Assignments in Mathematics Class X (Term II)

11. CONSTRUCTIONS

SUMMATIVE ASSESSMENT

MULTIPLE CHOICE QUESTIONS **[1 Mark]**

A. Important Questions

1. To divide a line segment AB in the ratio a : b (a, b are positive integers), first a ray AX is drawn such that $\angle BAX$ is acute and then at equal distances points are marked on the ray AX such that the minimum number of points is :

(a) a (b) b (c) a + b (d) a – b

2. To divide a line segment AB in the ratio 5 : 6, first a ray AX is drawn such that $\angle BAX$ is acute and then points P_1, P_2, P_3 .... are located at equal distances on ray AX. The point B is joined to :

(a) P_2 (b) P_5 (c) P_6 (d) P_{11}

3. To divide a line segment AB in the ratio 4 : 5, first a ray AX is drawn such that $\angle BAX$ is acute and then a ray BY parallel to AX is drawn. Then on ray AX and BY, respectively the points A_1, A_2, A_3 ... and B_1, B_2, B_3, ... are located at equal distances. Now, we join the points :

(a) A_4 and B_9 (b) A_5 and B_4
(c) A_9 and B_5 (d) A_4 and B_5

4. To divide a line segment AB in the ratio 3 : 8, first a ray AX is drawn such that $\angle BAX$ is acute and then at equal distances points A_1, A_2, A_3, ... are marked on ray AX. Then point B is joined to A_{11} and a line parallel to $A_{11}B$ is drawn through the point :

(a) A_3 (b) A_8 (c) A_5 (d) A_9

5. To draw a pair of tangents to a circle which are inclined to each other at an angle of 40°, it is required to draw tangents at the end points of those two radii of the circle, the angle between which is :

(a) 40° (b) 140° (c) 140° (d) 180°

6. To locate the centre of a circle we take any two non-parallel chords and then find the point of intersection of their :

(a) perpendicular bisectors (b) angle bisectors
(c) mid-points (d) none of these

7. The centre of a circle is not given and a point P outside the circle is given. From P, we :

(a) cannot draw the pair of tangents to the circle

(b) can always draw the pair of tangents to the circle

(c) can draw the pair of tangents to the circle only when its radius is known

(d) none of these

8. To construct a triangle similar to a given triangle ABC with its sides $\frac{3}{7}$ of the corresponding sides of ΔABC, first a ray AX is drawn such that $\angle CBX$ is acute and X lies on the opposite side of A with respect to BC. Then points B_1, B_2, B_3, ... on BX are located at equal distances and next step is to join :

(a) B_{10} to C (b) B_3 to C
(c) B_7 to C (d) B_4 to C

9. To construct a triangle similar to DABC with sides $\frac{5}{3}$ of the corresponding sides of DABC, first draw a ray BX such that $\angle CBX$ is acute and X is on the opposite side of A with respect to BC. The minimum number of points to be located on ray BX at equal distances is :

(a) 3 (b) 5 (c) 8 (d) 2

10. To construct a pair of tangents to a circle with centre O from a point P outside the circle, we first join OP. The next step is to :

(a) draw the perpendicular bisector of OP

(b) join P to any point on the circle

(c) draw ray PX such that $\angle OPX$ is acute

(d) none of these

B. Questions From CBSE Examination Papers

1. To draw a pair of tangents to a circle which are inclined to each other at an angle of 60°, it is required to draw the tangents at the end point of two radii inclined at an angle of : **[2011 (T-II)]**

(a) 120° (b) 60° (c) 30° (d) 90°

2. To divide a line segment AB in the ratio 3 : 4, we draw a ray AX, so that angle BAX is an acute angle, and then mark the point on the ray AX at equal distances such that the minimum number of these points is : **[2011 (T-II)]**

(a) 3 (b) 4 (c) 7 (d) 12

3. To divide a line segment AB in the ratio 4 : 7, a ray AX is drawn first such that $\angle BAX$ is an acute angle and then point A_1, A_2, A_3, ... are located at equal distances on the ray AX and the point B is joined to : **[2011 (T-II)]**

(a) A_{12} (b) A_{11} (c) A_{10} (d) A_9

4. Given a triangle with side AB = 8 cm. To get a line segment $A'B' = \frac{3}{4}$ of AB, it is required to divide the line segment AB in the ratio :

[2011 (T-II)]

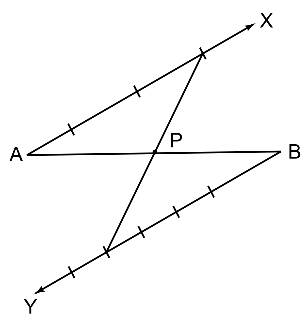
(a) 3 : 4 (b) 4 : 3 (c) 1 : 3 (d) 3 : 1

5. In drawing a triangle, it is given that AB = 3 cm, BC = 2 cm and AC = 6 cm. It is not possible to draw the triangle as : **[2011 (T-II)]**

(a) AB > AC (b) AB > BC

(c) AC > AB + BC (d) AB < AC + BC

6. In the figure, P divides AB internally in the ratio : **[2011 (T-II)]**



(a) 3 : 4 (b) 4 : 3 (c) 3 : 7 (d) 4 : 7

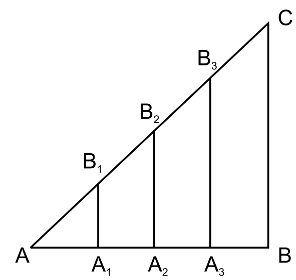
7. In the construction of triangle similar and larger to a given triangle as per given scale factor m : n, the construction is possible only when : **[2011 (T-II)]**

(a) $m > n$ (b) $m = n$

(c) $m < n$

(d) independent of scale factor

8. In the figure, $AA_1 = A_1A_2 = A_2A_3 = A_3B$. If $B_1A_1 \| CB$, then A_1 divides AB in the ratio : **[2011 (T-II)]**



(a) 1 : 2 (b) 1 : 3 (c) 1 : 4 (d) 1 : 1

9. The sides of a triangle (in cm) are given below: In which case, the construction of triangle is not possible ? **[2011 (T-II)]**

(a) 8, 7, 3 (b) 8, 6, 4 (c) 8, 4, 4 (d) 7, 6, 5

SHORT ANSWER TYPE QUESTIONS [2 Marks]

A. Important Questions

Write 'T' for true and 'F' for false statement. In each case, give reason for your answer.

1. We can always divide a line segment in the ratio $\sqrt{2} : \frac{1}{\sqrt{2}}$ by geometrical construction.

2. By geometrical construction, it is possible to divide a line segment in the ratio $3 + 2\sqrt{2} : 3 - 2\sqrt{2}$.

3. We can draw a tangent to a circle from a point which lies in the interior of the circle.

4. At any point on a circle, we can draw only one tangent.

5. From a point P which lies in the exterior of the circle, we can draw exactly two tangents to the circle.

6. A pair of tangents can be constructed to a circle inclined at an angle of 105°.

7. A pair of tangents can be constructed from a point P to a circle of radius 4 cm situated at a distance of 3.5 cm from the centre.

SHORT ANSWER TYPE QUESTIONS [3 Marks]

A. Important Questions

1. Draw a line segment AB = 7.5 cm. Find a point P on it which divides it in the ratio 2 : 7.

2. Draw a line segment of length 7.6 cm and divide it into the ratio 5 : 8. Measure the two parts.

3. Three sides PQ, QR and PR of ΔPQR are 5 cm, 6 cm and 7 cm respectively. Construct the ΔPQR. Construct a $\Delta PQ'R'$ such that each of its sides is $\frac{2}{3}$ of corresponding sides of ΔPQR.

4. Draw a right triangle ABC in which BC = 12 cm, AB = 5 cm and $\angle B = 90°$. Construct a triangle similar to it and of scale factor $\frac{5}{3}$ Is the new triangle also a right triangle ?

5. Construct a ΔABC in which AB = 5 cm, $\angle B = 60°$, altitude CD = 3 cm. Construct a ΔAQR similar to ΔABC such that each side of ΔAQR is 1.5 times that of the corresponding side of ΔABC.

6. Construct a tangent to a circle of radius 4 cm from a point which is at a distance of 6 cm from its centre.

7. At a point P on the circle, draw a tangent, without using the centre of the circle.

8. Draw a pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of 60°.

9. Two line segments AB and AC include an angle of 60°, where AB = 5 cm and AC = 7 cm. Locate points P and Q on AB and AC respectively such that AP = $\frac{3}{4}$ AB and AQ = $\frac{1}{4}$ AC. Join P and Q and measure the length of PQ.

10. Given a rhombus ABCD in which AB = 4 cm and $\angle ABC = 60°$, divide it into two triangles say ABC and ADC by the diagonal AC. Construct the $\Delta AB'C'$ similar to ΔABC with scale factor $\frac{3}{5}$ Draw a line segment C′D′ parallel to CD, where D′ lies on AD. Is AB′C′D′ a rhombus ?

11. Draw two concentric circles of radii 3 cm and 5 cm. Taking a point on outer circle construct the pair of tangents to the other.

12. Draw a circle of radius 3 cm. Take two points P and Q on one of its extended diameter each at a distance of 7 cm from its centre. Draw tangents to the circle from these two points P and Q.

13. Draw a circle of radius 5 cm. Construct a pair of tangents to it, the angle between which is 30°. Measure the distance between the centre of the circle and the point of intersection of the tangents.

14. Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 4 cm and taking B as centre, draw another circle of radius 3 cm. Construct tangents to each circle from the centre of the other circle.

15. Draw a parallelogram ABCD in which BC = 5 cm, AB = 3 cm and $\angle ABC = 60°$. Divide it into triangles BCD and ABD by the diagonal BD. Construct the $\Delta BD'C'$ similar to ΔBDC with scale factor $\frac{4}{3}$. Draw the line segment D′A′ parallel to DA, where A′ lies on extended side BA. Is A′BC′D′ a parallelogram?

B. Questions From CBSE Examination Papers

1. Construct a pair of tangents to a circle of radius 4 cm inclined at an angle of 45°. **[2011 (T-II)]**

2. Construct two circles of radii 3 cm and 4 cm whose centres are 8 cm apart. Draw the pair of tangents from the centre of each circle to the other circle. **[2011 (T-II)]**

3. Construct a triangle ABC in which AB = 5 cm, $\angle B = 60°$ and the altitude CD = 3 cm. Then Construct another triangle whose sides are $\frac{4}{5}$ times the corresponding sides of ΔABC. **[2011 (T-II)]**

4. Draw a circle of radius 6 cm. From a point 10 cm away from its centre, construct the pair of tangents to the circle and measure their lengths. **[2011 (T-II)]**

5. Draw a right triangle in which the sides (other than hypotenuse) are of lengths 4 cm and 3 cm. Then construct another triangle whose sides are $\frac{3}{5}$ times the corresponding sides of the given triangle. **[2011 (T-II)]**

6. Construct a triangle ABC, in which base BC = 6 cm, $\angle B = 60°$ and $\angle BAC = 90°$. Then construct another triangle whose sides are $\frac{3}{4}$ of the corresponding sides of ΔABC. **[2011 (T-II)]**

7. Draw a pair of tangents to a circle of radius 3.5 cm which are perpendicular to each other. **[2011 (T-II)]**

8. Draw a ΔABC with BC = 8 cm, $\angle ABC = 45°$ and $\angle BAC = 105°$. Then construct a triangle whose sides are $\frac{2}{3}$ times the corresponding sides of the ΔABC. **[2011 (T-II)]**

9. Draw a triangle ABC with side BC = 7 cm, $\angle B = 45°$, and $\angle A = 105°$. Then construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of ΔABC. **[2011 (T-II)]**

10. Draw a pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of 60°. **[2011 (T-II)]**

11. Draw a ΔABC with sides BC = 6 cm, AB = 5 cm and $\angle ABC = 60°$. Construct a $\Delta A'BC'$ similar to ΔABC such that sides of $\Delta A'BC$ are $\frac{3}{4}$ of the correspondings sides of ΔABC. **[2011 (T-II)]**

12. Draw two tangents to a circle of radius 3.5 cm from a point P at a distance of 6 cm from its centre O. **[2011 (T-II)]**

13. Draw a right triangle in which the sides (other than hypotenuse) are of lengths 8 cm and 6 cm. Then construct another triangle whose sides are 3/5 times the corresponding sides of the given triangle. **[2011 (T-II)]**

14. Construct a ΔABC in which BC = 6.5 cm, AB = 4.5 cm and $\angle ABC = 60°$. Construct a triangle similar to this triangle whose sides are $\frac{4}{5}$ of the corresponding sides of the triangle ABC. **[2009]**

15. Draw a circle of radius 3 cm. From a point P, 6 cm away from its centre, construct a pair of tangents to the circle. Measure the lengths of the tangents. **[2009]**

16. Construct a ΔABC in which BC = 9 cm, $\angle B = 60°$ and AB = 6 cm. Then construct another triangle whose sides are $\frac{2}{3}$ of the corresponding sides of ΔABC. **[2010]**

FORMATIVE ASSESSMENT

Converting a Triangle into a Square

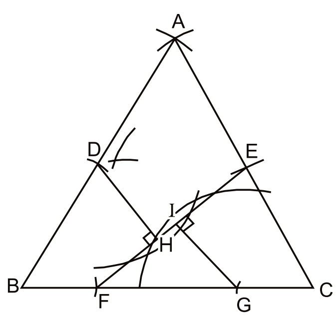
On a thick sheet of paper, construct an equilateral ΔABC.

Divide the triangle ABC into four pieces as shown in the figure,

Here, AD = BD, AE = CE

$BF = \frac{1}{4}BC, CG = \frac{1}{4}BC$

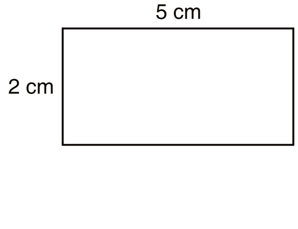
$DH \perp EF$ and $GI \perp EF$



Cut the pieces out and rearrange the pieces to form a square.

Converting a Rectangle into a Square

On a thick sheet of paper, draw a rectangle of dimensions 5 cm × 2 cm .



Using three straight cuts, divide the rectangle into 5 pieces such that these pieces when rearranged give a square.

Making Rectangle From Squares

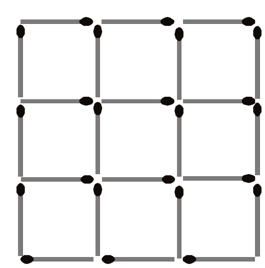
On thick sheets of paper, draw squares of sides 1 cm, 4 cm, 7 cm, 8 cm, 9 cm, 10 cm, 14 cm 15 cm and 18 cm.

Cut out each square.

Now rearrange these square pieces to form a rectangle. Paste the arrangement on a sheet of paper.

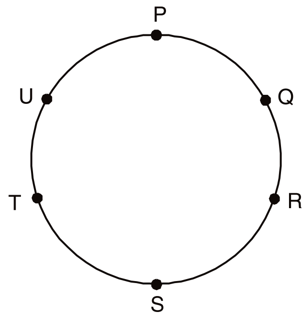
Matchstick Puzzle

A 3 × 3 array of matchsticks is given. From this array remove exactly four matchsticks to get five identical squares.



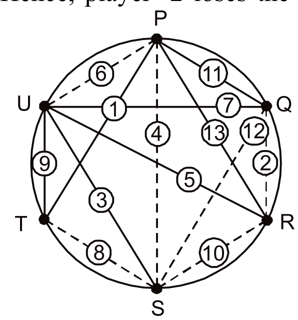
A Mathematical Game

This is a game for two players. Six points P, Q, R, S, T, U are marked on the circumference of a circle. Player-1 uses red coloured pencil and player-2 uses green coloured pencil. They take turns to join a pair of points with a straight line. Can you tell how many such lines are possible?



Obviously there are 6 sides and 9 diagonals i.e. 15 such lines are possible. But, here the aim of the game is to avoid making a triangle of your colour with the vertices on the triangle. The player who do so will loose.

The result of a game is shown here with numbers to indicate the order in which the lines were drawn. The solid lines are drawn by player-1 and the dotted lines are drawn by player -2. It's player-2's turn and the only possible moves are TR and TQ, which complete Δ TRS and Δ TQS. Hence, player -2 loses the game.



Exercise 11.1

Question 1:

Draw a line segment of length 7.6 cm and divide it in the ratio 5:8. Measure the two parts. Give the justification of the construction.

Answer:

A line segment of length 7.6 cm can be divided in the ratio of 5:8 as follows.

Step 1 Draw line segment AB of 7.6 cm and draw a ray AX making an acute angle with line segment AB.

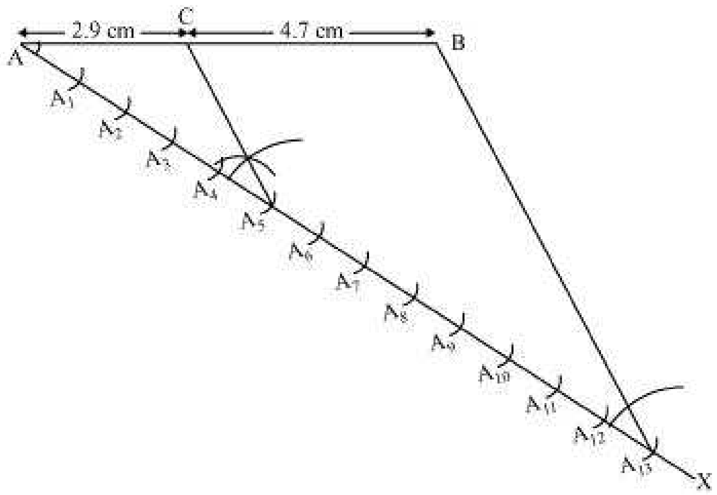
Step 2 Locate 13 (= 5 + 8) points, A_1, A_2, A_3, A_4 …….. A_{13}, on AX such that AA_1 = A_1A_2 = A_2A_3 and so on.

Step 3 Join BA_{13}.

Step 4 Through the point A_5, draw a line parallel to BA_{13} (by making an angle equal to $\angle AA_{13}B$) at A_5 intersecting AB at point C.

C is the point dividing line segment AB of 7.6 cm in the required ratio of 5:8.

The lengths of AC and CB can be measured. It comes out to 2.9 cm and 4.7 cm respectively.



Justification

The construction can be justified by proving that

$\frac{AC}{CB}=\frac{5}{8}$

By construction, we have $A_5C \parallel A_{13}B$. By applying Basic proportionality theorem for the triangle $AA_{13}B$, we obtain

$\frac{AC}{CB}=\frac{AA_5}{A_5A_{13}}$ … (1)

From the figure, it can be observed that AA_5 and A_5A_{13} contain 5 and 8 equal divisions of line segments respectively.

$\therefore \frac{AA_5}{A_5A_{13}}=\frac{5}{8}$ … (2)

On comparing equations (1) and (2), we obtain

$\frac{AC}{CB}=\frac{5}{8}$

This justifies the construction.

Question 2:

Construct a triangle of sides 4 cm, 5cm and 6cm and then a triangle similar to it whose sides are $\frac{2}{3}$ of the corresponding sides of the first triangle.

Give the justification of the construction.

Answer:

Step 1

Draw a line segment AB = 4 cm. Taking point A as centre, draw an arc of 5 cm radius. Similarly, taking point B as its centre, draw an arc of 6 cm radius. These arcs will intersect each other at point C. Now, AC = 5 cm and BC = 6 cm and ΔABC is the required triangle.

Step 2

Draw a ray AX making an acute angle with line AB on the opposite side of vertex C.

Step 3

Locate 3 points A_1, A_2, A_3 (as 3 is greater between 2 and 3) on line AX such that $AA_1 = A_1A_2 = A_2A_3$.

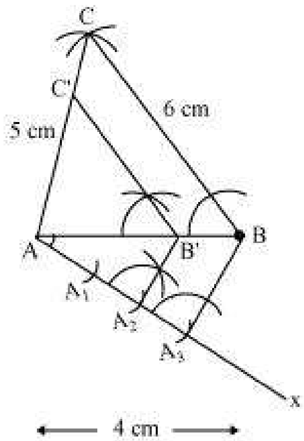
Step 4

Join BA_3 and draw a line through A_2 parallel to BA_3 to intersect AB at point B'.

Step 5

Draw a line through B' parallel to the line BC to intersect AC at C'.

ΔAB'C' is the required triangle.



Justification

The construction can be justified by proving that

$$AB' = \frac{2}{3}AB,\ B'C' = \frac{2}{3}BC,\ AC' = \frac{2}{3}AC$$

By construction, we have B'C' || BC

$\therefore \angle AB'C' = \angle ABC$ (Corresponding angles)

In ΔAB'C' and ΔABC,

$\angle AB'C' = \angle ABC$ (Proved above)

$\angle B'AC' = \angle BAC$ (Common)

$\therefore \Delta AB'C' \sim \Delta ABC$ (AA similarity criterion)

$\Rightarrow \frac{AB'}{AB} = \frac{B'C'}{BC} = \frac{AC'}{AC}$ ... (1)

In $\Delta AA_2B'$ and ΔAA_3B,

$\angle A_2AB' = \angle A_3AB$ (Common)

$\angle AA_2B' = \angle AA_3B$ (Corresponding angles)

$\therefore \Delta AA_2B' \sim \Delta AA_3B$ (AA similarity criterion)

$\Rightarrow \frac{AB'}{AB} = \frac{AA_2}{AA_3}$

$\Rightarrow \frac{AB'}{AB} = \frac{2}{3}$ ...(2)

From equations (1) and (2), we obtain

$\frac{AB'}{AB} = \frac{B'C'}{BC} = \frac{AC'}{AC} = \frac{2}{3}$

$\Rightarrow AB' = \frac{2}{3}AB, B'C' = \frac{2}{3}BC, AC' = \frac{2}{3}AC$

This justifies the construction.

Question 3:

Construct a triangle with sides 5 cm, 6 cm and 7 cm and then another triangle whose sides are $\frac{7}{5}$ of the corresponding sides of the first triangle.

Give the justification of the construction.

Answer:

Step 1

Draw a line segment AB of 5 cm. Taking A and B as centre, draw arcs of 6 cm and 5 cm radius respectively. Let these arcs intersect each other at point C. ΔABC is the required triangle having length of sides as 5 cm, 6 cm, and 7 cm respectively.

Step 2

Draw a ray AX making acute angle with line AB on the opposite side of vertex C.

Step 3

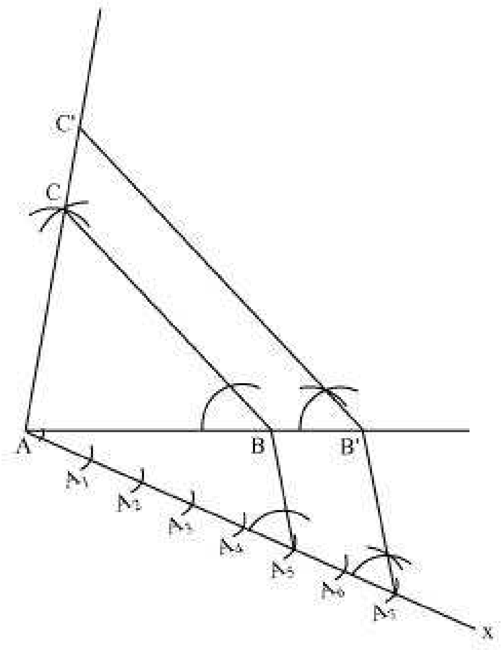
Locate 7 points, A_1, A_2, A_3, A_4 A_5, A_6, A_7 (as 7 is greater between 5and 7), on line AX such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = A_6A_7$.

Step 4

Join BA_5 and draw a line through A_7 parallel to BA_5 to intersect extended line segment AB at point B'.

Step 5

Draw a line through B' parallel to BC intersecting the extended line segment AC at C'. ΔAB'C' is the required triangle.



Justification

The construction can be justified by proving that

$$AB' = \frac{7}{5}AB, B'C' = \frac{7}{5}BC, AC' = \frac{7}{5}AC$$

In ΔABC and ΔAB'C',

$\angle ABC = \angle AB'C'$ (Corresponding angles)

$\angle BAC = \angle B'AC'$ (Common)

∴ $\Delta ABC \sim \Delta AB'C'$ (AA similarity criterion)

$$\Rightarrow \frac{AB}{AB'} = \frac{BC}{B'C'} = \frac{AC}{AC'} \quad \ldots (1)$$

In ΔAA_5B and $\Delta AA_7B'$,

$\angle A_5AB = \angle A_7AB'$ (Common)

$\angle AA_5B = \angle AA_7B'$ (Corresponding angles)

∴ $\Delta AA_5B \sim \Delta AA_7B'$ (AA similarity criterion)

$$\Rightarrow \frac{AB}{AB'} = \frac{AA_5}{AA_7}$$

$$\Rightarrow \frac{AB}{AB'} = \frac{5}{7} \quad \ldots(2)$$

On comparing equations (1) and (2), we obtain

$$\frac{AB}{AB'} = \frac{BC}{B'C'} = \frac{AC}{AC'} = \frac{5}{7}$$

$$\Rightarrow AB' = \frac{7}{5}AB,\ B'C' = \frac{7}{5}BC,\ AC' = \frac{7}{5}AC$$

This justifies the construction.

Question 4:

Construct an isosceles triangle whose base is 8 cm and altitude 4 cm and then another triangle whose side are $1\frac{1}{2}$ times the corresponding sides of the isosceles triangle.

Give the justification of the construction.

Answer:

Let us assume that ΔABC is an isosceles triangle having CA and CB of equal lengths, base AB of 8 cm, and AD is the altitude of 4 cm.

A $\Delta AB'C'$ whose sides are $\frac{3}{2}$ times of ΔABC can be drawn as follows.

Step 1

Draw a line segment AB of 8 cm. Draw arcs of same radius on both sides of the line segment while taking point A and B as its centre. Let these arcs intersect each other at O and O'. Join OO'. Let OO' intersect AB at D.

Step 2

Taking D as centre, draw an arc of 4 cm radius which cuts the extended line segment OO' at point C. An isosceles ΔABC is formed, having CD (altitude) as 4 cm and AB (base) as 8 cm.

Step 3

Draw a ray AX making an acute angle with line segment AB on the opposite side of vertex C.

Step 4

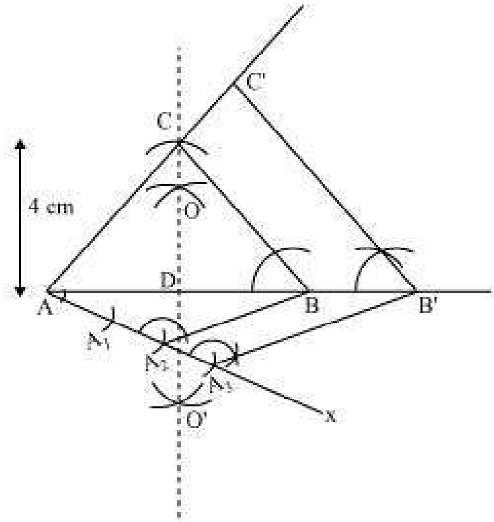
Locate 3 points (as 3 is greater between 3 and 2) A_1, A_2, and A_3 on AX such that $AA_1 = A_1A_2 = A_2A_3$.

Step 5

Join BA_2 and draw a line through A_3 parallel to BA_2 to intersect extended line segment AB at point B'.

Step 6

Draw a line through B' parallel to BC intersecting the extended line segment AC at C'. ΔAB'C' is the required triangle.



Justification

The construction can be justified by proving that

$$AB' = \frac{3}{2}AB,\ B'C' = \frac{3}{2}BC,\ AC' = \frac{3}{2}AC$$

In ΔABC and ΔAB'C',

∠ABC = ∠AB'C' (Corresponding angles)

∠BAC = ∠B'AC' (Common)

∴ ΔABC ~ ΔAB'C' (AA similarity criterion)

$$\Rightarrow \frac{AB}{AB'} = \frac{BC}{B'C'} = \frac{AC}{AC'} \quad \text{... (1)}$$

In ΔAA_2B and $\Delta AA_3B'$,

$\angle A_2AB = \angle A_3AB'$ (Common)

$\angle AA_2B = \angle AA_3B'$ (Corresponding angles)

∴ $\Delta AA_2B \sim \Delta AA_3B'$ (AA similarity criterion)

$$\Rightarrow \frac{AB}{AB'} = \frac{AA_2}{AA_3}$$

$$\Rightarrow \frac{AB}{AB'} = \frac{2}{3} \quad \text{...(2)}$$

On comparing equations (1) and (2), we obtain

$$\frac{AB}{AB'} = \frac{BC}{B'C'} = \frac{AC}{AC'} = \frac{2}{3}$$

$$\Rightarrow AB' = \frac{3}{2}AB,\ B'C' = \frac{3}{2}BC,\ AC' = \frac{3}{2}AC$$

This justifies the construction.

Question 5:

Draw a triangle ABC with side BC = 6 cm, AB = 5 cm and ∠ABC = 60°. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the triangle ABC. Give the justification of the construction.

Answer:

A ΔA'BC' whose sides are $\frac{3}{4}$ of the corresponding sides of ΔABC can be drawn as follows.

Step 1

Draw a ΔABC with side BC = 6 cm, AB = 5 cm and ∠ABC = 60°.

Step 2

Draw a ray BX making an acute angle with BC on the opposite side of vertex A.

Step 3

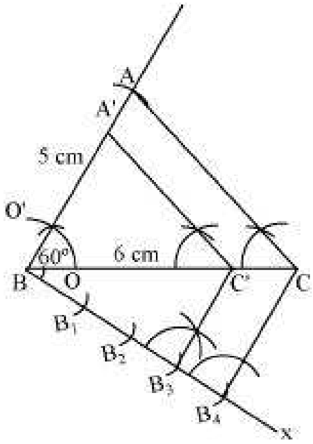
Locate 4 points (as 4 is greater in 3 and 4), B_1, B_2, B_3, B_4, on line segment BX.

Step 4

Join B_4C and draw a line through B_3, parallel to B_4C intersecting BC at C'.

Step 5

Draw a line through C' parallel to AC intersecting AB at A'. ΔA'BC' is the required triangle.



Justification

The construction can be justified by proving

$$A'B = \frac{3}{4}AB,\ BC' = \frac{3}{4}BC,\ A'C' = \frac{3}{4}AC$$

In $\Delta A'BC'$ and ΔABC,

$\angle A'C'B = \angle ACB$ (Corresponding angles)

$\angle A'BC' = \angle ABC$ (Common)

$\therefore \Delta A'BC' \sim \Delta ABC$ (AA similarity criterion)

$$\Rightarrow \frac{A'B}{AB} = \frac{BC'}{BC} = \frac{A'C'}{AC} \quad \dots (1)$$

In $\Delta BB_3C'$ and ΔBB_4C,

$\angle B_3BC' = \angle B_4BC$ (Common)

$\angle BB_3C' = \angle BB_4C$ (Corresponding angles)

$\therefore \Delta BB_3C' \sim \Delta BB_4C$ (AA similarity criterion)

$$\Rightarrow \frac{BC'}{BC} = \frac{BB_3}{BB_4}$$

$$\Rightarrow \frac{BC'}{BC} = \frac{3}{4} \quad \dots(2)$$

From equations (1) and (2), we obtain

$$\frac{A'B}{AB} = \frac{BC'}{BC} = \frac{A'C'}{AC} = \frac{3}{4}$$

$\Rightarrow A'B = \frac{3}{4}AB, BC' = \frac{3}{4}BC, A'C' = \frac{3}{4}AC$

This justifies the construction.

Question 6:

Draw a triangle ABC with side BC = 7 cm, ∠B = 45°, ∠A = 105°. Then, construct a triangle whose sides are $\frac{4}{3}$ times the corresponding side of ΔABC. Give the justification of the construction.

Answer:

∠B = 45°, ∠A = 105°

Sum of all interior angles in a triangle is 180°.

∠A + ∠B + ∠C = 180°

105° + 45° + ∠C = 180°

∠C = 180° − 150°

∠C = 30°

The required triangle can be drawn as follows.

Step 1

Draw a ΔABC with side BC = 7 cm, ∠B = 45°, ∠C = 30°.

Step 2

Draw a ray BX making an acute angle with BC on the opposite side of vertex A.

Step 3

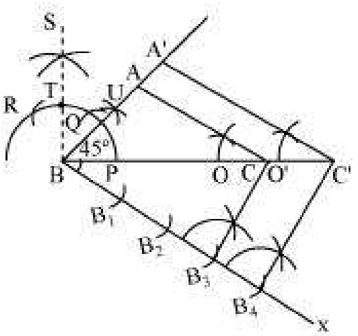
Locate 4 points (as 4 is greater in 4 and 3), B_1, B_2, B_3, B_4, on BX.

Step 4

Join B_3C. Draw a line through B_4 parallel to B_3C intersecting extended BC at C'.

Step 5

Through C', draw a line parallel to AC intersecting extended line segment at C'. ΔA'BC' is the required triangle.



Justification

The construction can be justified by proving that

$$A'B = \frac{4}{3}AB, \ BC' = \frac{4}{3}BC, \ A'C' = \frac{4}{3}AC$$

In ΔABC and ΔA'BC',

∠ABC = ∠A'BC' (Common)

∠ACB = ∠A'C'B (Corresponding angles)

∴ ΔABC ~ ΔA'BC' (AA similarity criterion)

$$\Rightarrow \frac{AB}{A'B} = \frac{BC}{BC'} = \frac{AC}{A'C'} \quad \ldots (1)$$

In ΔBB_3C and $\Delta BB_4C'$,

$\angle B_3BC = \angle B_4BC'$ (Common)

$\angle BB_3C = \angle BB_4C'$ (Corresponding angles)

∴ $\Delta BB_3C \sim \Delta BB_4C'$ (AA similarity criterion)

$$\Rightarrow \frac{BC}{BC'} = \frac{BB_3}{BB_4}$$

$$\Rightarrow \frac{BC}{BC'} = \frac{3}{4} \quad \ldots(2)$$

On comparing equations (1) and (2), we obtain

$$\frac{AB}{A'B} = \frac{BC}{BC'} = \frac{AC}{A'C'} = \frac{3}{4}$$

$$\Rightarrow A'B = \frac{4}{3}AB,\ BC' = \frac{4}{3}BC,\ A'C' = \frac{4}{3}AC$$

This justifies the construction.

Question 7:

Draw a right triangle in which the sides (other than hypotenuse) are of lengths 4 cm and 3 cm. the construct another triangle whose sides are $\frac{5}{3}$ times the corresponding sides of the given triangle. Give the justification of the construction.

Answer:

It is given that sides other than hypotenuse are of lengths 4 cm and 3 cm. Clearly, these will be perpendicular to each other.

The required triangle can be drawn as follows.

Step 1

Draw a line segment AB = 4 cm. Draw a ray SA making 90° with it.

Step 2

Draw an arc of 3 cm radius while taking A as its centre to intersect SA at C. Join BC. ΔABC is the required triangle.

Step 3

Draw a ray AX making an acute angle with AB, opposite to vertex C.

Step 4

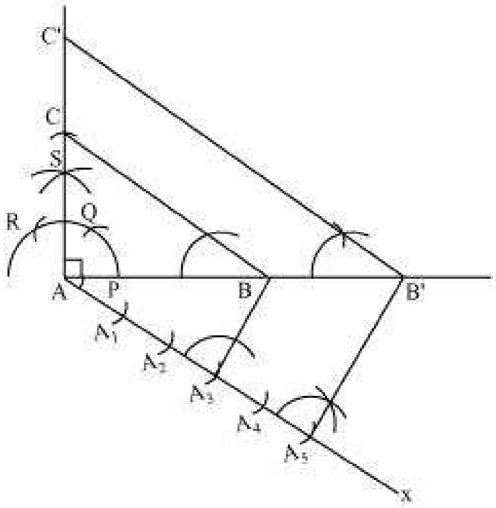
Locate 5 points (as 5 is greater in 5 and 3), A_1, A_2, A_3, A_4, A_5, on line segment AX such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$.

Step 5

Join A_3B. Draw a line through A_5 parallel to A_3B intersecting extended line segment AB at B'.

Step 6

Through B', draw a line parallel to BC intersecting extended line segment AC at C'. ΔAB'C' is the required triangle.



Justification

The construction can be justified by proving that

$$AB' = \frac{5}{3}AB,\ B'C' = \frac{5}{3}BC,\ AC' = \frac{5}{3}AC$$

In ΔABC and ΔAB'C',

∠ABC = ∠AB'C' (Corresponding angles)

∠BAC = ∠B'AC' (Common)

∴ ΔABC ~ ΔAB'C' (AA similarity criterion)

$$\Rightarrow \frac{AB}{AB'} = \frac{BC}{B'C'} = \frac{AC}{AC'} \quad \ldots (1)$$

In ΔAA_3B and $\Delta AA_5B'$,

$\angle A_3AB = \angle A_5AB'$ (Common)

$\angle AA_3B = \angle AA_5B'$ (Corresponding angles)

∴ $\Delta AA_3B \sim \Delta AA_5B'$ (AA similarity criterion)

$$\Rightarrow \frac{AB}{AB'} = \frac{AA_3}{AA_5}$$

$$\Rightarrow \frac{AB}{AB'} = \frac{3}{5} \qquad ...(2)$$

On comparing equations (1) and (2), we obtain

$$\frac{AB}{AB'} = \frac{BC}{B'C'} = \frac{AC}{AC'} = \frac{3}{5}$$

$$\Rightarrow AB' = \frac{5}{3}AB,\ B'C' = \frac{5}{3}BC,\ AC' = \frac{5}{3}AC$$

This justifies the construction.

Exercise 11.2

Question 1:

Draw a circle of radius 6 cm. From a point 10 cm away from its centre, construct the pair of tangents to the circle and measure their lengths. Give the justification of the construction.

Answer:

A pair of tangents to the given circle can be constructed as follows.

Step 1

Taking any point O of the given plane as centre, draw a circle of 6 cm radius. Locate a point P, 10 cm away from O. Join OP.

Step 2

Bisect OP. Let M be the mid-point of PO.

Step 3

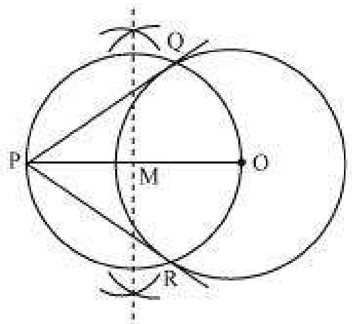
Taking M as centre and MO as radius, draw a circle.

Step 4

Let this circle intersect the previous circle at point Q and R.

Step 5

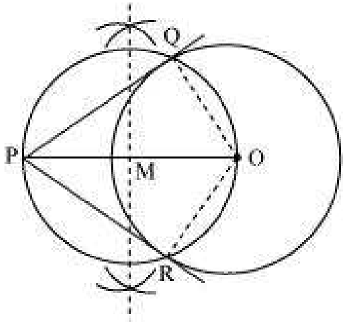
Join PQ and PR. PQ and PR are the required tangents.



The lengths of tangents PQ and PR are 8 cm each.

Justification

The construction can be justified by proving that PQ and PR are the tangents to the circle (whose centre is O and radius is 6 cm). For this, join OQ and OR.



$\angle PQO$ is an angle in the semi-circle. We know that angle in a semi-circle is a right angle.

$\therefore \angle PQO = 90°$

$\Rightarrow OQ \perp PQ$

Since OQ is the radius of the circle, PQ has to be a tangent of the circle. Similarly, PR is a tangent of the circle

Question 2:

Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm and measure its length. Also verify the measurement by actual calculation. Give the justification of the construction.

Answer:

Tangents on the given circle can be drawn as follows.

Step 1

Draw a circle of 4 cm radius with centre as O on the given plane.

Step 2

Draw a circle of 6 cm radius taking O as its centre. Locate a point P on this circle and join OP.

Step 3

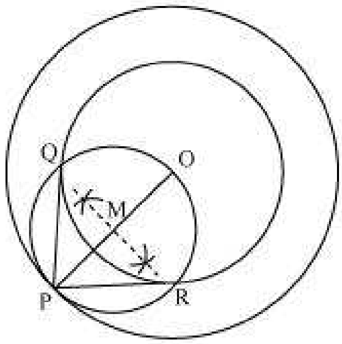
Bisect OP. Let M be the mid-point of PO.

Step 4

Taking M as its centre and MO as its radius, draw a circle. Let it intersect the given circle at the points Q and R.

Step 5

Join PQ and PR. PQ and PR are the required tangents.



It can be observed that PQ and PR are of length 4.47 cm each.

In ΔPQO,

Since PQ is a tangent,

$\angle PQO = 90°$

PO = 6 cm

QO = 4 cm

Applying Pythagoras theorem in ΔPQO, we obtain

$PQ^2 + QO^2 = PQ^2$

$PQ^2 + (4)^2 = (6)^2$

$PQ^2 + 16 = 36$

$PQ^2 = 36 - 16$

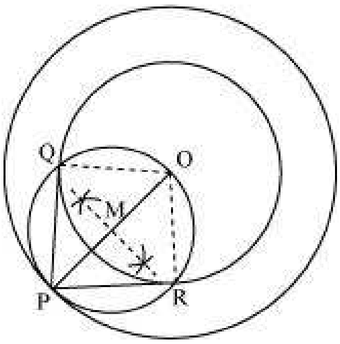
$PQ^2 = 20$

PQ $= 2\sqrt{5}$

PQ = 4.47 cm

Justification

The construction can be justified by proving that PQ and PR are the tangents to the circle (whose centre is O and radius is 4 cm). For this, let us join OQ and OR.



∠PQO is an angle in the semi-circle. We know that angle in a semi-circle is a right angle.

∴ ∠PQO = 90°

⇒ OQ ⊥ PQ

Since OQ is the radius of the circle, PQ has to be a tangent of the circle. Similarly, PR is a tangent of the circle

Question 3:

Draw a circle of radius 3 cm. Take two points P and Q on one of its extended diameter each at a distance of 7 cm from its centre. Draw tangents to the circle from these two points P and Q. Give the justification of the construction.

Answer:

The tangent can be constructed on the given circle as follows.

Step 1

Taking any point O on the given plane as centre, draw a circle of 3 cm radius.

Step 2

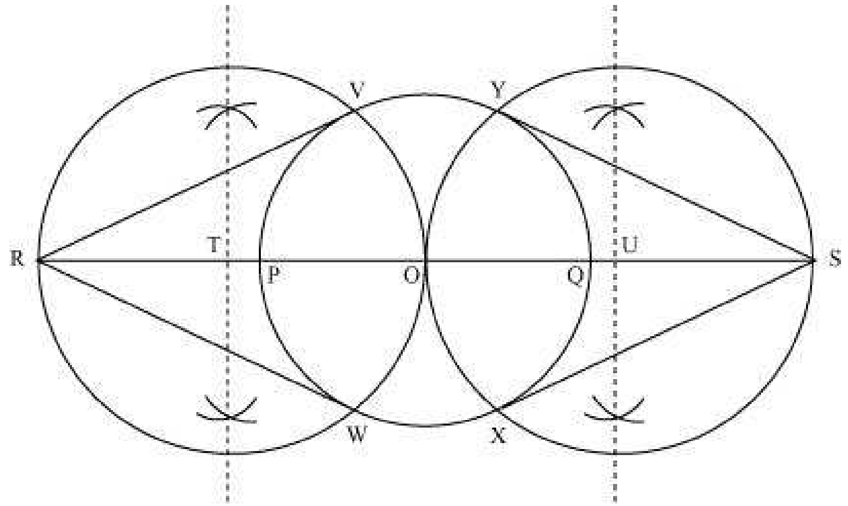
Take one of its diameters, PQ, and extend it on both sides. Locate two points on this diameter such that OR = OS = 7 cm

Step 3

Bisect OR and OS. Let T and U be the mid-points of OR and OS respectively.

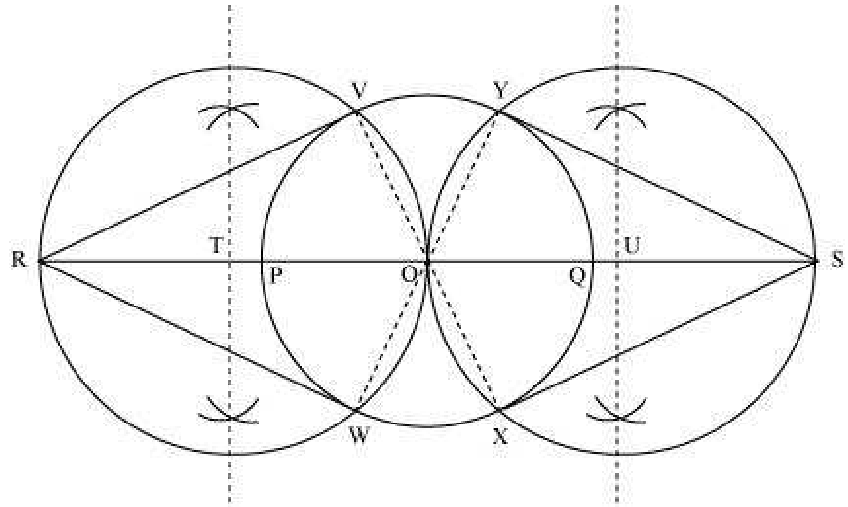
Step 4

Taking T and U as its centre and with TO and UO as radius, draw two circles. These two circles will intersect the circle at point V, W, X, Y respectively. Join RV, RW, SX, and SY. These are the required tangents.



Justification

The construction can be justified by proving that RV, RW, SY, and SX are the tangents to the circle (whose centre is O and radius is 3 cm). For this, join OV, OW, OX, and OY.



∠RVO is an angle in the semi-circle. We know that angle in a semi-circle is a right angle.

∴ ∠RVO = 90°

⇒ OV ⊥ RV

Since OV is the radius of the circle, RV has to be a tangent of the circle. Similarly, OW, OX, and OY are the tangents of the circle

Question 4:

Draw a pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of 60°. Give the justification of the construction.

Answer:

The tangents can be constructed in the following manner:

Step 1

Draw a circle of radius 5 cm and with centre as O.

Step 2

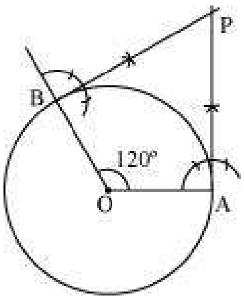
Take a point A on the circumference of the circle and join OA. Draw a perpendicular to OA at point A.

Step 3

Draw a radius OB, making an angle of 120° (180° − 60°) with OA.

Step 4

Draw a perpendicular to OB at point B. Let both the perpendiculars intersect at point P. PA and PB are the required tangents at an angle of 60°.



Justification

The construction can be justified by proving that ∠APB = 60°

By our construction

$\angle OAP = 90°$

$\angle OBP = 90°$

And $\angle AOB = 120°$

We know that the sum of all interior angles of a quadrilateral = 360°

$\angle OAP + \angle AOB + \angle OBP + \angle APB = 360°$

$90° + 120° + 90° + \angle APB = 360°$

$\angle APB = 60°$

This justifies the construction.

Question 5:

Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 4 cm and taking B as centre, draw another circle of radius 3 cm. Construct tangents to each circle from the centre of the other circle. Give the justification of the construction.

Answer:

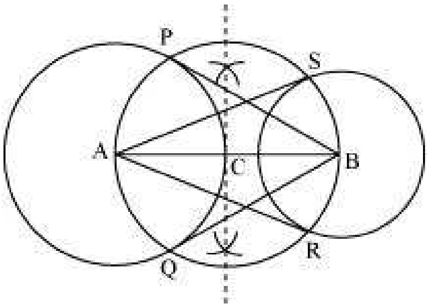
The tangents can be constructed on the given circles as follows.

Step 1

Draw a line segment AB of 8 cm. Taking A and B as centre, draw two circles of 4 cm and 3 cm radius.

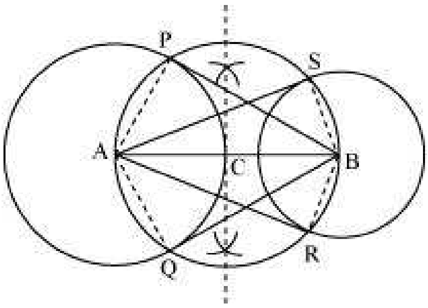
Step 2

Bisect the line AB. Let the mid-point of AB be C. Taking C as centre, draw a circle of AC radius which will intersect the circles at points P, Q, R, and S. Join BP, BQ, AS, and AR. These are the required tangents.



Justification

The construction can be justified by proving that AS and AR are the tangents of the circle (whose centre is B and radius is 3 cm) and BP and BQ are the tangents of the circle (whose centre is A and radius is 4 cm). For this, join AP, AQ, BS, and BR.



$\angle ASB$ is an angle in the semi-circle. We know that an angle in a semi-circle is a right angle.

$\therefore \angle ASB = 90°$

$\Rightarrow BS \perp AS$

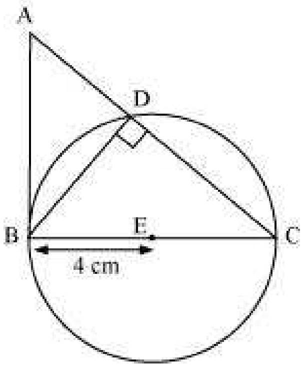
Since BS is the radius of the circle, AS has to be a tangent of the circle. Similarly, AR, BP, and BQ are the tangents.

Question 6:

Let ABC be a right triangle in which AB = 6 cm, BC = 8 cm and $\angle B = 90°$. BD is the perpendicular from B on AC. The circle through B, C, and D is drawn. Construct the tangents from A to this circle. Give the justification of the construction.

Answer:

Consider the following situation. If a circle is drawn through B, D, and C, BC will be its diameter as ∠BDC is of measure 90°. The centre E of this circle will be the mid-point of BC.



The required tangents can be constructed on the given circle as follows.

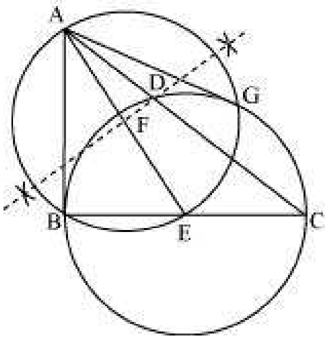
Step 1

Join AE and bisect it. Let F be the mid-point of AE.

Step 2

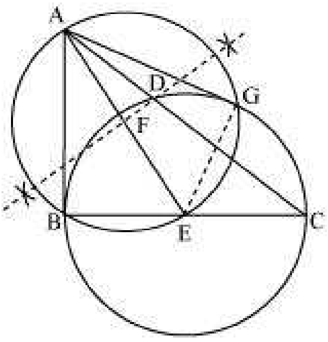
Taking F as centre and FE as its radius, draw a circle which will intersect the circle at point B and G. Join AG.

AB and AG are the required tangents.



Justification

The construction can be justified by proving that AG and AB are the tangents to the circle. For this, join EG.



$\angle AGE$ is an angle in the semi-circle. We know that an angle in a semi-circle is a right angle.

$\therefore \angle AGE = 90°$

$\Rightarrow EG \perp AG$

Since EG is the radius of the circle, AG has to be a tangent of the circle.

Already, $\angle B = 90°$

$\Rightarrow AB \perp BE$

Since BE is the radius of the circle, AB has to be a tangent of the circle.

Question 7:

Draw a circle with the help of a bangle. Take a point outside the circle. Construct the pair of tangents from this point to the circles. Give the justification of the construction.

Answer:

The required tangents can be constructed on the given circle as follows.

Step 1

Draw a circle with the help of a bangle.

Step 2

Take a point P outside this circle and take two chords QR and ST.

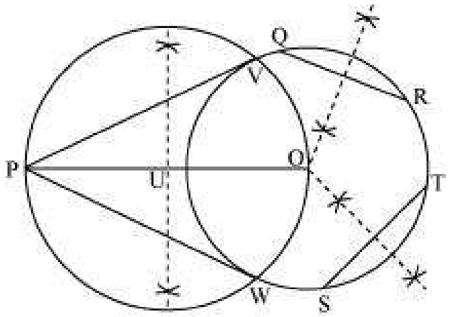
Step 3

Draw perpendicular bisectors of these chords. Let them intersect each other at point O.

Step 4

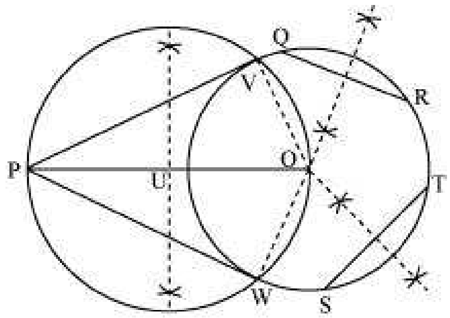
Join PO and bisect it. Let U be the mid-point of PO. Taking U as centre, draw a circle of radius OU, which will intersect the circle at V and W. Join PV and PW.

PV and PW are the required tangents.



Justification

The construction can be justified by proving that PV and PW are the tangents to the circle. For this, first of all, it has to be proved that O is the centre of the circle. Let us join OV and OW.



We know that perpendicular bisector of a chord passes through the centre. Therefore, the perpendicular bisector of chords QR and ST pass through the centre. It is clear that the intersection point of these perpendicular bisectors is the centre of the circle. ∠PVO is an angle in the semi-circle. We know that an angle in a semi-circle is a right angle.

$\therefore \angle PVO = 90°$

$\Rightarrow OV \perp PV$

Since OV is the radius of the circle, PV has to be a tangent of the circle. Similarly, PW is a tangent of the circle.