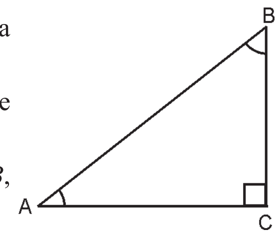
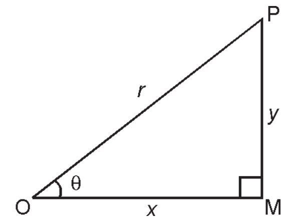
8. INTRODUCTION TO TRIGONOMETRY

IMPORTANT TERMS, DEFINITIONS AND RESULTS



- In trigonometry, we deal with relations between the sides and angles of a triangle.
- Ratios of the sides of a right angled triangle with respect to its acute angles, are called *trigonometric ratios of the angle*.
- For $\angle A$, AC is the base, BC the perpendicular and AB is the hypotenuse. For $\angle B$, BC is the base, AC the perpendicular and AB is the hypotenuse.



- **Six trigonometrical ratios**

(i) Sine $\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{y}{r}$. Sine θ is written as sin θ.

(ii) Cosine $\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{x}{r}$. Cosine θ is written as cos θ.

(iii) Tangent $\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{y}{x}$. Tangent θ is written as tan θ.

(iv) Cotangent $\theta = \frac{\text{Base}}{\text{Perpendicular}} = \frac{x}{y}$. Cotangent θ is written as cot θ.

(v) Secant $\theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{r}{x}$. Secant θ is written as sec θ.

(vi) Cosecant $\theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{r}{y}$. Cosecant θ is written as cosec θ.

- **Relations between trigonometric ratios**

(a) ***Reciprocal relations***

(i) $\text{cosec } \theta = \frac{1}{\sin\theta}$ or $\sin \theta = \frac{1}{\text{cosec}\,\theta}$ or $\sin \theta \text{ cosec } \theta = 1$

(ii) $\sec \theta = \frac{1}{\cos\theta}$ or $\cos \theta = \frac{1}{\sec\theta}$ or $\cos \theta \sec \theta = 1$

(iii) $\cot \theta = \frac{1}{\tan\theta}$ or $\tan \theta = \frac{1}{\cot\theta}$ or $\tan \theta \cot \theta = 1$

(b) ***Quotient relations***

(i) $\tan \theta = \frac{\sin\theta}{\cos\theta}$ (ii) $\cot \theta = \frac{\cos\theta}{\sin\theta}$

- sin A is a symbol which denotes the ratio $\frac{\text{perpendicular}}{\text{hypotenuse}}$. It does not mean the product of sin and A, i.e., $\sin A \neq \sin \times A$. In fact sin separated from A has no meaning. Similar interpretations follow for other trigonometric ratios.
- Table of values of various trigonometric ratios of 0°, 30°, 45°, 60° and 90°.

T- θ → ratios ↓	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cot θ	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cosec θ	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

Students may find easier to memorize the first row (values of sine ratio) as

sin	0°	30°	45°	60°	90°
	$\sqrt{\frac{0}{4}}$	$\sqrt{\frac{1}{4}}$	$\sqrt{\frac{2}{4}}$	$\sqrt{\frac{3}{4}}$	$\sqrt{\frac{4}{4}}$
	$= 0$	$= \frac{1}{2}$	$= \frac{1}{\sqrt{2}}$	$= \frac{\sqrt{3}}{2}$	$= 1$

- **Trigonometric ratios of complementary angles**

  (*i*) $\sin (90° - \theta) = \cos \theta$,

  $\cos (90° - \theta) = \sin \theta$

  (*ii*) $\tan (90° - \theta) = \cot \theta$,

  $\cot (90° - \theta) = \tan \theta$

  (*iii*) $\sec (90° - \theta) = \text{cosec } \theta$,

  $\text{cosec } (90° - \theta) = \sec \theta$

- **Trigonometric Identities**

  (a) An equation involving trigonometric ratios of an angle θ (say) is said to be a trigonometric identity, if it is satisfied for all values of θ for which the given trigonometric ratios are defined.

  (b) Some important trigonometric identities :

  (i) $\sin^2 \theta + \cos^2 \theta = 1$

  or $\sin^2 \theta = 1 - \cos^2 \theta$

  or $\cos^2 \theta = 1 - \sin^2 \theta$

  (ii) $\sec^2 \theta - \tan^2 \theta = 1$

  or $1 + \tan^2 \theta = \sec^2 \theta$

  or $\tan^2 \theta = \sec^2 \theta - 1$

  (iii) $\text{cosec}^2 \theta - \cot^2 \theta = 1$

  or $\text{cosec}^2 \theta = 1 + \cot^2 \theta$

  or $\cot^2 \theta = \text{cosec}^2 \theta - 1$

  (c) The following steps should be kept in mind while proving trigonometric identities :

  (i) Start with more complicated side of the identity and prove it equal to the other side.

  (ii) If the identity contains sine, cosine and other trigonometric ratios, then express all the ratios in terms of sine and cosine.

  (iii) If one side of an identity cannot be easily reduced to the other side value, then simplify both sides and prove them identically equal.

  (iv) While proving identities, never transfer terms from one side to another.

SUMMATIVE ASSESSMENT

MULTIPLE CHOICE QUESTIONS [1 Mark]

A. Important Questions

1. If $\cos A = \frac{4}{5}$, then the value of $\tan A$ is:

(a) $\frac{3}{5}$ (b) $\frac{3}{4}$ (c) $\frac{4}{3}$ (d) $\frac{5}{3}$

2. If $\sin\theta = \frac{a}{b}$, then $\cos\theta$ is equal to :

(a) $\frac{b}{\sqrt{b^2-a^2}}$ (b) $\frac{b}{a}$

(c) $\frac{\sqrt{b^2-a^2}}{b}$ (d) $\frac{a}{\sqrt{b^2-a^2}}$

3. The value of $\tan A$ is always less than 1.
(a) false
(b) true
(c) sometimes true, sometimes false
(d) none of the above

4. Maximum value of $\sin\theta$ is :
(a) more than 1 (b) less than 1
(c) equal to 1 (d) none of these

5. Minimum value of $\sin\theta$, where θ is acute, is:
(a) zero (b) more than 1
(c) equal to 1 (d) less than 1

6. If $4\tan\theta = 3$, then $\left(\frac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta}\right)$ is equal to :

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

7. If θ is an acute angle such that $\sec^2\theta = 3$, then the value of $\frac{\tan^2\theta - \text{cosec}^2\theta}{\tan^2\theta - \text{cosec}^2\theta}$ is:

(a) $\frac{4}{7}$ (b) $\frac{3}{7}$ (c) $\frac{2}{7}$ (d) $\frac{1}{7}$

8. $\sin\theta = \frac{4}{3}$ for some angle θ, is :

(a) true
(b) false
(c) it is not possible to say anything about it definitely
(d) neither (a) nor (b)

9. If $\cot\theta = \frac{4}{3}$, then $\cos^2\theta - \sin^2\theta$ is equal to :

(a) $\frac{7}{25}$ (b) 1 (c) $-\frac{7}{25}$ (d) $\frac{4}{25}$

10. If $\sin A = \frac{1}{2}$, then the value of $\cot A$ is :

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{\sqrt{3}}{2}$ (d) 1

11. If $a = b\tan\theta$, then $\frac{a\sin\theta + b\cos\theta}{a\sin\theta - b\cos\theta}$ is equal to :

(a) $\frac{a^2+b^2}{a^2-b^2}$ (b) $\frac{a^2-b^2}{a^2+b^2}$ (c) $\frac{a+b}{a-b}$ (d) $\frac{a-b}{a+b}$

12. If $\sin\theta = \frac{3}{5}$, then the value of $(\tan\theta + \sec\theta)^2$ is equal to :

(a) 1 (b) $\frac{1}{2}$ (c) 2 (d) –2

13. $\frac{1-\sin^2 45°}{1+\sin^2 45°}$ is equal to :

(a) cos 60° (b) sin 60° (c) tan 30° (d) sin 30°

14. The value of (sin 30° + cos 30°) – (sin 60° + cos 60°) is :
(a) –1 (b) 0 (c) 1 (d) 2

15. The value of (sin 45° + cos 45°) is :

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) 1

16. If x tan 45°.cos 60° = sin 60°.cot 60°, then x is equal to :

(a) 1 (b) $\sqrt{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{\sqrt{2}}$

17. The value of $\frac{\tan 30°}{\cos 60°}$ is :

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) 1

18. The value of $\frac{\sin 45°}{\text{cosec}\, 45°}$ is :

(a) 1 (b) $\frac{1}{2}$

(c) $\sqrt{2}$ (d) none of these

19. The value of (sin 45° cos 30° + cos 45° sin 30°) is :

(a) $\frac{\sqrt{3}+1}{\sqrt{2}}$ (b) $\frac{\sqrt{3}}{\sqrt{2}}$ (c) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ (d) $\frac{\sqrt{3}-1}{2\sqrt{2}}$

20. The value of (sin 30° cos 60° + cos 30° sin 60°) is :

(a) sin 90° (b) cos 90° (c) sin 0° (d) cos 30°

21. $\sqrt{\frac{1-\sin 60°}{2}}$ is equal to :

(a) sin 60° (b) sin 30° (c) sin 90° (d) sin 0°

22. The value of $3\sin 30° - 4\sin^3 30°$ is :

(a) 1 (b) 0 (c) 2 (d) $\frac{1}{2}$

23. The value of $\frac{\sin 18°}{\cos 72°}$ is :

(a) 1 (b) 0 (c) –1 (d) $\frac{1}{2}$

24. cos 48° – sin 42° is :

(a) 1 (b) 0 (c) –1 (d) $\frac{1}{2}$

25. The value of tan 80° . tan 75° . tan 15°. tan 10° is :

(a) –1 (b) 0
(c) 1 (d) none of these

26. The value of $\frac{\tan 26°}{\cot 64°}$ is :

(a) 0 (b) –1
(c) –1 (d) none of these

27. cosec 31° – sec 59° is equal to :

(a) 0 (b) 1 (c) –1 (d) $\frac{1}{2}$

28. The value of (tan 2° tan 4° tan 6° ... tan 88°) is:

(a) 1 (b) 0
(c) 2 (d) not defined

29. tan (40° + θ) – cot (40° – θ) is equal to:

(a) 1 (b) 0 (c) 2 (d) $\frac{1}{2}$

30. The value of sin (50° + θ) – cos (40° – θ) is :

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) 0

31. The value of the expression cosec (75° + θ) – sec (15° – θ) – tan (55° + θ) + cot (35° – θ) is :

(a) – 1 (b) 0 (c) 1 (d) $\frac{3}{2}$

32. sin (45° + θ) – cos (45° – θ) is equal to :

(a) 2 cosec θ (b) 0
(c) sin θ (d) 1

33. $9\sec^2\theta - 9\tan^2\theta$ is equal to :

(a) 1 (b) 9 (c) 8 (d) 0

34. If $\sin A = \frac{8}{17}$ and A is acute, then cot A is equal to :

(a) $\frac{15}{8}$ (b) $\frac{15}{17}$ (c) $\frac{8}{15}$ (d) $\frac{17}{8}$

35. $(\text{cosec}^2 72° - \tan^2 18°)$ is equal to :

(a) 0 (b) 1
(c) $\frac{3}{2}$ (d) none of these

36. If $x = \sec\theta + \tan\theta$, then tan θ is equal to :

(a) $\frac{x^2+1}{x}$ (b) $\frac{x^2-1}{x}$ (c) $\frac{x^2+14}{2x}$ (d) $\frac{x^2-1}{2x}$

37. $\tan^2\theta \sin^2\theta$ is equal to :

(a) $\tan^2\theta - \sin^2\theta$ (b) $\tan^2\theta + \sin^2\theta$
(c) $\frac{\tan^2\theta}{\sin^2\theta}$ (d) none of these

38. If cos θ – sin θ = 1, then the value of cos θ + sin θ is equal to :

(a) ± 4 (b) ± 3 (c) ± 2 (d) ± 1

39. $\frac{1+\tan^2\theta}{1+\cot^2\theta}$ is equal to :

(a) $\sec^2\theta$ (b) – 1 (c) $\cot^2\theta$ (d) $\tan^2\theta$

40. $(\sec^2 10° - \cot^2 80°)$ is equal to :

(a) 1 (b) 0 (c) 2 (d) $\frac{1}{2}$

41. The value of $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$ is :

(a) cot θ – cosec θ (b) cosec θ + cot θ
(c) $\text{cosec}^2\theta + \cot^2\theta$ (d) $\cot\theta + \text{cosec}^2\theta$

42. $\frac{\sin\theta}{1+\cos\theta}$ is equal to :

(a) $\frac{1+\cos\theta}{\sin\theta}$ (b) $\frac{1-\cot\theta}{\sin\theta}$
(c) $\frac{1-\cos\theta}{\sin\theta}$ (d) $\frac{1-\sin\theta}{\cos\theta}$

43. If $x = a\cos\alpha$ and $y = b\sin\alpha$, then $b^2x^2 + a^2y^2$ is equal to :

(a) a^2b^2 (b) ab (c) a^4b^4 (d) $a^2 + b^2$

44. $\sqrt{(1+\sin\theta)(1-\sin\theta)}$ is equal to :

(a) sin θ (b) $\sin^2\theta$ (c) $\cos^2\theta$ (d) cos θ

45. The value of the expression

$$\left[\frac{\sin^2 22^\circ+\sin^2 68^\circ}{\cos^2 22^\circ+\cos^2 68^\circ}+\sin^2 63^\circ+\cos 63^\circ \sin 27^\circ\right]$$ is :

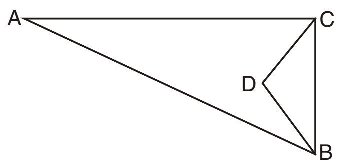
(a) 2 (b) 1
(c) 0 (d) none of these

46. If $\cos 9\alpha=\sin\alpha$ and $9\alpha<90^\circ$, then the value of $\tan 5\alpha$ is :

(a) 0 (b) 1
(c) $\sqrt{3}$ (d) cannot be determined

B. Questions From CBSE Examination Papers

1. In the given figure, ∠*ACB* = 90°, ∠*BDC* = 90°, *CD* – 4 cm, *BD* = 3 cm, *AC* – 12 cm, *cos A – sin A* is equal to : **[2010 (T-I)]**



(a) $\frac{5}{12}$ (b) $\frac{5}{13}$ (c) $\frac{7}{12}$ (d) $\frac{7}{13}$

2. If cot $A=\frac{12}{5}$, then the value of (sin *A* + cos *A*) × cosec *A* is : **[2010 (T-I)]**

(a) $\frac{13}{5}$ (b) $\frac{17}{5}$ (c) $\frac{14}{5}$ (d) 1

3. cos 1°, cos 2°, cos 3°, ... cos 180° is equal to : **[2010 (T-I)]**

(a) 1 (b) 0 (c) 1/2 (d) –1

4. 5 $\csc^2\theta$ – 5 $\cot^2\theta$ is equal to : **[2010 (T-I)]**

(a) 5 (b) 1 (c) 0 (d) –5

5. If sin θ = cos θ, then value of θ is : **[2010 (T-I)]**

(a) 0° (b) 45° (c) 30° (d) 90°

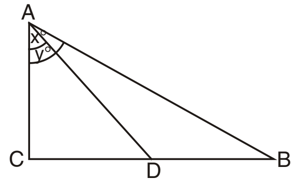
6. 9 $\sec^2\theta$ – 9 $\tan^2\theta$ is equal to : **[2010 (T-I)]**

(a) 1 (b) –1 (c) 9 (d) –9

7. If sin θ + $\sin^2\theta$ = 1, the value of ($\cos^2\theta$ + $\cos^4\theta$) is : **[2010 (T-I)]**

(a) 3 (b) 2 (c) 1 (d) 0

8. In the figure, if *D* is the mid-point of *BC,* the value of $\frac{\cot y^\circ}{\cot x^\circ}$ is : **[2010 (T-I)]**

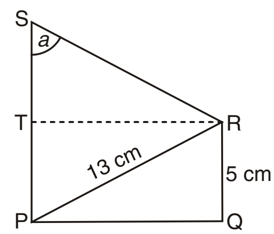


(a) 2 (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$

9. If $\csc\theta=\frac{3}{2}$, then 2 ($\csc^2\theta$ + $\cot^2\theta$) is : **[2010 (T-I)]**

(a) 3 (b) 7 (c) 9 (d) 5

10. In the figure, if *PS* = 14 cm, the value of tan *a* is equal to : **[2010 (T-I)]**



(a) $\frac{4}{3}$ (b) $\frac{14}{3}$ (c) $\frac{5}{3}$ (d) $\frac{13}{3}$

11. If *x* = 3 $\sec^2\theta$ – 1, *y* = $\tan^2\theta$ – 2, then *x* – 3*y* is equal to : **[2010 (T-I)]**

(a) 3 (b) 4 (c) 8 (d) 5

12. (sec *A* + tan *A*)(1 – sin *A*) is equal to : **[2010 (T-I)]**

(a) sec *A* (b) tan *A* (c) sin *A* (d) cos *A*

13. If sec θ – tan θ = $\frac{1}{3}$, the value of (sec θ + tan θ) is : **[2010 (T-I)]**

(a) 1 (b) 2 (c) 3 (d) 4

14. The value of $\frac{\cot 45^\circ}{\sin 30^\circ+\cos 60^\circ}$ is equal to : **[2010 (T-I)]**

(a) 1 (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{2}{3}$ (d) $\frac{1}{2}$

15. If $\cos 3\theta=\frac{\sqrt{3}}{2};0<\theta<20^\circ$, then the value of θ is : **[2010 (T-I)]**

(a) 15° (b) 10° (c) 0° (d) 12°

16. Δ*ABC* is a right angled at *A,* the value of tan *B* × tan *C* is : **[2010 (T-I)]**

(a) 0 (b) 1
(c) –1 (d) none of these

17. If $\sin\theta = \frac{1}{3}$, then the value of $2\cot^2\theta + 2$ is equal to : **[2010 (T-I)]**

(a) 6 (b) 9 (c) 4 (d) 18

18. The value of tan 1°.tan 2°.tan 3° .... tan 89° is : **[2010 (T-I)]**

(a) 0 (b) 1 (c) 2 (d) $\frac{1}{2}$

19. If $\sin(A-B) = \frac{1}{2}$ and $\cos(A+B) = \frac{1}{2}$, then the value of B is : **[2010 (T-I)]**

(a) 45° (b) 60° (c) 15° (d) 0°

20. Value of $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \text{cosec}\,\theta)$ is : **[2010 (T-I)]**

(a) 1 (b) –1 (c) 2 (d) –4

21. The value of $[\sin^2 20° + \sin^2 70° - \tan^2 45°]$ is : **[2010 (T-I)]**

(a) 0 (b) 1 (c) 2 (d) –1

22. Given that $\sin A = \frac{1}{2}$, and $\cos B = \frac{1}{\sqrt{2}}$, then the value of $(A + B)$ is : **[2010 (T-I)]**

(a) 30° (b) 45° (c) 75° (d) 15°

23. The value of $\left(\frac{\cos A}{\cot A} + \sin A\right)$ is : **[2010 (T-I)]**

(a) $\cot A$ (b) $2\sin A$ (c) $2\cos A$ (d) $\sec A$

24. If tan 2A = cot (A – 18°), then the value of A is : **[2010 (T-I)]**

(a) 18° (b) 36° (c) 24° (d) 27°

25. Expression of sin A in terms of cot A is : **[2010 (T-I)]**

(a) $\frac{\sqrt{1+\cot^2 A}}{\cot A}$ (b) $\frac{1}{\sqrt{1-\cot^2 A}}$

(c) $\frac{1}{\sqrt{1+\cot^2 A}}$ (d) $\frac{\sqrt{1-\cot^2 A}}{\cot A}$

26. If A is an acute angle in a right ΔABC, right angled at B, then the value of sin A + cos A is : **[2010 (T-I)]**

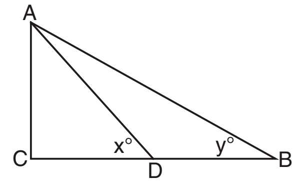
(a) equal to one (b) greater than one

(c) less than one (d) equal to two

27. If cos (α + β) = 0, then sin (α – β) can be reduced to : **[2010 (T-I)]**

(a) cos β (b) cos 2β (c) sin α (d) sin 2α

28. In the figure, if D is mid point of BC, then the value of $\frac{\tan x°}{\tan y°}$ is : **[2010 (T-I)]**



(a) 4 (b) 3 (c) 2 (d) 1

29. If $\text{cosec}\,\theta - \cot\theta = \frac{1}{3}$, the value of (cosec θ + cot θ) is : **[2010 (T-I)]**

(a) 1 (b) 2 (c) 3 (d) 4

30. If sin θ = cos θ, then the value of cosec θ is : **[2010 (T-I)]**

(a) 2 (b) 1 (c) $\frac{2}{\sqrt{3}}$ (d) $\sqrt{2}$

31. In sin 3θ = cos (θ – 26°), where 3θ and (θ – 26°) are acute angles, then value of θ is : **[2010 (T-I)]**

(a) 30° (b) 29° (c) 27° (d) 26°

32. If $\sin\alpha = \frac{1}{2}$ and α is acute, then $(3\cos\alpha - 4\cos^3\alpha)$ is equal to : **[2010 (T-I)]**

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{1}{6}$ (d) –1

33. If $\sec A = \text{cosec}\,B = \frac{12}{7}$, then $(A + B)$ is equal to : **[2010 (T-I)]**

(a) 0° (b) 90° (c) <90° (d) >90°

34. If $\cot A + \frac{1}{\cot A} = 1$, the value of $\cot^2 A + \frac{1}{\cot^2 A}$ is : **[2010 (T-I)]**

(a) 1 (b) 2 (c) –1 (d) –2

35. If sec θ + tan θ = x, then tan θ is : **[2010 (T-I)]**

(a) $\frac{x^2+1}{x}$ (b) $\frac{x^2-1}{x}$ (c) $\frac{x^2+1}{2x}$ (d) $\frac{x^2-1}{2x}$

36. If $2\sin 2\theta = \sqrt{3}$, then the value of θ is : **[2010 (T-I)]**

(a) 90° (b) 30° (c) 45° (d) 60°

37. If x cos A = 1 and tan A = y, then $x^2 - y^2$ is equal to : **[2010 (T-I)]**

(a) tan A (b) 1 (c) 0 (d) –tan A

38. $[\cos^4 A - \sin^4 A]$ is equal to : **[2010 (T-I)]**

(a) $2\cos^2 A + 1$ (b) $2\cos^2 A - 1$

(c) $2\sin^2 A - 1$ (d) $2\sin^2 A + 1$

39. The value of the expression $[(\sec^2\theta - 1)(1 - \text{cosec}^2\theta)]$ is : **[2010 (T-I)]**

(a) –1 (b) 1 (c) 0 (d) $\frac{1}{2}$

40. If $(A-B)=\frac{1}{\sqrt{3}}$ and $\sin A=\frac{1}{\sqrt{2}}$, then the value of B is : **[2010 (T-I)]**

(a) 45° (b) 60° (c) 0° (d) 15°

41. In ΔABC right angled at B, $\tan A = 1$, the value of $2 \sin A \cos A$ is : **[2010 (T-I)]**

(a) –1 (b) 2 (c) 3 (d) 1

42. If $\sqrt{2}\sin(60°-\alpha)=1$, then the value of α is : **[2010 (T-I)]**

(a) 45° (b) 15° (c) 60° (d) 30°

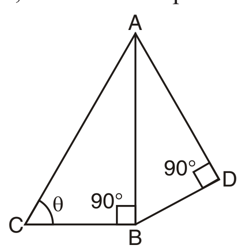
43. $\sin(60° + \theta) - \cos(30° - \theta)$ is equal to : **[2010 (T-I)]**

(a) $2 \cos \theta$ (b) $2 \sin \theta$ (c) 0 (d) 1

44. Given that $\cos\theta=\frac{1}{2}$, the value of $\frac{2\sec\theta}{1+\tan^2\theta}$ is : **[2010 (T-I)]**

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) 0

45. In the figure, AD = 3 cm, BD = 4 cm and CB = 12 cm, then $\tan \theta$ equals : **[2010 (T-I)]**



(a) $\frac{3}{4}$ (b) $\frac{5}{12}$ (c) $\frac{4}{3}$ (d) $\frac{12}{5}$

46. If $\cot\theta=\frac{7}{8}$, then the value of $\frac{(1+\cos\theta)}{(1-\sin\theta)}\frac{(1-\cos\theta)}{(1+\sin\theta)}$ is : **[2010 (T-I)]**

(a) $\frac{49}{64}$ (b) $\frac{8}{7}$ (c) $\frac{64}{49}$ (d) $\frac{7}{8}$

47. The value of $\sin \theta \cos (90° - \theta) + \cos \theta \sin (90° - \theta)$ is : **[2010 (T-I)]**

(a) 1 (b) 0 (c) 2 (d) –1

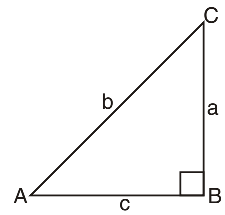
48. If $\tan \theta = \cot \theta$, then the value of $\sec \theta$ is : **[2010 (T-I)]**

(a) 2 (b) 1 (c) $\frac{2}{\sqrt{3}}$ (d) $\sqrt{2}$

49. If $\cos A + \cos^2 A = 1$, then $\sin^2 A + \sin^4 A$ is : **[2010 (T-I)]**

(a) –1 (b) 0 (c) 1 (d) 2

50. From the figure, the value of $\text{cosec } A + \cot A$ is : **[2010 (T-I)]**



(a) $\frac{b+c}{a}$ (b) $\frac{a+b}{c}$ (c) $\frac{a}{b+c}$ (d) $\frac{b}{a+c}$

51. If $a \cos \theta + b \sin \theta = 4$ and $a \sin \theta - b \cos \theta = 3$, then $a^2 + b^2$ is : **[2010 (T-I)]**

(a) 7 (b) 12 (c) 25 (d) none

52. If $\text{cosec}^2 \theta (1 + \cos \theta)(1 - \cos \theta) = \lambda$, then the value of λ is : **[2010 (T-I)]**

(a) 0 (b) $\cos^2 \theta$ (c) 1 (d) –1

53. If $x = 2 \sin^2 \theta$, $y = 2 \cos^2 \theta + 1$, then the value of $x + y$ is : **[2010 (T-I)]**

(a) 2 (b) 3 (c) $\frac{1}{2}$ (d) 1

54. In ΔABC, if $AB = 6\sqrt{3}$ cm, AC = 12 cm and BC = 6 cm, then angle B is equal to : **[2010 (T-I)]**

(a) 120° (b) 90° (c) 45° (d) 60°

55. The maximum value of $\frac{1}{\text{cosec}\,\theta}$ is : **[2010 (T-I)]**

(a) 0 (b) –1 (c) 1 (d) $\frac{\sqrt{3}}{2}$

56. If $\tan A=\frac{3}{4}$ and $A + B = 90°$, then the value of $\cot B$ is equal to : **[2010 (T-I)]**

(a) $\frac{4}{3}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) 1

57. If ΔPQR is right angled at R, then the value of $\cos (P + Q)$ is : **[2010 (T-I)]**

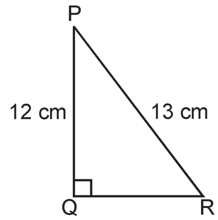
(a) 1 (b) 0 (c) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{2}$

58. Given that $\sin\alpha=\frac{1}{2}$ and $\cos\beta=\frac{1}{2}$, then the value of $\alpha + \beta$ is : **[2010 (T-I)]**

(a) 0° (b) 90° (c) 30° (d) 60°

A. Important Questions

1. In figure, find tan P – cot R.



2. If $\tan\theta + \frac{1}{\tan\theta} = 2$, find the value of $\tan^2\theta + \frac{1}{\tan^2\theta}$.

3. If $\sqrt{3}\tan\theta = 1$, then find the value of $\sin^2\theta - \cos^2\theta$.

4. In a right triangle ABC, right angled at C, if $\tan\theta = 1$, then verify that $2\sin\theta.\cos\theta = 1$.

5. State whether the following are true or false. Justify your answer.

(i) $\sin(A + B) = \sin A + \sin B$.

(ii) The value of $\sin\theta$ increases as θ increases.

(iii) The value of $\cos\theta$ increases as θ increases.

(iv) $\sin\theta = \cos\theta$ for all values of θ.

(v) $\cot A$ is not defined for $A = 0°$.

6. Find the value of θ in the following :

$\cos 2\theta = \cos 60° \cos 30° + \sin 60° \sin 30°$

7. If $A = 30°$ and $B = 60°$, verify that :

(i) $\sin(A + B) = \sin A.\cos B + \cos A.\sin B$

(ii) $\cos(A + B) = \cos A.\cos B - \sin A.\sin B$.

8. Using the formula, $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$, find the value of tan 60°.

9. Using the formula, $\cos A = \sqrt{\frac{1+\cos 2A}{2}}$, find the value of cos 30°.

10. If $\tan 2A = \cot(A - 18°)$, where $2A$ is an acute angle, find the value of A.

11. If $\sin 5A = \cos 4A$, where $5A$ and $4A$ are acute angles, find the value of A.

12. Express sin 67° + cos 75° in terms of trigonometric ratios of angles between 0° and 45°.

13. If $\tan A = \cot B$, prove that $A + B = 90°$.

Prove the following identies :

14. $\frac{\cos\theta + \tan^2\theta - 1}{\sin^2\theta} = \tan^2\theta$

15. $\cot\theta + \tan\theta = \operatorname{cosec}\theta\sec\theta$

16. $\frac{\sin^4 A - \cos^4 A}{\sin^2 A - \cos^2 A} = 1$

17. $\frac{\tan^3\theta - 1}{\tan\theta - 1} = \sec^2\theta + \tan\theta$

18. $\sec A + \tan A = \frac{1}{\sec A - \tan A}$

19. ΔABC is right angled at B and ΔPQR is right angled at Q. If $\cos A = \cos P$, show that $\angle A = \angle P$.

20. ΔABC is right angled at B and ΔDEF is right angled at E. If $\cos C = \cot F$, show that $\angle C = \angle F$.

21. If $60\sec A = 61$, find $\sin A$ and $\tan A$.

22. If $\cos A = 12/13$, find $\frac{13\sin A - 1}{12\tan A + 1}$.

23. If $8\cot A = 15$, find $\frac{16\cos A + 2\sin A}{24\cos A + 2\sin A}$

24. Evaluate : $\frac{\sin 30° + \cot 45° - \operatorname{cosec} 60°}{\sec 30 + \cos 60 + \tan 45}$.

25. Evaluate : $\frac{\sin 45°}{\operatorname{cosec} 30 + \sec 60}$

26. Evaluate : $\frac{4\cos^2 60° + 3\sec^2 30° - \cot^2 45°}{\cos^2 60 + \sin^2 60}$

27. Evaluate : $\frac{\sin^2 53° + \sin^2 37°}{\cos^2 27 + \cos^2 63}$

28. Evaluate : sin 15° cos 75° + cos 15° sin 75°

29. Evaluate : tan 38° tan 33° tan 52° tan 57°.

B. Questions From CBSE Examination Papers

1. Prove that $1 + \frac{\cot^2\theta}{1+\operatorname{cosec}\theta} = \frac{1}{\sin\theta}$. **[2010 (T-I)]**

2. If $\sec 4A = \operatorname{cosec}(A - 20°)$, where $4A$ is an acute angle, find the value of A. **[2010 (T-I)]**

3. If $5\tan\theta = 4$, find the value of $\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta}$.

4. If A, B and C are interior angles of ΔABC, then show that : $\tan\left(\frac{\angle A + \angle B}{2}\right) = \cot\frac{\angle C}{2}$ **[2010 (T-I)]**

5. In ABC, $ABC = 90°$, $AB = 5$ cm and $ACB = 30°$, find BC and AC. **[2010 (T-I)]**

6. If $\sin(A-B)=\dfrac{1}{2}, \cos(A+B)=\dfrac{1}{2}$ and $0 < A+B < 90°$ and $A > B$, then find the values of A and B. **[2010 (T-I)]**

7. If 3 cot A = 4, find the value of $\dfrac{\text{cosec}^2 A+1}{\text{cosec}^2 A-1}$ **[2010 (T-I)]**

8. Evaluate : **[2010 (T-I)]**
$$\frac{\text{cosec}^2(90°-\theta)-\tan^2\theta}{4(\cos^2 48+\cos^2 42°)}-\frac{2\tan^2 30°\sec^2 52°\sin^2 38°}{\text{cosec}^2 70°-\tan^2 20°}$$

9. Prove that **[2010 (T-I)]**
$$\frac{\sin\theta}{\sin(90°-\theta)}+\frac{\cos\theta}{\cos(90°-\theta)}=\sec\theta.\text{cosec}\,\theta$$

10. Evaluate : **[2010 (T-I)]**
$$\frac{\tan^2 60°+4\sin^2 45°+3\sec^2 30°+5\cos^2 90°}{\text{cosec}\,30°+\sec 60°-\cot^2 30°}$$

11. If tan (A + B) = $\sqrt{3}$, tan (A – B) = 1, where $A > B$ and A, B are acute angles, find the values of A and B. **[2010 (T-I)]**

12. If $\sqrt{3}\tan\theta=3\sin\theta$, then prove that $\sin^2\theta-\cos^2\theta=\dfrac{1}{3}$.

13. If 7 $\sin^2\theta$ + 3 $\cos^2\theta$ = 4, then prove that sec θ + cosec θ = 2 + $\dfrac{2}{\sqrt{3}}$.

14. Simplify : $\sin\theta\left\{\dfrac{1}{\sin\theta}-\dfrac{1}{\text{cosec}\,\theta}\right\}$. **[2010 (T-I)]**

15. If $\tan\theta=\dfrac{1}{\sqrt{7}}$, find the value of $\dfrac{\text{cosec}^2\theta+\sec^2\theta}{\text{cosec}^2\theta-\sec^2\theta}$. **[2010 (T-I)]**

16. If $\cot\theta=\dfrac{4}{3}$, evaluate $\dfrac{4\sin\theta+3\cos\theta}{4\sin\theta-3\cos\theta}$. **[2010 (T-I)]**

17. Find the value of k, if $\dfrac{\cos 20°}{\sin 70°}+\dfrac{2\cos\theta}{\sin(90°-\theta)}=\dfrac{k}{2}$ **[2010 (T-I)]**

18. If sin θ + cos θ = m and sec θ + cosec θ = n, then prove that $n(m^2-1)=2m$. **[2010 (T-I)]**

19. If $\cot\theta=\dfrac{7}{8}$, find the value of $\dfrac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$. **[2010 (T-I)]**

20. Simplify : (sec θ + tan θ)(1 – sin θ). **[2010 (T-I)]**

21. Simplify : $\left(\dfrac{1}{\cos\theta}+\dfrac{\sin\theta}{\cos\theta}\right)\left(\dfrac{1-\sin\theta}{\cos\theta}\right)$ **[2010 (T-I)]**

22. Given that sin (A + B) = sin A cos B + cos A sin B, find the value of sin 75°. **[2010 (T-I)]**

23. It cosec θ = $\dfrac{13}{12}$, find the value of cot θ + tan θ. **[2010 (T-I)]**

24. If $\tan A=\dfrac{5}{12}$, find the value of (sin A + cos A). sec A. **[2008]**

25. If $\cos A=\dfrac{7}{25}$, find the value of tan A + cot A. **[2008]**

26. If $\tan\theta=\dfrac{1}{\sqrt{3}}$, then evaluate $\left[\dfrac{\text{cosec}^2\theta-\sec^2\theta}{\text{cosec}^2\theta+\sec^2\theta}\right]$ **[2008 C]**

27. If $\sec^2\theta$ (1 + sin θ)(1 – sin θ) = k, then find the value of k. **[2009]**

28. Without using the trigonometric tables, evaluate : **[2008]**

(i) $\dfrac{11\sin 70°}{7\cos 20°}-\dfrac{4}{7}\dfrac{\cos 53°\text{cosec}\,37°}{\tan 15\ \ \tan 35\ \ \tan 55\ \ .\tan 75}$

(ii) ($\sin^2 25°$ + $\sin^2 65°$) + $\sqrt{3}$ (tan 5° tan 15° tan 30° tan 75° tan 85°)

(iii) ($\cos^2 25°$ + $\cos^2 65°$) + cosec θ.sec (90 – θ) – cot θ.tan (90° – θ)

29. In a ΔABC, right angled at A, if tan C = $\sqrt{3}$, find the value of sin B cos C + cos B sin C. **[2008]**

30. If 7 $\sin^2\theta$ + 3 $\cos^2\theta$ = 4, then show that $\tan\theta=\dfrac{1}{\sqrt{3}}$. **[2008]**

SHORT ANSWER TYPE QUESTIONS [3 Marks]

A. Important Questions

1. In triangle ABC, right-angled at B, if tan A = $\dfrac{1}{\sqrt{3}}$, find the value of :
cos A cos C – sin A sin C

2. If $\cot\theta=\dfrac{15}{8}$, then evaluate
$$\frac{(2+2\sin 9)(1-\sin\theta)}{(1+\cos\theta)(2-2\cos\theta)}$$

3. In a ΔABC, right-angled at C, if $\tan A = \frac{1}{\sqrt{3}}$, and $\tan B = \sqrt{3}$, show that $\sin A.\cos B + \cos A.\sin B = 1$. **[HOTS]**

4. Given that $16 \cot A = 12$, find the value of $\frac{\sin A + \cos A}{\sin A - \cos A}$.

5. If $\cot \theta = \frac{3}{4}$, prove that $\sqrt{\frac{\sec\theta - \text{cosec}\,\theta}{\sec\theta + \text{cosec}\,\theta}} = \frac{1}{\sqrt{7}}$.

6. If $\tan \theta = \frac{a}{b}$, prove that
$$\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a^2 - b^2}{a^2 + b^2}.$$ **[HOTS]**

7. Find acute angles A and B, if $\sin (A + 2B) = \frac{\sqrt{3}}{2}$ and $\cos (A + 4B) = 0°$. $A > B$.

8. Prove : $\tan^2 \theta + \cot^2 \theta = \sec^2 \theta \,\text{cosec}^2 \theta - 2$.

9. Prove :
$$(1 + \tan^2\theta) + \left(1 + \frac{1}{\tan^2\theta}\right) = \frac{1}{(\sin^2\theta - \sin^4\theta)}$$

10. Prove that $\frac{1 + \sec\theta - \tan\theta}{1 + \sec\theta + \tan\theta} = \frac{1 - \sin\theta}{\cos\theta}$

11. If $x \sin^3 \theta + y \cos^3 \theta = \sin \theta \cos \theta$ and $x \sin \theta = y \cos \theta$, prove that $x^2 + y^2 = 1$.

12. If $x \cos \theta - y \sin \theta = a$ and $x \sin \theta + y \cos \theta = b$, prove that $x^2 + y^2 = a^2 + b^2$.

Prove the following identities :

13. cosec A(1 – cos A)(cosec A + cot A) = 1.

14. $\frac{\tan A - \sin A}{\tan A + \sin A} = \frac{\sec A - 1}{\sec A + 1}$

15. $(\sec A - \tan A)^2 = \frac{1 - \sin A}{1 + \sin A}$

16. $\frac{1 + \text{cosec}\,A}{\text{cosec}\,A} = \frac{\cos^2 A}{1 - \sin A}$

17. $\sqrt{\frac{1 - \sin A}{1 + \sin A}} = \sec A - \tan A$

18. $\frac{\tan A}{1 + \sec A} - \frac{\tan A}{1 - \sec A} = 2\,\text{cosec}\,A$

19. $\frac{\cot^2 A}{1 + \text{cosec}\,A} + 1 = \text{cosec}\,A$

20. $\sin^6 A + \cos^6 A + 3 \sin^2 A \cos^2 A = 1$.

21. $(\sin^4 A - \cos^4 A + 1)\,\text{cosec}^2 A = 2$.

22. If A + B = 90°, show that
$$\sqrt{\cos A\,\text{cosec}\,B - \cos A \sin B} = \sin A$$

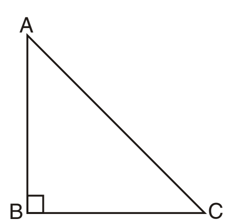
23. If $x = \gamma \cos \alpha \sin \beta$; $y = \gamma \cos \alpha \cos \beta$ and $z = \gamma \sin \alpha$, show that $x^2 + y^2 + z^2 = \gamma^2$

B. Questions From CBSE Examination Papers

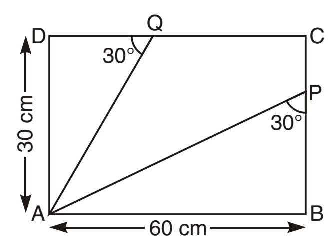
1. $\frac{\cos\alpha}{\cos\beta} = m$ and $\frac{\cos\alpha}{\sin\beta} = n$, then show that $(m^2 + n^2)\cos^2\beta = n^2$. **[2010 (T-I)]**

2. If $x = a\sec\theta + b\tan\theta$, $y = a\tan\theta + b\sec\theta$
prove that $x^2 - y^2 = a^2 - b^2$ **[2010 (T-I)]**

3. In the figure, ΔABC is right angled at B, BC = 7 cm and $AC - AB$ = 1 cm. Find the value of $\cos A - \sin A$. **[2010 (T-I)]**



4. In the figure, $ABCD$ is a rectangle in which segments AP and AQ are drawn. Find the length (AP + AQ). **[2010 (T-I)]**



5. Evaluate : sin (50° + θ) – cos (40° – θ) + tan 1° tan 10° tan 20° tan 70° tan 80°
tan 89° + sec (90° – θ).cosec θ – tan (90° – θ).cot θ.

6. Prove that $\frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2\sec A$ **[2010 (T-I)]**

7. Prove that $(\text{cosec}\,A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$ **[2010 (T-I)]**

8. Prove that $(\sin \theta + \text{cosec}\,\theta)^2 + (\cos \theta + \sec \theta)^2 = 7 + \tan^2 \theta + \cot^2 \theta$. **[2010 (T-I)]**

9. If A, B, C are interior angles of ΔABC, show that:

$$\operatorname{cosec}^2\left(\frac{B+C}{2}\right) - \tan^2\frac{A}{2} = 1$$ **[2010 (T-I)]**

10. Prove $\sec^2\theta + \cot^2(90° - \theta) = 2\operatorname{cosec}^2(90° - \theta) - 1$. **[2010 (T-I)]**

11. If A, B, C are interior angles of ΔABC, show that :

$$\sec^2\left(\frac{B+C}{2}\right) - 1 = \cot^2\frac{A}{2}$$ **[2010 (T-I)]**

12. Prove that :

$$\frac{\cos(90 - \theta)}{1+\sin(90°-\theta)} + \frac{1+\sin(90°-\theta)}{\cos(90°-\theta)} = 2\operatorname{cosec}\theta$$

13. Prove that : $\sqrt{\frac{\sec\theta-1}{\sec\theta+1}} + \sqrt{\frac{\sec\theta+1}{\sec\theta-1}} = 2\operatorname{cosec}\theta$ **[2010 (T-I)]**

14. If $\sin(A+B) = \frac{\sqrt{3}}{2}$ and $\cos(A-B) = 1, 0° < (A+B) < 90°, A \geq B$, find A and B. **[2010 (T-I)]**

15. Evaluate :

$$\frac{-\tan\theta.\cot(90-\theta) + \sec\theta.\operatorname{cosec}(90-\theta) + \sin^2 35° + \sin^2 55°}{\tan 10 .\tan 20 .\tan 30 .\tan 70 .\tan 80}$$

[2010 (T-I)]

16. Prove that $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta, \operatorname{cosec}\theta$ **[2010 (T-I)]**

17. Evaluate : **[2010 (T-I)]**

$$\frac{\operatorname{cosec}^2(90-\theta) - \tan^2\theta}{5(\cos^2 48° + \cos^2 42°)} + \frac{2}{5}\sin 48° \sec 42° - \frac{1}{5}\tan^2 60°.$$

18. Prove that $(1 + \cot\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta) = 2$. **[2010 (T-I)]**

19. Prove that : **[2010 (T-I)]**

$$\frac{\cot(90°-\theta)}{\tan\theta} + \frac{\operatorname{cosec}(90°-\theta)}{\tan(90°-\theta)}.\sin\theta = \sec^2\theta.$$

20. Prove that $(\operatorname{cosec}\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$ **[2010 (T-I)]**

21. If $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ and $\frac{x}{a}\sin\theta + \frac{y}{b}\cos\theta = 1$, prove that $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$. **[2010 (T-I)]**

22. If $\sin(2A + 45°) = \cos(30° - A)$, find A. **[2010 (T-I)]**

23. Prove that $\frac{\tan\theta - \cot\theta}{\sin\theta\cos\theta} = \tan^2\theta - \cot^2\theta$. **[2010 (T-I)]**

24. If $\tan A = n \tan B$ and $\sin A = m \sin B$, prove that $\cos^2 A = \frac{m^2-1}{n^2-1}$. **[2010 (T-I)]**

25. Prove that : $\frac{1+\sec A}{\sec A} - \frac{\sin 2A}{1-\cos A}$. **[2010 (T-I)]**

26. If $\sin\theta + \cos\theta = \sqrt{2}\sin(90-\theta)$, then find the value of $\tan\theta$.

27. Evaluate : **[2010 (T-I)]**

$\frac{\sin 39°}{\cos 51°} + 2\tan 11° \tan 31° \tan 45° \tan 59° \tan 79° - 3(\sin^2 21° + \sin^2 69°)$.

28. Prove that $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 2\sec\theta$. **[2010 (T-I)]**

29. If $m\sin\theta + n\cos\theta = p$ and $m\cos\theta - n\sin\theta = q$, then prove that $m^2 + n^2 = p^2 + q^2$ **[2010 (T-I)]**

30. In ΔPQR, right angled at Q, if $PR + QR = 25$ cm and $PQ = 5$ cm, determine the value of $\sin P$ and tan P.

31. Evaluate : **[2010 (T-I)]**

$$\frac{2\sin 68°}{\cos 22°} - \frac{2\tan(90°-15°)}{5\cot 15°} - \frac{3\tan 45°\tan 20°\tan 40°\tan 50°\tan 70°}{5(\sin^2 70° + \sin^2 20°)}.$$

32. Prove that $\frac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta} = \left(\frac{1+\sin\theta}{\cos\theta}\right)^2$ **[2010 (T-I)]**

33. Prove that **[2010 (T-I)]**

$$(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$$

34. If A + B = 90°, then prove that **[2010 (T-I)]**

$$\sqrt{\frac{\tan A\tan B + \tan A\cot B}{\sin A\sec B} - \frac{\sin^2 B}{\cos^2 A}} = \tan A$$

35. Prove that $2\sec^2\theta - \sec^4\theta - 2\operatorname{cosec}^2\theta + \operatorname{cosec}^4\theta = \cot^4\theta - \tan^4\theta$. **[2010 (T-I)]**

36. If $5x = \sec\theta$ and $\frac{5}{x} = \tan\theta$, find the value of $5\left(x^2 - \frac{1}{x^2}\right)$. **[2010]**

37. If $\sin\theta + \cos\theta = \sqrt{2}\sin(90°-\theta)$, show that $\cot\theta = \sqrt{2}+1$. **[2001 C]**

38. Prove : $\frac{\cos A}{1-\tan A} - \frac{\sin^2 A}{\cos A - \sin A} = \sin A + \cos A$ **[2003, 2007]**

39. Without using trigonometric tables evaluate : **[2007, 2008]**

$\tan 7^\circ \tan 23^\circ \tan 60^\circ \tan 67^\circ \tan 83^\circ$

$$+\frac{\cot 54^\circ}{\tan 36^\circ}+\sin 20^\circ.\sec 70^\circ-2.$$

40. Prove that : $\dfrac{\cot A-\cos A}{\cot A+\cos A}=\dfrac{\operatorname{cosec} A-1}{\operatorname{cosec} A+1}$ **[2008]**

41. Prove that : $(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$ **[2008]**

42. Prove that : $(1 + \cot A + \tan A)(\sin A - \cos A) = \sin A \tan A - \cot A.\cos A$. **[2008]**

43. Prove that : $\dfrac{1+\tan^2 A}{1+\cot^2 A}=\left(\dfrac{1-\tan A}{1-\cot A}\right)^2=\tan^2 A$

[2008 C]

44. Evaluate :

$$\frac{2}{3}\operatorname{cosec}^2 58^\circ-\frac{2}{3}\cot 58^\circ.\tan 32^\circ-\frac{5}{3}\tan 13^\circ.\tan 37^\circ.\tan 45^\circ.\tan 53^\circ.\tan 77^\circ$$ **[2009]**

45. If $\cos^2\theta - \sin^2\theta = \tan^2\phi$, prove that $\cos\phi=\dfrac{1}{\sqrt{2}\cos\theta}$. **[2002]**

46. If $\operatorname{cosec}\theta - \sin\theta = l$ and $\sec\theta - \cos\theta = m$, show that $l^2m^2 (l^2 + m^2 + 3) = 1$. **[2003]**

47. Evaluate :

$$\frac{\sin 70^\circ}{\cos 20^\circ}+\frac{\operatorname{cosec} 36^\circ}{\sec 54}-\frac{2\cos 43^\circ\operatorname{cosec} 47^\circ}{\tan 10\ \tan 40\ \tan 50\ \tan 80}$$

[2004 C]

LONG ANSWER TYPE QUESTIONS [4 Marks]

A. Important Questions

1. If $\dfrac{\cos\alpha}{\cos\beta}=m$ and $\dfrac{\sin\alpha}{\sin\beta}=n$, prove that $(n^2-m^2)\sin^2\beta=1-m^2$. **[HOTS]**

2. If $\sin\theta + \cos\theta = 1$, prove that $(\cos\theta - \sin\theta) = \pm 1$

3. If $\operatorname{cosec}\theta + \cot\theta = p$, show that $\cos\theta = \dfrac{p^2-1}{p^2+1}$ **[HOTS]**

4. If $1 + \sin^2\theta = 3\sin\theta\cos\theta$, then, prove that $\tan\theta = 1$ or $\dfrac{1}{2}$

5. If $\sin\theta + \sin^2\theta + \sin^3\theta = 1$, then prove that $\cos^6\theta - 4\cos^4\theta + 8\cos^2\theta = 4$.

6. In an acute angled triangle ABC, if $\sin 2(A + B - C) = 1$ and $\tan(B + C - A) = \sqrt{3}$, find the values of A, B and C. **[HOTS]**

7. If $\tan^2\theta=1+2\tan^2\phi$, prove that $2\sin^2\theta=1+\sin^2\phi$.

8. Prove : $\dfrac{\cot\theta}{\operatorname{cosec}\theta+1}+\dfrac{\operatorname{cosec}\theta+1}{\cot\theta}=2\sec\theta$

9. If $\sin\alpha = a\sin\beta$ and $\tan\alpha = b\tan\beta$, then prove that $\cos^2\alpha=\dfrac{a^2-1}{b^2-1}$.

B. Questions From CBSE Examination Papers

1. If $\sin\theta + \cos\theta = p$ and $\sec\theta + \operatorname{cosec}\theta = q$ then prove that $q(p^2 - 1) = 2p$. **[2010 (T-I)]**

2. Prove that : $\cos^4\theta - \cos^2\theta = \sin^4 - \sin^2\theta$. **[2010 (T-I)]**

3. Prove that : $\operatorname{cosec}^2 (90^\circ - \theta) - \tan^2\theta = \cos^2 (90^\circ - \theta) + \cos^2\theta$. **[2010 (T-I)]**

4. If $2\cos\theta - \sin\theta = x$ and $\cos\theta - 3\sin\theta = y$, prove that $2x^2 + y^2 - 2xy = 5$. **[2010 (T-I)]**

5. Without using trigonometric tables, evaluate the following :

$$\frac{\cos^2 20^\circ+\cos^2 70^\circ}{\cos^2 50^\circ+\cos^2 40^\circ}+2\operatorname{cosec}^2 58^\circ-2\cot 58^\circ\tan 32^\circ-4\tan 13^\circ\tan 37^\circ\tan 45^\circ\tan 53^\circ\tan 77^\circ$$

6. Prove that : **[2010 (T-I)]**

$$\frac{\sin A}{\sec A+\tan A-1}+\frac{\cos A}{\operatorname{cosec} A+\cot A-1}=1$$

7. Prove that : **[2010 (T-I)]**

$$\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}+\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}=\frac{2}{2\sin^2\theta-1}$$

8. Prove that : **[2010 (T-I)]**

$$\frac{2}{\cos^2\theta}-\frac{1}{\cos^4\theta}-\frac{2}{\sin^2\theta}+\frac{1}{\sin^4\theta}=\cot^4\theta-\tan^4\theta$$

9. Prove that : $(1 + \tan A\tan B)^2 + (\tan A - \tan B)^2 = \sec^2 A.\sec^2 B$. **[2010 (T-I)]**

10. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, prove that $m^2 - n^2 = 4\sqrt{mn}$. **[2010 (T-I)]**

11. Prove that :

$$\frac{1}{\operatorname{cosec} A - \cot A} - \frac{1}{\sin A} = \frac{1}{\sin A} - \frac{1}{\operatorname{cosec} A + \cot A}$$

12. Prove that : $\sec^2\theta - \dfrac{\sin^2\theta - 2\sin^4\theta}{2\cos^4\theta - \cos\theta^2} = 1.$ **[2010 (T-I)]**

13. If sec θ – tan θ = 4, then prove that cos θ $= \dfrac{8}{17}$. **[2010 (T-I)]**

14. Find the value of $\sin^2 5° + \sin^2 10° + \sin^2 80° + \sin^2 85°$. **[2010 (T-I)]**

15. Prove that $= \dfrac{1+\sec A}{\sec A} = \dfrac{\sin^2 A}{1-\cos A}$. **[2010 (T-I)]**

16. Prove that : $\dfrac{\tan\theta + 1 + \sec\theta}{\tan\theta + 1 - \sec\theta} = \dfrac{1}{\sec\theta - \tan\theta}$. **[2010 (T-I)]**

17. If $\sec\theta = x + \dfrac{1}{4x}$, then prove that

$\sec\theta + \tan\theta = 2x \text{ or } \dfrac{1}{2x}$. **[2010 (T-I)]**

18. Prove that $\dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta} = 1 + \tan\theta + \cot\theta.$ **[2010 (T-I)]**

19. Prove that : $\dfrac{\cot^2 A(\sec A - 1)}{1+\sin A} = \sec^2 A\left(\dfrac{1-\sin A}{1+\sec A}\right)$ **[2010 (T-I)]**

20. If sec θ + tan θ = p, show that $\dfrac{p^2-1}{p^2+1} = \sin\theta.$ **[2010 (T-I)]**

21. If a sin θ + b cos θ = c, then prove that a cos θ – b sin θ = $\sqrt{a^2+b^2-c^2}$. **[2010 (T-I)]**

22. Prove that **[2010 (T-I)]**

$$\frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A} = \frac{2}{\sin^2 A - \cos^2 A}.$$

23. Prove that $\dfrac{\cos A}{1-\sin A} + \dfrac{1-\sin A}{\cos A} = 2\sec A.$ **[2010 (T-I)]**

24. If $x = r \sin A \cos C$, $y = r \sin A \sin C$, $z = r \cos A$, prove that $r^2 = x^2 + y^2 + z^2$. **[2010 (T-I)]**

25. If $\tan A = \sqrt{2} - 1$, show that $\sin A \cos A = \dfrac{\sqrt{2}}{4}$. **[2010 (T-I)]**

26. Prove that $\dfrac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1} = \dfrac{1+\cos A}{\sin A}$. **[2010 (T-I)]**

27. Prove that $\sin^6\theta + \cos^6\theta = 3\sin^2\theta\cos^2\theta$.

28. Evaluate : **[2010 (T-I)]**

$$\sin(50° + \theta) - \cos(40° - \theta) + \frac{1}{4}\cot^2 30° + \frac{3\tan 45° \tan 20° \tan 40° \tan 50° \tan 70°}{5} + \frac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°}$$

29. Prove that : (cosec A – sin A)(sec A – cos A)(tan A + cot A) = 1. **[2010 (T-I)]**

30. Prove that : **[2010 (T-I)]**

$$\frac{1}{\sec\theta - \tan\theta} - \frac{1}{\cos\theta} = \frac{1}{\cos\theta} - \frac{1}{\sec\theta + \tan\theta}.$$

31. Prove that $\dfrac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \dfrac{1+\sin\theta}{\cos\theta}$ **[2007]**

32. Prove that $\dfrac{\cos A}{1-\tan A} + \dfrac{\sin A}{1-\cot A} = \sin A + \cos A$ **[2002]**

33. Prove that $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$ **[2008]**

34. Prove : $\dfrac{\sin\theta}{(\cot\theta + \operatorname{cosec}\theta)} = 2 + \dfrac{\sin\theta}{(\cot\theta - \operatorname{cosec}\theta)}$ **[2000]**

35. Prove : $\dfrac{\operatorname{cosec}\theta + \cot\theta}{\operatorname{cosec}\theta + \cot\theta} = (\operatorname{cosec}\theta + \cot\theta)^2$

$= 1 + 2\cot^2\theta + 2\operatorname{cosec}\theta\cot\theta$ **[2003]**

FORMATIVE ASSESSMENT

Objective : To solve a crossword puzzle with mathematical terms.

Clues down :

1. Collection of one or some outcomes of an experiment.

2. A group of 144 things.
3. A cumulative frequency curve.
4. The term which is used for the expression which is not defined.
5. A number which cannot be expressed in the form p/q, where p and q are integers and $q \neq 0$.
6. The value of the observation having maximum frequency.
7. Unit of length.
8. Figures having the same shape.

9		2	3		5			7	
1						6			
				4	10				8
	11								
12									
	13								
	14								
	15						16		

Clues Across :

9. A series of well defined steps which gives a procedure for solving a type of problem.
10. Solutions of equations.
11. Plural of radius.
12. An algebraic expression in which the variables involves have only non-negative integral powers.
13. A solid obtained by rolling a rectangular paper along its length or breadth.
14. Unit of area.
15. A solid having one vertex and two faces, one curved and one flat.
16. Part of a circle.

Exercise 8.1

Question 1:

In ΔABC right angled at B, AB = 24 cm, BC = 7 m. Determine

(i) sin A, cos A

(ii) sin C, cos C

Answer:

Applying Pythagoras theorem for ΔABC, we obtain

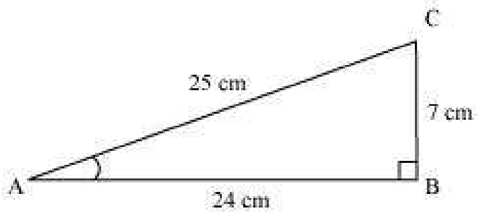
$AC^2 = AB^2 + BC^2$

$= (24\text{ cm})^2 + (7\text{ cm})^2$

$= (576 + 49)\text{ cm}^2$

$= 625\text{ cm}^2$

$\therefore AC = \sqrt{625}$ cm = 25 cm

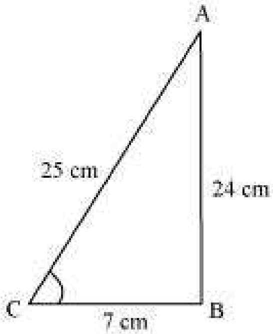


(i) sin A = $\frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC}$

$= \frac{7}{25}$

cos A = $\frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{24}{25}$

(ii)



sin C = $\frac{\text{Side opposite to } \angle C}{\text{Hypotenuse}} = \frac{AB}{AC}$

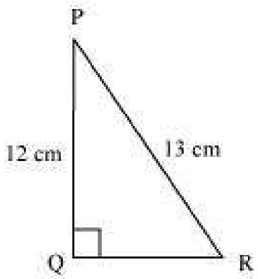
$= \frac{24}{25}$

cos C = $\frac{\text{Side adjacent to } \angle C}{\text{Hypotenuse}} = \frac{BC}{AC}$

$= \frac{7}{25}$

Question 2:

In the given figure find tan P − cot R



Answer:

Applying Pythagoras theorem for ΔPQR, we obtain

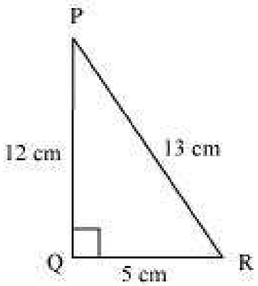
$PR^2 = PQ^2 + QR^2$

$(13\text{ cm})^2 = (12\text{ cm})^2 + QR^2$

$169\ cm^2 = 144\ cm^2 + QR^2$

$25\ cm^2 = QR^2$

QR = 5 cm



$$\tan P = \frac{\text{Side opposite to } \angle P}{\text{Side adjacent to } \angle P} = \frac{QR}{PQ}$$

$$= \frac{5}{12}$$

$$\cot R = \frac{\text{Side adjacent to } \angle R}{\text{Side opposite to } \angle R} = \frac{QR}{PQ}$$

$$= \frac{5}{12}$$

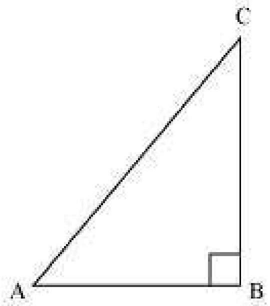
tan P − cot R = $\frac{5}{12} - \frac{5}{12} = 0$

Question 3:

If sin A = $\frac{3}{4}$, calculate cos A and tan A.

Answer:

Let ΔABC be a right-angled triangle, right-angled at point B.



Given that,

$$\sin A = \frac{3}{4}$$

$$\frac{BC}{AC} = \frac{3}{4}$$

Let BC be 3*k*. Therefore, AC will be 4*k*, where *k* is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$AC^2 = AB^2 + BC^2$

$(4k)^2 = AB^2 + (3k)^2$

$16k^2 - 9k^2 = AB^2$

$7k^2 = AB^2$

AB = $\sqrt{7}k$

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}}$$

$$= \frac{AB}{AC} = \frac{\sqrt{7}k}{4k} = \frac{\sqrt{7}}{4}$$

$$\tan A = \frac{\text{Side opposite to } \angle A}{\text{Side adjacent to } \angle A}$$

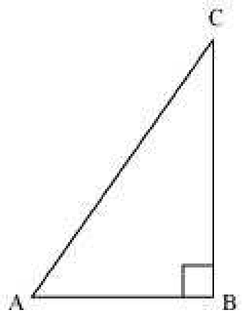
$$= \frac{BC}{AB} = \frac{3k}{\sqrt{7}k} = \frac{3}{\sqrt{7}}$$

Question 4:

Given 15 cot A = 8. Find sin A and sec A

Answer:

Consider a right-angled triangle, right-angled at B.



$$\cot A = \frac{\text{Side adjacent to } \angle A}{\text{Side opposite to } \angle A}$$

$$= \frac{AB}{BC}$$

It is given that,

cot A = $\frac{8}{15}$

$$\frac{AB}{BC} = \frac{8}{15}$$

Let AB be 8*k*.Therefore, BC will be 15*k*, where *k* is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$AC^2 = AB^2 + BC^2$

$= (8k)^2 + (15k)^2$

$= 64k^2 + 225k^2$

$= 289k^2$

AC = 17*k*

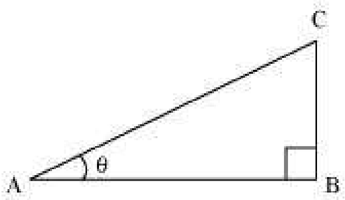
$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC}$$
$$= \frac{15k}{17k} = \frac{15}{17}$$
$$\sec A = \frac{\text{Hypotenuse}}{\text{Side adjacent to } \angle A}$$
$$= \frac{AC}{AB} = \frac{17}{8}$$

Question 5:

Given sec θ = $\frac{13}{12}$, calculate all other trigonometric ratios.

Answer:

Consider a right-angle triangle ΔABC, right-angled at point B.



$$\sec\theta = \frac{\text{Hypotenuse}}{\text{Side adjacent to } \angle\theta}$$
$$\frac{13}{12} = \frac{AC}{AB}$$

If AC is 13*k*, AB will be 12*k*, where *k* is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$(AC)^2 = (AB)^2 + (BC)^2$

$(13k)^2 = (12k)^2 + (BC)^2$

$169k^2 = 144k^2 + BC^2$

$25k^2 = BC^2$

BC = 5*k*

$$\sin\theta = \frac{\text{Side opposite to } \angle\theta}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}$$

$$\cos\theta = \frac{\text{Side adjacent to } \angle\theta}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{12k}{13k} = \frac{12}{13}$$

$$\tan\theta = \frac{\text{Side opposite to } \angle\theta}{\text{Side adjacent to } \angle\theta} = \frac{BC}{AB} = \frac{5k}{12k} = \frac{5}{12}$$

$$\cot\theta = \frac{\text{Side adjacent to } \angle\theta}{\text{Side opposite to } \angle\theta} = \frac{AB}{BC} = \frac{12k}{5k} = \frac{12}{5}$$

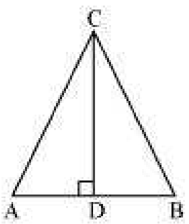
$$\text{cosec}\ \theta = \frac{\text{Hypotenuse}}{\text{Side opposite to } \angle\theta} = \frac{AC}{BC} = \frac{13k}{5k} = \frac{13}{5}$$

Question 6:

If ∠A and ∠B are acute angles such that cos A = cos B, then show that ∠A = ∠B.

Answer:

Let us consider a triangle ABC in which CD ⊥ AB.

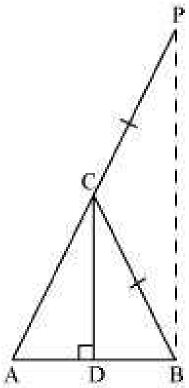


It is given that

cos A = cos B

$$\Rightarrow \frac{AD}{AC} = \frac{BD}{BC} \quad \ldots (1)$$

We have to prove ∠A = ∠B. To prove this, let us extend AC to P such that BC = CP.



From equation (1), we obtain

$$\frac{AD}{BD}=\frac{AC}{BC}$$

$$\Rightarrow \frac{AD}{BD}=\frac{AC}{CP} \qquad (\text{By construction, we have } BC = CP) \qquad \ldots (2)$$

By using the converse of B.P.T,

CD||BP

⇒∠ACD = ∠CPB (Corresponding angles) … (3)

And, ∠BCD = ∠CBP (Alternate interior angles) … (4)

By construction, we have BC = CP.

∴ ∠CBP **=** ∠CPB (Angle opposite to equal sides of a triangle) … (5)

From equations (3), (4), and (5), we obtain

∠ACD = ∠BCD … (6)

In ΔCAD and ΔCBD,

∠ACD = ∠BCD [Using equation (6)]

∠CDA = ∠CDB [Both 90°]

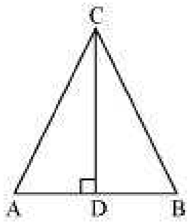
Therefore, the remaining angles should be equal.

∴∠CAD = ∠CBD

⇒ ∠A = ∠B

Alternatively,

Let us consider a triangle ABC in which CD $\perp$ AB.



It is given that,

cos A = cos B

$\Rightarrow \frac{AD}{AC} = \frac{BD}{BC}$

$\Rightarrow \frac{AD}{BD} = \frac{AC}{BC}$

Let $\frac{AD}{BD} = \frac{AC}{BC} = k$

$\Rightarrow$ AD = k BD … (1)

And, AC = k BC … (2)

Using Pythagoras theorem for triangles CAD and CBD, we obtain

$CD^2 = AC^2 - AD^2$ … (3)

And, $CD^2 = BC^2 - BD^2$ … (4)

From equations (3) and (4), we obtain

$AC^2 - AD^2 = BC^2 - BD^2$

$\Rightarrow (k\ BC)^2 - (k\ BD)^2 = BC^2 - BD^2$

$\Rightarrow k^2 (BC^2 - BD^2) = BC^2 - BD^2$

$\Rightarrow k^2 = 1$

$\Rightarrow k = 1$

Putting this value in equation (2), we obtain

AC = BC

$\Rightarrow \angle A = \angle B$(Angles opposite to equal sides of a triangle)

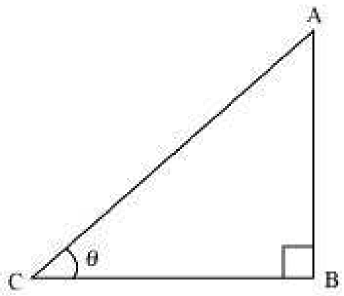
Question 7:

If $\cot\theta = \frac{7}{8}$, evaluate

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ (ii) $\cot^2\theta$

Answer:

Let us consider a right triangle ABC, right-angled at point B.



$$\cot\theta = \frac{\text{Side adjacent to } \angle\theta}{\text{Side opposite to } \angle\theta} = \frac{BC}{AB}$$

$$= \frac{7}{8}$$

If BC is $7k$, then AB will be $8k$, where k is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$AC^2 = AB^2 + BC^2$

$= (8k)^2 + (7k)^2$

$= 64k^2 + 49k^2$

$= 113k^2$

AC = $\sqrt{113}k$

$$\sin\theta = \frac{\text{Side opposite to } \angle\theta}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$= \frac{8k}{\sqrt{113}k} = \frac{8}{\sqrt{113}}$$

$$\cos\theta = \frac{\text{Side adjacent to } \angle\theta}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{7k}{\sqrt{113}k} = \frac{7}{\sqrt{113}}$$

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{(1-\sin^2\theta)}{(1-\cos^2\theta)}$

$$= \frac{1-\left(\frac{8}{\sqrt{113}}\right)^2}{1-\left(\frac{7}{\sqrt{113}}\right)^2} = \frac{1-\frac{64}{113}}{1-\frac{49}{113}}$$

$$= \frac{\frac{49}{113}}{\frac{64}{113}} = \frac{49}{64}$$

(ii) $\cot^2\theta = (\cot\theta)^2 = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$

Question 8:

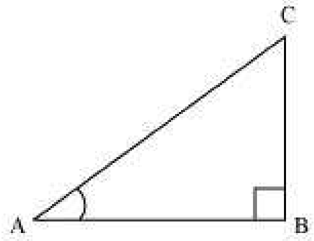
If 3 cot A = 4, Check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.

Answer:

It is given that 3cot A = 4

Or, cot A = $\frac{4}{3}$

Consider a right triangle ABC, right-angled at point B.



$$\cot A = \frac{\text{Side adjacent to } \angle A}{\text{Side opposite to } \angle A}$$

$$\frac{AB}{BC} = \frac{4}{3}$$

If AB is 4*k*, then BC will be 3*k*, where *k* is a positive integer.

In ΔABC,

$(AC)^2 = (AB)^2 + (BC)^2$

$= (4k)^2 + (3k)^2$

$= 16k^2 + 9k^2$

$= 25k^2$

$AC = 5k$

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$= \frac{4k}{5k} = \frac{4}{5}$$

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{3k}{5k} = \frac{3}{5}$$

$$\tan A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AB}$$

$$= \frac{3k}{4k} = \frac{3}{4}$$

$$\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}}$$

$$= \frac{\frac{7}{16}}{\frac{25}{16}} = \frac{7}{25}$$

$\cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

$$= \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

$$\therefore \frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$$

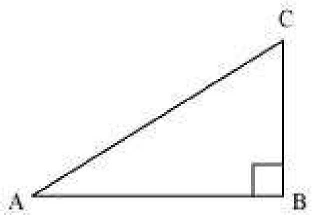
Question 9:

In ΔABC, right angled at B. If $\tan A = \frac{1}{\sqrt{3}}$, find the value of

(i) sin A cos C + cos A sin C

(ii) cos A cos C − sin A sin C

Answer:



$\tan A = \frac{1}{\sqrt{3}}$

$\frac{BC}{AB} = \frac{1}{\sqrt{3}}$

If BC is k, then AB will be $\sqrt{3}k$, where k is a positive integer.

In ΔABC,

$AC^2 = AB^2 + BC^2$

$= \left(\sqrt{3}k\right)^2 + (k)^2$

$= 3k^2 + k^2 = 4k^2$

$\therefore AC = 2k$

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\sin C = \frac{\text{Side opposite to } \angle C}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\cos C = \frac{\text{Side adjacent to } \angle C}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

(i) sin A cos C + cos A sin C

$$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{4} + \frac{3}{4}$$

$$= \frac{4}{4} = 1$$

(ii) cos A cos C − sin A sin C

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$$

Question 10:

In ΔPQR, right angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos P and tan P.

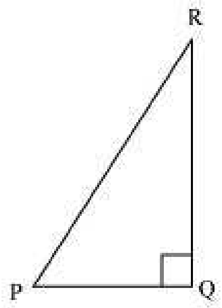
Answer:

Given that, PR + QR = 25

PQ = 5

Let PR be x.

Therefore, QR = 25 − x



Applying Pythagoras theorem in ΔPQR, we obtain

$PR^2 = PQ^2 + QR^2$

$x^2 = (5)^2 + (25 - x)^2$

$x^2 = 25 + 625 + x^2 - 50x$

$50x = 650$

$x = 13$

Therefore, PR = 13 cm

QR = (25 − 13) cm = 12 cm

$$\sin P = \frac{\text{Side opposite to } \angle P}{\text{Hypotenuse}} = \frac{QR}{PR} = \frac{12}{13}$$

$$\cos P = \frac{\text{Side adjacent to } \angle P}{\text{Hypotenuse}} = \frac{PQ}{PR} = \frac{5}{13}$$

$$\tan P = \frac{\text{Side opposite to } \angle P}{\text{Side adjacent to } \angle P} = \frac{QR}{PQ} = \frac{12}{5}$$

Question 11:

State whether the following are true or false. Justify your answer.

(i) The value of tan A is always less than 1.

(ii) sec A = $\frac{12}{5}$ for some value of angle A.

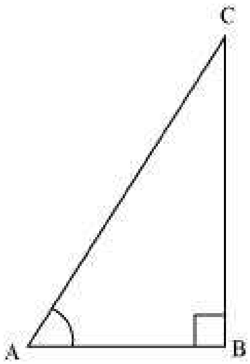
(iii) cos A is the abbreviation used for the cosecant of angle A.

(iv) cot A is the product of cot and A

(v) sin θ = $\frac{4}{3}$, for some angle θ

Answer:

(i) Consider a ΔABC, right-angled at B.



$$\tan A = \frac{\text{Side opposite to } \angle A}{\text{Side adjacent to } \angle A}$$

$$= \frac{12}{5}$$

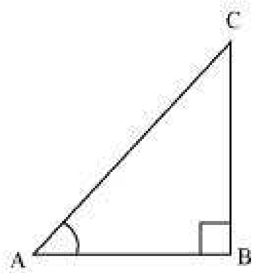
But $\frac{12}{5} > 1$

$\therefore$ tan A > 1

So, tan A < 1 is not always true.

Hence, the given statement is false.

(ii) $\sec A = \frac{12}{5}$



$$\frac{\text{Hypotenuse}}{\text{Side adjacent to } \angle A} = \frac{12}{5}$$

$$\frac{AC}{AB} = \frac{12}{5}$$

Let AC be $12k$, AB will be $5k$, where k is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$AC^2 = AB^2 + BC^2$

$(12k)^2 = (5k)^2 + BC^2$

$144k^2 = 25k^2 + BC^2$

$BC^2 = 119k^2$

$BC = 10.9k$

It can be observed that for given two sides AC = $12k$ and AB = $5k$,

BC should be such that,

AC − AB < BC < AC + AB

$12k - 5k < BC < 12k + 5k$

$7k < BC < 17\,k$

However, BC = 10.9*k*. Clearly, such a triangle is possible and hence, such value of sec A is possible.

Hence, the given statement is true.

(iii) Abbreviation used for cosecant of angle A is cosec A. And cos A is the abbreviation used for cosine of angle A.

Hence, the given statement is false.

(iv) cot A is not the product of cot and A. It is the cotangent of ∠A.

Hence, the given statement is false.

(v) sin θ = $\frac{4}{3}$

We know that in a right-angled triangle,

$$\sin\theta = \frac{\text{Side opposite to } \angle\theta}{\text{Hypotenuse}}$$

In a right-angled triangle, hypotenuse is always greater than the remaining two sides. Therefore, such value of sin θ is not possible.

Hence, the given statement is false

Exercise 8.2

Question 1:

Evaluate the following

(i) sin60° cos30° + sin30° cos 60°

(ii) $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

(iii) $\dfrac{\cos 45°}{\sec 30° + \operatorname{cosec} 30°}$

(iv) $\dfrac{\sin 30° + \tan 45° - \operatorname{cosec} 60°}{\sec 30° + \cos 60° + \cot 45°}$

(v) $\dfrac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$

Answer:

(i) sin60° cos30° + sin30° cos 60°

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$= \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$$

(ii) $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

$$= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= 2 + \frac{3}{4} - \frac{3}{4} = 2$$

(iii) $\dfrac{\cos 45°}{\sec 30° + \operatorname{cosec} 30°}$

$$=\frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}+2}=\frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}}$$

$$=\frac{\sqrt{3}}{\sqrt{2}\left(2+2\sqrt{3}\right)}=\frac{\sqrt{3}}{2\sqrt{2}+2\sqrt{6}}$$

$$=\frac{\sqrt{3}\left(2\sqrt{6}-2\sqrt{2}\right)}{\left(2\sqrt{6}+2\sqrt{2}\right)\left(2\sqrt{6}-2\sqrt{2}\right)}$$

$$=\frac{2\sqrt{3}\left(\sqrt{6}-\sqrt{2}\right)}{\left(2\sqrt{6}\right)^2-\left(2\sqrt{2}\right)^2}=\frac{2\sqrt{3}\left(\sqrt{6}-\sqrt{2}\right)}{24-8}=\frac{2\sqrt{3}\left(\sqrt{6}-\sqrt{2}\right)}{16}$$

$$=\frac{\sqrt{18}-\sqrt{6}}{8}=\frac{3\sqrt{2}-\sqrt{6}}{8}$$

(iv) $\dfrac{\sin 30°+\tan 45°-\text{cosec}60°}{\sec 30°+\cos 60°+\cot 45°}$

$$=\frac{\frac{1}{2}+1-\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}+\frac{1}{2}+1}=\frac{\frac{3}{2}-\frac{2}{\sqrt{3}}}{\frac{3}{2}+\frac{2}{\sqrt{3}}}$$

$$=\frac{\frac{3\sqrt{3}-4}{2\sqrt{3}}}{\frac{3\sqrt{3}+4}{2\sqrt{3}}}=\frac{\left(3\sqrt{3}-4\right)}{\left(3\sqrt{3}+4\right)}$$

$$=\frac{\left(3\sqrt{3}-4\right)\left(3\sqrt{3}-4\right)}{\left(3\sqrt{3}+4\right)\left(3\sqrt{3}-4\right)}=\frac{\left(3\sqrt{3}-4\right)^2}{\left(3\sqrt{3}\right)^2-\left(4\right)^2}$$

$$=\frac{27+16-24\sqrt{3}}{27-16}=\frac{43-24\sqrt{3}}{11}$$

(v) $\dfrac{5\cos^2 60°+4\sec^2 30°-\tan^2 45°}{\sin^2 30°+\cos^2 30°}$

$$=\frac{5\left(\frac{1}{2}\right)^2+4\left(\frac{2}{\sqrt{3}}\right)^2-(1)^2}{\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$=\frac{5\left(\frac{1}{4}\right)+\left(\frac{16}{3}\right)-1}{\frac{1}{4}+\frac{3}{4}}$$

$$=\frac{\frac{15+64-12}{12}}{\frac{4}{4}}=\frac{67}{12}$$

Question 2:

Choose the correct option and justify your choice.

(i) $\frac{2\tan 30°}{1+\tan^2 30°} =$

(A). sin60°

(B). cos60°

(C). tan60°

(D). sin30°

(ii) $\frac{1-\tan^2 45°}{1+\tan^2 45°} =$

(A). tan90°

(B). 1

(C). sin45°

(D). 0

(iii) sin2A = 2sinA is true when A =

(A). 0°

(B). 30°

(C). 45°

(D). 60°

(iv) $\frac{2\tan 30°}{1-\tan^2 30°} =$

(A). cos60°

(B). sin60°

(C). tan60°

(D). sin30°

Answer:

(i) $\frac{2\tan 30°}{1+\tan^2 30°}$

$$=\frac{2\left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)^2}=\frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}}=\frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}}$$

$$=\frac{6}{4\sqrt{3}}=\frac{\sqrt{3}}{2}$$

Out of the given alternatives, only $\sin 60°=\frac{\sqrt{3}}{2}$

Hence, (A) is correct.

(ii) $\frac{1-\tan^2 45°}{1+\tan^2 45°}$

$$=\frac{1-(1)^2}{1+(1)^2}=\frac{1-1}{1+1}=\frac{0}{2}=0$$

Hence, (D) is correct.

(iii)Out of the given alternatives, only A = 0° is correct.

As sin 2A = sin 0° = 0

2 sinA = 2sin 0° = 2(0) = 0

Hence, (A) is correct.

(iv) $\frac{2\tan 30°}{1-\tan^2 30°}$

$$=\frac{2\left(\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{1}{\sqrt{3}}\right)^2}=\frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}}=\frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}}$$

$$=\sqrt{3}$$

Out of the given alternatives, only tan 60° $= \sqrt{3}$

Hence, (C) is correct.

Question 3:

If $\tan(A+B)=\sqrt{3}$ and $\tan(A-B)=\frac{1}{\sqrt{3}}$;

0° < A + B ≤ 90°, A > B find A and B.

Answer:

$\tan(A+B)=\sqrt{3}$

$\Rightarrow \tan(A+B)=\tan 60$

$\Rightarrow$ A + B = 60 … (1)

$\tan(A-B)=\frac{1}{\sqrt{3}}$

$\Rightarrow$ tan (A − B) = tan30

$\Rightarrow$ A − B = 30 … (2)

On adding both equations, we obtain

2A = 90

$\Rightarrow$ A = 45

From equation (1), we obtain

45 + B = 60

B = 15

Therefore, ∠A = 45° and ∠B = 15°

Question 4:

State whether the following are true or false. Justify your answer.

(i) sin (A + B) = sin A + sin B

(ii) The value of sinθ increases as θ increases

(iii) The value of cos θ increases as θ increases

(iv) sinθ = cos θ for all values of θ

(v) cot A is not defined for A = 0°

Answer:

(i) sin (A + B) = sin A + sin B

Let A = 30° and B = 60°

sin (A + B) = sin (30° + 60°)

= sin 90°

= 1

sin A + sin B = sin 30° + sin 60°

$$=\frac{1}{2}+\frac{\sqrt{3}}{2}=\frac{1+\sqrt{3}}{2}$$

Clearly, sin (A + B) ≠ sin A + sin B

Hence, the given statement is false.

(ii) The value of sin θ increases as θ increases in the interval of 0° < θ < 90° as

sin 0° = 0

$$\sin 30° = \frac{1}{2} = 0.5$$

$$\sin 45° = \frac{1}{\sqrt{2}} = 0.707$$

$$\sin 60° = \frac{\sqrt{3}}{2} = 0.866$$

sin 90° = 1

Hence, the given statement is true.

(iii) cos 0° = 1

$\cos 30° = \frac{\sqrt{3}}{2} = 0.866$

$\cos 45° = \frac{1}{\sqrt{2}} = 0.707$

$\cos 60° = \frac{1}{2} = 0.5$

cos90° = 0

It can be observed that the value of cos θ does not increase in the interval of 0° < θ < 90°.

Hence, the given statement is false.

(iv) sin θ = cos θ for all values of θ.

This is true when θ = 45°

As $\sin 45° = \frac{1}{\sqrt{2}}$

$\cos 45° = \frac{1}{\sqrt{2}}$

It is not true for all other values of θ.

As $\sin 30° = \frac{1}{2}$ and $\cos 30° = \frac{\sqrt{3}}{2}$,

Hence, the given statement is false.

(v) cot A is not defined for A = 0°

As $\cot A = \frac{\cos A}{\sin A}$,

$\cot 0° = \frac{\cos 0°}{\sin 0°} = \frac{1}{0}$ = undefined

Hence, the given statement is true.

Exercise 8.3

Question 1:

Evaluate

(I) $\frac{\sin 18°}{\cos 72°}$

(II) $\frac{\tan 26°}{\cot 64°}$

(III) cos 48° − sin 42°

(IV)cosec 31° − sec 59°

Answer:

(I) $\frac{\sin 18°}{\cos 72°} = \frac{\sin(90° - 72°)}{\cos 72°}$

$= \frac{\cos 72°}{\cos 72°} = 1$

(II) $\frac{\tan 26°}{\cot 64°} = \frac{\tan(90° - 64°)}{\cot 64°}$

$= \frac{\cot 64°}{\cot 64°} = 1$

(III)cos 48° − sin 42° = cos (90°− 42°) − sin 42°

= sin 42° − sin 42°

= 0

(IV) cosec 31° − sec 59° = cosec (90° − 59°) − sec 59°

= sec 59° − sec 59°

= 0

Question 2:

Show that

(I) tan 48° tan 23° tan 42° tan 67° = 1

(II)cos 38° cos 52° − sin 38° sin 52° = 0

Answer:

(I) tan 48° tan 23° tan 42° tan 67°

= tan (90° − 42°) tan (90° − 67°) tan 42° tan 67°

= cot 42° cot 67° tan 42° tan 67°

= (cot 42° tan 42°) (cot 67° tan 67°)

= (1) (1)

= 1

(II) cos 38° cos 52° − sin 38° sin 52°

= cos (90° − 52°) cos (90°−38°) − sin 38° sin 52°

= sin 52° sin 38° − sin 38° sin 52°

= 0

Question 3:

If tan 2A = cot (A− 18°), where 2A is an acute angle, find the value of A.

Answer:

Given that,

tan 2A = cot (A− 18°)

cot (90° − 2A) = cot (A −18°)

90° − 2A = A− 18°

108° = 3A

A = 36°

Question 4:

If tan A = cot B, prove that A + B = 90°

Answer:

Given that,

tan A = cot B

tan A = tan (90° − B)

A = 90° − B

A + B = 90°

Question 5:

If sec 4A = cosec (A− 20°), where 4A is an acute angle, find the value of A.

Answer:

Given that,

sec 4A = cosec (A − 20°)

cosec (90° − 4A) = cosec (A − 20°)

90° − 4A= A− 20°

110° = 5A

A = 22°

Question 6:

If A, Band C are interior angles of a triangle ABC then show that

$$\sin\left(\frac{B+C}{2}\right)=\cos\frac{A}{2}$$

Answer:

We know that for a triangle ABC,

∠ A + ∠B + ∠C = 180°

∠B + ∠C= 180° − ∠A

$$\frac{\angle B+\angle C}{2}=90°-\frac{\angle A}{2}$$

$$\sin\left(\frac{B+C}{2}\right)=\sin\left(90°-\frac{A}{2}\right)$$

$$=\cos\left(\frac{A}{2}\right)$$

Question 7:

Express sin 67° + cos 75° in terms of trigonometric ratios of angles between 0° and 45°.

Answer:

sin 67° + cos 75°

= $\sin(90° - 23°) + \cos(90° - 15°)$

= $\cos 23° + \sin 15°$

Exercise 8.4

Question 1:

Express the trigonometric ratios sin A, sec A and tan A in terms of cot A.

Answer:

We know that,

$$\text{cosec}^2 A = 1 + \cot^2 A$$

$$\frac{1}{\text{cosec}^2 A} = \frac{1}{1+\cot^2 A}$$

$$\sin^2 A = \frac{1}{1+\cot^2 A}$$

$$\sin A = \pm \frac{1}{\sqrt{1+\cot^2 A}}$$

$\sqrt{1+\cot^2 A}$ will always be positive as we are adding two positive quantities.

Therefore, $\sin A = \frac{1}{\sqrt{1+\cot^2 A}}$

We know that, $\tan A = \frac{\sin A}{\cos A}$

However, $\cot A = \frac{\cos A}{\sin A}$

Therefore, $\tan A = \frac{1}{\cot A}$

Also, $\sec^2 A = 1 + \tan^2 A$

$$= 1 + \frac{1}{\cot^2 A}$$

$$= \frac{\cot^2 A + 1}{\cot^2 A}$$

$$\sec A = \frac{\sqrt{\cot^2 A + 1}}{\cot A}$$

Question 2:

Write all the other trigonometric ratios of ∠A in terms of sec A.

Answer:

We know that,

$$\cos A = \frac{1}{\sec A}$$

Also, $\sin^2 A + \cos^2 A = 1$

$\sin^2 A = 1 - \cos^2 A$

$$\sin A = \sqrt{1 - \left(\frac{1}{\sec A}\right)^2}$$

$$= \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}} = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

$\tan^2 A + 1 = \sec^2 A$

$\tan^2 A = \sec^2 A - 1$

$$\tan A = \sqrt{\sec^2 A - 1}$$

$$\cot A = \frac{\cos A}{\sin A} = \frac{\frac{1}{\sec A}}{\frac{\sqrt{\sec^2 A - 1}}{\sec A}}$$

$$= \frac{1}{\sqrt{\sec^2 A - 1}}$$

$$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

Question 3:

Evaluate

(i) $\dfrac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$

(ii) sin25° cos65° + cos25° sin65°

Answer:

(i) $\frac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°}$

$= \frac{[\sin(90° - 27°)]^2 + \sin^2 27°}{[\cos(90° - 73°)]^2 + \cos^2 73°}$

$= \frac{[\cos 27°]^2 + \sin^2 27°}{[\sin 73°]^2 + \cos^2 73°}$

$= \frac{\cos^2 27° + \sin^2 27°}{\sin^2 73° + \cos^2 73°}$

$= \frac{1}{1}$ (As $\sin^2 A + \cos^2 A = 1$)

= 1

(ii) sin25° cos65° + cos25° sin65°

$= (\sin 25°)\{\cos(90° - 25°)\} + \cos 25°\{\sin(90° - 25°)\}$

$= (\sin 25°)(\sin 25°) + (\cos 25°)(\cos 25°)$

$= \sin^2 25° + \cos^2 25°$

= 1 (As $\sin^2 A + \cos^2 A = 1$)

Question 4:

Choose the correct option. Justify your choice.

(i) $9 \sec^2 A - 9 \tan^2 A =$

(A) 1

(B) 9

(C) 8

(D) 0

(ii) (1 + tan θ + sec θ) (1 + cot θ − cosec θ)

(A) 0

(B) 1

(C) 2

(D) −1

(iii) (secA + tanA) (1 − sinA) =

(A) secA

(B) sinA

(C) cosecA

(D) cosA

(iv) $\frac{1+\tan^2 A}{1+\cot^2 A}$

(A) $\sec^2 A$

(B) −1

(C) $\cot^2 A$

(D) $\tan^2 A$

Answer:

(i) $9\sec^2 A - 9\tan^2 A$

$= 9(\sec^2 A - \tan^2 A)$

$= 9(1)$ [As $\sec^2 A - \tan^2 A = 1$]

= 9

Hence, alternative (B) is correct.

(ii)

$(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \text{cosec}\,\theta)$

$$= \left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)\left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)$$

$$= \left(\frac{\cos\theta+\sin\theta+1}{\cos\theta}\right)\left(\frac{\sin\theta+\cos\ \theta-1}{\sin\theta}\right)$$

$$= \frac{(\sin\theta+\cos\ \theta)^2-(1)^2}{\sin\theta\cos\theta}$$

$$= \frac{\sin^2\theta+\cos^2\theta+2\sin\theta\ \cos\ \theta-1}{\sin\theta\cos\ \theta}$$

$$= \frac{1+2\sin\theta\cos\ \theta-1}{\sin\theta\cos\ \theta}$$

$$= \frac{2\sin\theta\ \cos\ \theta}{\sin\ \theta\cos\ \theta} = 2$$

Hence, alternative (C) is correct.

(iii) (secA + tanA) (1 − sinA)

$$= \left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)(1-\sin A)$$

$$= \left(\frac{1+\sin A}{\cos A}\right)(1-\sin A)$$

$$= \frac{1-\sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A}$$

= cosA

Hence, alternative (D) is correct.

(iv) $$\frac{1+\tan^2 A}{1+\cot^2 A} = \frac{1+\frac{\sin^2 A}{\cos^2 A}}{1+\frac{\cos^2 A}{\sin^2 A}}$$

$$=\frac{\frac{\cos^2 A+\sin^2 A}{\cos^2 A}}{\frac{\sin^2 A+\cos^2 A}{\sin^2 A}}=\frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}$$

$$=\frac{\sin^2 A}{\cos^2 A}=\tan^2 A$$

Hence, alternative (D) is correct.

Question 5:

Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

Answer:

(i) $(\operatorname{cosec}\theta-\cot\theta)^2=\frac{1-\cos\theta}{1+\cos\theta}$

$$\text{L.H.S.}=(\operatorname{cosec}\theta-\cot\theta)^2$$

$$=\left(\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}\right)^2$$

$$=\frac{(1-\cos\theta)^2}{(\sin\theta)^2}=\frac{(1-\cos\theta)^2}{\sin^2\theta}$$

$$=\frac{(1-\cos\theta)^2}{1-\cos^2\theta}=\frac{(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)}=\frac{1-\cos\theta}{1+\cos\theta}$$

$=$R.H.S.

(ii) $\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}=2\sec A$

$$\text{L.H.S.} = \frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$$

$$= \frac{\cos^2 A + (1+\sin A)^2}{(1+\sin A)(\cos A)}$$

$$= \frac{\cos^2 A + 1 + \sin^2 A + 2\sin A}{(1+\sin A)(\cos A)}$$

$$= \frac{\sin^2 A + \cos^2 A + 1 + 2\sin A}{(1+\sin A)(\cos A)}$$

$$= \frac{1+1+2\sin A}{(1+\sin A)(\cos A)} = \frac{2+2\sin A}{(1+\sin A)(\cos A)}$$

$$= \frac{2(1+\sin A)}{(1+\sin A)(\cos A)} = \frac{2}{\cos A} = 2\sec A$$

$$= \text{R.H.S.}$$

(iii) $\dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\,\text{cosec}\theta$

$$\text{L.H.S.} = \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} + \frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$$

$$= \frac{1}{(\sin\theta-\cos\theta)}\left[\frac{\sin^2\theta}{\cos\theta} - \frac{\cos^2\theta}{\sin\theta}\right]$$

$$= \left(\frac{1}{\sin\theta-\cos\theta}\right)\left[\frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta}\right]$$

$$= \left(\frac{1}{\sin\theta-\cos\theta}\right)\left[\frac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{\sin\theta\cos\theta}\right]$$

$$= \frac{(1+\sin\theta\ \cos\theta)}{(\sin\theta\ \cos\theta)}$$

= secθ cosec θ +

= R.H.S.

(iv) $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$

$$\text{L.H.S.} = \frac{1+\sec A}{\sec A} = \frac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}}$$

$$= \frac{\frac{\cos A + 1}{\cos A}}{\frac{1}{\cos A}} = (\cos A + 1)$$

$$= \frac{(1-\cos A)(1+\cos A)}{(1-\cos A)}$$

$$= \frac{1-\cos^2 A}{1-\cos A} = \frac{\sin^2 A}{1-\cos A}$$

= R.H.S

(v) $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \text{cosec}A + \cot A$

Using the identity $\text{cosec}^2 A = 1 + \cot^2 A$,

L.H.S = $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$

$$= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} + \frac{1}{\sin A}}$$

$$= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$$

$$= \frac{\{(\cot A) - (1 - \operatorname{cosec} A)\}\{(\cot A) - (1 - \operatorname{cosec} A)\}}{\{(\cot A) + (1 - \operatorname{cosec} A)\}\{(\cot A) - (1 - \operatorname{cosec} A)\}}$$

$$= \frac{(\cot A - 1 + \operatorname{cosec} A)^2}{(\cot A)^2 - (1 - \operatorname{cosec} A)^2}$$

$$= \frac{\cot^2 A + 1 + \operatorname{cosec}^2 A - 2\cot A - 2\operatorname{cosec} A + 2\cot A \operatorname{cosec} A}{\cot^2 A - (1 + \operatorname{cosec}^2 A - 2\operatorname{cosec} A)}$$

$$= \frac{2\operatorname{cosec}^2 A + 2\cot A \operatorname{cosec} A - 2\cot A - 2\operatorname{cosec} A}{\cot^2 A - 1 - \operatorname{cosec}^2 A + 2\operatorname{cosec} A}$$

$$= \frac{2\operatorname{cosec} A(\operatorname{cosec} A + \cot A) - 2(\cot A + \operatorname{cosec} A)}{\cot^2 A - \operatorname{cosec}^2 A - 1 + 2\operatorname{cosec} A}$$

$$= \frac{(\operatorname{cosec} A + \cot A)(2\operatorname{cosec} A - 2)}{-1 - 1 + 2\operatorname{cosec} A}$$

$$= \frac{(\operatorname{cosec} A + \cot A)(2\operatorname{cosec} A - 2)}{(2\operatorname{cosec} A - 2)}$$

= cosec A + cot A

= R.H.S

(vi) $\sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$

$$\text{L.H.S.} = \sqrt{\frac{1+\sin A}{1-\sin A}}$$

$$= \sqrt{\frac{(1+\sin A)(1+\sin A)}{(1-\sin A)(1+\sin A)}}$$

$$= \frac{(1+\sin A)}{\sqrt{1-\sin^2 A}} \qquad = \frac{1+\sin A}{\sqrt{\cos^2 A}}$$

$$= \frac{1+\sin A}{\cos A} \qquad = \sec A + \tan A$$

$$= \text{R.H.S.}$$

(vii) $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

$$\text{L.H.S.} = \frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$$

$$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$$

$$= \frac{\sin\theta \times (1-2\sin^2\theta)}{\cos\theta \times \{2(1-\sin^2\theta) - 1\}}$$

$$= \frac{\sin\theta \times (1-2\sin^2\theta)}{\cos\theta \times (1-2\sin^2\theta)}$$

$$= \tan\theta = \text{R.H.S}$$

(viii) $(\sin A + \text{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

$$\text{L.H.S} = (\sin A + \text{cosec}A)^2 + (\cos A + \sec A)^2$$
$$= \sin^2 A + \text{cosec}^2 A + 2\sin A\,\text{cosec}A + \cos^2 A + \sec^2 A + 2\cos A \sec A$$
$$= (\sin^2 A + \cos^2 A) + (\text{cosec}^2 A + \sec^2 A) + 2\sin A\left(\frac{1}{\sin A}\right) + 2\cos A\left(\frac{1}{\cos A}\right)$$
$$= (1) + (1 + \cot^2 A + 1 + \tan^2 A) + (2) + (2)$$
$$= 7 + \tan^2 A + \cot^2 A$$
$$= \text{R.H.S}$$

(ix) $(\text{cosec}A - \sin A)(\sec A - \cos A) = \dfrac{1}{\tan A + \cot A}$

$$\text{L.H.S} = (\text{cosec}A - \sin A)(\sec A - \cos A)$$
$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$$
$$= \left(\frac{1 - \sin^2 A}{\sin A}\right)\left(\frac{1 - \cos^2 A}{\cos A}\right)$$
$$= \frac{(\cos^2 A)(\sin^2 A)}{\sin A \cos A}$$
$$= \sin A \cos A$$

$$\text{R.H.S} = \frac{1}{\tan A + \cot A}$$
$$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}} = \frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}}$$
$$= \frac{\sin A \cos A}{\sin^2 A + \cos^2 A} = \sin A \cos A$$

Hence, L.H.S = R.H.S

(x) $\left(\dfrac{1 + \tan^2 A}{1 + \cot^2 A}\right) = \left(\dfrac{1 - \tan A}{1 - \cot A}\right)^2 = \tan^2 A$

$$\frac{1+\tan^2 A}{1+\cot^2 A}=\frac{1+\frac{\sin^2 A}{\cos^2 A}}{1+\frac{\cos^2 A}{\sin^2 A}}=\frac{\frac{\cos^2 A+\sin^2 A}{\cos^2 A}}{\frac{\sin^2 A+\cos^2 A}{\sin^2 A}}$$

$$=\frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}=\frac{\sin^2 A}{\cos^2 A}$$

$$=\tan^2 A$$

$$\left(\frac{1-\tan A}{1-\cot A}\right)^2=\frac{1+\tan^2 A\ -\ 2\tan A}{1+\cot^2 A-2\cot A}$$

$$=\frac{\sec^2 A-\ 2\tan A}{\operatorname{cosec}^2 A-2\cot A}$$

$$=\frac{\frac{1}{\cos^2 A}-\frac{2\sin A}{\cos A}}{\frac{1}{\sin^2 A}-\frac{2\cos A}{\sin A}}=\frac{\frac{1-2\sin A\cos A}{\cos^2 A}}{\frac{1-2\sin A\cos A}{\sin^2 A}}$$

$$=\frac{\sin^2 A}{\cos^2 A}=\tan^2 A$$