Assignments in Mathematics Class X (Term II)

5. ARITHMETIC PROGRESSIONS

IMPORTANT TERMS, DEFINITIONS AND RESULTS

- Some numbers arranged in a definite order, according to a definite rule, are said to form a **sequence.**
- A sequence is called an **arithmetic progression** (AP), if the difference of any of its terms and the preceding term is always the same.
  i.e., $t_{n+1} - t_n$ = constant.
- The constant number is called the **common difference of the A.P.**
- If a is the first term and d the common difference of an AP, then the general form of the AP is $a, a + d, a + 2d, ...$
- Let a be the first term and d be the common difference of an AP, then, its nth term or general is given by

$$t_n = a + (n - 1)\, d$$

- If l is the last term of the AP, then nth term from the end is the nth term of an AP, whose first term is l and common difference is $-d$.
  $\therefore$ nth term from the end = Last term $+ (n - 1)(-d)$
  $\Rightarrow$ nth term from the end $= l - (n - 1)\, d$
- If $a, b, c,$ are in AP, then
  (i) $(a + k), (b + k), (c + k)$ are in AP.
  (ii) $(a - k), (b - k), (c - k)$ are in AP.
  (iii) ak, bk, ck, are in AP.
  (iv) $\frac{a}{k}, \frac{b}{k}, \frac{c}{k}$ are in AP $(k \neq 0)$
- Remember the following while working with consecutive terms in an AP.
  (i) *Three consecutive terms in an AP.*
  $a - d, a, a + d$
  First term $= a - d$, common difference $= d$
  Their sum $= a - d + a + a + d = 3a$
  (ii) *Four consecutive terms in an AP.*
  $a - 3d, a - d, a + d, a + 3d$
  First term : $a - 3d$, common difference $= 2d$
  Their sum $= a - 3d + a - d + a + d + a + 3d = 4a$
  (iii) *Five consecutive terms in an AP.*
  $a - 2d, a - d, a, a + d, a + 2d$
  First term $= a - 2d$, common difference $= d$
- The sum S_n up to n terms of an AP whose first term is a and common difference d is given by

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

- If the first term and the last term of an AP are t_1 and t_n, then

$$S_n = \frac{n}{2}(t_1 + t_n) = \frac{n}{2}\,(\text{first term} + \text{last term})$$

  If $t_1 = a$, the first term and $t_n = l$, the last term, then $S_n = \frac{n}{2}(a + l)$
- $S_n - S_{n-1} = t_n$

SUMMATIVE ASSESSMENT

MULTIPLE CHOICE QUESTIONS [1 Mark]

A. Important Questions

1. The common difference of the A.P. 3, 1, –1, –3 ... is :

(a) –2 (b) 2 (c) –1 (d) 3

2. The general form of an A.P. is :

(a) $a, a - d, a - 2d, a - 3d, ...$

(b) $a, a + d, a + 2d, a + 3d, ...$

(c) $a, 2d, 3d, 4d, ...$ (d) none of these

3. The common difference of the A.P. 8, 11, 14, 17, 20, ... is :

(a) 2 (b) –2 (c) 3 (d) –3

4. An A.P. whose first term is 10 and common difference is 3, is :

(a) 10, 13, 16, 19, ... (b) 5, 7, 9, 11, ...
(c) 8, 12, 16, 20, ... (d) all of these

5. The common difference of the A.P. –5, –1, 3, 7, ... is:

(a) 2 (b) 3 (c) –4 (d) 4

6. Which of the following list of numbers does form an A.P.?

(a) 2, 4, 8, 16, ... (b) $2, \frac{5}{2}, 3\frac{7}{2}$ .....

(c) 0.22, 0.22, 0.222, 0.2222, ...

(d) 1, 3, 9, 27, ...

7. The 10th term of the A.P. 5, 8, 11, 14, ... is :

(a) 32 (b) 35 (c) 38 (d) 185

8. The list of numbers – 10, – 6, – 2, 2 .... is :

(a) an A.P. with $a = -16$

(b) an A.P. with $a = -4$

(c) an A.P. with $a = 4$ (d) not an A.P.

9. In an A.P., if $a = -7.2$, $d = 3.6$ and $t_n = 7.2$, then n is :

(a) 1 (b) 3 (c) 4 (d) 5

10. In an A.P., if $t_n = 4$, $d = -4$, and $n = 7$, then a is :

(a) 6 (b) 7 (c) 20 (d) 28

11. In an A.P., if $a = 8.5$, $d = 0$, $n = 57$, then t_n will be :

(a) 0 (b) 8.5 (c) 103.5 (d) 150

12. The nth term of the A.P. 2, 5, 8, ... is :

(a) $3n - 1$ (b) $2n - 1$

(c) $3n - 2$ (d) $2n - 3$

13. If first term of an A.P. is 2 and common difference is – 2, then t_7 is :

(a) – 8 (b) – 10 (c) – 5 (d) 10

14. The 11th term of the A.P. $-5, -\frac{5}{2}, 0, \frac{5}{2}, ...$ is :

(a) – 20 (b) 20 (c) – 30 (d) 30

15. Which term of the A.P. 4, 9, 14, 19, ... is 109?

(a) 14th (b) 18th (c) 22nd (d) 16th

16. How many terms are there in the A.P. 1, 3, 5, ... 73, 75?

(a) 28 (b) 30 (c) 36 (d) 38

17. If the numbers a, b, c are in A.P., then :

(a) $b - a = c - b$ (b) $b + a = c + b$

(c) $a - b = b - c$ (d) none of these

18. How many terms are there in the A.P. 7, 10, 13, ..., 151?

(a) 50 (b) 55 (c) 45 (d) 49

19. Which term of the A.P. 72, 63, 54, ... is 0?

(a) 8th (b) 9th (c) 10th (d) 11th

20. The famous mathematician associated with finding the sum of first 100 natural numbers is :

(a) Pythagoras (b) Newton

(c) Gauss (d) Euclid

21. The sum of first 16 terms of the A.P. 10, 6, 2, ... is :

(a) – 320 (b) 320 (c) – 352 (d) – 400

22. The sum of first 5 multiples of 3 is :

(a) 45 (b) 55 (c) 65 (d) 75

23. The first term of an A.P. is – 5 and the common difference is 2. The sum of first 6 terms of this A.P. is:

(a) 0 (b) 5 (c) 6 (d) 15

24. In an A.P., if $a = 1$, $t_n = 20$ and $S_n = 399$, then n is :

(a) 19 (b) 21 (c) 38 (d) 42

25. The sum of first n natural numbers is :

(a) n^2 (b) $n(n + 1)$

(c) $\frac{n(n+1)}{2}$ (d) $\frac{n(n-1)}{2}$

26. If 4th term of an A.P. is 14 and its 12th term is 70, what is its first term?

(a) – 10 (b) – 7 (c) 7 (d) 10

27. The first term of an A.P. is 6 and its common difference is 5. What will be its 11th term?

(a) 56 (b) 41 (c) 46 (d) 50

28. The first four terms of an A.P. whose first term is – 2 and the common difference is – 2, are :

(a) – 2, 0, 2, 4 (b) – 2, 4, – 8, – 16

(c) – 2, – 4, – 6, – 8 (d) – 2, – 4, – 8, – 16

29. If the common difference of an A.P. is 5, then what is $t_{18} - t_{13}$?

(a) 5 (b) 20 (c) 25 (d) 30

30. The first two terms of an A.P. are – 3 and 4. The 21st term of the A.P. will be :

(a) 17 (b) 137 (c) 143 (d) – 143

31. Two A.P.s have the same common difference. The first term of one of these is – 2 and that of the other is – 10. The difference between their 10th terms is :

(a) 8 (b) – 8 (c) 4 (d) – 4

32. If 2nd term of an A.P. is 13 and the 5th term is 25, then its 7th term is :

(a) 30 (b) 33 (c) 37 (d) 38

33. How many numbers of 2-digits are divisible by 3?

(a) 10 (b) 20 (c) 30 (d) 40

34. If 11 times the 11th term of an A.P. is equal to 7 times its 7th term, then its 18th term will be :

(a) 7 (b) 11 (c) 18 (d) 0

35. What is the sum of all natural numbers from 1 to 100?

(a) 4550 (b) 4780 (c) 5050 (d) 5150

36. 51 + 52 + 53 + ... + 100 = ?

(a) 3775 (b) 4025 (c) 4275 (d) 5050

37. If the sum of p terms of an A.P. is q and the sum of q terms is p, then the sum of $(p + q)$ terms will be:

(a) 0 (b) $p - q$

(c) $p + q$ (d) $-(p + q)$

38. If the sum of n terms of an A.P. be $3n^2 - n$ and its common difference is 6, then its first term is:

(a) 2 (b) 3 (c) 1 (d) 4

39. If the sum of n terms of an A.P. be $3n^2 + 5n$, then which of its terms is 164?

(a) 26th (b) 27th

(c) 28th (d) none of these

40. If the sum of n terms of an A.P. is $2n^2 + 5n$, then its nth term is :

(a) $4n - 3$ (b) $3n - 4$ (c) $4n + 3$ (d) $3n + 4$

41. If the first term of an A.P. is 2 and common difference is 4, then the sum of its 40 terms is:

(a) 3200 (b) 1600 (c) 200 (d) 2800

42. $25 + 28 + 31 + ... + 100 = ?$

(a) 1625 (b) 1525 (c) 1725 (d)1650

43. The sum of first n positive integers is given by:

(a) $\frac{n(n-1)}{2}$ (b) $\frac{n(2n+1)}{2}$

(c) $\frac{n(n+1)}{2}$ (d) none of these

44. The sum of first n even integers is given by :

(a) $n(n + 1)$ (b) $\frac{n(n+1)}{2}$

(c) $(n + 1)(n - 1)$ (d) none of these

B. Questions From CBSE Examination Papers

1. Which term of the A.P. 24, 21, 18, ... is the first negative term? **[2011 (T-II)]**

(a) 8th (b) 9th (c) 10th (d) 12th

2. Which term of the A.P. 2, –1, –4, ... is –70? **[2011 (T-II)]**

(a) 15th (b) 18th (c) 25th (d) 30th

3. The 4th term from the end of A.P. –11, –8, –5, ... 49 is : **[2011 (T-II)]**

(a) 37 (b) 40 (c) 43 (d) 58

4. Which term of the A.P. 21, 42,, 63 .... is 210? **[2011 (T-II)]**

(a) 9th (b) 10th (c) 12th (d) 11th

5. What is the common difference of the A.P. in which $a_{18} - a_{14} = 32$? **[2011 (T-II)]**

(a) 8 (b) –8 (c) 4 (d) –4

6. The next term of the A.P. $\sqrt{27}, \sqrt{48}, \sqrt{75}$, ... is : **[2011 (T-II)]**

(a) $\sqrt{105}$ (b) $\sqrt{107}$ (c) $\sqrt{108}$ (d) $\sqrt{147}$

7. Which term of the A.P. 100, 90, 80....... is zero? **[2011 (T-II)]**

(a) 5th (b) 6th (c) 10th (d) 11th

8. Which of the following is not an A.P. **[2011 (T-II)]**

(a) 13, 8, 3, –2, –7, –12

(b) 10.8, 11.2, 11.6, 12, 12.4

(c) $8\frac{1}{7}, 18\frac{2}{7}, 28\frac{3}{7}, 48\frac{4}{7}, 58\frac{5}{7}$

(d) $8\frac{3}{23}, 11\frac{6}{23}, 14\frac{9}{23}, 17\frac{12}{23}$

9. Which term of the A.P. 92, 88, 84, 80, ... is 0? **[2011 (T-II)]**

(a) 23 (b) 32 (c) 22 (d) 24

10. For an A.P. if $a_{25} - a_{20} = 45$, then d equals to : **[2011 (T-II)]**

(a) 9 (b) –9 (c) 18 (d) 23

11. If the nth term of an A.P. is $\frac{3+n}{4}$, then its 8th term is : **[2011 (T-II)]**

(a) 11 (b) $\frac{11}{4}$ (c) $\frac{11}{2}$ (d) 22

12. If a, $a - 2$ and $3a$ are in A.P., then the value of a is : **[2011 (T-II)]**

(a) –3 (b) –2 (c) 3 (d) 2

13. If an A.P. has $a = 1$, $t_n = 20$ and $S_n = 399$, then value of n is : **[2011 (T-II)]**

(a) 20 (b) 32 (c) 38 (d) 40

14. If $a + 1$, $2a + 1$, $4a - 1$ are in A.P., then the value of a is : **[2011 (T-II)]**

(a) 1 (b) 2 (c) 3 (d) 4

15. The sum of the first n terms of an A.P. is $2n^2 + 5n$. Then its nth term is : **[2011 (T-II)]**

(a) $4n + 3$ (b) $4n - 3$

(c) $3n - 4$ (d) $3n + 4$

16. For what value of p are $2p + 1$, 13, $5p - 3$, three consecutive terms of an A.P.? **[2011 (T-II)]**

(a) 2 (b) –2 (c) 4 (d) –4

17. Which term of the A.P., 113, 108, 103, ... is the first negative term? **[2011 (T-II)]**

(a) 22nd term (b) 24th term

(c) 26th term (d) 28th term

18. 15th term of the A.P., $x - 7, x - 2, x + 3$,....is : **[2011 (T-II)]**

(a) $x + 63$ (b) $x + 73$ (c) $x + 83$ (d) $x + 53$

19. If $p - 1, p + 3, 3p - 1$ are in A.P., then p is equal to: **[2011 (T-II)]**

(a) 4 (b) – 4 (c) 2 (d) – 2

SHORT ANSWER TYPE QUESTIONS [2 Marks

A. Important Questions

1. For the A.P. – 3, – 7, – 11, ... can we find directly $t_{30} - t_{20}$ without actually finding t_{30} and t_{20}? Give reasons for your answer.

2. Is 68 a term of the A.P. 7, 10, 13, ...?

3. The 4th term of an A.P. is three times the first term and the 7th term exceeds twice the third term by 1. Find the first term and the common difference.

4. Shonal deposited Rs 1000 at compound interest at the rate of 10% p.a. The amounts at the end of first year, second year, third year .... form an A.P. Justify your answer. **[HOTS]**

5. The nth term of an A.P. cannot be $m^2 + 1$. Justify your answer.

6. Write the expression $t_n - t_m$ for the A.P. $a, a + d, a + 2d$, ... Hence, find the common difference of the A.P. for which 11th term is 5 and 13th term is 79.

7. Show that $x - y$, x and $x + y$ form consecutive terms of an A.P.

8. Justify whether it is true to say that $2n - 3$ is the nth term of an A.P.

9. Which term of the A.P. 32, 29, 26, ... is first negative term?

10. Is 0 a term of the A.P. 31, 28, 25, ... ? Justify your answer.

11. If the numbers of $x - 2$, $4x - 1$, and $5x + 2$ are in A.P find the value of x.

12. Verify that each of the following is an A.P. :

(a) $x + y, (x + 1) + y, (x + 1) + (y + 1)$, ...

(b) $x, 2x + 1, 3x + 2, 4x + 3$, ...

13. Find a, b and c, if the numbers a, 7, b, 23, c are in A.P..

14. Find the number of terms of the A.P. 17, 14.5, 12, ... – 38.

15. The sum of n terms of a sequence is $3n^2 + 4n$. Find the nth term and show that it is an A.P..

16. The sum of first n terms of an A.P. is given by $3n^2 - n$. Determine the A.P. and its 25th term.

17. How many terms of the A.P. $-6 - \frac{11}{2}, -5...$ are needed to give the sum – 25?

18. Find the sum of first n odd natural numbers.

19. Show that the sum of all odd integers between 1 and 100 which are divisible by 3 is 83667. **[HOTS]**

20. If $t_n = 3 - 4n$, show that t_1, t_2, t_3, ... form an A.P. Also, find S_{20}. **[HOTS]**

B. Questions From CBSE Examination Papers

1. Which term of the A.P., 6, 13, 20, 27, ... is 98 more than its 24th term ? **[2011 (T-II)]**

2. Calculate how many multiples of 7 are there between 100 and 300. **[2011 (T-II)]**

3. If S_n denotes the sum of n terms of an AP whose common difference is d and first term is a, find $S_n - 2S_{n-1} + S_{n-2}$. **[2011 (T-II)]**

4. Find the sum of all two digit positive numbers divisible by 3. **[2011 (T-II)]**

5. Determine the 2nd term of an A.P. whose 6th term is 12 and 8th term is 22. **[2011 (T-II)]**

6. Find the sum of first 10 terms of the sequence $\{a_n\}$ where $a_n = 5 - 6n$, where n is a natural number. **[2011 (T-II)]**

7. Find the sum of the first 50 odd natural numbers. **[2011 (T-II)]**

8. If the nth term of an A.P. is $(2n + 1)$, find the sum of first n terms of the A.P. **[2011 (T-II)]**

9. The 8th term of an A.P. is 37 and its 12th term is 57. Find the A.P. **[2011 (T-II)]**

10. Which term of the A.P. 3, 15, 27, 39,............. is 132 more than its 54th term? **[2011 (T-II)]**

11. In an A.P. the first term is – 4, the last term is 29 and the sum of all its terms is 150.Find the common difference of the A.P. **[2011 (T-II)]**

12. For an A.P. show that $a_p + a_{p+2} = 2a_{p+q}$ **[2011 (T-II)]**

13. Find the common difference of an A.P. whose first term is $\frac{1}{2}$ and the 8th term is $\frac{17}{6}$. Also write its 4th term. **[2011 (T-II)]**

14. Find the sum of first twelve multiples of 7. **[2011 (T-II)]**

15. Find the sum of the 25 terms of an A.P. whose nth term is given by $t_n = 7 - 3n$. **[2011 (T-II)]**

16. How many terms are there in A.P.

7, 16, 25, ..., 349? **[2011 (T-II)]**

17. Find the number of terms of the series:

$-5 + (-8) + (-11) + \ldots\ldots\ldots\ldots + (-230)$ **[2011 (T-II)]**

18. Which term of the arithmetic progression 3, 10, 17.......... will be 84 more than its 13th term ? **[2011 (T-II)]**

19. Find the number of all 2 digit numbers divisible by 3. **[2011 (T-II)]**

20. Find the value of p, if the numbers x, $2x + p$, $3x + 6$, are three consecutive terms of an A.P. **[2011 (T-II)]**

21. If 6th term of an A.P. is –10 and its 10th term is –26, then find the 15th term of the A.P. **[2011 (T-II)]**

22. If 8th term of an A.P. is 31 and 15th term is 16 more than 11th term, find the A.P. **[2011 (T-II)]**

23. If the sum of first n terms of an A.P. is given by $S_n = 4n^2 - 3n$, find the nth term of the A.P. **[2004C]**

24. If the sum of first n terms of an A.P. is given by $S_n = 2n^2 + 5n$, find the nth term of the A.P. **[2004C]**

25. Find the number of terms of the A.P. 54, 51, 48, ... so that their sum is 513. **[2005]**

26. Find the sum of the first 51 terms of the A.P. whose 2nd term is 2 and 4th term is 8. **[2005C]**

27. The sum of the first n terms of an A.P. is given by $S_n = 3n^2 - n$. Determine the A.P. and its 25th term. **[2005C]**

28. The sum of three numbers in A.P. is 27 and their product is 405. Find the numbers. **[2005C]**

29. How many terms are there in an A.P. whose first term and 6th term are – 12 and 8 respectively and sum of all its terms is 120? **[2006]**

30. In an A.P. the sum of first n terms is $\frac{5n^2}{2} + \frac{3n}{2}$. Find its 20th term. **[2006C]**

31. The first term, common difference and last term of an A.P. are 12, 6 and 252 respectively. Find the sum of all terms of this A.P. **[2007]**

SHORT ANSWER TYPE QUESTIONS [3 Marks]

A. Important Questions

1. Determine the A.P. whose 5th term is 19 and the difference of the eight term from the thirteeth term is 20.

2. The sum of first three terms of an A.P. is 21 and their product is 231. Find the numbers. **[Imp.]**

3. What are the middle most terms of the A.P. – 11, – 7, – 3, ... 49?

4. The 26th, 11th and the last term of an A.P. are 0, 3 and $-\frac{1}{5}$ respectively. Find the common difference and the number of terms.

5. If the 9th term of an A.P. is zero, prove that its 29th term is twice its 19th term.

6. The sum of first 6 terms of an A.P. is 42. The ratio of the 10th term to the 30th term is 1 : 3. Find the first term and the 11th term of the A.P.

7. If the nth terms of two A.Ps. 9, 7, 5, ... and 24, 21, 18, ... are same, find the value of n. Also, find that term.

8. How many numbers lie between 10 and 30 which when divided by 4 leave a remainder 3?

9. If $(m + 1)$th term of an A.P. is twice the $(n + 1)$ th term, prove that $(3m + 1)$th term is twice the $(m + n + 1)$th term.

10. The sum of first three terms of an A.P. is 33. If the product of the first and the third terms exceeds the second term by 29, find the A.P.

11. Find the sum $= \frac{a+b}{a-b} + \frac{3a-2b}{a+b} + \frac{5a-3b}{a+b} + \ldots$ to 11 terms.

12. If S_n denotes the sum of first n terms of an A.P., prove that $S_{12} = 3(S_8 - S_4)$.

13. If the numbers a, b, c, d, e, form an A.P., then find the value of $a - 4b + 6c - 4d + e$.

14. In an A.P. if the first term is 22, the common difference is – 4 and the sum of n terms is 64, find n.

15. Split 207 into three parts such that these are in A.P. and the product of the two smaller parts is 4623.

16. How many terms of the A.P. – 15, – 13, – 11, ... are needed to make the sum – 55? Explain the reason for the double answer. **[HOTS]**

17. The sum of first n terms of an A.P. whose first term is 8 the common difference is 20 is equal to the sum of first $2n$ terms of another A.P. whoe first term is – 30 and the common difference is 8. Find n.

18. If a, b, c are in A.P., show that $(a - c)^2 = 4(b^2 - ac)$.

19. Reshma saves Rs 32 during the first month, Rs 36 in the second month and Rs 40 in the third month. If she continues to save in this manner, in how many months will she save Rs 2000? **[HOTS]**

20. A manufacturer of radio sets produced 800 units in the third year and 700 units in the seventh year. Assuming that the product increases uniformly by a fixed number every year, find

(i) the production in the first year

(ii) the production in the 10th year.

21. The p^{th} term of an A.P is q and q^{th} term is p. Find its $(p + q)$th term.

22. If a, b, c are in A.P., show that

(i) $\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are in A.P.

(ii) $a^2(b + c)$, $b^2(c + a)$, $c^2(a + b)$ are in A.P.

23. If a^2, b^2, c^2 are in A.P., then prove that $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P. **[HOTS]**

24. If the sum of m terms of an A.P. is the same as the sum of n terms, show that the sum of its $(m + n)$ terms is 0. **[HOTS]**

B. Questions From CBSE Examination Papers

1. Which term of the A.P. : 3, 15, 27, 39, ... will be 120 more then its 21st term ? **[2011 (T-II)]**

2. The sum of the first n terms of an A.P. is $5n^2 - 3n$. Find the A.P. and hence find its 12th term. **[2011 (T-II)]**

3. The angles of a triangle are in A.P. The greatest angle is twice the least. Find all angles of the triangle. **[2011 (T-II)]**

4. Find the sum of all natural numbers between 200 and 1000 exactly divisible by 6. **[2011 (T-II)]**

5. Find the value of the middle most term(s) of the arithmetic progression : **[2011 (T-II)]**

$-11, -7, -3, \ldots\ldots\ldots 49.$

6. The sum of first six terms of an arithmetic progression is 42. The ratio of its 10th term to its 30th term is 1 : 3. Find the first and the thirteenth term of the A.P. **[2011 (T-II)]**

7. How many terms of the A.P. 9, 17, 25, ..., must be taken to get a sum of 450 ? **[2011 (T-II)]**

8. Find the sum of all two digit odd positive numbers. **[2011 (T-II)]**

9. If the sum of first fourteen terms, of an AP is 1050 and its first term is 10, find its 20th term. **[2011 (T-II)]**

10. If the sum of first n terms of an A.P. is $4n^2 - n$, find the 12th term. **[2011 (T-II)]**

11. The sum of first three terms of an A.P. is 33. If the product of the first and third term exceeds the second term by 29, find the A.P. **[2011 (T-II)]**

12. Which term is the first negative term in the given A.P. : $23, 21\frac{1}{2}, 20, \ldots\ldots ?$ **[2011 (T-II)]**

13. In an A.P. the first term is –4, the last term is 29 and the sum of all its term is 150. Find its common difference. **[2011 (T-II)]**

14. The ticket receipts at the show of a film amounted to Rs 6,500 on the first day and showed a drop of Rs 110 every succeeding day. If the operational expenses of the show are Rs 1000 a day, Find on which day the show ceases to be profitable. **[2011 (T-II)]**

15. Find the sum of all the two-digit natural numbers which are divisible by 4. **[2011 (T-II)]**

16. Determine 'a' so that $2a + 1, a^2 + a + 1$ and $3a^2 - 3a + 3$ are consecutive terms of an A.P. **[2011 (T-II)]**

17. If the sum of first m terms of an A.P. is n and the sum of first n terms in m, then show that the sum of its first $(m + n)$ terms is $-(m + n)$. **[2011 (T-II)]**

18. How many terms of the A.P. – 6, –11/2, –5, ... are needed to give the sum – 25 ? **[2011 (T-II)]**

19. If the sum of all the terms of an A.P. 1, 4, 7, 10, ..., x. is 287, find x. **[2011 (T-II)]**

20. How many terms of the A.P. 78, 71, 64, ... are needed to give the sum 465 ? Also find the last term of this A.P. **[2011 (T-II)]**

21. The 4th term of an A.P. is equal to 3 times the first term and the 7th term exceeds twice the 3rd term by 1. Find the first term and the common difference. **[2011 (T-II)]**

22. Find three numbers in A.P. whose sum is 15 and whose product is 105. **[2011 (T-II)]**

23. The sum of the 5th and 7th terms of an A.P. is 52 and its 10th term is 46. Find the A.P. **[2011 (T-II)]**

24. Sum of the first n terms of an A.P. is $5n^2 - 3n$. Find the A.P. and also find its 16th term. **[2011 (T-II)]**

25. Find the sum of all three digit numbers which leave the same remainder 2 when divided by 5. **[2011 (T-II)]**

26. For what value of n are the nth terms of two A.P. 63, 65, 67 ... and 3, 10, 17 ... equal ? **[2008]**

27. If the sum of first 7 terms of an A.P. is 49 and that of first 17 terms is 289, find the sum of first n terms. **[2008C]**

28. If the 8th term of an A.P. is 37 and the 15th term is 15 more than the 12th term, find the A.P. Hence find the sum of the first 15 terms of the A.P. **[2008C]**

29. The sum of first six terms of an A.P. is 42. The ratio of its 10th term to its 30th term is 1 : 3. Calculate the first and the thirteenth terms of the A.P. **[2009]**

30. The sum of the first sixteen terms of an A.P. is 112 and the sum of its next fourteen terms is 518. Find the A.P. **[2010]**

LONG ANSWER TYPE QUESTIONS [4 Marks]

A. Important Questions

1. The sum of four consecutive numbers in an A.P. is 32 and the ratio of the product of the first and the last terms to the product of the two middle terms is 7 : 15. Find the numbers.

2. If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P., show that :

(i) $\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ are in A.P.

(ii) bc, ca, ab are in A.P.

3. Find the sum of those integers :

(a) between 1 and 500 which are multiples of 2 as well as of 5.

(b) from 1 to 500 which are multiples of 2 as well as of 5.

(c) from 1 to 500 which are multiples of 2 or 5.

4. If pth, qth and rth terms of an A.P. are a, b and c respectively, then show that :

(a) $a(q - r) + b(r - p) + c(p - q) = 0$

(b) $(a - b)r + (b - c)p + (c - a)q = 0$

5. Solve the equation

$1 + 4 + 7 + 10 + ... + x = 287.$

6. The sums of x terms of three A.P.s are S_1, S_2 and S_3. The first term of each is unity and the common differences are 1, 2 and 3 respectively. Prove that $S_1 + S_3 = 2S_2$.

7. An A.P. consists of 37 terms. The sum of three middle most terms is 225 and the sum of the last three is 429. Find the A.P.

8. If m^{th} term of an A.P. is $\frac{1}{n}$ and the n^{th} term is $\frac{1}{m}$, show that the sum of its mn terms is $\frac{1}{2}(mn+1)$.

9. If S_1, S_2, S_3 be the sums of n, $2n$ and $3n$ terms respectively of an A.P, prove that $S_3 = 3(S_2 - S_1)$.

10. Prove that no matter what the real numbers a and b are, the sequence with n^{th} term $a + nb$ is always an A.P. What is the common difference? What is the sum of the first 20 terms? **[HOTS]**

11. A contractor employed 150 labourers to finish a piece of work in a certain number of days. 4 workers went away the second day, 4 more workers went away the third day and so on. If it took 8 more days to finish the work, find the number of days in which the work was completed. **[HOTS]**

12. The students of a school decided to beautify the school on the annual day by fixing colourful flags on the straight passage of the school. They have 27 flags to be fixed at interval of every 2m. The flags are stored in the position of the middle most flag. Ruchi was given the responsibility of placing the flags. Ruchi kept her books where the flags were stored. She could carry only one flag at a time. How much distance did she cover in completing this jobs and returning back to collect her books? What is the maximum distance she travelled carrying a flag? **[HOTS]**

B. Questions From CBSE Examination Papers

1. In an A.P. the sum of first ten terms is –80 and the sum of next ten terms is –280. Find the A.P. **[2011 (T-II)]**

2. The sum of first 7 terms of an A.P. is 49 and that of first 17 terms is 289. Find the sum of first n terms. **[2011 (T-II)]**

3. Kartik repays his total loan of Rs. 1,18,000 by paying every month starting with the first instalment of 1000. He increases the instalment by Rs 100 every month. What amount will be paid by him in the 30th instalment? What amount of loan does he still have to pay after the 30th instalment? **[2011 (T-II)]**

4. In an A.P., the sum of first n terms is given by $S_n = \frac{3n^2}{2} + \frac{5n}{2}$. Find the 25th term of the A.P.

5. A contractor on construction job specifies a penalty for delay of completion beyond a certain date as follows : Rs 200 for the first day, Rs 250 for the second day, Rs 300 for the third day etc. the penalty for each succeeding day being Rs 50 more than for the preceding day. How much money the contractor has to pay as penalty if he has delayed the work by 30 days ? **[2011 (T-II)]**

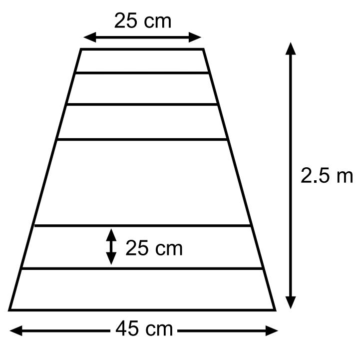
6. If a, b and c be the sums of first p, q and r terms respectively of an AP, show that

$$\frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$$ **[2011 (T-II)]**

7. In November 2009, the number of visitors to a zoo increased daily by 20. If a total of 12300 people visited the zoo in that month, find the number of visitors on 1st November 2009. **[2011 (T-II)]**

8. For what value of n, the nth terms of the A.P. 63, 65, 67, ... and 3, 10, 17, ... are equal? Also find that term. **[2011 (T-II)]**

9. A ladder has rungs 25 cm apart. The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top (see figure). If the top and bottom rungs are 2.5 m apart, what is the length of the wood required for the rungs? **[2011 (T-II)]**



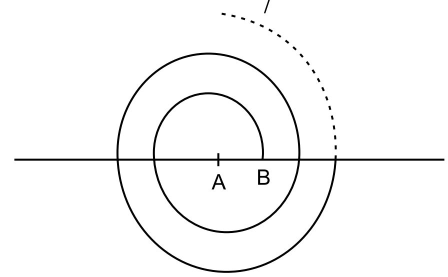
10. Find the sum of the first 31 terms of an A.P. whose nth term is given by $3 + \frac{2}{3}$ n. **[2011 (T-II)]**

11. The sum of the third and seventh term of an A.P. is 6 and their product is 8. Find the sum of the first sixteen terms of the A.P. **[2011 (T-II)]**

12. A sum of Rs 2700 is to be used to give eight cash prizes to students of a school for their overall academic performance. If each prize is Rs 25 more than its preceding prize find the value of each of the prizes. **[2011 (T-II)]**

13. In an A.P., prove that $a_{m+n} + a_{m-n} = 2a_m$, where a_a denotes nth term of the A.P. **[2011 (T-II)]**

14. A spiral is made up of successive semicircles, with centres alternatively at A and B, starting with centre at A of radii 0.5 cm, 1 cm, 1.5 cm, 2 cm.... as shown in the figure. What is the total length of such spiral made up of thirteen consecutive semicircles? (Take $\pi = \frac{22}{7}$) **[2011 (T-II)]**



15. A manufacturer of T.V. sets produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find

(a) The production in the first year

(b) The production in the 10th year

(c) The total production in first 7 years

[2011 (T-II)]

16. The houses of a row are numbered consecutively from 1 to 49. There is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the number of the houses following it. Find this value of x. **[2011 (T-II)]**

17. Show that the sum of all odd integers between 1 and 1000 which are divisible by 3 is 83667. **[2011 (T-II)]**

18. The 4th term of an A.P. is equal to 3 times the first term and the 7th term exceeds twice the 3rd term by 1. Find the A.P. **[2011 (T-II)]**

19. Find the sum of all multiples of 9 lying between 300 and 700. **[2011 (T-II)]**

20. A woman takes up a job of Rs 8000 per month with an annual increment of Rs 100. What will she earn over a period of 10 years ? **[2011 (T-II)]**

21. 228 logs are to be stacked in a store in the following manner: 30 logs in the bottom, 28 in the next row, then 26 and so on. In how many rows can these 228 logs be stacekd? How many logs are there in the last row? **[2011 (T-II)]**

22. Neera saves Rs 1600 during the first year, Rs 2100 in the second year, Rs 2600 in the third year. If she continues her savings in this pattern, in how many years will she save Rs 38500? **[2011 (T-II)]**

23. If m times the mth term of an A.P. is equal to n times its nth term, show that the $(m+n)$th term of the A.P. is zero ($m \neq n$). **[2011 (T-II)]**

FORMATIVE ASSESSMENT

Activity-1

Objective : To verify that the given sequence is an arithmetic progression by paper cutting and pasting method.

Materials Requried : Squared paper, colour pencils, geometry box, etc.

Procedure :

Case 1. Let the given sequence be 1, 4, 7, 10, 13, ...

1. Take a squared paper. Using a colour pencil, colour 1 square (1 is the first term of the given sequence).
2. Using a different colour pencils, colour 4 squares vertically (4 is the 2nd term of the given sequence).
3. Colour 7 squares vertically (7 is the 3rd term of the sequence).
4. Colour 10 squares vertically (10 is the 4th term of the sequence) and so on.

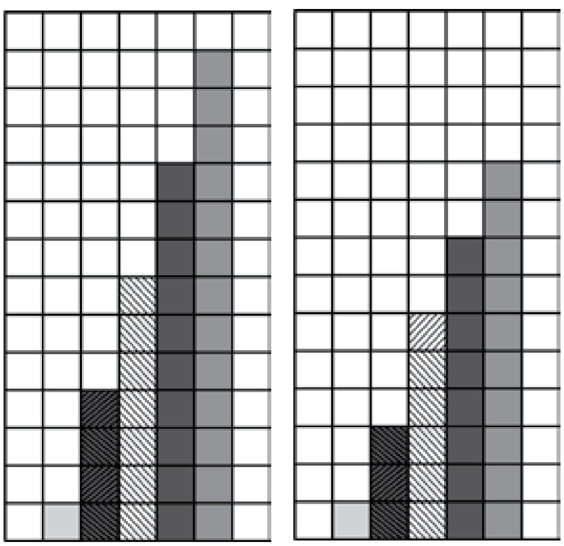


Figure 1 *Figure* 2

Case 2. Let the sequence be 1, 3, 6, 8, 10, 12, …

1. Take a squared paper. Using a colour pencil, colour 1 square (1 is the first term of the sequence).
2. Using a different colour pencil, colour 3 squares vertically (3 is the second term of the sequence).
3. Next colour 6 squares vertically (6 is the third term of the sequence).
4. Colour 8 squares vertically (8 is the fourth term of the sequence) and so on.

Observations :

1. **For figure 1:**

   The difference between the heights of first and second strips = 3 cm.

   The difference between the heights of second and third strips = 3 cm.

   The difference between the heights of third and fourth strips = 3 cm.

   The difference between the heights of fourth and the fifth strips = 3 cm.
2. If we consider figure 1 as a ladder, then we can say that the difference between the heights of adjoining steps of this ladder is constant, which is 3 cm.
3. **For figure 2 :**

   The difference between the heights of first and second strips = 2 cm.

   The difference between the heights of second and third strips = 3 cm.

   The difference between the heights of third and fourth strips = 2 cm.

   The difference between the heights of fourth and fifth strips = 2 cm.
4. If we consider figure 2 as a ladder, then we can say that the difference between the heights of adjoining steps of this ladder is not constant.

Conclusion : Squence 1 is an AP, as the difference between its consecutive terms is constant. Sequence 2 is not an AP, as the difference between its consecutive terms is not constant.

Activity-2

Objective : To verify that the sum of first n natural numbers is $\frac{n(n+1)}{2}$ by activity method.

Materials Required : Squared paper, colour pencils, geomety box, a pair of scissors, white sheets of paper, gluestick, etc.

Procedure : Let us find the sum of first 12 natural numbers, ie, 1 + 2 + 3 + 4 + … + 12.

1. Take a squared paper of size 12 cm × 12 cm. With the help of colour pencils, shade rectangles of length 1 cm, 2 cm, 3 cm, 4 cm, … 12 cm and of width 1 cm each, as shown in figure 3. Mark the rectangles as 1, 2, 3, … 12 respectivety from top to bottom.

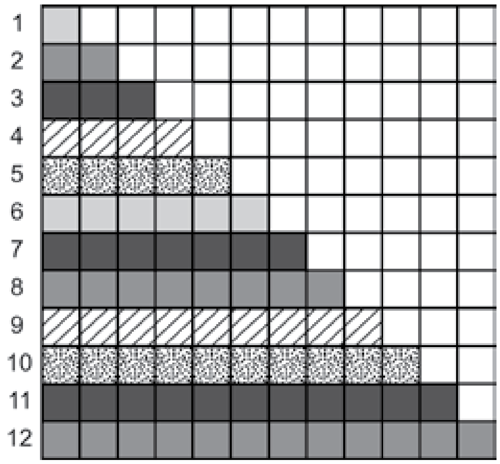


Figure 3

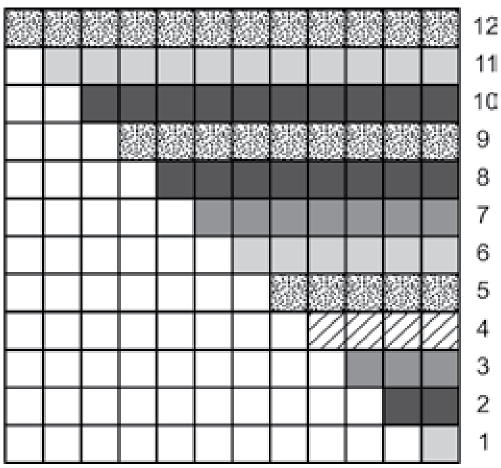


Figure 4

2. Take another squared paper of size 12 cm ×12 cm. Shade rectangles of length 1 cm, 2 cm, 3 cm, … 12 cm and of width 1 cm each, in riverse order as shown in figure 4. Mark the rectangles as 1, 2, 3, … 12 respectivety from bottom to top.
3. Now, cut out the coloured portions of figures 3 and 4 and paste them together on a white sheet of paper as shown in figure 5.

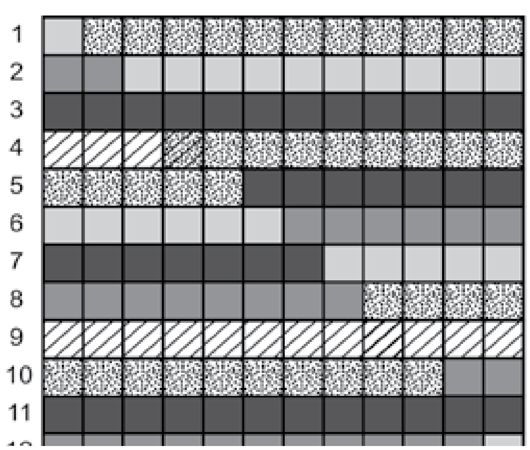


Figure 5

Observations :

1. The area of the shaded region in figure 3
$= (1×1 + 1×2 + 1×3 + 1×4 + \ldots + 1×12)\ \text{cm}^2$
$= (1 + 2 + 3 + 4 + \ldots + 12)\ \text{cm}^2$
2. The area of the shaded region in figure 4
$= (1×1 + 1×2 + 1×3 + 1×4 + \ldots + 1×12)\ \text{cm}^2 = (1 + 2 + 3 + 4 + \ldots + 12)\ \text{cm}^2$
3. Since, figure 5 is comprised of shaded regions of figures 1 and 2 so, area of figure 5
$= 2\ (1 + 2 + 3 + 4 + \ldots + 12)\ \text{cm}^2$
4. But, figure 5 is a rectangle of dimensions 12 cm × 13 cm or 12 cm × (12 + 1) cm
So, its area = $12(12 + 1)\ \text{cm}^2$
5. From 3 and 4 above, $2\ (1 + 2 + 3 + 4 + \ldots + 12) = 12 \times (12 + 1)$
$\Rightarrow 1 + 2 + 3 + 4 + \ldots + 12 = \dfrac{12\times(12+1)}{2}$.
$\Rightarrow$ sum of first 12 natural numbers = $\dfrac{12(12+1)}{2}$.
So, sum of first n natural numbers = $1 + 2 + 3 + \ldots + n = \dfrac{n(n+1)}{2}$.

Do Yourself :

Repeat the above activity by considering the sum of first 20 natural numbers.

Activity-3

Objective : To verify that the sum of first n odd natural numbers is given by n^2 by an activity method.

Materials Required : Squared paper, colour pencils, white sheets of paper, a pair of scissors, geometry box, gluestick, etc.

Procedure :

1. Take a squared paper of size 10 cm × 10 cm. Using a colour pencil, shade its top left square.
2. Colour the three squares adjacent to the above square, using a different colour.
3. Colour the five squares, which are adjacent to the previous squares using a different colour.
4. Colour seven squares which are adjacent to the previous squares, using a different colour.
5. Continue this process till you shade all the squares.

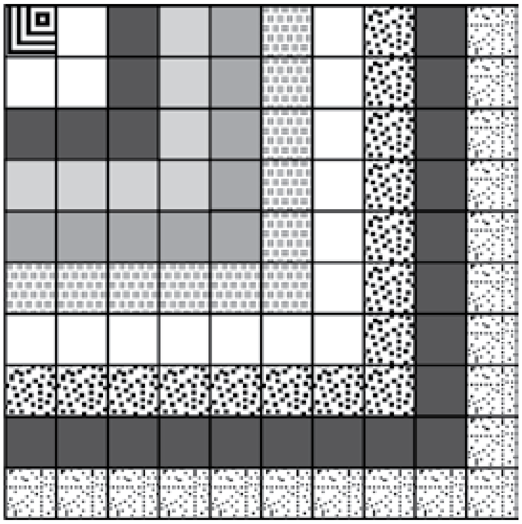


Figure 6

Observations :

1. Area of the square shaded in step 1 = 1 cm^2
2. Total area of the squares shaded in steps 1 and 2 = (1 + 3) cm^2
3. But, the squares shaded in steps 1 and 2 together form another square of side 2 cm.

   So, its area = 2^2 cm^2.
4. So, from 2 and 3 above, $1 + 3 = 2^2$.
5. Area of squares shaded in steps 1, 2, and 3 = (1 + 3 + 5) cm^2.
6. But, the squares shaded in steps 1, 2 and 3 together form another square of side 3 cm.

   So, its area = 3^2 cm^2.
7. So, from 5 and 6 above, $1 + 3 + 5 = 3^2$.
8. Area of the squares shaded in steps 1, 2, 3 and 4 = (1 + 3 + 5 + 7) cm^2.
9. But, the squares shaded in steps 1, 2, 3 and 4 together form another square of side 4 cm.

   So, its area = 4^2 cm^2.
10. From 8 and 9 above, $1 + 3 + 5 + 7 = 4^2$.
11. Similarly, we can say that $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 = 10^2$

    or $1 + 3 + 5 + \ldots$ up to 10 terms = 10^2

    or $1 + 3 + 5 \ldots$ up to n terms = n^2

Conclusion : From the above activity, we can say that the sum of first n odd natural numbers is n^2.

Activity-4

Objective : To find the formula for nth term of an arithmetic progression.

Materials Required : Squared paper, colour pencils, geometry box, etc.

Procedure : Let the AP be 4, 6, 8, 10, 12, 14, 16, 18, 20.

Here, $a = 4$, $d = 2$, and $t_9 = 20$

1. Take a squared paper of dimensions 9 cm × 20 cm.
2. Shade 4 squares of all the 9 rows as shown in figure 7.

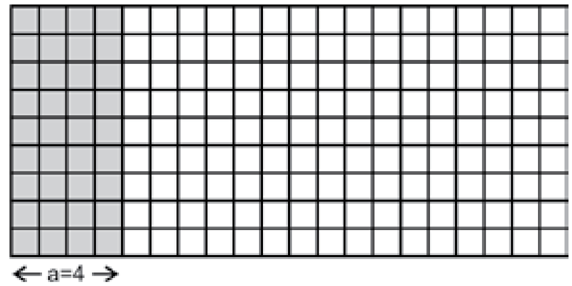


Figure 7

3. Shade 1 × 2 = 2 more squares of second row using different colour as shown in the figure 8.

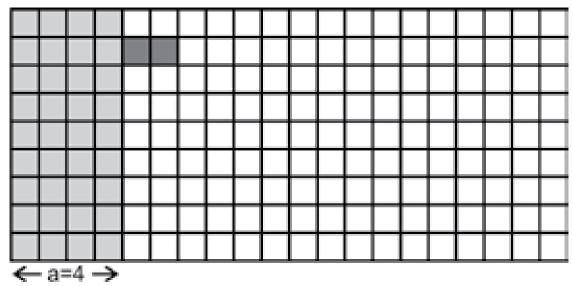


Figure 8

4. Shade 2 × 2 = 4 more squares of third row as shown in figure 9.

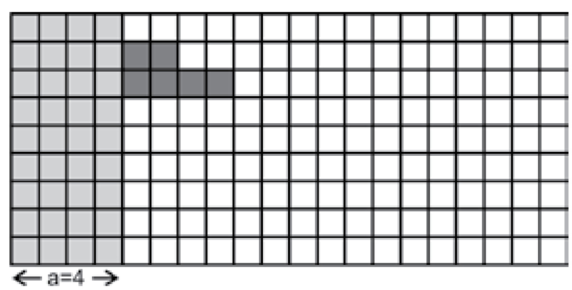


Figure 9

5. Shade 3 × 2 = 6 more squares of fourth row. Continue this process up to the 9th row as shown below.

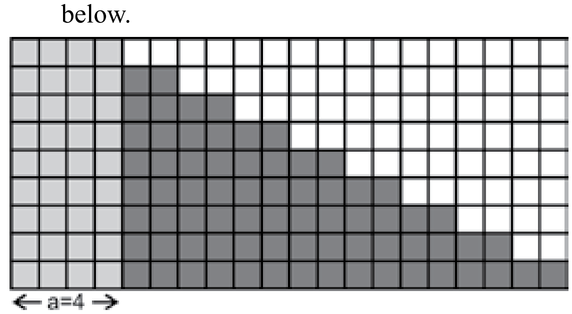


Figure 10

Observations :

1. In figure 10,

   the first row represents $T_1 = a = 4$

   the second row represents $T_2 = a + d = 4 + 2$

   the third row represents $T_3 = a + 2d = 4 + 2 \times 2$

   the fourth row represents $T_4 = a + 3d = 4 + 3 \times 2$

   the fifth row represents $T_5 = a + 4d = 4 + 4 \times 2$

   the sixth row represents $T_6 = a + 5d = 4 + 5 \times 2$

   the seventh row represents $T_7 = a + 6d = 4 + 6 \times 2$

   the eighth row represents $T_8 = a + 7d = 4 + 7 \times 2$

   the ninth row represents $T_9 = a + 8d = 4 + 8 \times 2$

2. From 1 above, we have, $T_1 = a$, $T_2 = a + d = a + (2 - 1)d$,

   $T_3 = a + 2d = a + (3 - 1)d$, ... $T_9 = a + 8d = a + (9 - 1)d$

   If we continue the pattern, we can write $T_n = a + (n - 1)d$

Conclusion : From the above activity, we find the *n*th term of an AP whose first term is *a* and common difference *d* is given by $T_n = a + (n - 1)d$.

Do Yourself : Find the formula for *n*th term of an AP by activity method taking the following AP's

(*i*) 5, 8, 11, 14, 17, … (*ii*) 10, 11, 12, 13, 14, …

Activity-5

Objective : To find the formula for the sum of *n* terms of an arithmetic progression.

Materials Required : Squared paper, colour pencils, geometry box, a pair of scissors, white sheets of paper, gluestick, etc.

Procedure : Let us take the AP 1, 4, 7, 10, 13, 16, …

Here $a = 1$, $d = 3$

1. Take a squared paper of size 16 cm × 6 cm. With the help of colour pencils, shade rectangles of length 1 cm, 4 cm, 7 cm, 10 cm, 13 cm, 16 cm and of width 1 cm each as shown in the figure.

$T_1 = 1$

$T_2 = 1 + 3 = 4$

$T_3 = 1 + 2 \times 3 = 7$

$T_4 = 1 + 3 \times 3 = 10$

$T_5 = 1 + 4 \times 3 = 13$

$T_6 = 1 + 5 \times 3 = 16$

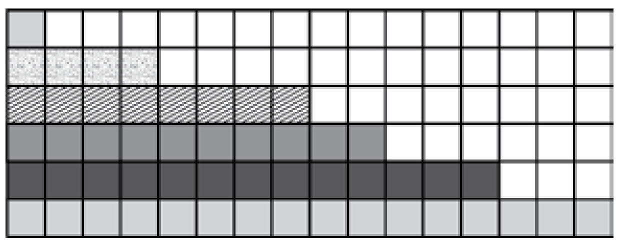


Figure 11

2. Take another squared paper of size 16 cm × 6 cm. Shade rectangles of length 1 cm, 4 cm, 7 cm, 10 cm, 13 cm, 16 cm and of width 1 cm each in the reverse order as shown in figure 12.

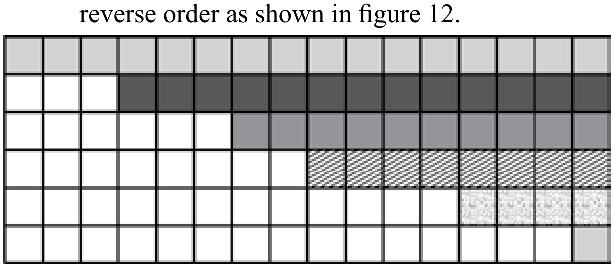


Figure 12

3. Now, cut the coloured portions of figures 11 and 12 and paste them on a white sheet of paper as shown in figure.

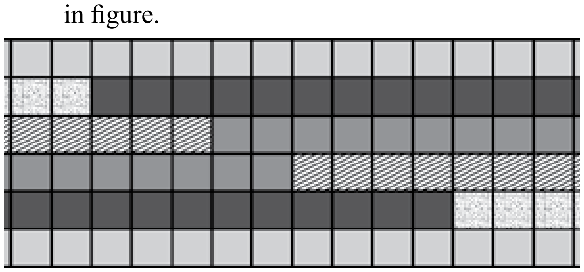


Figure 13

Observations :

1. The area of the shaded region in the figure 11

   $= (1\times1 + 1\times4 + 1\times7 + 1\times10 + 1\times13 + 1\times16)\ cm^2$

   $= (1 + 4 + 7 + 10 + 13 + 16)\ cm^2$

2. The area of the shaded region in the figure 12

   $= (1\times1 + 1\times4 + 1\times7 + 1\times10 + 1\times13 + 1\times16)\ cm^2$

   $= (1 + 4 + 7 + 10 + 13 + 16)\ cm^2$

3. Since, figure 13 is made up of the shaded regions of figures 11 and 12, so area of figure 13

   $= 2\ (1 + 4 + 7 + 10 + 13 + 16)\ cm^2$

4. But, figure 13 is a rectangle of dimensions 6 cm × 17 cm

   So, its area = $(6 \times 17)\ cm^2$

5. From 3 and 4 above we have,

   $1 + 4 + 7 + 10 + 13 + 16 = \frac{6}{2} \times 17 = \frac{6}{2}\ (1 + 16) = \frac{6}{2}\ [1 + (1 + 5 \times 3)]\ [\because\ T_6 = 16 = 1 + 5 \times 3]$

   $= \frac{6}{2}\ [2 + 5 \times 3] = \frac{6}{2}\ [2 + (6 - 1)\ 3]$

   $\Rightarrow T_1 + T_2 + T_3 + T_4 + T_5 + T_6$

$= \frac{6}{2}$ $[2a + (6 - 1)d]$

[Replacing 1 by a and 3 by d]

$\Rightarrow S_6 = T_1 + T_2 + T_3 + T_4 + T_5 + T_6 = \frac{6}{2}$ $[2a + (6 - 1)d]$

$\Rightarrow S_n = T_1 + T_2 + T_3 + \ldots T_n = \frac{n}{2}$ $[2a + (n - 1)d]$

[Extending the pattern for n terms]

6. From figure 13, the area of first row = $1 \times (1 + 16)$ cm^2

There are 6 rows in figure 13. So, area of firure 13 = $6 \times (1 + 16)$ cm^2

7. From 3 and 6, we have

$2(1 + 4 + 7 + 10 + 13 + 16) = 6\,(1 + 16)$

$\Rightarrow 1 + 4 + 7 + 10 + 13 + 16 = \frac{6}{2}$ $(1+16)$

$\Rightarrow T_1 + T_2 + T_3 + T_4 + T_5 + T_6 = \frac{6}{2}$ $(1 + 16)$ $= \frac{6}{2}$ $(T_1 + T_6)$

$\Rightarrow S_6 = \frac{6}{2}$ $(T_1 + T_6) \Rightarrow S_n = \frac{n}{2}$ $[T_1 + T_n]$

[Extending the same pattern for n terms]

Conclusion :

1. From the above activity, it is verified that

(*a*) the sum of first n terms of an AP whose first term is a and common difference is d, is given by $S_n = \frac{n}{2}$ $[2a + (n - 1)d]$

(*b*) the sum of first n terms of an AP whose first term is T_1 and the last term is T_n, is given by $S_n = \frac{n}{2}$ $[T_1 + T_n]$

Exercise 5.1

Question 1:

In which of the following situations, does the list of numbers involved make as arithmetic progression and why?

(i) The taxi fare after each km when the fare is Rs 15 for the first km and Rs 8 for each additional km.

(ii) The amount of air present in a cylinder when a vacuum pump removes $\frac{1}{4}$ of the air remaining in the cylinder at a time.

(iii) The cost of digging a well after every metre of digging, when it costs Rs 150 for the first metre and rises by Rs 50 for each subsequent metre.

(iv)The amount of money in the account every year, when Rs 10000 is deposited at compound interest at 8% per annum.

Answer:

(i) It can be observed that

Taxi fare for 1^{st} km = 15

Taxi fare for first 2 km = 15 + 8 = 23

Taxi fare for first 3 km = 23 + 8 = 31

Taxi fare for first 4 km = 31 + 8 = 39

Clearly 15, 23, 31, 39 … forms an A.P. because every term is 8 more than the preceding term.

(ii) Let the initial volume of air in a cylinder be *V* lit. In each stroke, the vacuum pump removes $\frac{1}{4}$ of air remaining in the cylinder at a time.

In other words, after every stroke, only $1-\frac{1}{4}=\frac{3}{4}$th part of air will remain.

Therefore, volumes will be $V,\left(\frac{3V}{4}\right),\left(\frac{3}{4}V\right)^2,\left(\frac{3}{4}V\right)^3\ldots$

Clearly, it can be observed that the adjacent terms of this series do not have the same difference between them. Therefore, this is not an A.P.

(iii) Cost of digging for first metre = 150

Cost of digging for first 2 metres = 150 + 50 = 200

Cost of digging for first 3 metres = 200 + 50 = 250

Cost of digging for first 4 metres = 250 + 50 = 300

Clearly, 150, 200, 250, 300 … forms an A.P. because every term is 50 more than the preceding term.

(iv) We know that if Rs P is deposited at *r*% compound interest per annum for *n* years, our money will be $\mathrm{P}\left(1+\frac{r}{100}\right)^n$ after *n* years.

Therefore, after every year, our money will be

$$10000\left(1+\frac{8}{100}\right),10000\left(1+\frac{8}{100}\right)^2,10000\left(1+\frac{8}{100}\right)^3,10000\left(1+\frac{8}{100}\right)^4,\ldots$$

Clearly, adjacent terms of this series do not have the same difference between them. Therefore, this is not an A.P.

Question 2:

Write first four terms of the A.P. when the first term *a* and the common difference *d* are given as follows

(i) $a = 10$, $d = 10$

(ii) $a = -2$, $d = 0$

(iii) $a = 4$, $d = -3$

(iv) $a = -1$ $d = \frac{1}{2}$

(v) $a = -1.25$, $d = -0.25$

Answer:

(i) $a = 10$, $d = 10$

Let the series be $a_1, a_2, a_3, a_4, a_5 \ldots$

$a_1 = a = 10$

$a_2 = a_1 + d = 10 + 10 = 20$

$a_3 = a_2 + d = 20 + 10 = 30$

$a_4 = a_3 + d = 30 + 10 = 40$

$a_5 = a_4 + d = 40 + 10 = 50$

Therefore, the series will be 10, 20, 30, 40, 50 …

First four terms of this A.P. will be 10, 20, 30, and 40.

(ii) $a = -2$, $d = 0$

Let the series be $a_1, a_2, a_3, a_4 \ldots$

$a_1 = a = -2$

$a_2 = a_1 + d = -2 + 0 = -2$

$a_3 = a_2 + d = -2 + 0 = -2$

$a_4 = a_3 + d = -2 + 0 = -2$

Therefore, the series will be −2, −2, −2, −2 …

First four terms of this A.P. will be −2, −2, −2 and −2.

(iii) $a = 4$, $d = -3$

Let the series be a_1, a_2, a_3, a_4 …

$a_1 = a = 4$

$a_2 = a_1 + d = 4 - 3 = 1$

$a_3 = a_2 + d = 1 - 3 = -2$

$a_4 = a_3 + d = -2 - 3 = -5$

Therefore, the series will be 4, 1, −2 −5 …

First four terms of this A.P. will be 4, 1, −2 and −5.

(iv) $a = -1$, $d = \frac{1}{2}$

Let the series be a_1, a_2, a_3, a_4 …

$$a_1 = a = -1$$

$$a_2 = a_1 + d = -1 + \frac{1}{2} = -\frac{1}{2}$$

$$a_3 = a_2 + d = -\frac{1}{2} + \frac{1}{2} = 0$$

$$a_4 = a_3 + d = 0 + \frac{1}{2} = \frac{1}{2}$$

Clearly, the series will be

$-1, -\frac{1}{2}, 0, \frac{1}{2}$ ………….

First four terms of this A.P. will be $-1, -\frac{1}{2}, 0 \text{ and } \frac{1}{2}$.

(v) $a = -1.25$, $d = -0.25$

Let the series be a_1, a_2, a_3, a_4 …

$a_1 = a = -1.25$

$a_2 = a_1 + d = -1.25 - 0.25 = -1.50$

$a_3 = a_2 + d = -1.50 - 0.25 = -1.75$

$a_4 = a_3 + d = -1.75 - 0.25 = -2.00$

Clearly, the series will be 1.25, −1.50, −1.75, −2.00 ……..

First four terms of this A.P. will be −1.25, −1.50, −1.75 and −2.00.

Question 3:

For the following A.P.s, write the first term and the common difference.

(i) 3, 1, − 1, − 3 …

(ii) − 5, − 1, 3, 7 …

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3} \ldots$

(iv) 0.6, 1.7, 2.8, 3.9 …

Answer:

(i) 3, 1, −1, −3 …

Here, first term, $a = 3$

Common difference, d = Second term − First term

= 1 − 3 = −2

(ii) −5, −1, 3, 7 …

Here, first term, $a = -5$

Common difference, d = Second term − First term

= (−1) − (−5) = − 1 + 5 = 4

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3} \ldots$

Here, first term, $a = \frac{1}{3}$

Common difference, d = Second term − First term

$= \frac{5}{3} - \frac{1}{3} = \frac{4}{3}$

(iv) 0.6, 1.7, 2.8, 3.9 …

Here, first term, a = 0.6

Common difference, d = Second term − First term

= 1.7 − 0.6

= 1.1

Exercise 5.2

Question 1:

Fill in the blanks in the following table, given that a is the first term, d the common difference and a_n the n^{th} term of the A.P.

	a	d	n	a_n
I	7	3	8	……
II	− 18	…..	10	0
III	…..	− 3	18	− 5
IV	− 18.9	2.5	…..	3.6
V	3.5	0	105	…..

Answer:

I. $a = 7$, $d = 3$, $n = 8$, $a_n = ?$

We know that,

For an A.P. $a_n = a + (n - 1)\, d$

$= 7 + (8 - 1)\, 3$

$= 7 + (7)\, 3$

$= 7 + 21 = 28$

Hence, $a_n = 28$

II. Given that

$a = -18$, $n = 10$, $a_n = 0$, $d = ?$

We know that,

$a_n = a + (n - 1)\,d$

$0 = -18 + (10 - 1)\,d$

$18 = 9d$

$d = \frac{18}{9} = 2$

Hence, common difference, $d = 2$

III. Given that

$d = -3$, $n = 18$, $a_n = -5$

We know that,

$a_n = a + (n - 1)\,d$

$-5 = a + (18 - 1)(-3)$

$-5 = a + (17)(-3)$

$-5 = a - 51$

$a = 51 - 5 = 46$

Hence, $a = 46$

IV. $a = -18.9$, $d = 2.5$, $a_n = 3.6$, $n = ?$

We know that,

$a_n = a + (n - 1)\,d$

$3.6 = -18.9 + (n - 1)\,2.5$

$3.6 + 18.9 = (n - 1)\,2.5$

$22.5 = (n - 1)\,2.5$

$(n-1) = \frac{22.5}{2.5}$

$n - 1 = 9$

$n = 10$

Hence, $n = 10$

V. $a = 3.5$, $d = 0$, $n = 105$, $a_n = ?$

We know that,

$a_n = a + (n - 1) d$

$a_n = 3.5 + (105 - 1) 0$

$a_n = 3.5 + 104 \times 0$

$a_n = 3.5$

Hence, $a_n = 3.5$

Question 2:

Choose the correct choice in the following and justify

I. 30th term of the A.P: 10, 7, 4, …, is

A. 97 **B.** 77 **C.** − 77 **D.** − 87

II 11th term of the A.P. $-3, -\frac{1}{2}, 2, \ldots$ is

A. 28 **B.** 22 **C.** − 38 **D.** $-48\frac{1}{2}$

Answer:

I. Given that

A.P. 10, 7, 4, …

First term, $a = 10$

Common difference, $d = a_2 - a_1 = 7 - 10$

$= -3$

We know that, $a_n = a + (n - 1) d$

$a_{30} = 10 + (30 - 1) (-3)$

$a_{30} = 10 + (29) (-3)$

$a_{30} = 10 - 87 = -77$

Hence, the correct answer is **C**.

II. Given that, A.P. $-3, -\frac{1}{2}, 2, \ldots$

First term $a = -3$

Common difference, $d = a_2 - a_1$

$= -\frac{1}{2} - (-3)$

$= -\frac{1}{2} + 3 = \frac{5}{2}$

We know that,

$a_n = a + (n-1)d$

$a_{11} = -3 + (11-1)\left(\frac{5}{2}\right)$

$a_{11} = -3 + (10)\left(\frac{5}{2}\right)$

$a_{11} = -3 + 25$

$a_{11} = 22$

Hence, the answer is **B**.

Question 3:

In the following APs find the missing term in the boxes

I. $2, \square, 26$

II. $\square, 13, \square, 3$

III. $5, \square, \square, 9\frac{1}{2}$

IV. $-4, \square, \square, \square, \square, 6$

V. $\square, 38, \square, \square, \square, -22$

Answer:

I. $2, \square, 26$

For this A.P.,

$a = 2$

$a_3 = 26$

We know that, $a_n = a + (n - 1) d$

$a_3 = 2 + (3 - 1) d$

$26 = 2 + 2d$

$24 = 2d$

$d = 12$

$a_2 = 2 + (2 - 1) 12$

$= 14$

Therefore, 14 is the missing term.

II. $\square, 13, \square, 3$

For this A.P.,

$a_2 = 13$ and

$a_4 = 3$

We know that, $a_n = a + (n - 1) d$

$a_2 = a + (2 - 1) d$

$13 = a + d$ (I)

$a_4 = a + (4 - 1) d$

$3 = a + 3d$ (II)

On subtracting (I) from (II), we obtain

$-10 = 2d$

$d = -5$

From equation (I), we obtain

$13 = a + (-5)$

$a = 18$

$a_3 = 18 + (3 - 1)(-5)$

$= 18 + 2(-5) = 18 - 10 = 8$

Therefore, the missing terms are 18 and 8 respectively.

III. $5, \square, \square, 9\frac{1}{2}$

For this A.P.,

$a = 5$

$a_4 = 9\frac{1}{2} = \frac{19}{2}$

We know that,

$$a_n = a + (n-1)d$$
$$a_4 = a + (4-1)d$$
$$\frac{19}{2} = 5 + 3d$$
$$\frac{19}{2} - 5 = 3d$$
$$\frac{9}{2} = 3d$$
$$d = \frac{3}{2}$$
$$a_2 = a + d = 5 + \frac{3}{2} = \frac{13}{2}$$
$$a_3 = a + 2d = 5 + 2\left(\frac{3}{2}\right) = 8$$

Therefore, the missing terms are $\frac{13}{2}$ and 8 respectively.

IV. $-4, \square, \square, \square, \square, 6$

For this A.P.,

$a = -4$ and

$a_6 = 6$

We know that,

$a_n = a + (n - 1)\, d$

$a_6 = a + (6 - 1)\, d$

$6 = -4 + 5d$

$10 = 5d$

$d = 2$

$a_2 = a + d = -4 + 2 = -2$

$a_3 = a + 2d = -4 + 2(2) = 0$

$a_4 = a + 3d = -4 + 3(2) = 2$

$a_5 = a + 4d = -4 + 4(2) = 4$

Therefore, the missing terms are −2, 0, 2, and 4 respectively.

v. $\square, 38, \square, \square, \square, -22$

For this A.P.,

$a_2 = 38$

$a_6 = -22$

We know that

$a_n = a + (n - 1)d$

$a_2 = a + (2 - 1)d$

$38 = a + d$ (1)

$a_6 = a + (6 - 1)d$

$-22 = a + 5d$ (2)

On subtracting equation (1) from (2), we obtain

$-22 - 38 = 4d$

$-60 = 4d$

$d = -15$

$a = a_2 - d = 38 - (-15) = 53$

$a_3 = a + 2d = 53 + 2(-15) = 23$

$a_4 = a + 3d = 53 + 3(-15) = 8$

$a_5 = a + 4d = 53 + 4(-15) = -7$

Therefore, the missing terms are 53, 23, 8, and −7 respectively.

Question 4:

Which term of the A.P. 3, 8, 13, 18, … is 78?

Answer:

3, 8, 13, 18, …

For this A.P.,

$a = 3$

$d = a_2 - a_1 = 8 - 3 = 5$

Let n^{th} term of this A.P. be 78.

$a_n = a + (n - 1) d$

$78 = 3 + (n - 1) 5$

$75 = (n - 1) 5$

$(n - 1) = 15$

$n = 16$

Hence, 16^{th} term of this A.P. is 78.

Question 5:

Find the number of terms in each of the following A.P.

I. 7, 13, 19, …, 205

II. $18, 15\frac{1}{2}, 13, \ldots, -47$

Answer:

I. 7, 13, 19, …, 205

For this A.P.,

$a = 7$

$d = a_2 - a_1 = 13 - 7 = 6$

Let there are n terms in this A.P.

$a_n = 205$

We know that

$a_n = a + (n - 1) d$

Therefore, $205 = 7 + (n - 1) 6$

$198 = (n - 1) 6$

$33 = (n - 1)$

$n = 34$

Therefore, this given series has 34 terms in it.

II. $18,\ 15\frac{1}{2}, 13, \ldots, -47$

For this A.P.,

$a = 18$

$d = a_2 - a_1 = 15\frac{1}{2} - 18$

$d = \frac{31 - 36}{2} = -\frac{5}{2}$

Let there are n terms in this A.P.

Therefore, $a_n = -47$ and we know that,

$$a_n = a + (n-1)d$$
$$-47 = 18 + (n-1)\left(-\frac{5}{2}\right)$$
$$-47 - 18 = (n-1)\left(-\frac{5}{2}\right)$$
$$-65 = (n-1)\left(-\frac{5}{2}\right)$$
$$(n-1) = \frac{-130}{-5}$$
$$(n-1) = 26$$
$$n = 27$$

Therefore, this given A.P. has 27 terms in it.

Question 6:

Check whether − 150 is a term of the A.P. 11, 8, 5, 2, …

Answer:

For this A.P.,

$a = 11$

$d = a_2 - a_1 = 8 - 11 = -3$

Let −150 be the n^{th} term of this A.P.

We know that,

$$a_n = a + (n-1)d$$
$$-150 = 11 + (n-1)(-3)$$
$$-150 = 11 - 3n + 3$$
$$-164 = -3n$$
$$n = \frac{164}{3}$$

Clearly, n is not an integer.

Therefore, −150 is not a term of this A.P.

Question 7:

Find the 31st term of an A.P. whose 11th term is 38 and the 16th term is 73

Answer:

Given that,

$a_{11} = 38$

$a_{16} = 73$

We know that,

$a_n = a + (n - 1)d$

$a_{11} = a + (11 - 1)d$

$38 = a + 10d$ (1)

Similarly,

$a_{16} = a + (16 - 1)d$

$73 = a + 15d$ (2)

On subtracting (1) from (2), we obtain

$35 = 5d$

$d = 7$

From equation (1),

$38 = a + 10 \times (7)$

$38 - 70 = a$

$a = -32$

$a_{31} = a + (31 - 1)d$

$= -32 + 30(7)$

$= -32 + 210$

$= 178$

Hence, 31^{st} term is 178.

Question 8:

An A.P. consists of 50 terms of which 3^{rd} term is 12 and the last term is 106. Find the 29^{th} term

Answer:

Given that,

$a_3 = 12$

$a_{50} = 106$

We know that,

$a_n = a + (n - 1)\,d$

$a_3 = a + (3 - 1)\,d$

$12 = a + 2d$ (I)

Similarly, $a_{50} = a + (50 - 1)\,d$

$106 = a + 49d$ (II)

On subtracting (I) from (II), we obtain

$94 = 47d$

$d = 2$

From equation (I), we obtain

$12 = a + 2\,(2)$

$a = 12 - 4 = 8$

$a_{29} = a + (29 - 1)\,d$

$a_{29} = 8 + (28)2$

$a_{29} = 8 + 56 = 64$

Therefore, 29^{th} term is 64.

Question 9:

If the 3rd and the 9th terms of an A.P. are 4 and − 8 respectively.

Which term of this A.P. is zero.

Answer:

Given that,

$a_3 = 4$

$a_9 = -8$

We know that,

$a_n = a + (n - 1)\, d$

$a_3 = a + (3 - 1)\, d$

$4 = a + 2d$ (I)

$a_9 = a + (9 - 1)\, d$

$-8 = a + 8d$ (II)

On subtracting equation (I) from (II), we obtain

$-12 = 6d$

$d = -2$

From equation (I), we obtain

$4 = a + 2\,(-2)$

$4 = a - 4$

$a = 8$

Let n^{th} term of this A.P. be zero.

$a_n = a + (n - 1)\, d$

$0 = 8 + (n - 1)\,(-2)$

$0 = 8 - 2n + 2$

$2n = 10$

$n = 5$

Hence, 5th term of this A.P. is 0.

Question 10:

If 17th term of an A.P. exceeds its 10th term by 7. Find the common difference.

Answer:

We know that,

For an A.P., $a_n = a + (n - 1)\, d$

$a_{17} = a + (17 - 1)\, d$

$a_{17} = a + 16d$

Similarly, $a_{10} = a + 9d$

It is given that

$a_{17} - a_{10} = 7$

$(a + 16d) - (a + 9d) = 7$

$7d = 7$

$d = 1$

Therefore, the common difference is 1.

Question 11:

Which term of the A.P. 3, 15, 27, 39, … will be 132 more than its 54th term?

Answer:

Given A.P. is 3, 15, 27, 39, …

$a = 3$

$d = a_2 - a_1 = 15 - 3 = 12$

$a_{54} = a + (54 - 1)\, d$

$= 3 + (53)(12)$

$= 3 + 636 = 639$

$132 + 639 = 771$

We have to find the term of this A.P. which is 771.

Let n^{th} term be 771.

$a_n = a + (n - 1) d$

$771 = 3 + (n - 1) 12$

$768 = (n - 1) 12$

$(n - 1) = 64$

$n = 65$

Therefore, 65^{th} term was 132 more than 54^{th} term.

Alternatively,

Let n^{th} term be 132 more than 54^{th} term.

$$n = 54 + \frac{132}{12}$$

$$= 54 + 11 = 65^{th} \text{ term}$$

Question 12:

Two APs have the same common difference. The difference between their 100^{th} term is 100, what is the difference between their 1000^{th} terms?

Answer:

Let the first term of these A.P.s be a_1 and a_2 respectively and the common difference of these A.P.s be d.

For first A.P.,

$a_{100} = a_1 + (100 - 1) d$

$= a_1 + 99d$

$a_{1000} = a_1 + (1000 - 1)\ d$

$a_{1000} = a_1 + 999d$

For second A.P.,

$a_{100} = a_2 + (100 - 1)\ d$

$= a_2 + 99d$

$a_{1000} = a_2 + (1000 - 1)\ d$

$= a_2 + 999d$

Given that, difference between

100^{th} term of these A.P.s = 100

Therefore, $(a_1 + 99d) - (a_2 + 99d) = 100$

$a_1 - a_2 = 100$ (1)

Difference between 1000^{th} terms of these A.P.s

$(a_1 + 999d) - (a_2 + 999d) = a_1 - a_2$

From equation (1),

This difference, $a_1 - a_2 = 100$

Hence, the difference between 1000^{th} terms of these A.P. will be 100.

Question 13:

How many three digit numbers are divisible by 7

Answer:

First three-digit number that is divisible by 7 = 105

Next number = 105 + 7 = 112

Therefore, 105, 112, 119, …

All are three digit numbers which are divisible by 7 and thus, all these are terms of an A.P. having first term as 105 and common difference as 7.

The maximum possible three-digit number is 999. When we divide it by 7, the remainder will be 5. Clearly, 999 − 5 = 994 is the maximum possible three-digit number that is divisible by 7.

The series is as follows.

105, 112, 119, …, 994

Let 994 be the nth term of this A.P.

$a = 105$

$d = 7$

$a_n = 994$

$n = ?$

$a_n = a + (n - 1)\, d$

$994 = 105 + (n - 1)\, 7$

$889 = (n - 1)\, 7$

$(n - 1) = 127$

$n = 128$

Therefore, 128 three-digit numbers are divisible by 7.

Question 14:

How many multiples of 4 lie between 10 and 250?

Answer:

First multiple of 4 that is greater than 10 is 12. Next will be 16.

Therefore, 12, 16, 20, 24, …

All these are divisible by 4 and thus, all these are terms of an A.P. with first term as 12 and common difference as 4.

When we divide 250 by 4, the remainder will be 2. Therefore, 250 − 2 = 248 is divisible by 4.

The series is as follows.

12, 16, 20, 24, …, 248

Let 248 be the n^{th} term of this A.P.

$$a = 12$$
$$d = 4$$
$$a_n = 248$$
$$a_n = a + (n-1)d$$
$$248 = 12 + (n-1)4$$
$$\frac{236}{4} = n - 1$$
$$59 = n - 1$$
$$n = 60$$

Therefore, there are 60 multiples of 4 between 10 and 250.

Question 15:

For what value of *n*, are the n^{th} terms of two APs 63, 65, 67, and 3, 10, 17, … equal

Answer:

63, 65, 67, …

$a = 63$

$d = a_2 - a_1 = 65 - 63 = 2$

n^{th} term of this A.P. $= a_n = a + (n - 1)\, d$

$a_n = 63 + (n - 1)\, 2 = 63 + 2n - 2$

$a_n = 61 + 2n$ (1)

3, 10, 17, …

$a = 3$

$d = a_2 - a_1 = 10 - 3 = 7$

n^{th} term of this A.P. $= 3 + (n - 1)\,7$

$a_n = 3 + 7n - 7$

$a_n = 7n - 4$ (2)

It is given that, n^{th} term of these A.P.s are equal to each other.

Equating both these equations, we obtain

$61 + 2n = 7n - 4$

$61 + 4 = 5n$

$5n = 65$

$n = 13$

Therefore, 13^{th} terms of both these A.P.s are equal to each other.

Question 16:

Determine the A.P. whose third term is 16 and the 7^{th} term exceeds the 5^{th} term by 12.

Answer:

$=a_3 = 16$

$a + (3 - 1)\,d = 16$

$a + 2d = 16$ (1)

$a_7 - a_5 = 12$

$[a + (7 - 1)\,d] - [a + (5 - 1)\,d] = 12$

$(a + 6d) - (a + 4d) = 12$

$2d = 12$

$d = 6$

From equation (1), we obtain

$a + 2\,(6) = 16$

$a + 12 = 16$

$a = 4$

Therefore, A.P. will be

4, 10, 16, 22, …

Question 17:

Find the 20th term from the last term of the A.P. 3, 8, 13, …, 253

Answer:

Given A.P. is

3, 8, 13, …, 253

Common difference for this A.P. is 5.

Therefore, this A.P. can be written in reverse order as

253, 248, 243, …, 13, 8, 5

For this A.P.,

$a = 253$

$d = 248 - 253 = -5$

$n = 20$

$a_{20} = a + (20 - 1)\,d$

$a_{20} = 253 + (19)\,(-5)$

$a_{20} = 253 - 95$

$a = 158$

Therefore, 20th term from the last term is 158.

Question 18:

The sum of 4th and 8th terms of an A.P. is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the A.P.

Answer:

We know that,

$a_n = a + (n - 1)\,d$

$a_4 = a + (4 - 1)\,d$

$a_4 = a + 3d$

Similarly,

$a_8 = a + 7d$

$a_6 = a + 5d$

$a_{10} = a + 9d$

Given that, $a_4 + a_8 = 24$

$a + 3d + a + 7d = 24$

$2a + 10d = 24$

$a + 5d = 12$ (1)

$a_6 + a_{10} = 44$

$a + 5d + a + 9d = 44$

$2a + 14d = 44$

$a + 7d = 22$ (2)

On subtracting equation (1) from (2), we obtain

$2d = 22 - 12$

$2d = 10$

$d = 5$

From equation (1), we obtain

$a + 5d = 12$

$a + 5(5) = 12$

$a + 25 = 12$

$a = -13$

$a_2 = a + d = -13 + 5 = -8$

$a_3 = a_2 + d = -8 + 5 = -3$

Therefore, the first three terms of this A.P. are −13, −8, and −3.

Question 19:

Subba Rao started work in 1995 at an annual salary of Rs 5000 and received an increment of Rs 200 each year. In which year did his income reach Rs 7000?

Answer:

It can be observed that the incomes that Subba Rao obtained in various years are in A.P. as every year, his salary is increased by Rs 200.

Therefore, the salaries of each year after 1995 are

5000, 5200, 5400, …

Here, $a = 5000$

$d = 200$

Let after n^{th} year, his salary be Rs 7000.

Therefore, $a_n = a + (n - 1)d$

$7000 = 5000 + (n - 1) 200$

$200(n - 1) = 2000$

$(n - 1) = 10$

$n = 11$

Therefore, in 11th year, his salary will be Rs 7000.

Question 20:

Ramkali saved Rs 5 in the first week of a year and then increased her weekly saving by Rs 1.75. If in the n^{th} week, her week, her weekly savings become Rs 20.75, find *n.*

Answer:

Given that,

$a = 5$

$d = 1.75$

$a_n = 20.75$

$n = ?$

$a_n = a + (n - 1)\, d$

$$20.75 = 5 + (n-1)1.75$$

$$15.75 = (n-1)1.75$$

$$(n-1) = \frac{15.75}{1.75} = \frac{1575}{175}$$

$$= \frac{63}{7} = 9$$

$n - 1 = 9$

$n = 10$

Hence, n is 10.

Exercise 5.3

Question 1:

Find the sum of the following APs.

(i) 2, 7, 12 ,…., to 10 terms.

(ii) − 37, − 33, − 29 ,…, to 12 terms

(iii) 0.6, 1.7, 2.8 ,…….., to 100 terms

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}$,………, to 11 terms

Answer:

(i)2, 7, 12 ,…, to 10 terms

For this A.P.,

$a = 2$

$d = a_2 - a_1 = 7 - 2 = 5$

$n = 10$

We know that,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{10} = \frac{10}{2}\left[2(2) + (10-1)5\right]$$

$$= 5\left[4 + (9)\times(5)\right]$$

$$= 5 \times 49 = 245$$

(ii)−37, −33, −29 ,…, to 12 terms

For this A.P.,

$a = -37$

$d = a_2 - a_1 = (-33) - (-37)$

$= -33 + 37 = 4$

n = 12

We know that,

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$S_{12} = \frac{12}{2}\left[2(-37)+(12-1)4\right]$$

$$=6\left[-74+11\times 4\right]$$

$$=6\left[-74+44\right]$$

$$=6(-30)=-180$$

(iii) 0.6, 1.7, 2.8 ,…, to 100 terms

For this A.P.,

a = 0.6

$d = a_2 - a_1$ = 1.7 − 0.6 = 1.1

n = 100

We know that,

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$S_{100} = \frac{100}{2}\left[2(0.6)+(100-1)1.1\right]$$

$$=50\left[1.2+(99)\times(1.1)\right]$$

$$=50\left[1.2+108.9\right]$$

$$=50\left[110.1\right]$$

$$=5505$$

(iv) $\frac{1}{15},\frac{1}{12},\frac{1}{10},$. …….. , to 11 terms

For this A.P.,

$a = \frac{1}{15}$

n = 11

$d = a_2 - a_1 = \frac{1}{12} - \frac{1}{15}$

$= \frac{5-4}{60} = \frac{1}{60}$

We know that,

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$S_{11} = \frac{11}{2}\left[2\left(\frac{1}{15}\right) + (11-1)\frac{1}{60}\right]$

$= \frac{11}{2}\left[\frac{2}{15} + \frac{10}{60}\right]$

$= \frac{11}{2}\left[\frac{2}{15} + \frac{1}{6}\right] = \frac{11}{2}\left[\frac{4+5}{30}\right]$

$= \left(\frac{11}{2}\right)\left(\frac{9}{30}\right) = \frac{33}{20}$

Question 2:

Find the sums given below

(i) 7 + $10\frac{1}{2}$ + 14 + ………… + 84

(ii) 34 + 32 + 30 + ……….. + 10

(iii) − 5 + (− 8) + (− 11) + ………… + (− 230)

Answer:

(i)7 + $10\frac{1}{2}$ + 14 + …………+ 84

For this A.P.,

$a = 7$

$l = 84$

$$d = a_2 - a_1 = 10\frac{1}{2} - 7 = \frac{21}{2} - 7 = \frac{7}{2}$$

Let 84 be the n^{th} term of this A.P.

$l = a + (n - 1)d$

$$84 = 7 + (n-1)\frac{7}{2}$$

$$77 = (n-1)\frac{7}{2}$$

$22 = n - 1$

$n = 23$

We know that,

$$S_n = \frac{n}{2}(a+l)$$

$$S_n = \frac{23}{2}[7+84]$$

$$= \frac{23 \times 91}{2} = \frac{2093}{2}$$

$$= 1046\frac{1}{2}$$

(ii)34 + 32 + 30 + ……….. + 10

For this A.P.,

$a = 34$

$d = a_2 - a_1 = 32 - 34 = -2$

$l = 10$

Let 10 be the n^{th} term of this A.P.

$l = a + (n - 1)\, d$

$10 = 34 + (n - 1)(-2)$

$-24 = (n - 1)(-2)$

$12 = n - 1$

$n = 13$

$$S_n = \frac{n}{2}(a+l)$$

$$= \frac{13}{2}(34+10)$$

$$= \frac{13 \times 44}{2} = 13 \times 22$$

$$= 286$$

(iii)$(-5) + (-8) + (-11) + \ldots\ldots\ldots + (-230)$

For this A.P.,

$a = -5$

$l = -230$

$d = a_2 - a_1 = (-8) - (-5)$

$= -8 + 5 = -3$

Let -230 be the n^{th} term of this A.P.

$l = a + (n - 1)d$

$-230 = -5 + (n - 1)(-3)$

$-225 = (n - 1)(-3)$

$(n - 1) = 75$

$n = 76$

And, $S_n = \frac{n}{2}(a+l)$

$= \frac{76}{2}\left[(-5)+(-230)\right]$
$= 38(-235)$
$= -8930$

Question 3:

In an AP

(i) Given $a = 5$, $d = 3$, $a_n = 50$, find n and S_n.

(ii) Given $a = 7$, $a_{13} = 35$, find d and S_{13}.

(iii) Given $a_{12} = 37$, $d = 3$, find a and S_{12}.

(iv) Given $a_3 = 15$, $S_{10} = 125$, find d and a_{10}.

(v) Given $d = 5$, $S_9 = 75$, find a and a_9.

(vi) Given $a = 2$, $d = 8$, $S_n = 90$, find n and a_n.

(vii) Given $a = 8$, $a_n = 62$, $S_n = 210$, find n and d.

(viii) Given $a_n = 4$, $d = 2$, $S_n = -14$, find n and a.

(ix) Given $a = 3$, $n = 8$, $S = 192$, find d.

(x)Given $l = 28$, $S = 144$ and there are total 9 terms. Find a.

Answer:

(i) Given that, $a = 5$, $d = 3$, $a_n = 50$

As $a_n = a + (n - 1)d$,

$\therefore 50 = 5 + (n - 1)3$

$45 = (n - 1)3$

$15 = n - 1$

$n = 16$

$$S_n = \frac{n}{2}[a + a_n]$$

$$S_{16} = \frac{16}{2}[5 + 50]$$

$$= 8 \times 55$$

$$= 440$$

(ii) Given that, $a = 7$, $a_{13} = 35$

As $a_n = a + (n - 1)\, d$,

$\therefore\ a_{13} = a + (13 - 1)\, d$

$35 = 7 + 12\, d$

$35 - 7 = 12d$

$28 = 12d$

$$d = \frac{7}{3}$$

$$S_n = \frac{n}{2}[a + a_n]$$

$$S_{13} = \frac{n}{2}[a + a_{13}]$$

$$= \frac{13}{2}[7 + 35]$$

$$= \frac{13 \times 42}{2} = 13 \times 21$$

$$= 273$$

(iii)Given that, $a_{12} = 37$, $d = 3$

As $a_n = a + (n - 1)d$,

$a_{12} = a + (12 - 1)3$

$37 = a + 33$

$a = 4$

$$S_n = \frac{n}{2}\left[a + a_n\right]$$
$$S_n = \frac{12}{2}\left[4 + 37\right]$$
$$S_n = 6(41)$$
$$S_n = 246$$

(iv) Given that, $a_3 = 15$, $S_{10} = 125$

As $a_n = a + (n - 1)d$,

$a_3 = a + (3 - 1)d$

$15 = a + 2d$ (i)

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
$$S_{10} = \frac{10}{2}\left[2a + (10-1)d\right]$$
$$125 = 5(2a + 9d)$$
$$25 = 2a + 9d \qquad \text{(ii)}$$

On multiplying equation (1) by 2, we obtain

$30 = 2a + 4d$ (iii)

On subtracting equation (iii) from (ii), we obtain

$-5 = 5d$

$d = -1$

From equation (i),

$15 = a + 2(-1)$

$15 = a - 2$

$a = 17$

$a_{10} = a + (10 - 1)d$

$a_{10} = 17 + (9)(-1)$

$a_{10} = 17 - 9 = 8$

(v)Given that, $d = 5$, $S_9 = 75$

As $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$,

$S_9 = \frac{9}{2}\left[2a + (9-1)5\right]$

$75 = \frac{9}{2}(2a + 40)$

$25 = 3(a + 20)$

$25 = 3a + 60$

$3a = 25 - 60$

$a = \frac{-35}{3}$

$a_n = a + (n - 1)d$

$a_9 = a + (9 - 1)(5)$

$= \frac{-35}{3} + 8(5)$

$= \frac{-35}{3} + 40$

$= \frac{-35 + 120}{3} = \frac{85}{3}$

(vi) Given that, $a = 2$, $d = 8$, $S_n = 90$

As $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$,

$90 = \frac{n}{2}\left[4 + (n-1)8\right]$

$90 = n[2 + (n - 1)4]$

$90 = n[2 + 4n - 4]$

$90 = n(4n - 2) = 4n^2 - 2n$

$4n^2 - 2n - 90 = 0$

$4n^2 - 20n + 18n - 90 = 0$

$4n(n - 5) + 18(n - 5) = 0$

$(n - 5)(4n + 18) = 0$

Either $n - 5 = 0$ or $4n + 18 = 0$

$n = 5$ or $n = -\frac{18}{4} = \frac{-9}{2}$

However, n can neither be negative nor fractional.

Therefore, $n = 5$

$a_n = a + (n - 1)d$

$a_5 = 2 + (5 - 1)8$

$= 2 + (4)(8)$

$= 2 + 32 = 34$

(vii) Given that, $a = 8$, $a_n = 62$, $S_n = 210$

$$S_n = \frac{n}{2}[a + a_n]$$

$$210 = \frac{n}{2}[8 + 62]$$

$$210 = \frac{n}{2}(70)$$

$n = 6$

$a_n = a + (n - 1)d$

$62 = 8 + (6 - 1)d$

$62 - 8 = 5d$

$54 = 5d$

$d = \frac{54}{5}$

(viii) Given that, $a_n = 4$, $d = 2$, $S_n = -14$

$a_n = a + (n - 1)d$

$4 = a + (n - 1)2$

$4 = a + 2n - 2$

$a + 2n = 6$

$a = 6 - 2n$ (i)

$S_n = \frac{n}{2}[a + a_n]$

$-14 = \frac{n}{2}[a + 4]$

$-28 = n(a + 4)$

$-28 = n(6 - 2n + 4)$ {From equation (i)}

$-28 = n(-2n + 10)$

$-28 = -2n^2 + 10n$

$2n^2 - 10n - 28 = 0$

$n^2 - 5n - 14 = 0$

$n^2 - 7n + 2n - 14 = 0$

$n(n - 7) + 2(n - 7) = 0$

$(n - 7)(n + 2) = 0$

Either $n - 7 = 0$ or $n + 2 = 0$

$n = 7$ or $n = -2$

However, n can neither be negative nor fractional.

Therefore, $n = 7$

From equation (i), we obtain

$a = 6 - 2n$

$a = 6 - 2(7)$

$= 6 - 14$

$= -8$

(ix)Given that, $a = 3$, $n = 8$, $S = 192$

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$192 = \frac{8}{2}\left[2\times3+(8-1)d\right]$$

$192 = 4\ [6 + 7d]$

$48 = 6 + 7d$

$42 = 7d$

$d = 6$

(x)Given that, $l = 28$, $S = 144$ and there are total of 9 terms.

$$S_n = \frac{n}{2}(a+l)$$

$$144 = \frac{9}{2}(a+28)$$

$(16) \times (2) = a + 28$

$32 = a + 28$

$a = 4$

Question 4:

How many terms of the AP. 9, 17, 25 … must be taken to give a sum of 636?

Answer:

Let there be n terms of this A.P.

For this A.P., $a = 9$

$d = a_2 - a_1 = 17 - 9 = 8$

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$636 = \frac{n}{2}\left[2\times a+(n-1)8\right]$$

$$636 = \frac{n}{2}\left[18+(n-1)8\right]$$

$636 = n\,[9 + 4n - 4]$

$636 = n\,(4n + 5)$

$4n^2 + 5n - 636 = 0$

$4n^2 + 53n - 48n - 636 = 0$

$n\,(4n + 53) - 12\,(4n + 53) = 0$

$(4n + 53)\,(n - 12) = 0$

Either $4n + 53 = 0$ or $n - 12 = 0$

$n = \frac{-53}{4}$ or $n = 12$

n cannot be $-\frac{53}{4}$. As the number of terms can neither be negative nor fractional, therefore, $n = 12$ only.

Question 5:

The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Answer:

Given that,

$a = 5$

$l = 45$

$S_n = 400$

$$S_n = \frac{n}{2}(a+l)$$

$$400 = \frac{n}{2}(5+45)$$

$$400 = \frac{n}{2}(50)$$

$n = 16$

$l = a + (n - 1)\ d$

$45 = 5 + (16 - 1)\ d$

$40 = 15d$

$$d = \frac{40}{15} = \frac{8}{3}$$

Question 6:

The first and the last term of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Answer:

Given that,

$a = 17$

$l = 350$

$d = 9$

Let there be n terms in the A.P.

$l = a + (n - 1)\ d$

$350 = 17 + (n - 1)9$

333 = $(n - 1)9$

$(n - 1) = 37$

$n = 38$

$$S_n = \frac{n}{2}(a + l)$$

$$\Rightarrow S_n = \frac{38}{2}(17 + 350) = 19(367) = 6973$$

Thus, this A.P. contains 38 terms and the sum of the terms of this A.P. is 6973.

Question 7:

Find the sum of first 22 terms of an AP in which $d = 7$ and 22nd term is 149.

Answer:

$d = 7$

$a_{22} = 149$

$S_{22} = ?$

$a_n = a + (n - 1)d$

$a_{22} = a + (22 - 1)d$

$149 = a + 21 \times 7$

$149 = a + 147$

$a = 2$

$$S_n = \frac{n}{2}(a + a_n)$$

$$= \frac{22}{2}(2 + 149)$$

$$= 11(151) = 1661$$

Question 8:

Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Answer:

Given that,

$a_2 = 14$

$a_3 = 18$

$d = a_3 - a_2 = 18 - 14 = 4$

$a_2 = a + d$

$14 = a + 4$

$a = 10$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{51} = \frac{51}{2}\left[2 \times 10 + (51-1)4\right]$$

$$= \frac{51}{2}\left[20 + (50)(4)\right]$$

$$= \frac{51(220)}{2} = 51(110)$$

$= 5610$

Question 9:

If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Answer:

Given that,

$S_7 = 49$

$S_{17} = 289$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_7 = \frac{7}{2}\left[2a + (7-1)d\right]$$

$$49 = \frac{7}{2}(2a + 6d)$$

7 = (a + 3d)

a + 3d = 7 (i)

Similarly, $S_{17} = \frac{17}{2}\left[2a + (17-1)d\right]$

$$289 = \frac{17}{2}\left[2a + 16d\right]$$

17 = (a + 8d)

a + 8d = 17 (ii)

Subtracting equation (i) from equation (ii),

5d = 10

d = 2

From equation (i),

a + 3(2) = 7

a + 6 = 7

a = 1

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$= \frac{n}{2}\left[2(1)+(n-1)(2)\right]$$

$$= \frac{n}{2}(2+2n-2)$$

$$= \frac{n}{2}(2n)$$

$= n^2$

Question 10:

Show that $a_1, a_2 \ldots, a_n, \ldots$ form an AP where a_n is defined as below

(i) $a_n = 3 + 4n$

(ii) $a_n = 9 - 5n$

Also find the sum of the first 15 terms in each case.

Answer:

(i) $a_n = 3 + 4n$

$a_1 = 3 + 4(1) = 7$

$a_2 = 3 + 4(2) = 3 + 8 = 11$

$a_3 = 3 + 4(3) = 3 + 12 = 15$

$a_4 = 3 + 4(4) = 3 + 16 = 19$

It can be observed that

$a_2 - a_1 = 11 - 7 = 4$

$a_3 - a_2 = 15 - 11 = 4$

$a_4 - a_3 = 19 - 15 = 4$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, this is an AP with common difference as 4 and first term as 7.

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{15} = \frac{15}{2}\left[2(7) + (15-1)4\right]$$

$$= \frac{15}{2}\left[(14) + 56\right]$$

$$= \frac{15}{2}(70)$$

= 15 × 35

= 525

(ii) $a_n = 9 - 5n$

$a_1 = 9 - 5 \times 1 = 9 - 5 = 4$

$a_2 = 9 - 5 \times 2 = 9 - 10 = -1$

$a_3 = 9 - 5 \times 3 = 9 - 15 = -6$

$a_4 = 9 - 5 \times 4 = 9 - 20 = -11$

It can be observed that

$a_2 - a_1 = -1 - 4 = -5$

$a_3 - a_2 = -6 - (-1) = -5$

$a_4 - a_3 = -11 - (-6) = -5$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, this is an A.P. with common difference as −5 and first term as 4.

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{15} = \frac{15}{2}\left[2(4) + (15-1)(-5)\right]$$

$$= \frac{15}{2}\left[8 + 14(-5)\right]$$

$$= \frac{15}{2}(8-70)$$

$$= \frac{15}{2}(-62) = 15(-31)$$

$= -465$

Question 11:

If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly find the 3rd, the10th and the nth terms.

Answer:

Given that,

$S_n = 4n - n^2$

First term, $a = S_1 = 4(1) - (1)^2 = 4 - 1 = 3$

Sum of first two terms = S_2

$= 4(2) - (2)^2 = 8 - 4 = 4$

Second term, $a_2 = S_2 - S_1 = 4 - 3 = 1$

$d = a_2 - a = 1 - 3 = -2$

$a_n = a + (n - 1)d$

$= 3 + (n - 1)(-2)$

$= 3 - 2n + 2$

$= 5 - 2n$

Therefore, $a_3 = 5 - 2(3) = 5 - 6 = -1$

$a_{10} = 5 - 2(10) = 5 - 20 = -15$

Hence, the sum of first two terms is 4. The second term is 1. 3rd, 10th, and nth terms are −1, −15, and $5 - 2n$ respectively.

Question 12:

Find the sum of first 40 positive integers divisible by 6.

Answer:

The positive integers that are divisible by 6 are

6, 12, 18, 24 …

It can be observed that these are making an A.P. whose first term is 6 and common difference is 6.

$a = 6$

$d = 6$

$S_{40} = ?$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{40} = \frac{40}{2}\left[2(6) + (40-1)6\right]$$

= 20[12 + (39) (6)]

= 20(12 + 234)

= 20 × 246

= 4920

Question 13:

Find the sum of first 15 multiples of 8.

Answer:

The multiples of 8 are

8, 16, 24, 32…

These are in an A.P., having first term as 8 and common difference as 8.

Therefore, $a = 8$

$d = 8$

$S_{15} = ?$

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$
$$= \frac{15}{2}\left[2(8)+(15-1)8\right]$$
$$= \frac{15}{2}\left[16+14(8)\right]$$
$$= \frac{15}{2}(16+112)$$
$$= \frac{15(128)}{2} = 15\times 64$$

= 960

Question 14:

Find the sum of the odd numbers between 0 and 50.

Answer:

The odd numbers between 0 and 50 are

1, 3, 5, 7, 9 … 49

Therefore, it can be observed that these odd numbers are in an A.P.

$a = 1$

$d = 2$

$l = 49$

$l = a + (n - 1)\ d$

$49 = 1 + (n - 1)2$

$48 = 2(n - 1)$

$n - 1 = 24$

$n = 25$

$$S_n = \frac{n}{2}(a + l)$$

$$S_{25} = \frac{25}{2}(1 + 49)$$

$$= \frac{25(50)}{2} = (25)(25)$$

$= 625$

Question 15:

A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs. 200 for the first day, Rs. 250 for the second day, Rs. 300 for the third day, etc., the penalty for each succeeding day being Rs. 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days.

Answer:

It can be observed that these penalties are in an A.P. having first term as 200 and common difference as 50.

$a = 200$

$d = 50$

Penalty that has to be paid if he has delayed the work by 30 days = S_{30}

$=\frac{30}{2}\left[2(200)+(30-1)50\right]$

= 15 [400 + 1450]

= 15 (1850)

= 27750

Therefore, the contractor has to pay Rs 27750 as penalty.

Question 16:

A sum of Rs 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs 20 less than its preceding prize, find the value of each of the prizes.

Answer:

Let the cost of 1st prize be P.

Cost of 2nd prize = $P - 20$

And cost of 3rd prize = $P - 40$

It can be observed that the cost of these prizes are in an A.P. having common difference as −20 and first term as P.

$a = P$

$d = -20$

Given that, $S_7 = 700$

$\frac{7}{2}\left[2a+(7-1)d\right]=700$

$\frac{\left[2a+(6)(-20)\right]}{2}=100$

$a + 3(-20) = 100$

$a - 60 = 100$

$a = 160$

Therefore, the value of each of the prizes was Rs 160, Rs 140, Rs 120, Rs 100, Rs 80, Rs 60, and Rs 40.

Question 17:

In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of class I will plant 1 tree, a section of class II will plant 2 trees and so on till class XII. There are three sections of each class. How many trees will be planted by the students?

Answer:

It can be observed that the number of trees planted by the students is in an AP.

1, 2, 3, 4, 5………………..12

First term, $a = 1$

Common difference, $d = 2 - 1 = 1$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{12} = \frac{12}{2}\left[2(1) + (12-1)(1)\right]$$

$= 6 (2 + 11)$

$= 6 (13)$

$= 78$

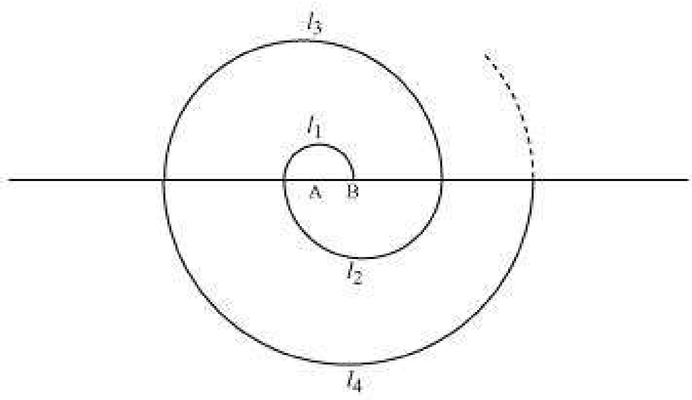
Therefore, number of trees planted by 1 section of the classes = 78

Number of trees planted by 3 sections of the classes = 3 × 78 = 234

Therefore, 234 trees will be planted by the students.

Question 18:

A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A of radii 0.5, 1.0 cm, 1.5 cm, 2.0 cm, ……… as shown in figure. What is the total length of such a spiral made up of thirteen consecutive semicircles? $\left(\text{Take } \pi = \frac{22}{7}\right)$



Answer:

Semi-perimeter of circle = π*r*

$I_1 = \pi(0.5) = \frac{\pi}{2}$ cm

$I_2 = \pi(1) = \pi$ cm

$I_3 = \pi(1.5) = \frac{3\pi}{2}$ cm

Therefore, I_1, I_2, I_3 ,i.e. the lengths of the semi-circles are in an A.P.,

$\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, \ldots\ldots\ldots\ldots$

$a = \frac{\pi}{2}$

$d = \pi - \frac{\pi}{2} = \frac{\pi}{2}$

$S_{13} = ?$

We know that the sum of *n* terms of an a A.P. is given by

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$= \frac{13}{2}\left[2\left(\frac{\pi}{2}\right) + (13-1)\left(\frac{\pi}{2}\right)\right]$$

$$= \frac{13}{2}\left[\pi + \frac{12\pi}{2}\right]$$

$$= \left(\frac{13}{2}\right)(7\pi)$$

$$= \frac{91\pi}{2}$$

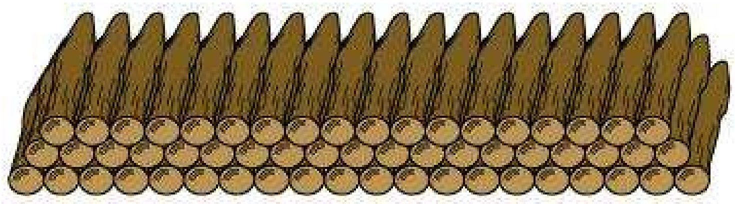
$$= \frac{91 \times 22}{2 \times 7} = 13 \times 11$$

= 143

Therefore, the length of such spiral of thirteen consecutive semi-circles will be 143 cm.

Question 19:

200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on. In how many rows are the 200 logs placed and how many logs are in the top row?



Answer:

It can be observed that the numbers of logs in rows are in an A.P.

20, 19, 18…

For this A.P.,

$a = 20$

$d = a_2 - a_1 = 19 - 20 = -1$

Let a total of 200 logs be placed in n rows.

$S_n = 200$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$200 = \frac{n}{2}\left[2(20) + (n-1)(-1)\right]$$

$400 = n(40 - n + 1)$

$400 = n(41 - n)$

$400 = 41n - n^2$

$n^2 - 41n + 400 = 0$

$n^2 - 16n - 25n + 400 = 0$

$n(n - 16) - 25(n - 16) = 0$

$(n - 16)(n - 25) = 0$

Either $(n - 16) = 0$ or $n - 25 = 0$

$n = 16$ or $n = 25$

$a_n = a + (n - 1)d$

$a_{16} = 20 + (16 - 1)(-1)$

$a_{16} = 20 - 15$

$a_{16} = 5$

Similarly,

$a_{25} = 20 + (25 - 1)(-1)$

$a_{25} = 20 - 24$

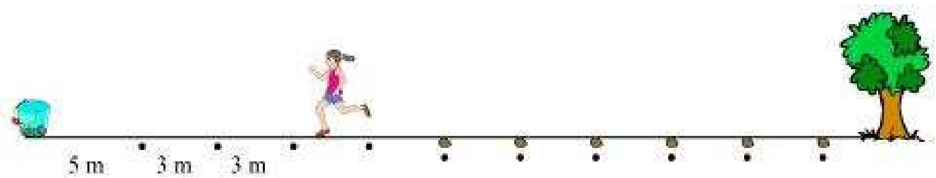
$= -4$

Clearly, the number of logs in 16^{th} row is 5. However, the number of logs in 25^{th} row is negative, which is not possible.

Therefore, 200 logs can be placed in 16 rows and the number of logs in the 16^{th} row is 5.

Question 20:

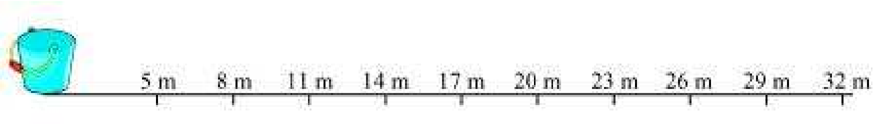
In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato and other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line.



A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

[Hint: to pick up the first potato and the second potato, the total distance (in metres) run by a competitor is 2 × 5 + 2 ×(5 + 3)]

Answer:



The distances of potatoes are as follows.

5, 8, 11, 14…

It can be observed that these distances are in A.P.

$a = 5$

$d = 8 - 5 = 3$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{10} = \frac{10}{2}\left[2(5) + (10-1)3\right]$$

= 5[10 + 9 × 3]

= 5(10 + 27) = 5(37)

= 185

As every time she has to run back to the bucket, therefore, the total distance that the competitor has to run will be two times of it.

Therefore, total distance that the competitor will run = 2 × 185

= 370 m

Alternatively,

The distances of potatoes from the bucket are 5, 8, 11, 14…

Distance run by the competitor for collecting these potatoes are two times of the distance at which the potatoes have been kept. Therefore, distances to be run are

10, 16, 22, 28, 34,……….

$a = 10$

$d = 16 - 10 = 6$

$S_{10} =?$

$$S_{10} = \frac{10}{2}\left[2\times10+(10-1)6\right]$$

= 5[20 + 54]

= 5 (74)

= 370

Therefore, the competitor will run a total distance of 370 m.

Exercise 5.4

Question 1:

Which term of the A.P. 121, 117, 113 … is its first negative term?

[Hint: Find n for $a_n < 0$]

Answer:

Given A.P. is 121, 117, 113 …

$a = 121$

$d = 117 - 121 = -4$

$a_n = a + (n - 1) d$

$= 121 + (n - 1)(-4)$

$= 121 - 4n + 4$

$= 125 - 4n$

We have to find the first negative term of this A.P.

$$\text{Therefore}, a_n < 0$$
$$125 - 4n < 0$$
$$125 < 4n$$
$$n > \frac{125}{4}$$
$$n > 31.25$$

Therefore, 32nd term will be the first negative term of this A.P.

Question 2:

The sum of the third and the seventh terms of an A.P is 6 and their product is 8. Find the sum of first sixteen terms of the A.P.

Answer:

We know that,

$a_n = a + (n - 1)d$

$a_3 = a + (3 - 1)d$

$a_3 = a + 2d$

Similarly, $a_7 = a + 6d$

Given that, $a_3 + a_7 = 6$

$(a + 2d) + (a + 6d) = 6$

$2a + 8d = 6$

$a + 4d = 3$

$a = 3 - 4d$ (*i*)

Also, it is given that $(a_3) \times (a_7) = 8$

$(a + 2d) \times (a + 6d) = 8$

From equation (*i*),

$$(3-4d+2d)\times(3-4d+6d)=8$$

$$(3-2d)\times(3+2d)=8$$

$$9-4d^2=8$$

$$4d^2=9-8=1$$

$$d^2=\frac{1}{4}$$

$$d=\pm\frac{1}{2}$$

$$d=\frac{1}{2}\text{ or }-\frac{1}{2}$$

From equation (*i*),

$$\left(\text{When } d \text{ is } \frac{1}{2}\right)$$

$$a=3-4d$$

$$a = 3 - 4\left(\frac{1}{2}\right)$$
$$= 3 - 2 = 1$$

$$\left(\text{When } d \text{ is } -\frac{1}{2}\right)$$

$$a = 3 - 4\left(-\frac{1}{2}\right)$$
$$a = 3 + 2 = 5$$

$$S_n = \frac{n}{2}\left[2a(n-1)d\right]$$

$$\left(\text{When } a \text{ is} 1 \text{ and } d \text{ is } \frac{1}{2}\right)$$

$$S_{16} = \frac{16}{2}\left[2(1) + (16-1)\left(\frac{1}{2}\right)\right]$$
$$= 8\left[2 + \frac{15}{2}\right]$$
$$= 4(19) = 76$$

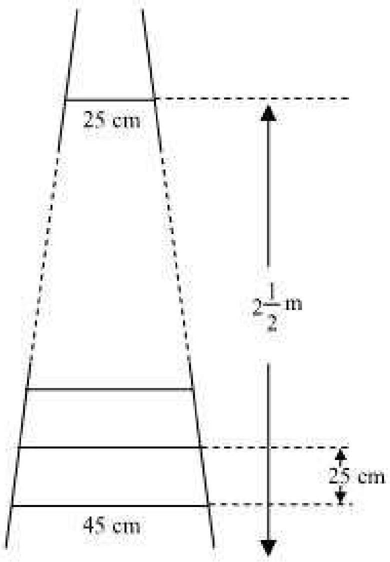
$$\left(\text{When } a \text{ is } 5 \text{ and } d \text{ is } -\frac{1}{2}\right)$$

$$S_{16} = \frac{16}{2}\left[2(5) + (16-1)\left(-\frac{1}{2}\right)\right]$$
$$= 8\left[10 + (15)\left(-\frac{1}{2}\right)\right]$$
$$= 8\left(\frac{5}{2}\right)$$
$$= 20$$

Question 3:

A ladder has rungs 25 cm apart. (See figure). The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and bottom rungs are $2\frac{1}{2}$ m apart, what is the length of the wood required for the rungs?

[Hint: number of rungs $=\frac{250}{25}$]



Answer:

It is given that the rungs are 25 cm apart and the top and bottom rungs are $2\frac{1}{2}$ m apart.

$\therefore$ Total number of rungs $=\dfrac{2\frac{1}{2}\times100}{25}+1=\dfrac{250}{25}+1=11$

Now, as the lengths of the rungs decrease uniformly, they will be in an A.P.

The length of the wood required for the rungs equals the sum of all the terms of this A.P.

First term, a = 45

Last term, l = 25

n = 11

$$S_n = \frac{n}{2}(a+l)$$

$$\therefore S_{10} = \frac{11}{2}(45+25) = \frac{11}{2}(70) = 385 \text{ cm}$$

Therefore, the length of the wood required for the rungs is 385 cm.

Question 4:

The houses of a row are number consecutively from 1 to 49. Show that there is a value of x such that the sum of numbers of the houses preceding the house numbered x is equal to the sum of the number of houses following it.

Find this value of x.

[Hint $S_{x-1} = S_{49} - S_x$]

Answer:

The number of houses was

1, 2, 3 … 49

It can be observed that the number of houses are in an A.P. having a as 1 and d also as 1.

Let us assume that the number of x^{th} house was like this.

We know that,

Sum of n terms in an A.P. $=\frac{n}{2}\left[2a+(n-1)d\right]$

Sum of number of houses preceding x^{th} house = S_{x-1}

$$=\frac{(x-1)}{2}\left[2a+(x-1-1)d\right]$$
$$=\frac{x-1}{2}\left[2(1)+(x-2)(1)\right]$$
$$=\frac{x-1}{2}[2+x-2]$$
$$=\frac{(x)(x-1)}{2}$$

Sum of number of houses following x^{th} house = $S_{49}-S_x$

$$=\frac{49}{2}\left[2(1)+(49-1)(1)\right]-\frac{x}{2}\left[2(1)+(x-1)(1)\right]$$
$$=\frac{49}{2}(2+49-1)-\frac{x}{2}(2+x-1)$$
$$=\left(\frac{49}{2}\right)(50)-\frac{x}{2}(x+1)$$
$$=25(49)-\frac{x(x+1)}{2}$$

It is given that these sums are equal to each other.

$$\frac{x(x-1)}{2}=25(49)-x\left(\frac{x+1}{2}\right)$$
$$\frac{x^2}{2}-\frac{x}{2}=1225-\frac{x^2}{2}-\frac{x}{2}$$
$$x^2=1225$$
$$x=\pm 35$$

However, the house numbers are positive integers.

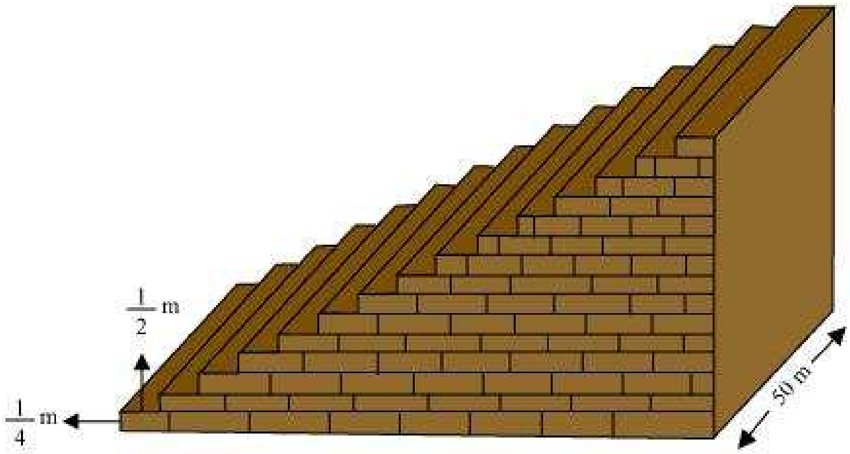
The value of x will be 35 only.

Therefore, house number 35 is such that the sum of the numbers of houses preceding the house numbered 35 is equal to the sum of the numbers of the houses following it.

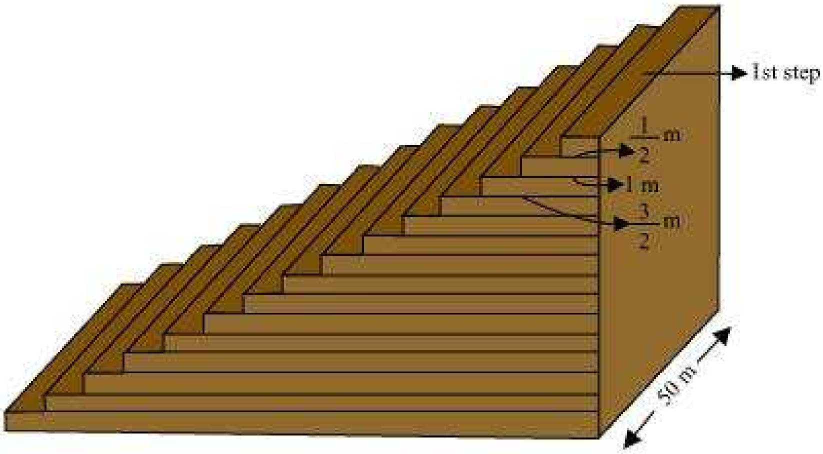
Question 5:

A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete.

Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m (See figure) calculate the total volume of concrete required to build the terrace.



Answer:



From the figure, it can be observed that

1^{st} step is $\frac{1}{2}$ m wide,

2^{nd} step is 1 m wide,

3^{rd} step is $\frac{3}{2}$ m wide.

Therefore, the width of each step is increasing by $\frac{1}{2}$ m each time whereas their height $\frac{1}{4}$ m and length 50 m remains the same.

Therefore, the widths of these steps are

$\frac{1}{2}, 1, \frac{3}{2}, 2, \ldots$

Volume of concrete in 1^{st} step $= \frac{1}{4} \times \frac{1}{2} \times 50 = \frac{25}{4}$

Volume of concrete in 2^{nd} step $= \frac{1}{4} \times 1 \times 50 = \frac{25}{2}$

Volume of concrete in 3rd step $= \frac{1}{4} \times \frac{3}{2} \times 50 = \frac{75}{4}$

It can be observed that the volumes of concrete in these steps are in an A.P.

$$\frac{25}{4}, \frac{25}{2}, \frac{75}{4}, \ldots$$

$$a = \frac{25}{4}$$

$$d = \frac{25}{2} - \frac{25}{4} = \frac{25}{4}$$

and $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$$S_{15} = \frac{15}{2}\left[2\left(\frac{25}{4}\right) + (15-1)\frac{25}{4}\right]$$

$$= \frac{15}{2}\left[\frac{25}{2} + \frac{(14)25}{4}\right]$$

$$= \frac{15}{2}\left[\frac{25}{2} + \frac{175}{2}\right]$$

$$= \frac{15}{2}(100) = 750$$

Volume of concrete required to build the terrace is 750 m^3.