II)]**



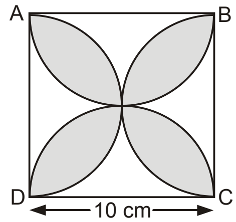
5. In the figure, OPQR is a rhombus whose three vertices, P, Q, R lie on a circle of radius 8 cm. Find the area of the shaded region. **[2011 (T-II)]**



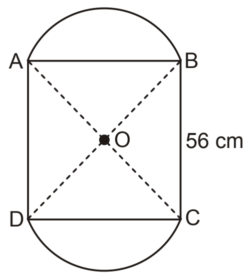
6. In the figure, AC = 24 cm, BC = 10 cm and O is the centre of the circle. Find the area of the shaded region. (Use π = 3.14) **[2011 (T-II)]**



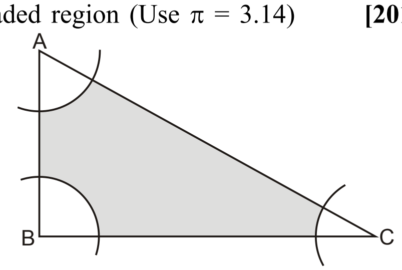
7. In the figure, find the area of the shaded design, where ABCD is a square of side 10 cm and semi circles are drawn with each side of the square as diameter. (Use π = 3.14) **[2011 (T-II)]**



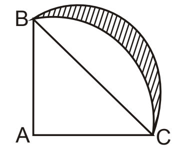
8. In the figure, two circular flower beds have been shown on two sides of a square lawn ABCD of side 56 m. If the centre of each circular flower bed is the point of intersection O of the diagonals of the square lawn, find the sum of the areas of the lawn and flower beds. **[2011 (T-II)]**



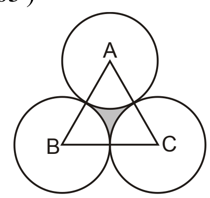
9. With the vertices A, B and C of a triangle ABC as centres, arcs are drawn with radii 5 cm each as shown in the figure. If AB = 14 cm, BC = 48 cm and CA = 50 cm, then find the area of the shaded region (Use π = 3.14) **[2011 (T-II)]**



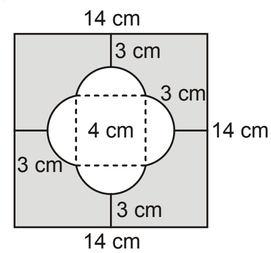
10. In the figure, ABC is a quadrant of a circle of radius 14 cm and a semi circle is drawn with BC as diameter. Find the area of shaded region. **[2011 (T-II)]**



11. The area of an equilateral triangle ABC is 17320.5 cm^2. With each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle. Find the area of the shaded region. (use π = 3.14 and $\sqrt{3} = 1.73205$) **[2011 (T-II)]**



12. Find the area of shaded region in the figure, in term of π. **[2011 (T-II)]**



FORMATIVE ASSESSMENT

Activity

Objective : To derive the formula for area of sector of a circle.

Materials Required : Glaze paper, geometry box, a pair of scissors, fevistick etc.

Procedure :

1. Draw some circles of any radius (say 3 cm) on a glaze paper. Cut these out and paste them on a drawing sheet.
2. Mark two points P and Q on the circumference of one of the circles. Join OP and OQ. The region OPQ is called the sector of a circle. Mark $\angle POQ = \theta$ (Figure 1). POQ is the angle of the sector.

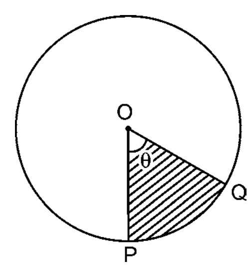


Figure 1

3. Now on other circles, make different sectors of 45°, 60°, 90° and 120° (Figure 2).
4. Circle C_1 with sector of 45°, circle C_2 with sectors of 60°, circle C_3 with sector of 90° and circle C_4 with sector of 120°.

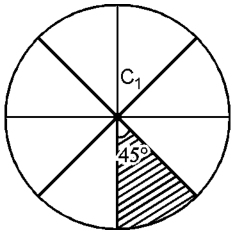


Figure 2(a)

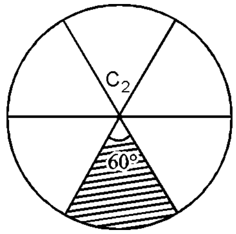


Figure 2(b)

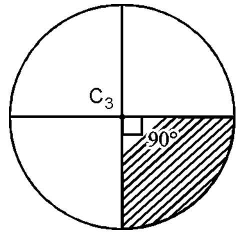


Figure 2(c)

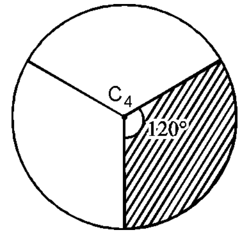


Figure 2(d)

5. Calculate the areas of sectors of C_1, C_2, C_3 and C_4, record your observations in the following table.

Circle	Angle of the sector = θ	No. of equal sectors in the circle	Area of one sector
C_1	45°	8	$\frac{1}{8} \times \pi r^2 = \frac{45°}{360} \times \pi r^2$
C_2	60°	6	$\frac{1}{6} \times \pi r^2 = \frac{60°}{360°} \times \pi r^2$
C_3	90°	4	$\frac{1}{4} \times \pi r^2 = \frac{90°}{360°} \times \pi r^2$
C_4	120°	3	$\frac{1}{3} \times \pi r^2 = \frac{120°}{360°} \times \pi r^2$

Observations : We see from the above table that area of a sector of angle $\theta = \frac{\theta}{360°} \times \pi r^2$.

Exercise 12.1

Question 1:

The radii of two circles are 19 cm and 9 cm respectively. Find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles.

Answer:

Radius (r_1) of 1st circle = 19 cm

Radius (r_2) or 2nd circle = 9 cm

Let the radius of 3rd circle be r.

Circumference of 1st circle = $2\pi r_1 = 2\pi\ (19) = 38\pi$

Circumference of 2nd circle = $2\pi r_2 = 2\pi\ (9) = 18\pi$

Circumference of 3rd circle = $2\pi r$

Given that,

Circumference of 3rd circle = Circumference of 1st circle + Circumference of 2nd circle

$2\pi r = 38\pi + 18\pi = 56\pi$

$$r = \frac{56\pi}{2\pi} = 28$$

Therefore, the radius of the circle which has circumference equal to the sum of the circumference of the given two circles is 28 cm.

Question 2:

The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having area equal to the sum of the areas of the two circles.

Answer:

Radius (r_1) of 1st circle = 8 cm

Radius (r_2) of 2nd circle = 6 cm

Let the radius of 3rd circle be r.

Area of 1st circle $= \pi r_1^2 = \pi(8)^2 = 64\pi$

Area of 2nd circle $= \pi r_2^2 = \pi(6)^2 = 36\pi$

Given that,

Area of 3rd circle = Area of 1st circle + Area of 2nd circle

$\pi r^2 = \pi r_1^2 + \pi r_2^2$

$\pi r^2 = 64\pi + 36\pi$

$\pi r^2 = 100\pi$

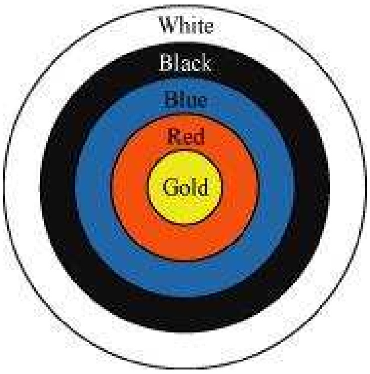
$r^2 = 100$

$r = \pm 10$

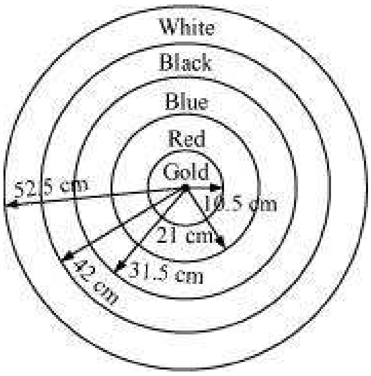
However, the radius cannot be negative. Therefore, the radius of the circle having area equal to the sum of the areas of the two circles is 10 cm.

Question 3:

Given figure depicts an archery target marked with its five scoring areas from the centre outwards as Gold, Red, Blue, Black and White. The diameter of the region representing Gold score is 21 cm and each of the other bands is 10.5 cm wide. Find the area of each of the five scoring regions. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer:



Radius (r_1) of gold region (i.e., 1st circle) $= \frac{21}{2} = 10.5 \text{ cm}$

Given that each circle is 10.5 cm wider than the previous circle.

Therefore, radius (r_2) of 2nd circle = 10.5 + 10.5

21 cm

Radius (r_3) of 3rd circle = 21 + 10.5

= 31.5 cm

Radius (r_4) of 4th circle = 31.5 + 10.5

= 42 cm

Radius (r_5) of 5th circle = 42 + 10.5

= 52.5 cm

Area of gold region = Area of 1st circle $= \pi r_1^2 = \pi(10.5)^2 = 346.5 \text{ cm}^2$

Area of red region = Area of 2nd circle − Area of 1st circle

$=\pi r_2^2 - \pi r_1^2$

$= \pi(21)^2 - \pi(10.5)^2$

$= 441\pi - 110.25\pi = 330.75\pi$

$= 1039.5 \text{ cm}^2$

Area of blue region = Area of 3rd circle − Area of 2nd circle

$=\pi r_3^2 - \pi r_1^2$

$=\pi (31.5)^2 - \pi (21)^2$

$= 992.25\pi - 441\pi = 551.25\pi$

$= 1732.5 \text{ cm}^2$

Area of black region = Area of 4th circle − Area of 3rd circle

$=\pi r_4^2 - \pi r_3^2$

$= \pi(42)^2 - \pi(31.5)^2$

$= 1764\pi - 992.25\pi$

$= 771.75\pi = 2425.5 \text{ cm}^2$

Area of white region = Area of 5th circle − Area of 4th circle

$= \pi r_5^2 - \pi r_4^2$

$= \pi(52.5)^2 - \pi(42)^2$

$= 2756.25\pi - 1764\pi$

$= 992.25\pi = 3118.5 \text{ cm}^2$

Therefore, areas of gold, red, blue, black, and white regions are 346.5 cm^2, 1039.5 cm^2, 1732.5 cm^2, 2425.5 cm^2, and 3118.5 cm^2 respectively.

Question 4:

The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is traveling at a speed of 66 km per hour? $\left[\text{Use } \pi = \frac{22}{7}\right]$

Answer:

Diameter of the wheel of the car = 80 cm

Radius (r) of the wheel of the car = 40 cm

Circumference of wheel = $2\pi r$

$= 2\pi (40) = 80\pi$ cm

Speed of car = 66 km/hour

$= \frac{66 \times 100000}{60}$ cm/min

$= 110000$ cm/min

Distance travelled by the car in 10 minutes

= 110000 × 10 = 1100000 cm

Let the number of revolutions of the wheel of the car be *n*.

n × Distance travelled in 1 revolution (i.e., circumference)

= Distance travelled in 10 minutes

$n \times 80\pi = 1100000$

$n = \frac{1100000 \times 7}{80 \times 22}$

$= \frac{35000}{8} = 4375$

Therefore, each wheel of the car will make 4375 revolutions.

Question 5:

Tick the correct answer in the following and justify your choice: If the perimeter and the area of a circle are numerically equal, then the radius of the circle is

(A) 2 units (B) π units (C) 4 units (D)7 units

Answer:

Let the radius of the circle be *r*.

Circumference of circle = $2\pi r$

Area of circle = πr^2

Given that, the circumference of the circle and the area of the circle are equal.

This implies $2\pi r = \pi r^2$

$2 = r$

Therefore, the radius of the circle is 2 units.

Hence, the correct answer is A.

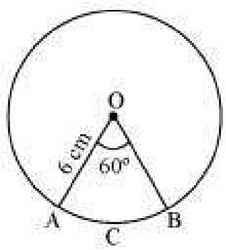
Exercise 12.2

Question 1:

Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60°.

$\left[\text{Use } \pi = \frac{22}{7}\right]$

Answer:



Let OACB be a sector of the circle making 60° angle at centre O of the circle.

Area of sector of angle θ = $\frac{\theta}{360°} \times \pi r^2$

Area of sector OACB = $\frac{60°}{360°} \times \frac{22}{7} \times (6)^2$

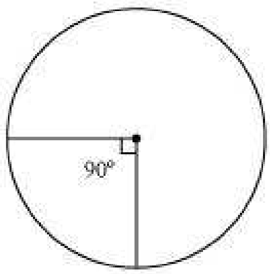
$= \frac{1}{6} \times \frac{22}{7} \times 6 \times 6 = \frac{132}{7} \text{ cm}^2$

Therefore, the area of the sector of the circle making 60° at the centre of the circle is $\frac{132}{7} \text{ cm}^2$

Question 2:

Find the area of a quadrant of a circle whose circumference is 22 cm. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Answer:



Let the radius of the circle be *r*.

Circumference = 22 cm

$2\pi r = 22$

$$r = \frac{22}{2\pi} = \frac{11}{\pi}$$

Quadrant of circle will subtend 90° angle at the centre of the circle.

Area of such quadrant of the circle $= \frac{90°}{360°} \times \pi \times r^2$

$$= \frac{1}{4\pi} \times \pi \times \left(\frac{11}{\ }\right)^2$$

$$= \frac{121}{4\pi} = \frac{121 \times 7}{4 \times 22}$$

$$= \frac{77}{8}\ \text{cm}^2$$

Question 3:

The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Answer:

We know that in 1 hour (i.e., 60 minutes), the minute hand rotates 360°.

In 5 minutes, minute hand will rotate = $\frac{360°}{60} \times 5 = 30°$

Therefore, the area swept by the minute hand in 5 minutes will be the area of a sector of 30° in a circle of 14 cm radius.

Area of sector of angle θ = $\frac{\theta}{360°} \times \pi r^2$

Area of sector of 30° $= \frac{30°}{360°} \times \frac{22}{7} \times 14 \times 14$

$= \frac{22}{12} \times 2 \times 14$

$= \frac{11 \times 14}{3}$

$= \frac{154}{3}\ \text{cm}^2$

Therefore, the area swept by the minute hand in 5 minutes is $\frac{154}{3}\ \text{cm}^2$.

Question 4:

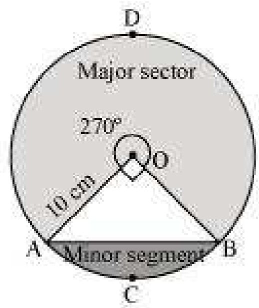
A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding:

(i) Minor segment

(ii) Major sector

[Use π = 3.14]

Answer:



Let AB be the chord of the circle subtending 90° angle at centre O of the circle.

Area of major sector OADB = $\left(\frac{360°-90°}{360°}\right)\times \pi r^2 = \left(\frac{270°}{360°}\right)\pi r^2$

$= \frac{3}{4}\times 3.14\times 10\times 10$

$= 235.5 \text{ cm}^2$

Area of minor sector OACB = $\frac{90°}{360°}\times \pi r^2$

$= \frac{1}{4}\times 3.14\times 10\times 10$

$= 78.5 \text{ cm}^2$

Area of ΔOAB = $\frac{1}{2}\times \text{OA}\times \text{OB} = \frac{1}{2}\times 10\times 10$

= 50 cm^2

Area of minor segment ACB = Area of minor sector OACB −

Area of ΔOAB = 78.5 − 50 = 28.5 cm^2

Question 5:

In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find:

(i) The length of the arc

(ii) Area of the sector formed by the arc

(iii) Area of the segment forced by the corresponding chord

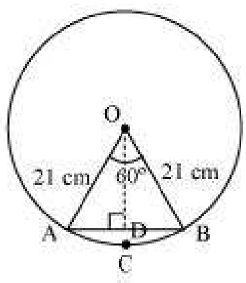
$\left[\text{Use } \pi = \dfrac{22}{7}\right]$

Answer:

Radius (r) of circle = 21 cm

Angle subtended by the given arc = 60°

Length of an arc of a sector of angle θ = $\dfrac{\theta}{360^\circ} \times 2\pi r$



Length of arc ACB = $\dfrac{60^\circ}{360^\circ} \times 2 \times \dfrac{22}{7} \times 21$

$= \dfrac{1}{6} \times 2 \times 22 \times 3$

= 22 cm

Area of sector OACB = $\dfrac{60^\circ}{360^\circ} \times \pi r^2$

$= \dfrac{1}{6} \times \dfrac{22}{7} \times 21 \times 21$

$= 231 \text{ cm}^2$

In ΔOAB,

∠OAB = ∠OBA (As OA = OB)

∠OAB + ∠AOB + ∠OBA = 180°

2∠OAB + 60° = 180°

∠OAB = 60°

Therefore, ΔOAB is an equilateral triangle.

Area of ΔOAB = $\frac{\sqrt{3}}{4}\times(\text{Side})^2$

$=\frac{\sqrt{3}}{4}\times(21)^2=\frac{441\sqrt{3}}{4}\ \text{cm}^2$

Area of segment ACB = Area of sector OACB − Area of ΔOAB

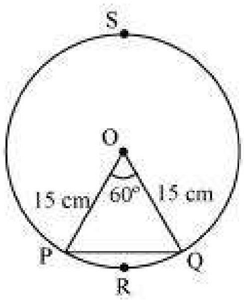
$=\left(231-\frac{441\sqrt{3}}{4}\right)\ \text{cm}^2$

Question 6:

A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle.

[Use π = 3.14 and $\sqrt{3}=1.73$]

Answer:



Radius (r) of circle = 15 cm

Area of sector OPRQ = $\frac{60^\circ}{360^\circ}\times\pi r^2$

$=\frac{1}{6}\times3.14\times(15)^2$

$=117.75\ \text{cm}^2$

In ΔOPQ,

∠OPQ = ∠OQP (As OP = OQ)

$\angle OPQ + \angle OQP + \angle POQ = 180°$

$2\angle OPQ = 120°$

$\angle OPQ = 60°$

ΔOPQ is an equilateral triangle.

Area of $\Delta OPQ = \frac{\sqrt{3}}{4}\times(\text{side})^2$

$= \frac{\sqrt{3}}{4}\times(15)^2 = \frac{225\sqrt{3}}{4}\text{ cm}^2$

$= 56.25\sqrt{3}$

$= 97.3125\text{ cm}^2$

Area of segment PRQ = Area of sector OPRQ − Area of ΔOPQ

= 117.75 − 97.3125

= 20.4375 cm^2

Area of major segment PSQ = Area of circle − Area of segment PRQ

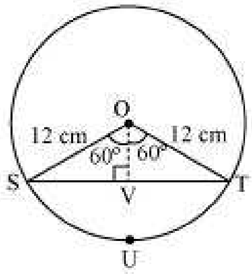
$$= \pi(15)^2 - 20.4375$$
$$= 3.14\times 225 - 20.4375$$
$$= 706.5 - 20.4375$$
$$= 686.0625\text{ cm}^2$$

Question 7:

A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle.

[Use π = 3.14 and $\sqrt{3} = 1.73$]

Answer:



Let us draw a perpendicular OV on chord ST. It will bisect the chord ST.

SV = VT

In ΔOVS,

$$\frac{OV}{OS} = \cos 60^\circ$$

$$\frac{OV}{12} = \frac{1}{2}$$

$$OV = 6 \text{ cm}$$

$$\frac{SV}{SO} = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\frac{SV}{12} = \frac{\sqrt{3}}{2}$$

$$SV = 6\sqrt{3} \text{ cm}$$

$$ST = 2SV = 2 \times 6\sqrt{3} = 12\sqrt{3} \text{ cm}$$

Area of ΔOST = $\frac{1}{2} \times ST \times OV$

$$= \frac{1}{2} \times 12\sqrt{3} \times 6$$

$$= 36\sqrt{3} = 36 \times 1.73 = 62.28 \text{ cm}^2$$

Area of sector OSUT = $\frac{120^\circ}{360^\circ} \times \pi (12)^2$

$$= \frac{1}{3} \times 3.14 \times 144 = 150.72 \text{ cm}^2$$

Area of segment SUT = Area of sector OSUT − Area of ΔOST

= 150.72 − 62.28

= 88.44 cm^2

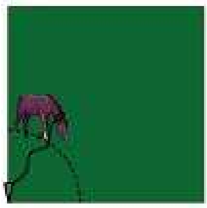
Question 8:

A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope (see the given figure). Find

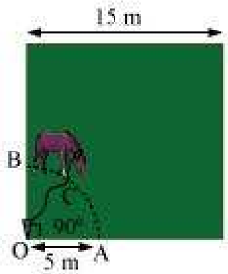
(i) The area of that part of the field in which the horse can graze.

(ii) The increase in the grazing area of the rope were 10 m long instead of 5 m.

[Use π = 3.14]



Answer:



From the figure, it can be observed that the horse can graze a sector of 90° in a circle of 5 m radius.

Area that can be grazed by horse = Area of sector OACB

$$= \frac{90°}{360°}\pi r^2$$

$$= \frac{1}{4} \times 3.14 \times (5)^2$$

$$= 19.625 \text{ m}^2$$

Area that can be grazed by the horse when length of rope is 10 m long

$= \frac{90°}{360°} \times \pi \times (10)^2$

$= \frac{1}{4} \times 3.14 \times 100$

$= 78.5 \text{ m}^2$

Increase in grazing area = (78.5 − 19.625) m^2

= 58.875 m^2

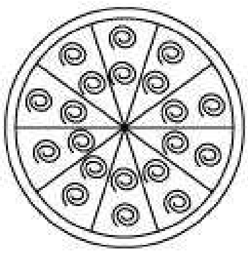
Question 9:

A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in figure. Find.

(i) The total length of the silver wire required.

(ii) The area of each sector of the brooch

$\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer:

Total length of wire required will be the length of 5 diameters and the circumference of the brooch.

Radius of circle = $\frac{35}{2}$ mm

Circumference of brooch = 2π*r*

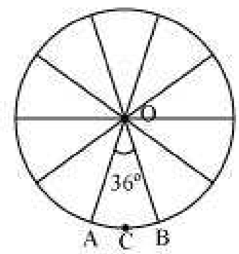
$= 2 \times \frac{22}{7} \times \left(\frac{35}{2}\right)$

= 110 mm

Length of wire required = 110 + 5 × 35

= 110 + 175 = 285 mm

It can be observed from the figure that each of 10 sectors of the circle is subtending 36° at the centre of the circle.



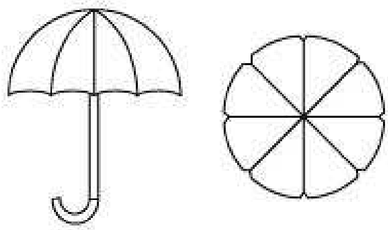
Therefore, area of each sector = $\frac{36°}{360°} \times \pi r^2$

$= \frac{1}{10} \times \frac{22}{7} \times \left(\frac{35}{2}\right) \times \left(\frac{35}{2}\right)$

$= \frac{385}{4}\ \text{mm}^2$

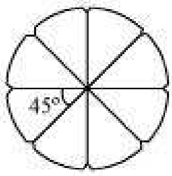
Question 10:

An umbrella has 8 ribs which are equally spaced (see figure). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer:

There are 8 ribs in an umbrella. The area between two consecutive ribs is subtending $\frac{360^\circ}{8} = 45^\circ$ at the centre of the assumed flat circle.



Area between two consecutive ribs of circle = $\frac{45^\circ}{360^\circ} \times \pi r^2$

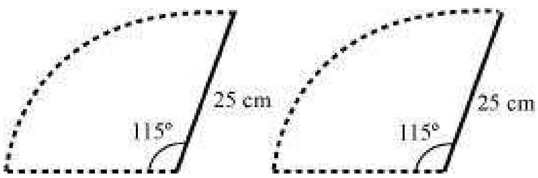
$$= \frac{1}{8} \times \frac{22}{7} \times (45)^2$$

$$= \frac{11}{28} \times 2025 = \frac{22275}{28} \text{ cm}^2$$

Question 11:

A car has two wipers which do not overlap. Each wiper has blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Answer:



It can be observed from the figure that each blade of wiper will sweep an area of a sector of 115° in a circle of 25 cm radius.

Area of such sector = $\frac{115°}{360°}\times\pi\times(25)^2$

$=\frac{23}{72}\times\frac{22}{7}\times25\times25$

$=\frac{158125}{252}\text{ cm}^2$

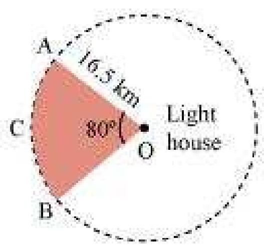
Area swept by 2 blades = $2\times\frac{158125}{252}$

$=\frac{158125}{126}\text{ cm}^2$

Question 12:

To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships warned. [Use π = 3.14]

Answer:



It can be observed from the figure that the lighthouse spreads light across a sector of 80° in a circle of 16.5 km radius.

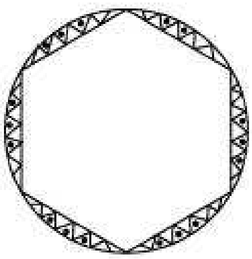
Area of sector OACB = $\frac{80°}{360°}\times\pi r^2$

$= \frac{2}{9} \times 3.14 \times 16.5 \times 16.5$

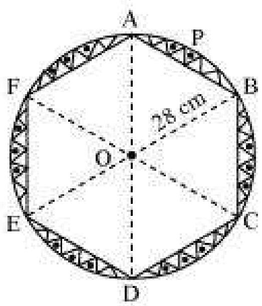
$= 189.97 \text{ km}^2$

Question 13:

A round table cover has six equal designs as shown in figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of Rs.0.35 per cm^2. [Use $\sqrt{3} = 1.7$]



Answer:



It can be observed that these designs are segments of the circle.

Consider segment APB. Chord AB is a side of the hexagon. Each chord will substitute $\frac{360°}{6} = 60°$ at the centre of the circle.

In ΔOAB,

$\angle OAB = \angle OBA$ (As OA = OB)

$\angle AOB = 60°$

$\angle OAB + \angle OBA + \angle AOB = 180°$

$2\angle OAB = 180° - 60° = 120°$

$\angle OAB = 60°$

Therefore, ΔOAB is an equilateral triangle.

Area of ΔOAB = $\frac{\sqrt{3}}{4}\times(\text{side})^2$

$= \frac{\sqrt{3}}{4}\times(28)^2 = 196\sqrt{3} = 196\times 1.7$ = 333.2 cm^2

Area of sector OAPB = $\frac{60°}{360°}\times \pi r^2$

$= \frac{1}{6}\times\frac{22}{7}\times 28\times 28$

$= \frac{1232}{3}\text{ cm}^2$

Area of segment APB = Area of sector OAPB − Area of ΔOAB

$= \left(\frac{1232}{3} - 333.2\right)\text{cm}^2$

$$\text{Therefore, area of designs} = 6\times\left(\frac{1232}{3} - 333.2\right)\text{cm}^2$$
$$= (2464 - 1999.2)\text{cm}^2$$
$$= 464.8\text{ cm}^2$$

Cost of making 1 cm^2 designs = Rs 0.35

Cost of making 464.76 cm^2 designs = 464.8×0.35 = Rs 162.68

Therefore, the cost of making such designs is Rs 162.68.

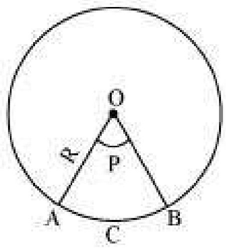
Question 14:

Tick the correct answer in the following:

Area of a sector of angle *p* (in degrees) of a circle with radius R is

(A) $\frac{p}{180} \times 2\pi R$, (B) $\frac{p}{180} \times \pi R^2$, (C) $\frac{p}{360} \times 2\pi R$, (D) $\frac{p}{720} \times 2\pi R^2$

Answer:



We know that area of sector of angle θ = $\frac{\theta}{360°} \times \pi R^2$

Area of sector of angle P = $\frac{p}{360°}\left(\pi R^2\right)$

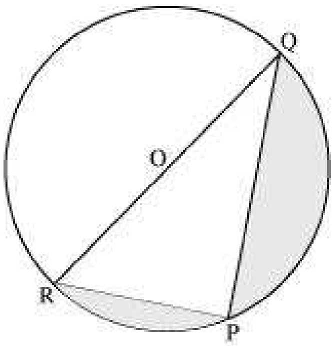
$= \left(\frac{p}{720°}\right)\left(2\pi R^2\right)$

Hence, (D) is the correct answer.

Exercise 12.3

Question 1:

Find the area of the shaded region in the given figure, if PQ = 24 cm, PR = 7 cm and O is the centre of the circle. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer:

It can be observed that RQ is the diameter of the circle. Therefore, ∠RPQ will be 90º.

By applying Pythagoras theorem in ΔPQR,

$RP^2 + PQ^2 = RQ^2$

$(7)^2 + (24)^2 = RQ^2$

$RQ = \sqrt{625} = 25$

Radius of circle, $OR = \frac{RQ}{2} = \frac{25}{2}$

Since RQ is the diameter of the circle, it divides the circle in two equal parts.

$$\text{Area of semi-circle RPQOR} = \frac{1}{2}\pi r^2$$
$$= \frac{1}{2}\pi\left(\frac{25}{2}\right)^2$$
$$= \frac{1}{2}\times\frac{22}{7}\times\frac{625}{4}$$
$$= \frac{6875}{28}\text{ cm}^2$$

Area of ΔPQR $= \frac{1}{2}\times \text{PQ}\times\text{PR}$

$= \frac{1}{2}\times 24\times 7$

$= 84 \text{ cm}^2$

Area of shaded region = Area of semi-circle RPQOR − Area of ΔPQR

$= \frac{6875}{28} - 84$

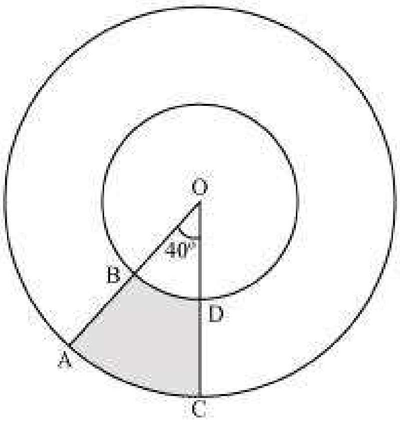
$= \frac{6875-2352}{28}$

$= \frac{4523}{28}$ cm^2

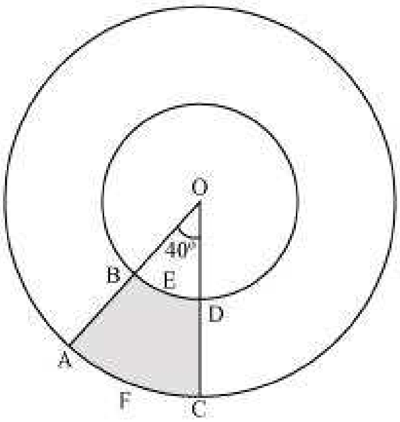
Question 2:

Find the area of the shaded region in the given figure, if radii of the two concentric circles with centre O are 7 cm and 14 cm respectively and ∠AOC = 40°.

$\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer:



Radius of inner circle = 7 cm

Radius of outer circle = 14 cm

Area of shaded region = Area of sector OAFC – Area of sector OBED

$$= \frac{40^\circ}{360^\circ} \times \pi (14)^2 = \frac{40^\circ}{360^\circ} \times \pi (7)^2$$

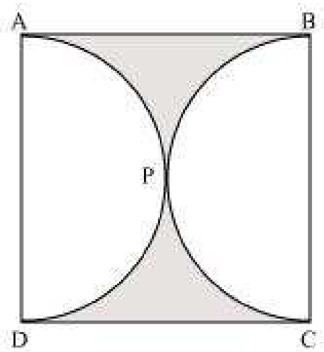
$$= \frac{1}{9} \times \frac{22}{7} \times 14 \times 14 - \frac{1}{9} \times \frac{22}{7} \times 7 \times 7$$

$$= \frac{616}{9} - \frac{154}{9} = \frac{462}{9}$$

$$= \frac{154}{3} \text{cm}^2$$

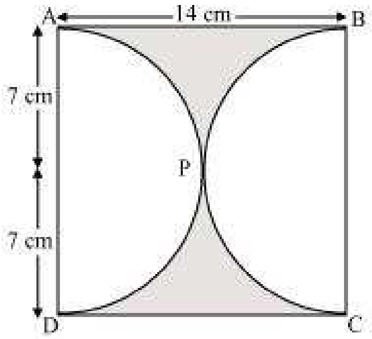
Question 3:

Find the area of the shaded region in the given figure, if ABCD is a square of side 14 cm and APD and BPC are semicircles. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer:

It can be observed from the figure that the radius of each semi-circle is 7 cm.



Area of each semi-circle = $\frac{1}{2}\pi r^2$

$=\frac{1}{2}\times\frac{22}{7}\times(7)^2$

$= 77\ \text{cm}^2$

Area of square ABCD = $(\text{Side})^2 = (14)^2 = 196\ \text{cm}^2$

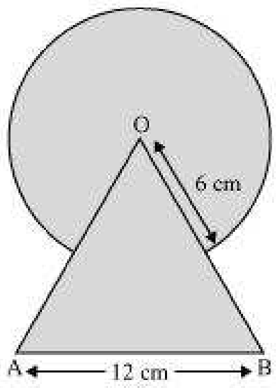
Area of the shaded region

= Area of square ABCD − Area of semi-circle APD − Area of semi-circle BPC

= 196 − 77 − 77 = 196 − 154 = 42 cm^2

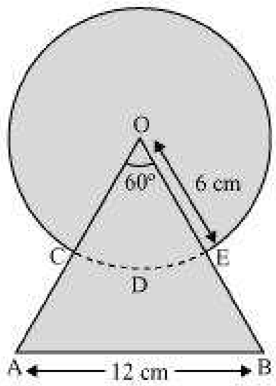
Question 4:

Find the area of the shaded region in the given figure, where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer:

We know that each interior angle of an equilateral triangle is of measure 60°.



Area of sector OCDE $= \frac{60°}{360°}\pi r^2$

$= \frac{1}{6}\times\frac{22}{7}\times 6\times 6$

$= \frac{132}{7}\ \text{cm}^2$

Area of $\Delta OAB = \frac{\sqrt{3}}{4}(12)^2 = \frac{\sqrt{3}\times 12\times 12}{4} = 36\sqrt{3}\ \text{cm}^2$

Area of circle = $\pi r^2 = \frac{22}{7}\times 6\times 6 = \frac{792}{7}\ \text{cm}^2$

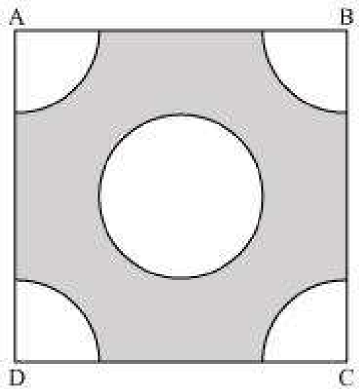
Area of shaded region = Area of ΔOAB + Area of circle − Area of sector OCDE

$$= 36\sqrt{3} + \frac{792}{7} - \frac{132}{7}$$

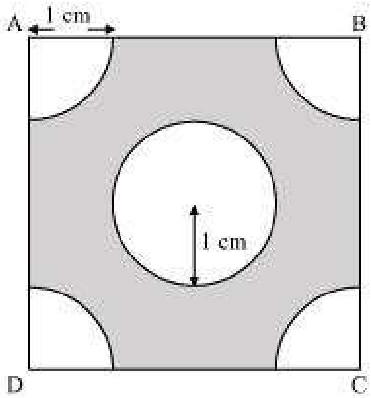
$$= \left(36\sqrt{3} + \frac{660}{7}\right)\text{ cm}^2$$

Question 5:

From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in the given figure. Find the area of the remaining portion of the square. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer:



Each quadrant is a sector of 90° in a circle of 1 cm radius.

Area of each quadrant $= \frac{90°}{360°}\pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times (1)^2 = \frac{22}{28} \text{cm}^2$

Area of square = $(\text{Side})^2 = (4)^2 = 16 \text{ cm}^2$

Area of circle = $\pi r^2 = \pi\,(1)^2$

$= \frac{22}{7} \text{ cm}^2$

Area of the shaded region = Area of square − Area of circle − 4 × Area of quadrant

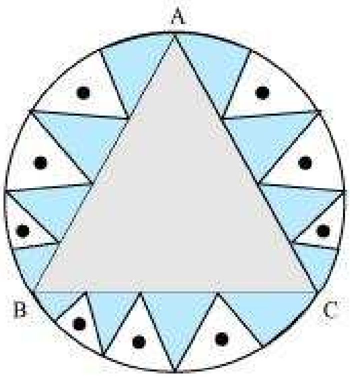
$= 16 - \frac{22}{7} - 4 \times \frac{22}{28}$

$= 16 - \frac{22}{7} - \frac{22}{7} = 16 - \frac{44}{7}$

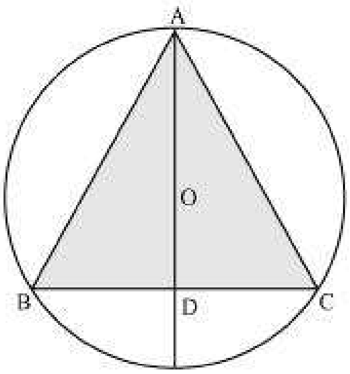
$= \frac{112 - 44}{7} = \frac{68}{7} \text{ cm}^2$

Question 6:

In a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in the given figure. Find the area of the design (Shaded region). $\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer:



Radius (r) of circle = 32 cm

AD is the median of ΔABC.

$$AO = \frac{2}{3}AD = 32$$

AD = 48 cm

In ΔABD,

$AB^2 = AD^2 + BD^2$

$$AB^2 = (48)^2 + \left(\frac{AB}{2}\right)^2$$

$$\frac{3AB^2}{4} = (48)^2$$

$$AB = \frac{48 \times 2}{\sqrt{3}} = \frac{96}{\sqrt{3}}$$

$$= 32\sqrt{3} \text{ cm}$$

Area of equilateral triangle, $\Delta ABC = \frac{\sqrt{3}}{4}\left(32\sqrt{3}\right)^2$

$$= \frac{\sqrt{3}}{4} \times 32 \times 32 \times 3 = 96 \times 8 \times \sqrt{3}$$

$$= 768\sqrt{3} \text{ cm}^2$$

Area of circle = πr^2

$$= \frac{22}{7} \times (32)^2$$

$$= \frac{22}{7} \times 1024$$

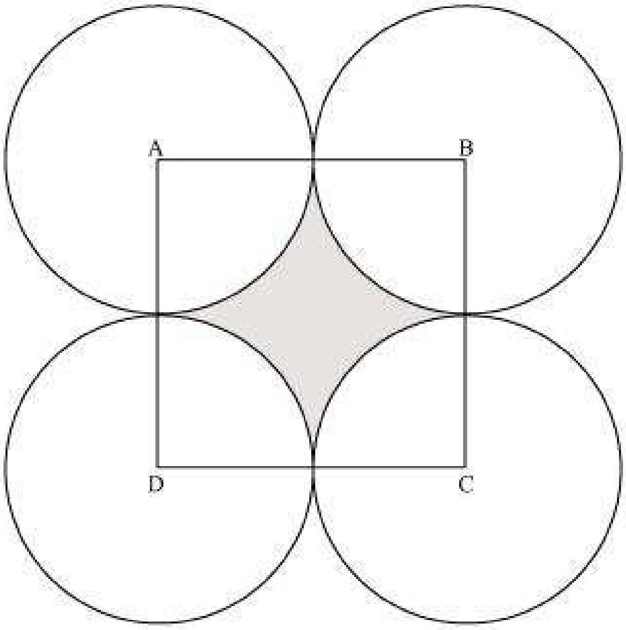
$$= \frac{22528}{7} \text{ cm}^2$$

Area of design = Area of circle − Area of ΔABC

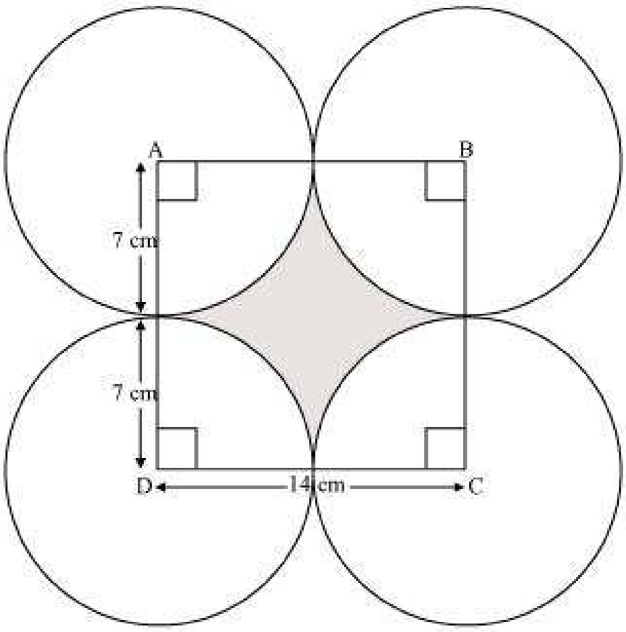
$$= \left(\frac{22528}{7} - 768\sqrt{3}\right) \text{ cm}^2$$

Question 7:

In the given figure, ABCD is a square of side 14 cm. With centres A, B, C and D, four circles are drawn such that each circle touches externally two of the remaining three circles. Find the area of the shaded region. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer:



Area of each of the 4 sectors is equal to each other and is a sector of 90° in a circle of 7 cm radius.

Area of each sector $= \frac{90°}{360°} \times \pi (7)^2$

$= \frac{1}{4} \times \frac{22}{7} \times 7 \times 7$

$= \frac{77}{2}\ \text{cm}^2$

Area of square ABCD = $(\text{Side})^2 = (14)^2 = 196\ \text{cm}^2$

Area of shaded portion = Area of square ABCD − 4 × Area of each sector

$= 196 - 4 \times \frac{77}{2} = 196 - 154$

$= 42\ \text{cm}^2$

Therefore, the area of shaded portion is 42 cm^2.

Question 8:

Thegivenfigure depicts a racing track whose left and right ends are semicircular.

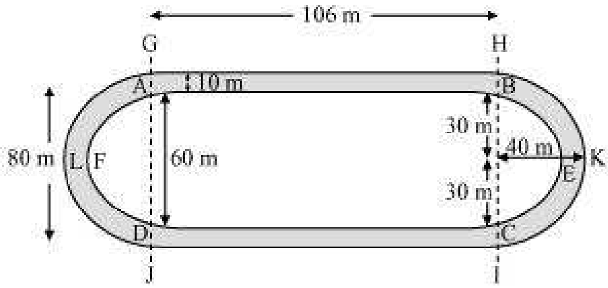
The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide, find:

(i) The distance around the track along its inner edge

(ii) The area of the track

$\left[\text{Use } \pi = \frac{22}{7}\right]$

Answer:



Distance around the track along its inner edge = AB + arc BEC + CD + arc DFA

$$= 106 + \frac{1}{2} \times 2\pi r + 106 + \frac{1}{2} \times 2\pi r$$

$$= 212 + \frac{1}{2} \times 2 \times \frac{22}{7} \times 30 + \frac{1}{2} \times 2 \times \frac{22}{7} \times 30$$

$$= 212 + 2 \times \frac{22}{7} \times 30$$

$$= 212 + \frac{1320}{7}$$

$$= \frac{1484 + 1320}{7} = \frac{2804}{7} \text{ m}$$

Area of the track = (Area of GHIJ − Area of ABCD) + (Area of semi-circle HKI − Area of semi-circle BEC) + (Area of semi-circle GLJ − Area of semi-circle AFD)

$$= 106 \times 80 - 106 \times 60 + \frac{1}{2} \times \frac{22}{7} \times (40)^2 - \frac{1}{2} \times \frac{22}{7} \times (30)^2 + \frac{1}{2} \times \frac{22}{7} \times (40)^2 - \frac{1}{2} \times \frac{22}{7} \times (30)^2$$

$$= 106(80 - 60) + \frac{22}{7} \times (40)^2 - \frac{22}{7} \times (30)^2$$

$$= 106(20) + \frac{22}{7}\left[(40)^2 - (30)^2\right]$$

$$= 2120 + \frac{22}{7}(40 - 30)(40 + 30)$$

$$= 2120 + \left(\frac{22}{7}\right)(10)(70)$$

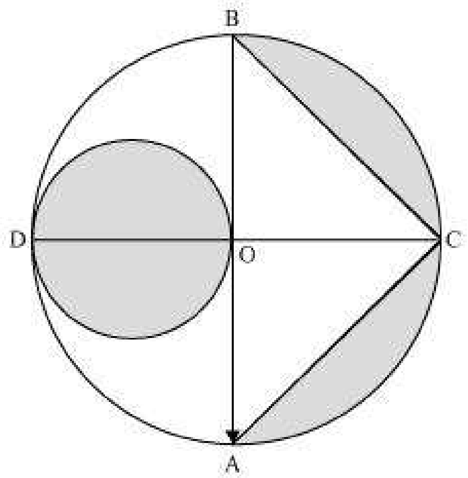
$$= 2120 + 2200$$

$$= 4320 \text{ m}^2$$

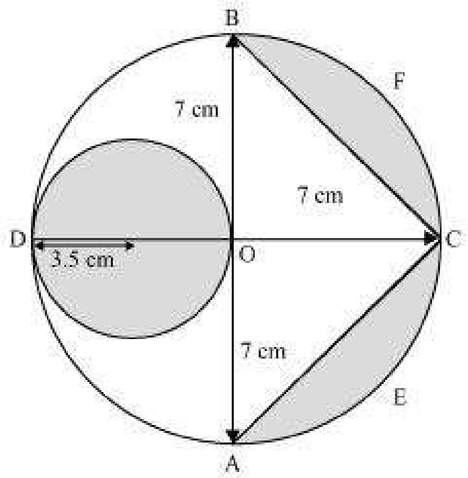
Therefore, the area of the track is 4320 m^2.

Question 9:

In the given figure, AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer:



Radius (r_1) of larger circle = 7 cm

Radius (r_2) of smaller circle $= \frac{7}{2}$ cm

Area of smaller circle $= \pi r_1^2$

$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

$= \frac{77}{2}$ cm^2

Area of semi-circle AECFB of larger circle $= \frac{1}{2} \pi r_2^2$

$= \frac{1}{2} \times \frac{22}{7} \times (7)^2$

$= 77$ cm^2

Area of $\Delta ABC = \frac{1}{2} \times AB \times OC$

$= \frac{1}{2} \times 14 \times 7 = 49$ cm^2

Area of the shaded region

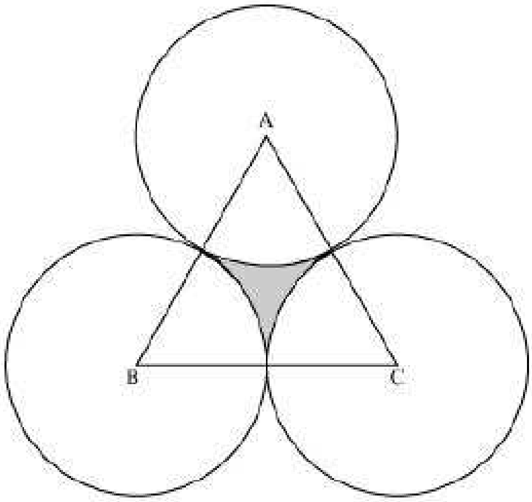
= Area of smaller circle + Area of semi-circle AECFB − Area of ΔABC

$= \frac{77}{2} + 77 - 49$

$= 28 + \frac{77}{2} = 28 + 38.5 = 66.5$ cm^2

Question 10:

The area of an equilateral triangle ABC is 17320.5 cm^2. With each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle (See the given figure). Find the area of shaded region. [Use π = 3.14 and $\sqrt{3} = 1.73205$]



Answer:

Let the side of the equilateral triangle be *a*.

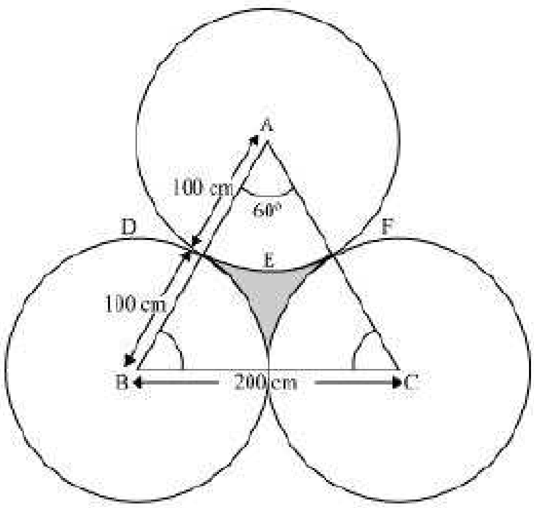
Area of equilateral triangle = 17320.5 cm^2

$$\frac{\sqrt{3}}{4}(a)^2 = 17320.5$$

$$\frac{1.73205}{4}a^2 = 17320.5$$

$$a^2 = 4\times10000$$

$$a = 200 \text{ cm}$$



Each sector is of measure 60°.

Area of sector ADEF $= \frac{60°}{360°} \times \pi \times r^2$

$= \frac{1}{6} \times \pi \times (100)^2$

$= \frac{3.14 \times 10000}{6}$

$= \frac{15700}{3}\ \text{cm}^2$

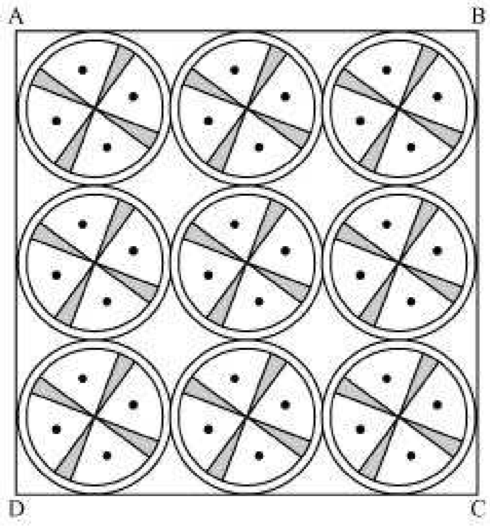
Area of shaded region = Area of equilateral triangle − 3 × Area of each sector

$= 17320.5 - 3 \times \frac{15700}{3}$

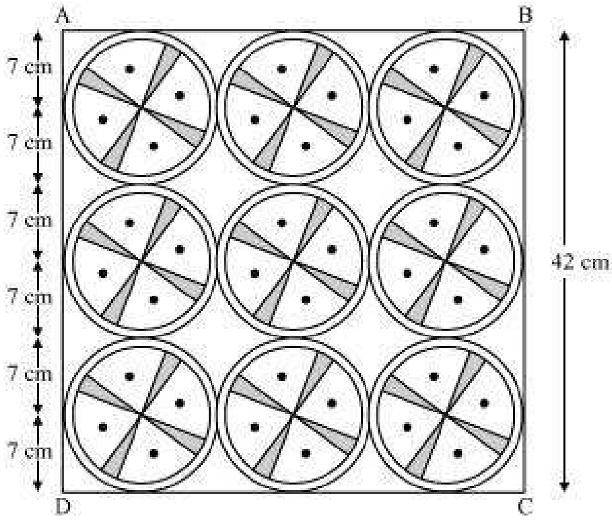
$= 17320.5 - 15700 = 1620.5\ \text{cm}^2$

Question 11:

On a square handkerchief, nine circular designs each of radius 7 cm are made (see the given figure). Find the area of the remaining portion of the handkerchief. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer:



From the figure, it can be observed that the side of the square is 42 cm.

Area of square = $(\text{Side})^2 = (42)^2 = 1764 \text{ cm}^2$

Area of each circle = $\pi r^2 = \frac{22}{7} \times (7)^2 = 154 \text{ cm}^2$

Area of 9 circles = $9 \times 154 = 1386 \text{ cm}^2$

Area of the remaining portion of the handkerchief = 1764 − 1386 = 378 cm^2

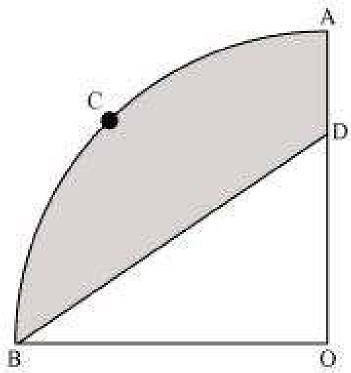
Question 12:

In the given figure, OACB is a quadrant of circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the

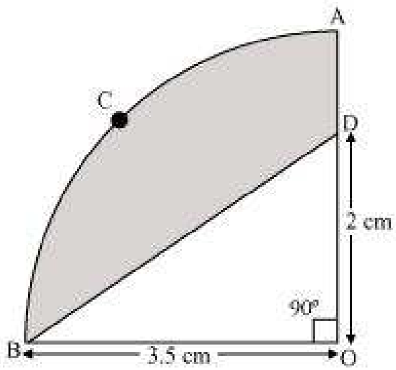
(i) Quadrant OACB

(ii) Shaded region

$\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer:



(i) Since OACB is a quadrant, it will subtend 90° angle at O.

Area of quadrant OACB $= \frac{90°}{360°} \times \pi r^2$

$$= \frac{1}{4} \times \frac{22}{7} \times (3.5)^2 = \frac{1}{4} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2$$

$$= \frac{11 \times 7 \times 7}{2 \times 7 \times 2 \times 2} = \frac{77}{8} \text{ cm}^2$$

(ii) Area of ΔOBD $= \frac{1}{2} \times \text{OB} \times \text{OD}$

$$= \frac{1}{2} \times 3.5 \times 2$$

$$= \frac{1}{2} \times \frac{7}{2} \times 2$$

$$= \frac{7}{2} \text{ cm}^2$$

Area of the shaded region = Area of quadrant OACB − Area of ΔOBD

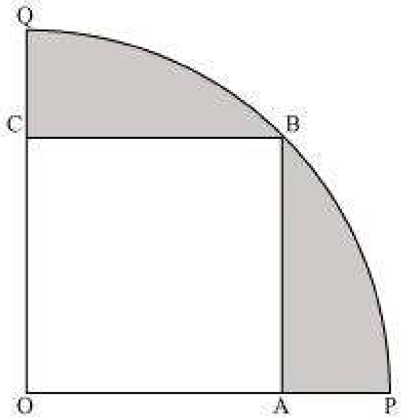
$$= \frac{77}{8} - \frac{7}{2}$$

$$= \frac{77 - 28}{8}$$

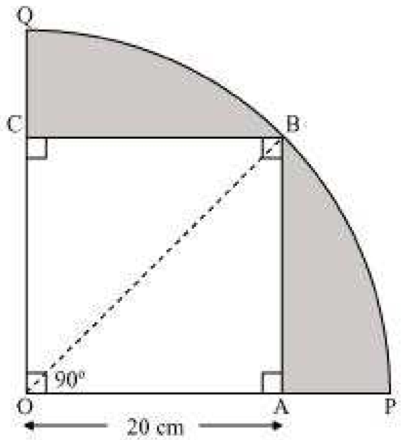
$$= \frac{49}{8} \text{ cm}^2$$

Question 13:

In the given figure, a square OABC is inscribed in a quadrant OPBQ. If OA = 20 cm, find the area of the shaded region. [Use π = 3.14]



Answer:



In ΔOAB,

$OB^2 = OA^2 + AB^2$

$= (20)^2 + (20)^2$

$OB = 20\sqrt{2}$

Radius (r) of circle $= 20\sqrt{2}$ cm

Area of quadrant OPBQ $= \dfrac{90^\circ}{360^\circ} \times 3.14 \times \left(20\sqrt{2}\right)^2$

$= \frac{1}{4} \times 3.14 \times 800$

$= 628 \text{ cm}^2$

Area of OABC = $(\text{Side})^2 = (20)^2 = 400 \text{ cm}^2$

Area of shaded region = Area of quadrant OPBQ − Area of OABC

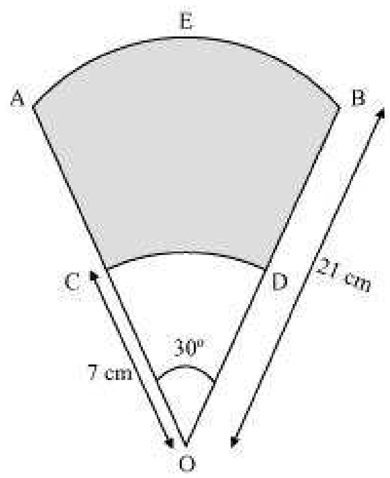
$= (628 - 400) \text{ cm}^2$

$= 228 \text{ cm}^2$

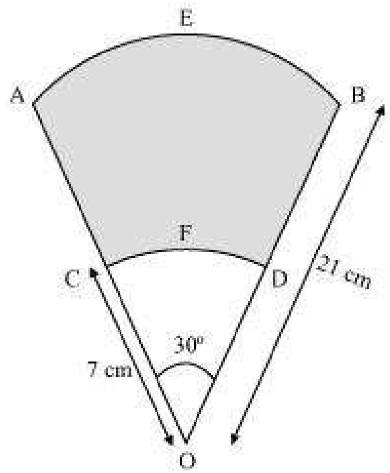
Question 14:

AB and CD are respectively arcs of two concentric circles of radii 21 cm and 7 cm and centre O (see the given figure). If $\angle AOB = 30°$, find the area of the shaded region.

$\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer:



Area of the shaded region = Area of sector OAEB − Area of sector OCFD

$$= \frac{30°}{360°} \times \pi \times (21)^2 - \frac{30°}{360°} \times \pi \times (7)^2$$

$$= \frac{1}{12} \times \pi \left[(21)^2 - (7)^2 \right]$$

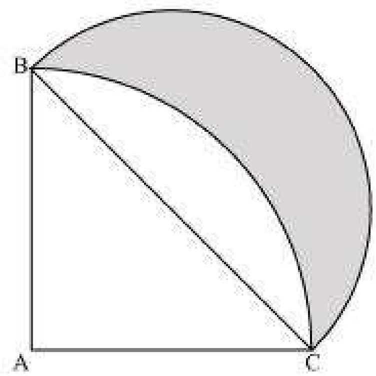
$$= \frac{1}{12} \times \frac{22}{7} \times \left[(21-7)(21+7) \right]$$

$$= \frac{22 \times 14 \times 28}{12 \times 7}$$

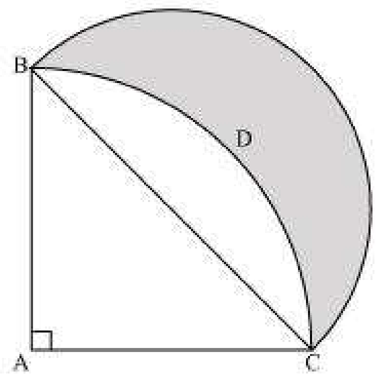
$$= \frac{308}{3} \text{ cm}^2$$

Question 15:

In the given figure, ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region. $\left[\text{Use } \pi = \frac{22}{7} \right]$



Answer:



As ABC is a quadrant of the circle, ∠BAC will be of measure 90º.

In ΔABC,

$BC^2 = AC^2 + AB^2$

$= (14)^2 + (14)^2$

$BC = 14\sqrt{2}$

Radius (r_1) of semi-circle drawn on $BC = \frac{14\sqrt{2}}{2} = 7\sqrt{2}$ cm

Area of $\Delta ABC = \frac{1}{2} \times AB \times AC$

$= \frac{1}{2} \times 14 \times 14$

$= 98 \text{ cm}^2$

Area of sector $ABDC = \frac{90°}{360°} \times \pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times 14 \times 14$

$= 154 \text{ cm}^2$

Area of semi-circle drawn on BC $= \frac{1}{2} \times \pi \times r_1^2 = \frac{1}{2} \times \frac{22}{7} \times \left(7\sqrt{2}\right)^2$

$= \frac{1}{2} \times \frac{22}{7} \times 98 = 154 \text{ cm}^2$

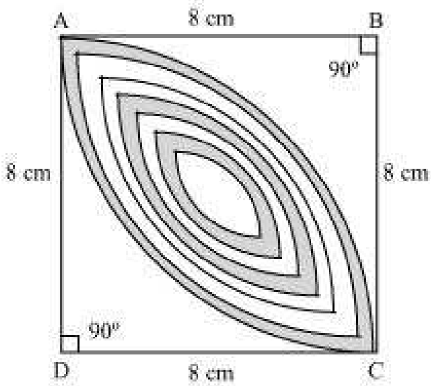
Area of shaded region = Area of semi-circle − (Area of sector ABDC − Area of ΔABC) =

154 − (154 − 98)

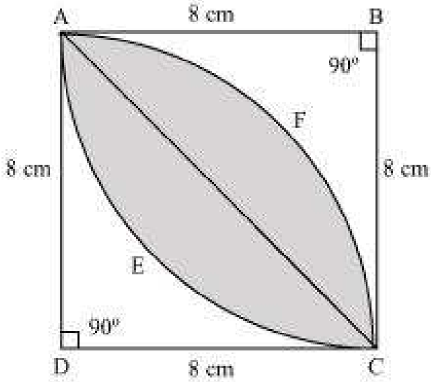
= 98 cm^2

Question 16:

Calculate the area of the designed region in the given figure common between the two quadrants of circles of radius 8 cm each. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer:



The designed area is the common region between two sectors BAEC and DAFC.

Area of sector $BAEC = \frac{90°}{360°} \times \frac{22}{7} \times (8)^2$

$= \frac{1}{4} \times \frac{22}{7} \times 64$

$= \frac{22 \times 16}{7}$

$= \frac{352}{7}\ \text{cm}^2$

Area of ΔBAC $= \frac{1}{2} \times BA \times BC$

$= \frac{1}{2} \times 8 \times 8 = 32\ \text{cm}^2$

Area of the designed portion = 2 × (Area of segment AEC)

= 2 × (Area of sector BAEC − Area of ΔBAC)

$$= 2\times\left(\frac{352}{7}-32\right) = 2\left(\frac{352-224}{7}\right)$$

$$= \frac{2\times 128}{7}$$

$$= \frac{256}{7}\ \text{cm}^2$$