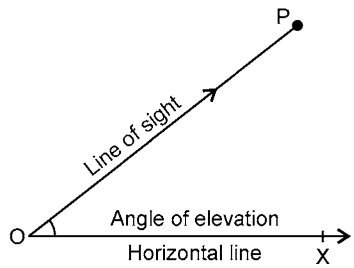
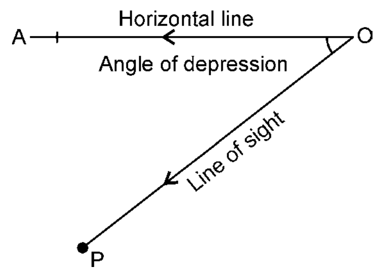
9. SOME APPLICATIONS OF TRIGONOMETRY

IMPORTANT TERMS, DEFINITIONS AND RESULTS

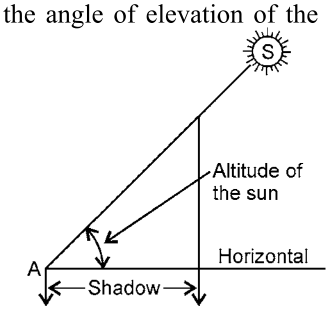
- **Line of sight :** When an observer looks from a point O at an object P, then the line OP is called the *line of sight.*
- **Angle of elevation :** The angle which the line of sight makes with the horizontal line through O is called the *angle of elevation* of P, as seen from O; i.e., $\angle XOP$.
  Or the angle of elevation of an object viewed, is the angle formed by the line of sight with the horizontal when it is above the horizontal level. i.e. the case when we raise our head to look the object.



- **Angle of depression :** The angle which the line of sight makes with the horizontal line through O is called the *angle of depression* of P, as seen from O.
  Or the angle of depression of an object viewed, is the angle formed by the line of sight with the horizontal when it is below the horizontal level. i.e., the case when we lower our head to look at the object.



- **Altitude of the Sun :** The altitude of the sun is simply the angle of elevation of the sun.



SUMMATIVE ASSESSMENT

MULTIPLE CHOICE QUESTIONS [1 Mark]

A. Important Questions

1. The angles of elevation of the top of a tower from two points distant a and b from the base and in the same straight line with it, are complementary, then the height of the tower is :

(a) ab (b) $\sqrt{ab}$ (c) $\frac{a}{b}$ (d) $\sqrt{\frac{a}{b}}$

2. The ratio of the length of a rod and its shadow is $1 : \sqrt{3}$. The altitude of the sun is :

(a) 30° (b) 45° (c) 60° (d) 90°

3. The angle of elevation of a cloud from a point h metres above a lake is θ. The angle of depression of its reflection in the lake is 45°. The height of the cloud is :

(a) $\frac{h(1+\tan\theta)}{(1-\tan\theta)}$ (b) $\frac{h(1-\tan\theta)}{(1+\tan\theta)}$

(c) $\frac{h(1+\tan\theta)}{\tan\theta}$ (d) none of these

4. The height of a tower is 50 m. When the sun's altitude changes from 30° to 45°, the shadow of the tower becomes x metres less. The value of x is :

(a) 50 m (b) $50\sqrt{3}$ m

(c) $50\left(\sqrt{3}-1\right)$ m (d) $\frac{50}{\sqrt{3}}$ m

5. If the angle of elevation of a building from a distance of 100 m from its foot is 60°, then the height of the building is :

(a) $100\sqrt{3}$ m (b) $\frac{100}{\sqrt{3}}$ m

(c) $50\sqrt{3}$ m (d) $\frac{200}{\sqrt{3}}$ m

6. From the top of a cliff 50 m high the angle of elevation of a tower is found to be equal to the angle of depression of the foot of the tower. The height of the tower is :

(a) 50 m (b) 100 m (c) 125 m (d) 150 m

7. The altitude of the sun is 60°. The height of a tower which casts a shadow of length 30 m is :

(a) $30\sqrt{3}$ m (.b) 15 m

(c) $\frac{30}{\sqrt{3}}$ m (d) $15\sqrt{2}$ m

8. A pole subtends an angle of 30° at a point on the same level as its foot. At a second point *h* metres above the first, the depression of the foot of the pole is 60°. The height of the pole is :

(a) $\frac{h}{2}$ m (b) $\sqrt{3}$ m (c) $\frac{h}{3}$ m (d) $\frac{h}{\sqrt{3}}$ m

9. The angle of elevation of the top of a tower from a point P on the ground is α. After walking a distance *d* towards the foot of the tower, angle of elevation is found to be β. The height of the tower is :

(a) $\frac{d}{\cot\alpha + \cot\beta}$ (b) $\frac{d}{\cot\alpha - \cot\beta}$

(c) $\frac{d}{\tan\alpha + \tan\beta}$ (d) $\frac{d}{\tan\beta - \tan\alpha}$

10. The length of the shadow of a tower standing on level plane is found to be $2x$ metres longer when the sun's altitude is 30° than when it was 45°. The height of the tower is :

(a) $x\left(\sqrt{3} - 1\right)$ m (b) $x\left(\sqrt{3} + 1\right)$ m

(c) $\frac{x}{\sqrt{3} - 1}$ m (d) $\frac{\sqrt{3} - 1}{x}$ m

11. As observed from the top of a 75 m high light-house from the sea-level, the angles of depression of two ships are 30° and 45°. One ship is exactly behind the other on the same side of the light-house. The distance between the two ships is :

(a) 75 m (b) $75\left(\sqrt{3} + 1\right)$ m

(c) $75\left(\sqrt{3}\right)$ m (d) $75\left(\sqrt{3} - 1\right)$ m

12. Two persons are *a* metres apart and the height of one is double that of the other. If from the middle point of the line joining their feet, an observer finds the angular elevation of their tops to be complementary, then the height of the short person is :

(a) $\frac{a}{4}$ (b) $\frac{a}{\sqrt{2}}$ (c) $a\sqrt{2}$ (d) $\frac{a}{2\sqrt{2}}$

B. Questions From CBSE Examination Papers

1. If the altitude of the sun is 60°, the height of the tower which casts a shadow of length 30 m is : **[2011 (T-II)]**

(a) $30\sqrt{3}$ m (b) $\frac{30}{3}\sqrt{3}$ m

(c) $15\sqrt{3}$ m (d) 15 m

2. The tops of two poles of heights 20 m and 14 m are connected by a wire. If the wire makes an angle of 30° with the horizontal, then the length of the wire is : **[2011 (T-II)]**

(a) 34 m (b) 12 m (c) 6 m (d) 17 m

3. The length of the string of a kite flying at 100 m above the ground with the elevation of 60° is : **[2011 (T-II)]**

(a) 100 m (b) $100\sqrt{2}$ m

(c) $\frac{200}{\sqrt{3}}$ (d) 200 m

4. A ladder of 10 m length touches a wall at height of 5 m. The angle θ made by it with the horizontal is : **[2011 (T-II)]**

(a) 90° (b) 60° (c) 45° (d) 30°

5. The measure of angle of elevation of top of a tower $75\sqrt{3}$ m high from a point at a distance of 75 m from foot of the tower in a horizontal plane is : **[2011 (T-II)]**

(a) 30° (b) 60° (c) 90° (d) 45°

6. A pole 10 m high casts a shadow 10 m long on the ground, then the sun's elevation is : **[2011 (T-II)]**

(a) 60° (b) 45° (c) 30° (d) 90°

7. The angle of deperssion from the top of a tower 12 m high, at a point on the ground is 30°. The distance of the point from the top of the tower is : **[2011 (T-II)]**

(a) 12 m (b) 6 m (c) $12\sqrt{3}$ m (d) 24 m

8. A tree casts a shadow 4 m long on the ground, when the angle of elevation of the sun is 45°. The height of the tree (in metres) is : **[2011 (T-II)]**

(a) 3 (b) 4 (c) 4.5 (d) 5.2

9. If sun's elevation is 60°, then a pole of height 6 m will cast a shadow of length : **[2011 (T-II)]**

(a) $6\sqrt{3}$ m (b) $\sqrt{3}$ m

(c) $2\sqrt{3}$ m (d) $3\sqrt{2}$ m

10. The angle of elevation of the top of a building 50 m high, from a point on the ground is 45°. The distance of the point from the foot of the building is : **[2011 (T-II)]**

(a) 100 m (b) 50 m (c) 45 m (d) 60 m

11. A tree 6 m tall casts a 4 m long shadow. At the same time a pole casts a shadow 10 m long. The height of the pole is : **[2011 (T-II)]**

(a) 40 m (b) 20 m (c) 15 m (d) 10 m

12. If the angle of elevation of top of a tower from a point at a distance of 100 m from its foot is 60°, then the height of the tower is : **[2011 (T-II)]**

(a) $50\sqrt{3}$ m (b) $\frac{200}{\sqrt{3}}$ m

(c) $\frac{100}{\sqrt{3}}$ m (d) $100\sqrt{3}$ m

13. The angle formed by the line of sight with the horizontal, when the point being viewed is above the horizontal level is called : **[2011 (T-II)]**

(a) vertical angle (b) angle of depression

(c) angle of elevation (d) obtuse angle

14. If the ratio of height of a tower and the length of its shadow on the ground is $\sqrt{3}:1$, then the angle of elevation of the sun is : **[2011 (T-II)]**

(a) 60° (b) 45° (c) 30° (d) 90°

15. If the height and length of the shadow of a man are the same, then the angle of elevation of the sun is : **[2011 (T-II)]**

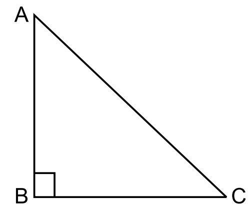
(a) 30° (b) 60° (c) 45° (d) 15°

16. If AB = 4 m and AC = 8 m, then angle of observation of A as observed from C is :

[2011 (T-II)]

(a) 60° (b) 30°

(c) 45° (d) cannot be determined



17. If the angle of depression of an object from a 75 m high tower is 30°, then the distance of the object from the base of tower is : **[2011 (T-II)]**

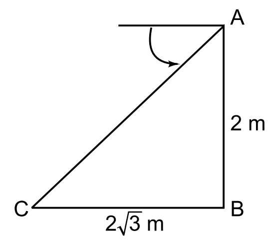
(a) $25\sqrt{3}$ m (b) $50\sqrt{3}$ m

(c) $75\sqrt{3}$ m (d) 150 m

18. The figure, shows the observation of point C from point A. The angle of depression from A is :

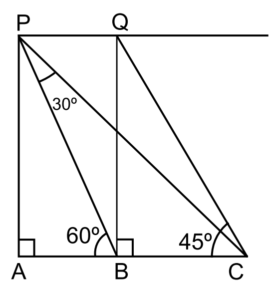
[2011 (T-II)]

(a) 60° (b) 30° (c) 45° (d) 75°



19. Form the figure, the angle of depression of point C from the point P is : **[2011 (T-II)]**

(a) 90° (b) 60° (c) 30° (d) 45°



SHORT ANSWER TYPE QUESTIONS [2 Marks]

B. Questions From CBSE Examination Papers

1. A tower stands vertically on the ground. From a point on the ground which is 60 m away from foot of the tower, the angle of elevation of the top of the tower is found to be 60°. Find the height of the tower. **[2011 (T-II)]**

2. A ladder 15 m long just reaches the top of a vertical wall. If the ladder makes an angle of 60° with the wall, find the height of the wall. **[2011 (T-II]**

3. A tower stands vertically on the ground. From a point on the ground which is 15 m away from the foot of the tower, the angle of elevation of the top of the tower is found to be 60°. Find the height of the tower. **[2011 (T-II)]**

SHORT ANSWER TYPE QUESTIONS [3 Marks]

B. Questions From CBSE Examination Papers

1. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree. **[2011 (T-II)]**
2. A kite is flying at a height of 90 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string assuming that there is no slack in the string. [Take $\sqrt{3} = 1.732$] **[2011 (T-II)]**
3. A player sitting on the top of a tower of height 20 m observes the angle of depression of a ball lying on the ground as 60°. Find the distance between the foot of the tower and the ball. **[2011 (T-II)]**
4. The shadow of a tower is 30 m long, when the sun's elevation is 30°. What is the length of the shadow, when sun's elevation is 60° ? **[2011 (T-II)]**
5. From a point P on the ground, the angle of elevation of the top of a 10 m tall building is 30°. A flag is hosted at the top of the building and the angle of elevation of the top of the flagstaff from P is 45°. Find the length of the flagstaff. (Take $\sqrt{3} = 1.732$) **[2011 (T-II)]**
6. A person standing on the bank of the river observes that the angle subtended by a tree on the opposite bank is 60°, when he retreats 20 m from the bank, he finds the angle to be 30°, find the height of the tree. **[2011 (T-II)]**
7. The angle of elevation of the top of a tower from two points distant a and b from the base and in the same straight line with it are complementary. Prove that the height of tower is $\sqrt{ab}$. **[2011 (T-II)]**
8. A tower is 60 m high. From the top of it the angles of depression of the top and the bottom of a tree are found to be 30° and 60° respectively. Find the height of the tree and its distance from the tower. **[2011 (T-II)]**
9. A man standing on the top of a multi-storey building, which is 30 m high, observes the angle of elevation of the top of a tower as 60° and the angle of depression of the base of the tower as 30°. Find the horizontal distance between the building and the tower. Also, find the height of the tower. **[2011 (T-II)]**
10. An aeroplane, when 3000 m high, passes vertically above another plane at an instant when the angles of elevation of the two aeroplanes from the same point on the ground are 60° and 45° respectively. Find the vertical distance between the two aeroplanes. **[2011 (T-II)]**
11. A circus artist is climbing a rope 12 m long which is tightly stretched and tied from the top of a vertical pole to the ground. find the height of the pole if the angle made by the rope with the ground is 30°. **[2011 (T-II)]**
12. The length of the shadow of a tower standing on level ground is found to be $2x$ metres longer when the sun's altitude is 30° than when it was 45°. Prove that the height of tower is $(\sqrt{3}+1)x$ metres. **[2011 (T-II)]**
13. From the top of a 10 m tall tower the angle of depression of a point on a ground was found to be 60°. How far is the point from the base of the tower? **[2011 (T-II)]**
14. An observer 1.5 m tall is 28.5 m away from a tower. The angle of elevation of the top of the tower from his eyes is 45°. What is the height of the tower ? **[2011 (T-II)]**

LONG ANSWER TYPE QUESTIONS [4 Marks]

A. Important Questions

1. Two poles of equal heights are standing opposite to each other on either side of a road, which is 100 metres wide. From a point between them on the road, the angles of elevation of their tops are 30° and 60°. Find the position of the point and also, the heights of the poles.
2. A tree breaks due to the storm and the broken part bends so that the top of the tree touches ground making an angle of 30° with the ground. The distance from the foot of the tree to the point where the top touches the ground, is 10 metres. Find the height of the tree.
3. A boy is standing on the ground and flying a kite with 100 m of string at an elevation of 30°. Another boy is standing on the roof of a 20 m

high building and is flying his kite at an elevation of 45°. Both the boys are on the opposite sides of both the kites. Find the length of the string that the second boy must have so that the two kites meet.

4. At the foot of a mountain the elevation of its summit is 45°. After ascending 1000 m towards the mountain up a slope of 30° inclination, the elevation is found to be 60°. Find the height of the mountain.

5. The angle of elevation of the top Q of a vertical tower PQ from a point X on the ground is 60°. At a point Y, 40 m vertically above X, the angle of elevation is 45°. Find the height of the tower PQ and the distance XQ.

6. A round balloon of radius r subtends an angle α at the eye of the observer while the angle of elevation of its centre is β. Prove that the height of the centre of the balloon is $r \sin \beta \operatorname{cosec} \alpha/2$.

7. The angle of elevation of a cliff from a fixed point is θ. After going up a distance of k metres towards the top of the cliff at an angle of ϕ, it is found that the angle of elevation is α. Show that the height of the cliff is

$$\frac{k(\cos\phi - \sin\phi\cot\alpha)}{\cot\theta - \cot\alpha} \text{ metres.}$$

8. If the angle of elevation of a cloud from a point h metres above a lake is α and the angle of depression of its reflection in the lake from the same point is β, prove that the height of the cloud is $\frac{h(tan\beta + tan\alpha)}{(tan\beta - tan\alpha)}$.

9. A man on a cliff observes a boat at an angle of depression of 30° which is approaching the shore to the point immediately beneath the observer with a uniform speed. Six minutes later, the angle of depression of the boat is found to be 60°. Find the time taken by the boat to reach the shore.

10. A ladder rests against a wall at an angle α to the horizontal. Its foot is pulled away from the wall through a distance a, so that it slides a distance b down the wall making an angle β with the horizontal. Show that $\frac{a}{b} = \frac{\cos\alpha - \cos\beta}{\sin\beta - \sin\alpha}$.

11. The angles of elevation of the top of a hill at the city centres of two towns on either side of the hill are observed to be 30° and 60°. If the distance up-hill from the first city centre is 9 km, find in kilometres the distance up-hill from the other city centre.

12. An aeroplane when 3000 m high passes vertically above another aeroplane at an instance when their angles of elevation at the same observation point are 60° and 45° respectively. How many metres higher is the one than the other ?

13. The distance between two vertical poles is 60 m. The height of one of the poles is double the height of the other. The angles of elevation of the tops of the poles from the middle point of the line segment joining their feet are complementary to each other. Find the heights of the poles.

14. The angles of elevation of a cloud from a point h metres above a lake is 30° and the angle of depression of its reflection in the lake is 45°. If the height of the cloud be 200 m, find h.

15. A bird is perched on the top of a tree 20 m high and its angle of elevation from a point on the ground 45°. The bird flies off horizontally straight away from the observer and in one second the angle of elevation of the bird reduces to 30°. Find the speed of the bird.

16. From the top of a tower, the angles of depression of two objects on the same side of the tower are found to be α and β ($\alpha > \beta$). If the disance between the objects is a metres, show that the height of the tower is $\frac{a\tan\alpha\tan\beta}{\tan\alpha - \tan\beta}$.

17. A vertical tower stands on a horizontal plane and is surmounted by a vertical flagstaff of height h. At a point on the plane, the angles of elevation of the bottom and the top of the flagstaff are α and β respectively. Prove that the height of the tower is $\frac{h\tan\alpha}{\tan\beta - \tan\alpha}$.

18. A tree standing on a horizontal plane is leaning towards east. At two points situated at distance a and b exactly due west of it, the angles of elevation of the top are α and β respectively. Prove that the height of the top of the tree from the ground is $\frac{(b-a)\tan\alpha\tan\beta}{\tan\alpha - \tan\beta}$.

19. At a point on a level plane, a tower subtends an angle α and a man a metres tall standing on its top subtends an angle β. Prove that the height of the tower is $\frac{a\sin\alpha\cos(\alpha+\beta)}{\sin\beta}$.

B. Questions From CBSE Examination Papers

1. An aeroplane flying horizontally 1 km above the ground is observed at an elevation of 60°. After a flight of 10 seconds, its angle of elevation is observed to be 30° from the same point on the ground. Find the speed of the aeroplane in km/hour. **[2011 (T-II)]**

2. The angle of elevation of the top of a tower from two points P and Q at distance of 4 m and 9 m respectively from the base of the tower and in the same straight line with it are 60° and 30°. Prove that the height of the tower is 6 m. **[2011 (T-II)]**

3. A person standing on the bank of a river observes that the angle of elevation of the top of the tree standing on the opposite bank is 60°. When he moves 30 m away from the bank, he finds the angle of elevation to be 30°. Find the height of the tree and the width of the river. **[2011 (T-II)]**

4. The angle of elevation of a cloud from a point 60 m above the lake is 30° and the angle of depression of its reflection in the lake is 60°. Find the height of the cloud above the lake. **[2011 (T-II)]**

5. A straight highways leads to the foot of tower. A man standing at the top of tower observes a car at an angle of depression 30° which is approaching the foot of the tower with a uniform speed. Six second later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point. **[2011 (T-II)]**

6. As observed from the top of a light house, 100 m high above sea-level, the angle of depression of a ship sailing directly towerds it, changes from 30° to 60°. Detremine the distance travelled by the ship during the period of observation $\left(\sqrt{3} = 1.732\right)$ **[2011 (T-II)]**

7. A man standing on the deck of the ship which is 10 m above the sea level, observes the angle of elevation of the top of the cloud as 30° and angle of depression of its reflection in the sea was found to be 60°. Find the height of the cloud and also the distance of the cloud from the ship. **[2011 (T-II)]**

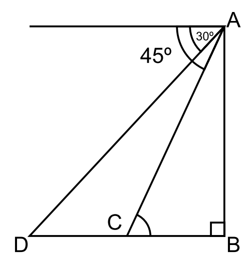
8. A vertical tower stands on a horizontal plane and is surmounted by a vertical flagstaff of height 'h'. At a point on the plane, the angles of elevation of the bottom and the top of the flagstaff are 45° and 60° respectively. Find the height of the tower. **[2011 (T-II)]**

9. From the top of a light house the angle of depression of a ship sailing towards it was found to be 30°. After 10 seconds the angle of depression changes to 60°. Assuming that the ship is sailing at uniform speed, find how much time it will take to reach the light house. **[2011 (T-II)]**

10. From a point on the ground the angle of elevation of the bottom and the top of a flagstaff situated on the top of a 120 m tall house, was found to be 30° and 45° respectively. Find the height of the flagstaff. **[2011 (T-II)]**

11. A vertical tower is surmounted by a flag staff of height 5 metres. At a point on the ground, the angles of elevation of bottom and top of flag staff are 45° and 60° respectively. Find the height of the tower. **[2011 (T-II)]**

12. A man on the top of a vertical tower observes a car moving towards the tower. If it takes 12 minutes for the angle of depression to change from 30° to 45°, how soon after this the car will reach the tower ? **[2011 (T-II)]**



13. An aeroplane at an altitude of 200 m observes the angles of depression of two opposite points on two banks of the river to be 45° and 60°. Find, in metres, the width of the river. $\left(\text{Use } \sqrt{3} = 1.732\right)$ **[2011 (T-II)]**

14. From a window, 60 m high above the ground, of a house in a street, the angles of elevation and depression of the top and foot of another house on the opposite side of the street are 60° and 45° respectively. Show that the height of the opposite house is $60\,(1 + \sqrt{3})$ metres. **[2011 (T-II)]**

15. Two pillars of equal heights are on either side of a road, which is 100 m wide. The angles of elevation of the top of the pillars are 60° and 30° at a point on the road between the pillars. Find the position of the point between the pillars on the road and the height of the pillars. **[2011 (T-II)]**

16. From the top of a building 60 m high the angles of depression of the top and the bottom of a tower are observed to be 30° and 60° respectively. Find the height of the tower. **[2011 (T-II)]**

17. Find the height of a mountain if the elevation of its top at an unknown distance from the base is 60° and at a distance 10 km further off from the mountain, along the same line, the angle of elevation is 30°. **[2011 (T-II)]**

18. From an aeroplane vertically above a straight horizontal road, the angles of depression of two consecutive kilometre stones on opposite sides of the areoplane are observed to be 60° and 30°. Show that height (in metres) of aeroplane above the road is $\frac{\sqrt{3}}{4}$ km **[2011 (T-II)]**

19. From the top and foot of a tower 40 m high, the angle of elevation of the top of a light house are found to be 30° and 60° respectively. Find the height of the light house. Also, find the distance of the top of the light house from the foot of the tower. **[2011 (T-II)]**

20. The angle of elevation of the top of a building from the foot of a tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building. **[2011 (T-II)]**

21. The angles of depression of the top and the bottom of an 8 m tall building from the top of a multi-storied building are 30° and 45° respectively. Find the height of the multistoried building. **[2011 (T-II)]**

22. From a point on the ground, the angles of elevation of the bottom and top of a transmission tower fixed at the top of 20 m high building are 45° and 60° respectively. Find the height of the transmission tower. **[2011 (T-II)]**

23. An aircraft is flying at a constant height with a speed of 360 km/hour. From a point on the ground, the angle of elevation at an instant was observed to be 45°. After 20 seconds, the angle of elevation was observed to be 30°. Determine the height at which the aircraft is flying. $\left(\text{Use}\sqrt{3} = 1.732\right)$ **[2011 (T-II)]**

24. A boy 2 m tall is standing at some distance from a 20 m tall building. The angle of elevation from his eyes of the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building. **[2011 (T-II)]**

25. Two men on either side of a cliff, 60 m high, observe the angles of elevation of the top of the cliff to be 45° and 60° respectively. Find the distance between two men. **[2011 (T-II)]**

26. The angle of elevation θ of the top of a light-house, as seen by a person on the ground, is such that $\tan\theta = \frac{5}{12}$. When the person moves a distance of 240 m towards the light-house, the angle of elevation becomes ϕ such that $\tan\phi = \frac{3}{4}$. Find the height of the light-house. **[2002, 2006]**

27. If the angle of elevation of a cloud from a point h metres above a lake is α and the angle of depression of its reflection in the lake is β, prove that the distance of the cloud from the point of observation is $\frac{2h\sec\alpha}{\tan\beta - \tan\alpha}$. **[2004]**

28. A man standing on the deck of a ship, which is 10 m above water level, observes the angle of elevation of the top of a hill as 60° and angle of depression of the base of the hill as 30°. Find the distance of the hill from the ship and height of the hill. **[2004, 2005, 2006]**

29. A bird is sitting on the top of a tree, which is 80 m high. The angle of elevation of the bird, from a point on the ground is 45°. The bird flies away from the point of observation horizontally and remains at a constant height. After 2 seconds, the angle of elevation of the bird from the point of observation becomes 30°. Find the speed of flying of the bird. **[2004]**

30. On a horizontal plane there is a vertical tower with a flag pole on the top of the tower. At a point 9 metres away from the foot of the tower the angle of elevation of the top and bottom of the flag pole are 60° and 30° respectively. Find the height of the tower and flag pole mounted on it. **[2005]**

31. The angle of elevation of the top of a tower as observed from a point on the ground is 'α' and on moving 'a' metres towards the tower, the angle of elevation is 'β'. Prove that the height of the tower is $\frac{a\tan\alpha\tan\beta}{\tan\beta - \tan\alpha}$. **[2006]**

32. A boy standing on a horizontal plane finds a bird flying at a distnce of 100 m from him at an elevation of 30°. A girl standing on the roof of 20 metre high building, finds the angle of elevation of the same bird to be 45°. Both the boy and the girl are on opposite sides of the bird. Find the distance of bird from the girl. **[2007]**

33. A statue 1.46 m tall stand on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point, the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal. **[2008]**

34. A straight highway leads to the foot of the tower. A man standing at the top of the tower observes a car at angle of depression of 30°, which is approaching the foot of the tower with a uniform speed, 6 seconds later the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point. **[2008, 2009]**

35. From a point 100 m above a lake, the angle of elevation of a stationary helicopter is 30° and the angle of depression of reflection of the helicopter in the lake is 60°. Find the height of the helicopter. **[2008C]**

36. The angles of depression of the top and bottom of a 8 m tall building from the top of a multi-storeyed building are 30° and 45° respectively. Find the height of the multistoreyed building and the distance between the two buildings. **[2009]**

37. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at that instant is 60°. After some time the angle of elevation reduces to 30°. Find the distance travelled by the balloon during the interval. **[2009]**

38. The angle of elevation of the top of a building from the foot of a tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high. Find the height of the building. **[2009]**

39. A tower stand vertically on a bank of a canal from a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°, from another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower and the width of the canal. **[2009]**

FORMATIVE ASSESSMENT

Activity-1

Objective : To make a height estimator and use it to measure the approximate height of a tall object.

Materials Required : 45°-45°-90° set square, 30°-60°-90° set square, small pipe or drinking straw, sellotape, a weight (a metal washer is ideal), thread.

Procedure :

Case I. Using a 45°-45°-90° set square

1. Take a 45°-45°-90° set square and fix a viewing tube (drinking straw) along the longest side (hypotenuse) using sellotape. Fix one end of a piece of thread at C and suspend a weight at the other end of the thread so that the thread passes through CB as shown in figure 1.

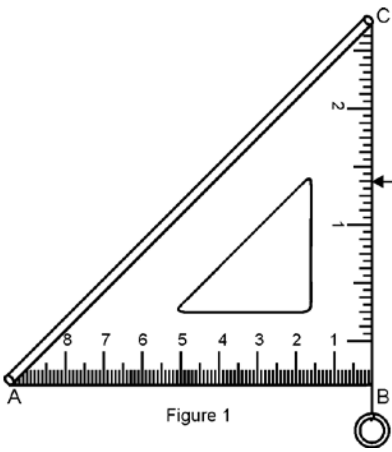


Figure 1

2. Suppose we want to estimate the height of a tall building. Take the height estimator and look through the straw at the top of the building. Move forward or backward to make sure that the top of the building is clearly visible.

   Also, keep the side BA of the height estimator horizontal, or the thread should always pass through edge CB.

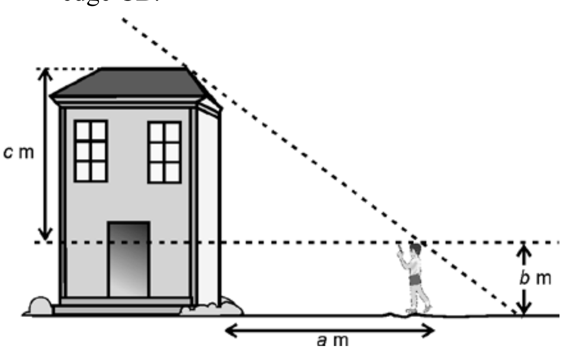


Figure 2

Case II. Using a 30°-60°-90° set square

1. Take a 30°-60°-90° set square and repeat steps 1 and 2 of case I.

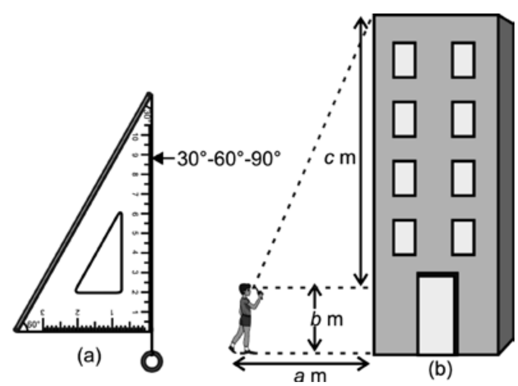


Figure 3

Observations :

1. In figure 2, let the horizontal distance between your foot and the foot of the building be *a* m, your height up to the eye level be *b* m and the height of the building above the eye level be *c* m.

2. Figure 2 can be represented as shown in the figure 4.

   So, $\tan 45° = \frac{c}{a} \Rightarrow c = a$

   So, height of the building = $c + b = a + b$.

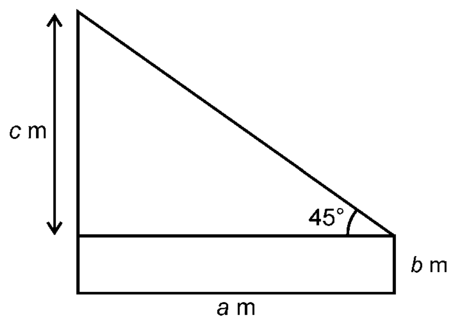


Figure 4

3. Since, we can measure *a* and *b* easily, so the approximate height of the building can be calculated.

4. In figure 3(b), let the horizontal distance between your foot and the foot of the building be *a* m, your height up to the eye level be *b* m and the height of the building above the eye level be *c* m.

5. Figure 3(b) can be represented as shown in the figure 5.

   Here there are two situations.

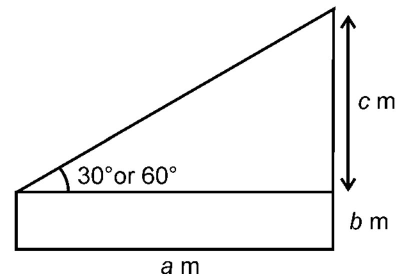


Figure 5

(a) When the 30° end of the estimator is kept near eye.

In this case, $\tan 30° = \frac{c}{a} \Rightarrow c = \frac{a}{\sqrt{3}}$

So, height of the building = $c + b = \frac{a}{\sqrt{3}} + b$

(b) When the 60° end of the estimator is kept near eye.

In this case, $\tan 60° = \frac{c}{a} \Rightarrow c = a\sqrt{3}$

So, height of the building = $c + b = a\sqrt{3} + b$

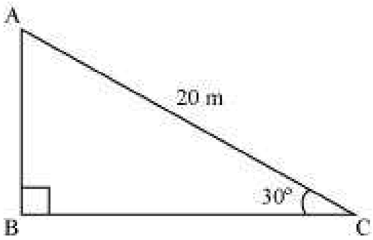
6. Since we can measure *a* and *b* easily, so the approximate height of the building can be calculated.

Conclusion : We can estimate the approximate height of a tall object using a height estimator and then applying trigonometric ratios.

Exercise 9.1

Question 1:

A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30 °.



Answer:

It can be observed from the figure that AB is the pole.

In ΔABC,

$$\frac{AB}{AC} = \sin 30°$$

$$\frac{AB}{20} = \frac{1}{2}$$

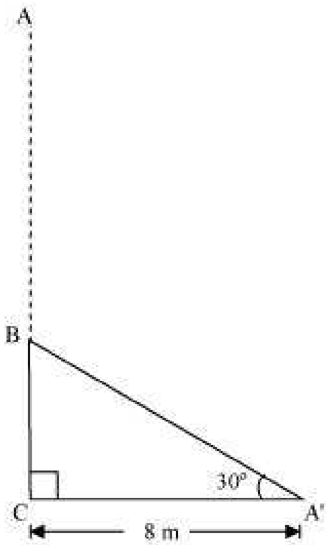
$$AB = \frac{20}{2} = 10$$

Therefore, the height of the pole is 10 m.

Question 2:

A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30 ° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

Answer:



Let AC was the original tree. Due to storm, it was broken into two parts. The broken part $A'B$ is making 30° with the ground.

In $\Delta A'BC$,

$$\frac{BC}{A'C} = \tan 30°$$

$$\frac{BC}{8} = \frac{1}{\sqrt{3}}$$

$$BC = \left(\frac{8}{\sqrt{3}}\right)\text{m}$$

$$\frac{A'C}{A'B} = \cos 30°$$

$$\frac{8}{A'B} = \frac{\sqrt{3}}{2}$$

$$A'B = \left(\frac{16}{\sqrt{3}}\right)\text{m}$$

Height of tree = $A'B$ + BC

$$= \left(\frac{16}{\sqrt{3}} + \frac{8}{\sqrt{3}}\right) m = \frac{24}{\sqrt{3}} \text{ m}$$
$$= 8\sqrt{3} \text{ m}$$

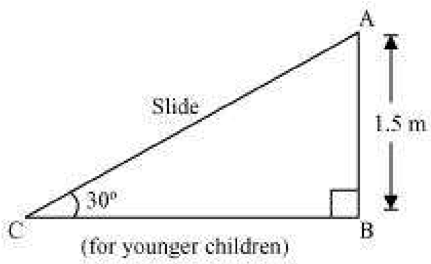
Hence, the height of the tree is $8\sqrt{3}$ m

Question 3:

A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m, and is inclined at an angle of 30 ° to the ground, where as for the elder children she wants to have a steep side at a height of 3 m, and inclined at an angle of 60 ° to the ground. What should be the length of the slide in each case?

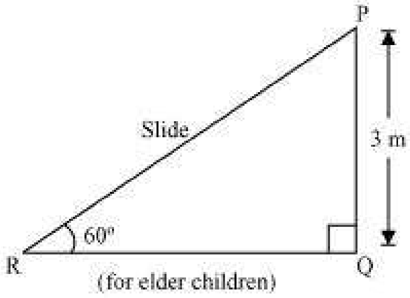
Answer:

It can be observed that AC and PR are the slides for younger and elder children respectively.



In ΔABC,

$$\frac{AB}{AC} = \sin 30°$$
$$\frac{1.5}{AC} = \frac{1}{2}$$
$$AC = 3 \text{ m}$$



In ΔPQR,

$$\frac{PQ}{PR} = \sin 60$$

$$\frac{3}{PR} = \frac{\sqrt{3}}{2}$$

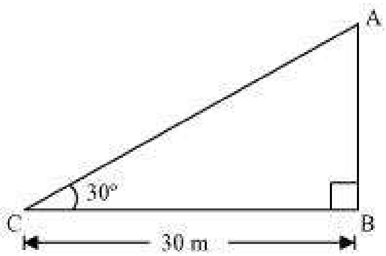
$$PR = \frac{6}{\sqrt{3}} = 2\sqrt{3}\ \text{m}$$

Therefore, the lengths of these slides are 3 m and $2\sqrt{3}$ m

Question 4:

The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower is 30°. Find the height of the tower.

Answer:



Let AB be the tower and the angle of elevation from point C (on ground) is 30°.

In ΔABC,

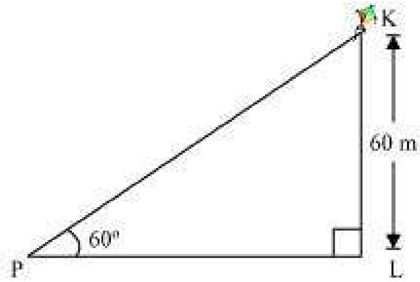
$$\frac{AB}{BC} = \tan 30°$$
$$\frac{AB}{30} = \frac{1}{\sqrt{3}}$$
$$AB = \frac{30}{\sqrt{3}} = 10\sqrt{3} \text{ m}$$

Therefore, the height of the tower is $10\sqrt{3}$ m.

Question 5:

A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.

Answer:



Let K be the kite and the string is tied to point P on the ground.

In ΔKLP,

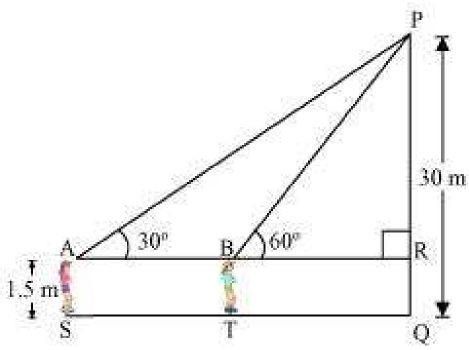
$$\frac{KL}{KP} = \sin 60°$$
$$\frac{60}{KP} = \frac{\sqrt{3}}{2}$$
$$KP = \frac{120}{\sqrt{3}} = 40\sqrt{3} \text{ m}$$

Hence, the length of the string is $40\sqrt{3}$ m.

Question 6:

A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

Answer:



Let the boy was standing at point S initially. He walked towards the building and reached at point T.

It can be observed that

PR = PQ − RQ

$= (30 - 1.5)\text{ m} = 28.5\text{ m} = \dfrac{57}{2}\text{ m}$

In ΔPAR,

$$\frac{PR}{AR} = \tan 30^\circ$$

$$\frac{57}{2AR} = \frac{1}{\sqrt{3}}$$

$$AR = \left(\frac{57}{2}\sqrt{3}\right)\text{m}$$

In ΔPRB,

$$\frac{PR}{BR} = \tan 60°$$

$$\frac{57}{2\,BR} = \sqrt{3}$$

$$BR = \frac{57}{2\sqrt{3}} = \left(\frac{19\sqrt{3}}{2}\right) m$$

ST = AB

$$= AR - BR = \left(\frac{57\sqrt{3}}{2} - \frac{19\sqrt{3}}{2}\right) m$$

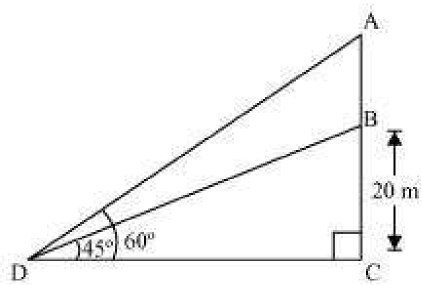
$$= \left(\frac{38\sqrt{3}}{2}\right) m = 19\sqrt{3}\ m$$

Hence, he walked $19\sqrt{3}$ m towards the building.

Question 7:

From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

Answer:



Let BC be the building, AB be the transmission tower, and D be the point on the ground from where the elevation angles are to be measured.

In ΔBCD,

$$\frac{BC}{CD} = \tan 45°$$
$$\frac{20}{CD} = 1$$
$$CD = 20 \text{ m}$$

In ΔACD,

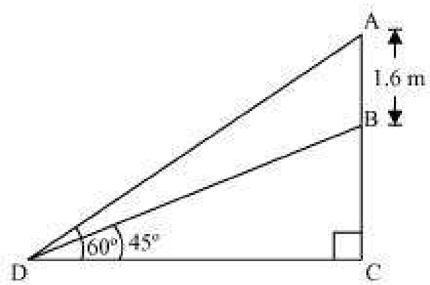
$$\frac{AC}{CD} = \tan 60°$$
$$\frac{AB + BC}{CD} = \sqrt{3}$$
$$\frac{AB + 20}{20} = \sqrt{3}$$
$$AB = \left(20\sqrt{3} - 20\right) \text{m}$$
$$= 20\left(\sqrt{3} - 1\right) \text{m}$$

Therefore, the height of the transmission tower is $20\left(\sqrt{3}-1\right)$ m.

Question 8:

A statue, 1.6 m tall, stands on a top of pedestal, from a point on the ground, the angle of elevation of the top of statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45 °. Find the height of the pedestal.

Answer:



Let AB be the statue, BC be the pedestal, and D be the point on the ground from where the elevation angles are to be measured.

In ΔBCD,

$$\frac{BC}{CD} = \tan 45°$$

$$\frac{BC}{CD} = 1$$

$$BC = CD$$

In ΔACD,

$$\frac{AB + BC}{CD} = \tan 60°$$

$$\frac{AB + BC}{BC} = \sqrt{3}$$

$$1.6 + BC = BC\sqrt{3}$$

$$BC\left(\sqrt{3} - 1\right) = 1.6$$

$$BC = \frac{(1.6)\left(\sqrt{3}+1\right)}{\left(\sqrt{3}-1\right)\left(\sqrt{3}+1\right)}$$

$$= \frac{1.6\left(\sqrt{3}+1\right)}{\left(\sqrt{3}\right)^2 - (1)^2}$$

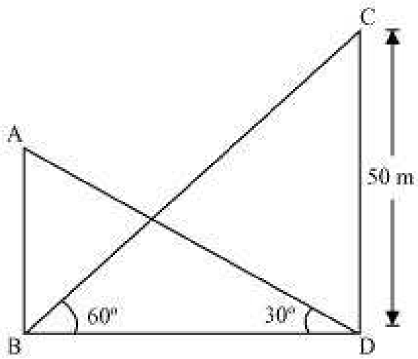
$$= \frac{1.6\left(\sqrt{3}+1\right)}{2} = 0.8\left(\sqrt{3}+1\right)$$

Therefore, the height of the pedestal is 0.8$\left(\sqrt{3}+1\right)$ m.

Question 9:

The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.

Answer:



Let AB be the building and CD be the tower.

In ΔCDB,

$$\frac{CD}{BD} = \tan 60°$$

$$\frac{50}{BD} = \sqrt{3}$$

$$BD = \frac{50}{\sqrt{3}}$$

In ΔABD,

$$\frac{AB}{BD} = \tan 30°$$

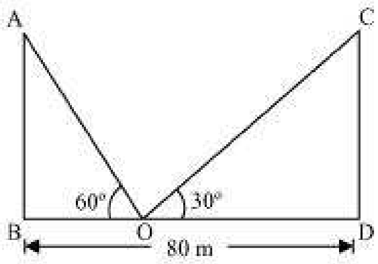
$$AB = \frac{50}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{50}{3} = 16\frac{2}{3}$$

Therefore, the height of the building is $16\frac{2}{3}$ m.

Question 10:

Two poles of equal heights are standing opposite each other an either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30º, respectively. Find the height of poles and the distance of the point from the poles.

Answer:



Let AB and CD be the poles and O is the point from where the elevation angles are measured.

In ΔABO,

$$\frac{AB}{BO} = \tan 60^\circ$$

$$\frac{AB}{BO} = \sqrt{3}$$

$$BO = \frac{AB}{\sqrt{3}}$$

In ΔCDO,

$$\frac{CD}{DO} = \tan 30^\circ$$

$$\frac{CD}{80 - BO} = \frac{1}{\sqrt{3}}$$

$$CD\sqrt{3} = 80 - BO$$

$$CD\sqrt{3} = 80 - \frac{AB}{\sqrt{3}}$$

$$CD\sqrt{3} + \frac{AB}{\sqrt{3}} = 80$$

Since the poles are of equal heights,

CD = AB

$$\text{CD}\left[\sqrt{3}+\frac{1}{\sqrt{3}}\right]=80$$

$$\text{CD}\left(\frac{3+1}{\sqrt{3}}\right)=80$$

$$\text{CD}=20\sqrt{3}\text{ m}$$

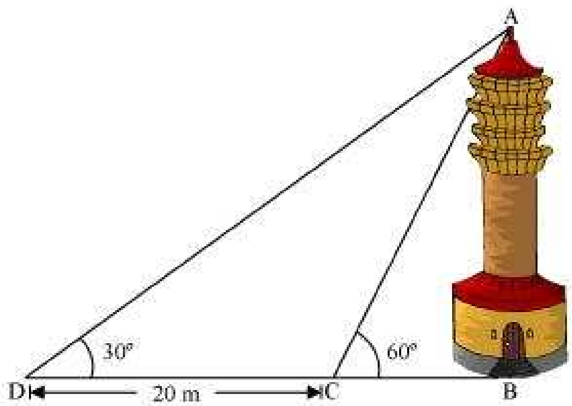
$$\text{BO}=\frac{\text{AB}}{\sqrt{3}}=\frac{\text{CD}}{\sqrt{3}}=\left(\frac{20\sqrt{3}}{\sqrt{3}}\right)\text{m}=20\text{ m}$$

DO = BD − BO = (80 − 20) m = 60 m

Therefore, the height of poles is $20\sqrt{3}$ m and the point is 20 m and 60 m far from these poles.

Question 11:

A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower and the width of the canal.



Answer:

In ΔABC,

$$\frac{AB}{BC} = \tan 60°$$
$$\frac{AB}{BC} = \sqrt{3}$$
$$BC = \frac{AB}{\sqrt{3}}$$

In ΔABD,

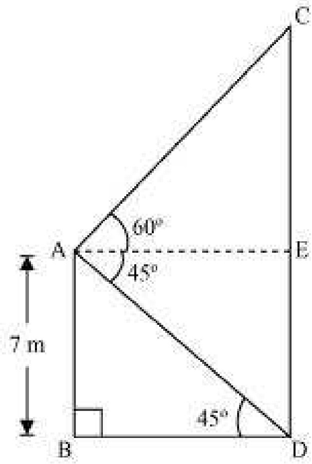
$$\frac{AB}{BD} = \tan 30°$$
$$\frac{AB}{BC+CD} = \frac{1}{\sqrt{3}}$$
$$\frac{AB}{\frac{AB}{\sqrt{3}}+20} = \frac{1}{\sqrt{3}}$$
$$\frac{AB\sqrt{3}}{AB+20\sqrt{3}} = \frac{1}{\sqrt{3}}$$
$$3AB = AB + 20\sqrt{3}$$
$$2AB = 20\sqrt{3}$$
$$AB = 10\sqrt{3}\text{ m}$$
$$BC = \frac{AB}{\sqrt{3}} = \left(\frac{10\sqrt{3}}{\sqrt{3}}\right)\text{m} = 10\text{ m}$$

Therefore, the height of the tower is $10\sqrt{3}$ m and the width of the canal is 10 m.

Question 12:

From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

Answer:



Let AB be a building and CD be a cable tower.

In ΔABD,

$$\frac{AB}{BD} = \tan 45°$$

$$\frac{7}{BD} = 1$$

$$BD = 7 \text{ m}$$

In ΔACE,

AC = BD = 7 m

$$\frac{CE}{AE} = \tan 60°$$

$$\frac{CE}{7} = \sqrt{3}$$

$$CE = 7\sqrt{3} \text{ m}$$

$$CD = CE + ED = \left(7\sqrt{3} + 7\right)\text{m}$$

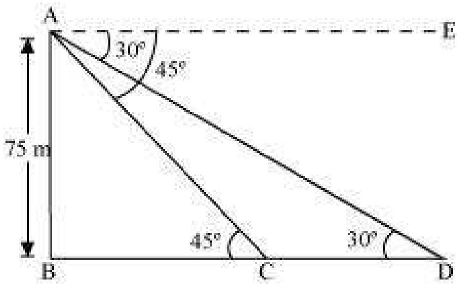
$$= 7\left(\sqrt{3} + 1\right)\text{m}$$

Therefore, the height of the cable tower is $7\left(\sqrt{3}+1\right)$ m.

Question 13:

As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

Answer:



Let AB be the lighthouse and the two ships be at point C and D respectively.

In ΔABC,

$$\frac{AB}{BC} = \tan 45°$$

$$\frac{75}{BC} = 1$$

$$BC = 75 \text{ m}$$

In ΔABD,

$$\frac{AB}{BD} = \tan 30°$$

$$\frac{75}{BC + CD} = \frac{1}{\sqrt{3}}$$

$$\frac{75}{75 + CD} = \frac{1}{\sqrt{3}}$$

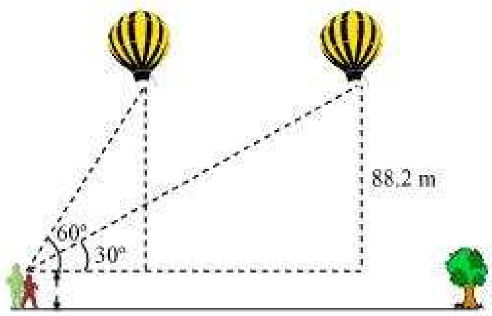
$$75\sqrt{3} = 75 + CD$$

$$75\left(\sqrt{3} - 1\right)\text{m} = CD$$

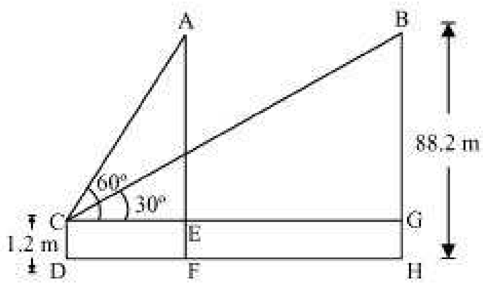
Therefore, the distance between the two ships is $75\left(\sqrt{3}-1\right)$ m.

Question 14:

A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After some time, the angle of elevation reduces to 30°. Find the distance travelled by the balloon during the interval.



Answer:



Let the initial position A of balloon change to B after some time and CD be the girl.

In ΔACE,

$$\frac{AE}{CE} = \tan 60°$$

$$\frac{AF - EF}{CE} = \tan 60°$$

$$\frac{88.2 - 1.2}{CE} = \sqrt{3}$$

$$\frac{87}{CE} = \sqrt{3}$$

$$CE = \frac{87}{\sqrt{3}} = 29\sqrt{3} \text{ m}$$

In ΔBCG,

$$\frac{BG}{CG} = \tan 30°$$

$$\frac{88.2 - 1.2}{CG} = \frac{1}{\sqrt{3}}$$

$$87\sqrt{3} \text{ m} = CG$$

Distance travelled by balloon = EG = CG − CE

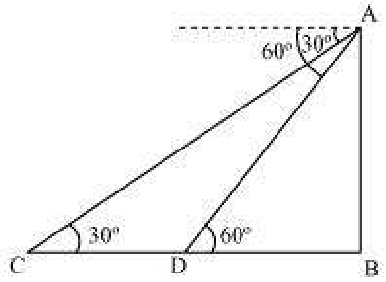
$$= \left(87\sqrt{3} - 29\sqrt{3}\right)\text{m}$$

$$= 58\sqrt{3} \text{ m}$$

Question 15:

A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car as an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

Answer:



Let AB be the tower.

Initial position of the car is C, which changes to D after six seconds.

In ΔADB,

$$\frac{AB}{DB} = \tan 60^\circ$$

$$\frac{AB}{DB} = \sqrt{3}$$

$$DB = \frac{AB}{\sqrt{3}}$$

In ΔABC,

$$\frac{AB}{BC} = \tan 30^\circ$$

$$\frac{AB}{BD + DC} = \frac{1}{\sqrt{3}}$$

$$AB\sqrt{3} = BD + DC$$

$$AB\sqrt{3} = \frac{AB}{\sqrt{3}} + DC$$

$$DC = AB\sqrt{3} - \frac{AB}{\sqrt{3}} = AB\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)$$

$$= \frac{2AB}{\sqrt{3}}$$

Time taken by the car to travel distance DC $\left(\text{i.e., } \frac{2AB}{\sqrt{3}}\right)$ = 6 seconds

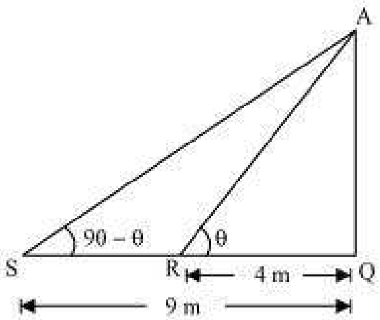
Time taken by the car to travel distance DB $\left(\text{i.e., } \frac{AB}{\sqrt{3}}\right) = \frac{6}{\frac{2AB}{\sqrt{3}}} \times \frac{AB}{\sqrt{3}}$

$= \frac{6}{2} = 3 \text{ seconds}$

Question 16:

The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m. from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.

Answer:



Let AQ be the tower and R, S are the points 4m, 9m away from the base of the tower respectively.

The angles are complementary. Therefore, if one angle is θ, the other will be 90 − θ.

In ΔAQR,

$$\frac{AQ}{QR} = \tan\theta$$

$$\frac{AQ}{4} = \tan\theta \qquad ...(i)$$

In ΔAQS,

$$\frac{AQ}{SQ} = \tan(90 - \theta)$$

$$\frac{AQ}{9} = \cot\theta \qquad ...(ii)$$

On multiplying equations (*i*) and (*ii*), we obtain

$$\left(\frac{AQ}{4}\right)\left(\frac{AQ}{9}\right) = (\tan\theta).(\cot\theta)$$

$$\frac{AQ^2}{36} = 1$$

$$AQ^2 = 36$$

$$AQ = \sqrt{36} = \pm 6$$

However, height cannot be negative.

Therefore, the height of the tower is 6 m.