Assignments in Mathematics Class X (Term I)

2. POLYNOMIALS

IMPORTANT TERMS, DEFINITIONS AND RESULTS

- An expression of the form

$$p(x) = a_a + a_1x + a_2x^2 + \ldots.a_nx^n,$$

where $ax^2 + bx + c,$ is called a polynomial in x of degree n.
Here, $a_o, a_1, a_2, \ldots a_n,$ are real numbers and each power of x is a non-negative integer.
- The exponent of the highest degree term in a polynomial is known as its degree. A polynomial of degree 0 is called a **constant polynomial.**
- A polynomial of degree 1 is called a **linear polynomial.** A linear polynomial is of the form $p(x) = ax + b,$ where $a \neq 0$,.
- A polynomial of degree 2 is called a **quadratic polynomial.** A quadratic polynomial is of the form $p(x) = ax^2 + bx + c,$ where $a \neq 0$,.
- A polynomial of degree 3 is called a **cubic polynomial.** A cubic polynomial is of the form $p(x) = ax^3 + bx^2 + cx + d,$ where $a \neq 0$,.
- A polynomial of degree 4 is called a **biquadratic polynomial.** A biquadratic polynomial is of the form $p(x) = ax^4 + bx^3 + cx^2 + dx + e,$ where $a \neq 0$,.
- If $p(x)$ is a polynomial in x and if α is any real number, then the value obtained by putting $x = \alpha$ in $p(x)$ is called the value of $p(x)$ at $x = \alpha$. The value of $p(x)$ at $x = \alpha$ is denoted by $p(\alpha)$.
- A real number α is called a zero of the polynomial $p(x)$, if $p(\alpha) = 0.$
- A polynomial of degree n can have at most n real zeroes.
- Geometrically the zeroes of a polynomial $p(x)$ are the x-coordinates of the points, where the graph of $p(\alpha) = 0.$ intersects x-axis.
- Zero of the linear polynomial $ax + b$ is

$$-\frac{b}{a} = \frac{-\text{constant term}}{\text{coefficient of } x}$$

- If α and β are the zeroes of a quadratic polynomial $p(x) = ax^2 + bx + c,\ a \neq 0,$ then

$$\alpha + \beta = -\frac{b}{a} = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2},$$

$$\alpha\beta = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

- If α, β and γ are the zeroes of a cubic polynomial $p(x) = ax^3 + bx^2 + cx + d,\ a \neq 0,$ then

$$\alpha + \beta + \gamma = \frac{-b}{a} = -\frac{\text{coefficient of } x^2}{\text{coefficient of } x^3}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{\text{coefficient of } x}{\text{coefficient of } x^3}$$

$$\alpha\beta\gamma = -\frac{d}{a} = -\frac{\text{constant term}}{\text{coefficient of } x^3}$$

- A quadratic polynomial whose zeroes are α, β is given by $p(x) = x^2 - (\alpha + \beta)x + \alpha\beta = x^2 -$ (sum of the zeroes) x + product of the zeroes.
- A cubic polynomial whose zeroes are α, β, γ is given by

$$p(x) = x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$$

$= x^3 -$ (sum of the zeroes)x^2
+ (sum of the products of the zeroes taken two at a time)x
– product of the zeroes.
- The division algorithm states that given any polynomial $p(x)$ and any non-zero polynomial $g(x)$, there are polynomial $q(x)$ and $r(x)$ such that $p(x) = g(x)q(x) + r(x)$, where $r(x) = 0$ or degree $r(x) <$ degree $g(x)$.

SUMMATIVE ASSESSMENT

MULTIPLE CHOICE QUESTIONS [1 Mark]

A. Important Questions

1. Which of the following is a polynomial?

(a) $x^2 - 6\sqrt{x} + 2$ (b) $\sqrt{x} + \frac{1}{\sqrt{x}}$

(c) $\frac{5}{x^2 - 3x + 1}$ (d) none of these

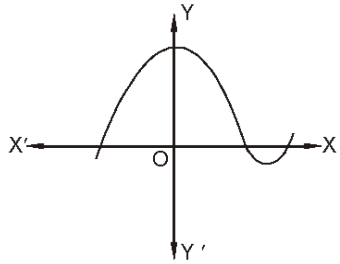
2. If $p(x) = 2x^2 - 3x + 5,$ then $p(-1)$ is equal to :

(a) 7 (b) 8 (c) 9 (d) 10

3. The zero of the polynomial $3x + 2$ is :

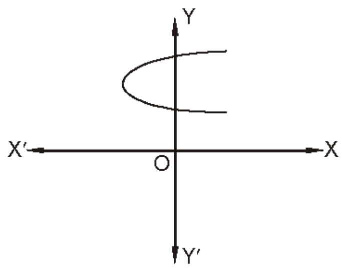
(a) $-\frac{2}{3}$ (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) $-\frac{3}{2}$

4. The following figure shows the graph of $y = p(x)$, where $p(x)$ is a polynomial. $p(x)$ has :



(a) 1 zero (b) 2 zeroes
(c) 3 zeroes (d) 4 zeroes

5. The following figure shows the graph of $y = p(x)$, where $p(x)$ is a polynomial. $p(x)$ has :



(a) no zero (b) 1 zero
(c) 2 zeroes (d) 3 zeroes

6. If zeroes of the quadratic polynomial $2x^2 - 8x - m$ are $\frac{5}{2}$ and $\frac{3}{2}$ respectively, then the value of m is

(a) $-\frac{15}{2}$ (b) $\frac{15}{2}$ (c) 2 (d) 15

7. If one zero of the quadratic polynomial $2x^2 - 8x - m$ is $\frac{5}{2}$, then the other zero is:

(a) $\frac{2}{3}$ (b) $-\frac{2}{3}$ (c) $\frac{3}{2}$ (d) $\frac{-15}{2}$

8. If α and β are zeroes of $x^2 + 5x + 8$, then the value of $\alpha + \beta$ is :

(a) 5 (b) –5 (c) 8 (d) –8

9. The sum and product of the zeroes of a quadratic polynomial are 2 and –15 respectively. The quadratic polynomial is :

(a) $x^2 - 2x + 15$ (b) $x^2 - 2x - 15$
(c) $x^2 + 2x - 15$ (d) $x^2 + 2x + 15$

10. If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - x - 4$, then the value of $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$ is :

(a) $\frac{15}{4}$ (b) $\frac{-15}{4}$ (c) 4 (d) 15

11. If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - p(x + 1) - c$, then $(\alpha + 1)(\beta + 1)$ is equal to :

(a) $1 + c$ (b) $1 - c$ (c) $c - 1$ (d) $2 + c$

12. If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - 5x + k$ such that $\alpha - \beta = 1$, then value of k is :

(a) 6 (b) 0 (c) 1 (d) –1

13. If α and β are the zeroes of the polynomial $f(x) = x^2 - p(x + 1) - c$ such that $(\alpha - 1)(\beta + 1) = 0$, then c is equal to:

(a) 1 (b) 0 (c) –1 (d) 2

14. The value of k such that the quadratic polynomial $x^2 - (k + 6)x + (2k + 1)$ has sum of the zeroes as half of their product is :

(a) 2 (b) 3 (c) –5 (d) 5

15. If α and β are the zeroes of the polynomial $p(x) = 4x^2 - 5x - 1$, then value of $\alpha^2\beta + \alpha\beta^2$ is :

(a) $-\frac{1}{4}$ (b) $\frac{1}{4}$ (c) $\frac{5}{16}$ (d) $-\frac{5}{16}$

16. If sum of the squares of zeroes of the quadratic polynomial $f(x) = x^2 - 8x + k$ is 40, the value of k is :

(a) 10 (b) 12 (c) 14 (d) 16

17. The graph of the polynomial $p(x)$ cuts the x-axis 5 times and touches it 3 times. The number of zeroes of $p(x)$ is :

(a) 5 (b) 3 (c) 8 (d) 2

18. If the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$ are 2 and –3, then :

(a) $a = -7, b = -1$ (b) $a = 5, b = -1$
(c) $a = 2, b = -6$ (d) $a = 0, b = -6$

19. The zeroes of the quadratic polynomial $x^2 + 89x + 720$ are :

(a) both are negative
(b) both are positive
(c) one is positive and one is negative
(d) both are equal

20. If the zeroes of the quadratic polynomial $ax^2 + bx + c, c \neq 0$, are equal, then :

(a) c and a have opposite signs
(b) c and b have opposite sign
(c) c and a have the same sign
(d) c and b have the same sign

21. If one of the zeroes of a quadratic polynomial of the form $x^2 + ax + b$ is the negative of the other, then it :

(a) has no linear term and the constant term is positive.

(b) has no linear term and the constant term is negative.

(c) can have a linear term but the constant term is negative.

(d) can have a linear term but the constant term is positive.

22. If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is :

(a) 10 (b) –10 (c) 5 (d) –5

23. A polynomial of degree 7 is divided by a polynomial of degree 4. Degree of the quotient is :

(a) less than 3 (b) 3

(c) more than 3 (d) more than 5

24. The number of zeroes, the polynomial $f(x) = (x - 3)^2 + 1$ can have is :

(a) 0 (b) 1 (c) 2 (d) 3

25. A polynomial of degree 7 is divided by a polynomial of degree 3. Degree of the remainder is :

(a) less than 2 (b) 3

(c) more than 2 (d) 2 or less than 2

26. If one of the zeroes of the quadratic polynomial $(k+1)x^2 + kx - 1$ is –3, then the value of k is :

(a) $\frac{4}{3}$ (b) $\frac{-4}{3}$ (c) $\frac{2}{3}$ (d) $\frac{-2}{3}$

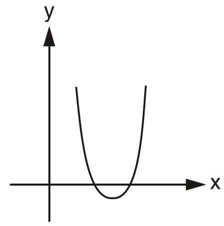
27. The graph of $y = f(x)$, where $f(x)$ is a quadratic polynomial meets the x-axis at $A(-2, 0)$ and $B(-3, 0)$, then the expression for $f(x)$ is :

(a) $x^2 + 5x + 6$ (b) $x^2 - 5x + 6$

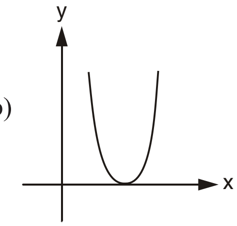
(c) $x^2 + 5x - 6$ (d) $x^2 - 5x - 6$

28. The graphs of $y = f(x)$, where $f(x)$ is a polynomial in x are given below. In which case $f(x)$ is not a quadratic polynomial?

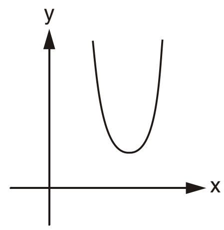
(a)



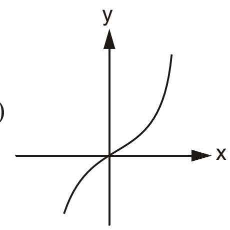
(b)



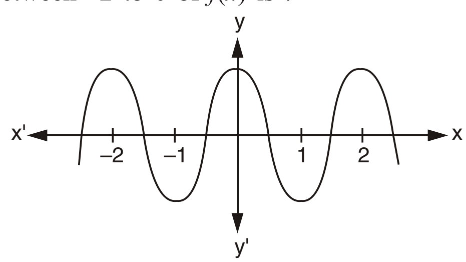
(c)



(d)



29. The graph of $y = f(x)$, where $f(x)$ is a polynomial in x is given below. The number of zeroes lying between –2 to 0 of $f(x)$ is :



(a) 3 (b) 6 (c) 2 (d) 4

B. Questions From CBSE Examination Papers

1. If one of the zeroes of the quadratic polynomial $(k - 1)x^2 + kx + 1$ is (–3), then k equal to : **[2010 (T-I)]**

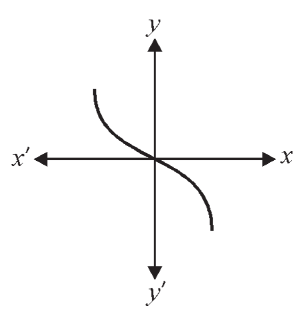
(a) $\frac{4}{3}$ (b) $-\frac{4}{3}$ (c) $\frac{2}{3}$ (d) $-\frac{2}{3}$

2. If α and β are the zeroes of the polynomial $5x^2 - 7x + 2$, then sum of their reciprocals is : **[2010 (T-I)]**

(a) $\frac{7}{2}$ (b) $\frac{7}{5}$ (c) $\frac{2}{5}$ (d) $\frac{14}{25}$

3. The graph of $y = f(x)$ is shown. The number of zeroes of $f(x)$ is : **[2010 (T-I)]**

(a) 3 (b) 1 (c) 0 (d) 2

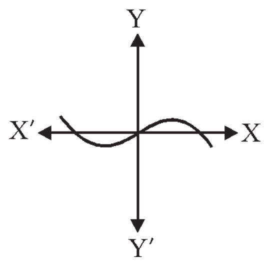


4. If α and β are the zeroes of the polynomial $4x^2 + 3x + 7$, then $\frac{1}{\alpha} + \frac{1}{\beta}$ is equal to : **010 (T-I)]**

(a) $\frac{7}{3}$ (b) $-\frac{7}{3}$ (c) $\frac{3}{7}$ (d) $-\frac{3}{7}$

5. The quadratic polynomial $p(x)$ with –81 and 3 as product and one of the zeroes respectively is : **[2010 (T-I)]**
(a) $x^2 + 24x - 81$ (b) $x^2 - 24x - 81$
(c) $x^2 - 24x + 81$ (d) $x^2 + 24x + 81$

6. The graph of $y = p(x)$, where $p(x)$ is a polynomial is shown. The number of zeroes of $p(x)$ is : **[2010 (T-I)]**



(a) 1 (b) 2 (c) 3 (d) 4

7. If α, β are zeroes of the polynomial $f(x) = x^2 + px + q$, then polynomial having $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ as its zeroes is : **[2010 (T-I)]**
(a) $x^2 + qx + p$ (b) $x^2 - px + q$
(c) $qx^2 + px + 1$ (d) $px^2 + qx + 1$

8. If α and β are zeroes of $x^2 - 4x + 1$, then $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$ is : **[2010 (T-I)]**
(a) 3 (b) 5 (c) –5 (d) –3

9. The quadratic polynomial having zeroes as 1 and –2 is : **[2010 (T-I)]**
(a) $x^2 - x + 2$ (b) $x^2 - x - 2$
(c) $x^2 + x - 2$ (d) $x^2 + x + 2$

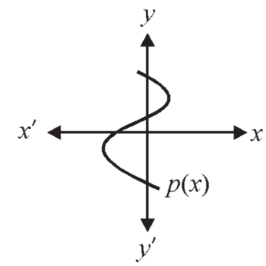
10. The value of p for which the polynomial $x^3 + 4x^2 - px + 8$ is exactly divisible by $(x - 2)$ is : **[2010 (T-I)]**
(a) 0 (b) 3 (c) 5 (d) 16

11. If 1 is a zero of the polynomial $p(x) = ax^2 - 3(a - 1)x - 1$, then the value of a is : **[2010 (T-I)]**
(a) 1 (b) –1 (c) 2 (d) –2

12. If –4 is a zero of the polynomial $x^2 - x - (2 + 2k)$, then the value of k is : **[2010 (T-I)]**
(a) 3 (b) 9 (c) 6 (d) –9

13. The degree of the polynomial $(x + 1)(x^2 - x - x^4 + 1)$ is : **[2010 (T-I)]**
(a) 2 (b) 3 (c) 4 (d) 5

14. The graph of $y = p(x)$, where $p(x)$ is a polynomial is shown. The number of zeroes of $p(x)$ is : **[2010 (T-I)]**



(a) 3 (b) 4 (c) 1 (d) 2

15. If α, β are zeroes of $x^2 - 6x + k$, what is the value of k if $3\alpha + 2\beta = 20$? **[2010 (T-I)]**
(a) –16 (b) 8 (c) 2 (d) –8

16. If one zero of $2x^2 - 3x + k$ is reciprocal to the other, then the value of k is : **[2010 (T-I)]**
(a) 2 (b) $\frac{-2}{3}$ (c) $\frac{-3}{2}$ (d) –3

17. The quadratic polynomial whose sum of zeroes is 3 and product of zeroes is –2 is : **[2010 (T-I)]**
(a) $x^2 + 3x - 2$ (b) $x^2 - 2x + 3$
(c) $x^2 - 3x + 2$ (d) $x^2 - 3x - 2$

18. If $(x + 1)$ is a factor of $x^2 - 3ax + 3a - 7$, then the value of a is : **[2010 (T-I)]**
(a) 1 (b) –1 (c) 0 (d) –2

19. The number of polynomials having zeroes –2 and 5 is : **[2010 (T-I)]**
(a) 1 (b) 2
(c) 3 (d) more than 3

20. The quadratic polynomial $p(y)$ with –15 and –7 as sum and one of the zeroes respectively is : **[2010 (T-I)]**
(a) $y^2 - 15y - 56$ (b) $y^2 - 15y + 56$
(c) $y^2 + 15y + 56$ (d) $y^2 + 15y - 56$

SHORT ANSWER TYPE QUESTIONS [2 Marks]

A. Important Questions

1. The graph of $y = f(x)$ cuts the x-axis at (1, 0) and $\left(\frac{-3}{2}, 0\right)$. Find all the zeroes of $f(x)$.

2. Show that 1, –1 and 3 are the zeroes of the polynomial $x^3 - 3x^2 - x + 3$.

3. For what value of k, (–4) is a zero of the polynomial $x^2 - x - (2k + 2)$?

4. If 1 is a zero of the polynomial $p(x) = ax^2 - 3(a - 1)x - 1$, then find the value of a.

5. Write the polynomial, the product and sum of whose zeroes are $\frac{-9}{2}$ and $\frac{-3}{2}$ respectively.

6. If $(x + a)$ is a factor of $2x^2 + 2ax + 5x + 10$, find a.
7. If α and β are the zeroes of the quadratic polynomial $f(t) = t^2 - 4t + 3$, find the value of $\alpha^4\beta^3 + \alpha^3\beta^4$
8. Write the zeroes of the polynomial $x^2 - x - 6$.
9. Find a quadratic polynomial, the sum and product of whose zeroes are 3 and 2 respectively.
10. Find a quadratic polynomial, the sum and product of whose zeroes are $\sqrt{2}$ and $\frac{1}{3}$ respectively.
11. Find a quadratic polynomial, the sum and product of whose zeroes are 0 and $\sqrt{5}$ respectively.

B. Questions From CBSE Examination Papers

1. Divide $6x^3 + 13x^2 + x - 2$ by $2x + 1$, and find the quotient and remainder. **[2010 (T-I)]**
2. Divide $x^4 - 3x^2 + 4x + 5$ by $x^2 - x + 1$, find quotient and remainder. **[2010 (T-I)]**
3. α, β are the roots of the quadratic polynomial $p(x) = x^2 - (k - 6)x + (2k + 1)$. Find the value of k, if $\alpha + \beta = \alpha\beta$. **[2010 (T-I)]**
4. α, β are the roots of the quadratic polynomial $p(x) = x^2 - (k + 6)x + 2(2k - 1)$. Find the value of k, if $\alpha + \beta = \frac{1}{2}\alpha\beta$. **[2010 (T-I)]**
5. Find the zeroes of the polynomial $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$. **[2010 (T-I)]**
6. Find a quadratic polynomial whose zeroes are $3 + \sqrt{5}$ and $3 - \sqrt{5}$. **[2010 (T-I)]**
7. What must be added to polynomial $f(x) = x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polynomial is exactly divisible by $x^2 + 2x - 3$. **[2010 (T-I)]**
8. Find a quadratic polynomial, the sum of whose zeroes is 7 and their product is 12. Hence find the zeroes of the polynomial. **[2010 (T-I)]**
9. Find a quadratic polynomial whose zeroes are 2 and –6. Verify the relation betweeen the coefficients and zeroes of the polynomial. **[2010 (T-I)]**
10. If α and $\frac{1}{\alpha}$ are the zeroes of the polynomial $4x^2 - 2x + (k - 4)$, find the value of k. **[2010 (T-I)]**
11. Find the zeroes of the polynomial $100x^2 - 81$. **[2010 (T-I)]**
12. Divide the polynomial $p(x) = 3x^2 - x^3 - 3x + 5$ by $g(x) = x - 1 - x^2$ and find its quotient and remainder. **[2010 (T-I)]**
13. Can $(x + 3)$ be the remainder on the division of a polynomial $p(x)$ by $(2x - 5)$? Justify your answer. **[2010 (T-I)]**
14. Can $(x - 3)$ be the remainder on division of a polynomial $p(x)$ by $(3x + 2)$? Justify your answer. **[2010 (T-I)]**
15. Find the zeroes of the polynomial $2x^2 - 7x + 3$ and hence find the sum of product of its zeroes. **[2010 (T-I)]**
16. It being given that 1 is one of the zeros of the polynomial $7x - x^3 - 6$. Find its other zeros. **[2010 (T-I)]**
17. Find the zeroes of the quadratic polynomial $\sqrt{3}x^2 - 8x + 4\sqrt{3}$. **[2010 (T-I)]**
18. Check whether $x^2 + 3x + 1$ is a factor of $3x^4 + 5x^3 - 7x^2 + 2x + 2$. **[2010 (T-I)]**
19. Check whether $x^2 - x + 1$ is a factor of $x^3 - 3x^2 + 3x - 2$. **[2010 (T-I)]**
20. Find the zeroes of the quadratic polynomial $x^2 + 7x + 12$ and verify the relationship between the zeroes and its coefficients. **[2010 (T-I)]**
21. Divide $(2x^2 + x - 20)$ by $(x + 3)$ and verify division algorithm. **[2010 (T-I)]**
22. If α and β are the zeroes of $x^2 + 7x + 12$, then find the value of $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$. **[2010 (T-I)]**
23. For what value of k, is –2 a zero of the polynomial $3x^2 + 4x + 2k$? **[2010 (T-I)]**
24. For what value of k, is –3 a zero of the polynomial $x^2 + 11x + k$? **[2010 (T-I)]**
25. If α and β are the zeroes of the polynomial $2y^2 + 7y + 5$, write the value of $\alpha + \beta + \alpha\beta$. **[2010 (T-I)]**
26. For what value of k, is 3 a zero of the polynomial $2x^2 + x + k$? **[2010 (T-I)]**
27. If the product of zeroes of the polynomial $ax^2 - 6x - 6$ is 4, find the value of a. **[2008]**
28. Find the quadratic polynomial, sum of whose zeroes is 8 and their product is 12. Hence, find the zeroes of the polynomial. **[2008]**
29. If one zero of the polynomial $(a^2 + 9)x^2 + 13x + 6a$ is reciprocal of the other, find the value of a. **[2008]**

SHORT ANSWER TYPE QUESTIONS [3 Marks]

A. Important Questions

1. Find the zeroes of the quadratic polynomial $f(x) = abx^2 + (b^2 - ac)x - bc$ and verify the relationship between the zeroes and its coefficients.

2. Find the zeroes of the quadratic polynomial $p(x) = x^2 - (\sqrt{3}+1)x+\sqrt{3}$ and verify the relationship between the zeroes and its coefficients.

3. Find a cubic polynomial with the sum, sum of the products of its zeroes taken two at a time and product of its zeroes as 3, –1 and –3 respectively.

4. If α and β are zeroes of the quadratic polynomial $f(x) = x^2 - 1$, find a quadratic polynomial whose zeroes and $\frac{2\alpha}{\beta}$ and $\frac{2\beta}{\alpha}$.

5. If α and β are zeroes of the quadratic polynomial $f(x) = kx^2 + 4x + 4$ such that $\alpha^2 + \beta^2 = 24$, find the value of k.

6. If the square of the difference of the zeroes of the quadratic polynomial $f(x) = x^2 + px + 45$ is equal to 144, find the value of p.

7. If the sum of the zeroes of the quadratic polynomial $f(t) = kt^2 + 2t + 3k$ is equal to their product, find the value of k.

8. If one zero of the quadratic polynomial $f(x) = 4x^2 - 8kx - 9$ is negative of the other, find the value of k.

9. Find the zeroes of the quadratic polynomial $x^2+\frac{7}{2}x+\frac{3}{4}$, and verify relationship between the zeroes and the coefficients.

10. Find the zeroes of the polynomial $x^2 - 5$ and verify the relationship between the zeroes and the coefficients.

11. Find the zeroes of the polynomial $4x^2+5\sqrt{2x}-3$ and verify the relationship between the zeroes and the coefficients.

12. Find the zeroes of the quadratic polynomial $3x^2-6-7x$ and verify relationship between the zeroes and the coefficients.

B. Questions From CBSE Examination Papers

1. If α and β are zeroes of the quadratic polynomial $x^2 - 6x + a$; find the value of a if $3\alpha+2\beta=20$. **[2010 (T-I)]**

2. Divide $(6 + 19x + x^2 - 6x^3)$ by $(2 + 5x - 3x^2)$ and verify the division algorithm. **[2010 (T-I)]**

3. If α, β, γ are zeroes of the polynomial $6x^3 + 3x^2 - 5x + 1$, then find the value of $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$. **[2010 (T-I)]**

4. If the zeroes of the polynomial $x^3 - 3x^2 + x + 1$ are $a - b$, a and $a + b$, find the values of a and b. **[2010 (T-I)]**

5. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $-2x+4$ respectively. Find $g(x)$. **[2010 (T-I)]**

6. If α, β are zeroes of the polynomial $x^2 - 2x - 8$, then form a quadratic polynomial whose zeroes are 2α and 2β. **[2010 (T-I)]**

7. If α, β are the zeroes of the polynomial $6y^2 - 7y + 2$, find a quadratic polynomial whose zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$. **[2010 (T-I)]**

8. If α, β are zeroes of the polynomial $x^2 - 4x + 3$, then form a quadratic polynomial whose zeroes are 3α and 3β. **[2010 (T-I)]**

9. Obtain all zeroes of $f(x) = x^4 - 3x^3 - x^2 + 9x - 6$ if two of its zeroes are $(-\sqrt{3})$ and $\sqrt{3}$. **[2010 (T-I)]**

10. Check whether the polynomial $g(x) = x^3 - 3x + 1$ is the factor of polynomial $p(x)=x^5-4x^3+x^2+3x+1$ **[2010 (T-I)]**

11. Find the zeroes of the quadratic polynomial $6x^2 - 3 - 7x$, and verify the relationship between the zeroes and the coefficients. **[2010 (T-I)]**

12. Find the zeroes of $4\sqrt{3}x^2+5x-2\sqrt{3}$ and verify the relation between the zeroes and coefficients of the polynomial. **[2010 (T-I)]**

13. If α, β are the zeroes of the polynomial $25p^2 - 15p + 2$, find a quadratic polynomial whose zeroes are $\frac{1}{2\alpha}$ and $\frac{1}{2\beta}$. **[2010 (T-I)]**

14. Divide $3x^2 - x^3 - 3x + 5$ by $x - 1 - x^2$ and verify the division algorithm.

15. If α, β are the zeroes of the polynomial $21y^2 - y - 2$, find a quadratic polynomial whose zeroes are 2α and 2β. **[2010 (T-I)]**

16. Find the zeroes of $3\sqrt{2}x^2+13x+6\sqrt{2}$ and verify the relation between the zeroes and coefficients of the polynomial. **[2010 (T-I)]**

17. Find the zeroes of $4\sqrt{5}x^2+17x+3\sqrt{5}$ and verify the relation between the zeroes and coefficients of the polynomial. **[2010 (T-I)]**

18. If the polynomial $6x^4 + 8x^3 + 17x^2 + 21x + 7$ is divided by another polynomial $3x^2 + 4x + 1$, the remainder comes out to be $(ax + b)$, find a and b. **[2009]**

19. If the polynomial $x^4 + 2x^3 + 8x^2 + 12x + 18$ is divided by another polynomial $x^2 + 5$, the remainder comes out to be $px + q$. Find the values of p and q. **[2009]**

20. Find all the zeroes of the polynomial $x^3 + 3x^2 - 2x - 6$, if two of its zeroes are $-\sqrt{2}$ and $\sqrt{2}$. **[2009]**

21. Find all the zeroes of the polynomial $2x^3 + x^2 - 6x - 3$, if two of its zeroes are $-\sqrt{3}$ and $\sqrt{3}$. **[2009]**

LONG ANSWER TYPE QUESTIONS [4 Marks]

A. Important Questions

1. If α and β are the zeroes of the quadratic polynomial $p(s) = 3s^2 - 6s + 4$, find the value of $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+3\alpha\beta.$

2. If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - px + q$, prove that $\frac{\alpha^2}{\beta^2}+\frac{\beta^2}{\alpha^2}=\frac{p^4}{q^2}-\frac{4p^2}{q}+2.$

3. If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 + px + q$, form a polynomial whose zeroes are $(\alpha + \beta)^2$ and $(\alpha - \beta)^2$.

4. If α and β are the zeroes of the polynomial $f(x) = x^2 - 2x + 3$, find a polynomial whose zeroes are $\alpha+2$ and $\alpha+\beta$

5. Obtain all the zeroes of the polynomial $f(x) = 2x^4 + x^3 - 14x^2 - 19x - 6$, if two of its zeroes are -2 and -1.

6. Find the value of k for which the polynomial $x^4 + 10x^3 + 25x^2 + 15x + k$ is exactly divisible by $x+7$.

7. Find the value of p for which the polynomial $x^3 + 4x^2 - px + 8$ is exactly divisible by $x - 2$.

8. What must be added to $6x^5 + 5x^4 + 11x^3 - 3x^2 + x + 5$ so that it may be exactly divisible by $3x^2-2x+4$?

9. What must be subtracted from the polynomial $f(x) = x^4 + 2x^3 - 13x^2 - 12x + 21$ so that the resulting polynomial is exactly divisible by $g(x) = x^2 - 4x + 3$?

B. Questions From CBSE Examination Papers

1. What must be added to the polynomial $f(x) = x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polynomial is exactly divisible by $x^2 + 2x - 3$? **[2010 (T-I)]**

2. Find the other zeroes of the polynomial $2x^4 - 3x^3 - 3x^2 + 6x - 2$, if $\sqrt{}$ and $\sqrt{}$ are the zeroes of the given polynomial. **[2010 (T-I)]**

3. If the remainder on division of $x^3 + 2x^2 + kx + 3$ by $x - 3$ is 21, find the quotient and the value of k. Hence, find the zeroes of the cubic polynomial $x^3 + 2x^2 + kx - 18$. **[2010 (T-I)]**

4. If two zeroes of $p(x) = x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2\pm\sqrt{3}$, find the other zeroes. **[2010 (T-I)]**

5. If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be $(x + a)$, find the values of k and a. **[2010 (T-I)]**

6. Find all the zeroes of the polynomial $2x^4 + 7x^3 - 19x^2 - 14x + 30$, if two of its zeros are $\sqrt{2},-\sqrt{2}$. **[2010 (T-I)]**

7. Find other zeroes of the polynomial $x^4 + x^3 - 9x^2 - 3x + 18$, if it is given that two of its zeroes are $\sqrt{3}$ and $-\sqrt{3}$. **[2010 (T-I)]**

8. Divide $2x^4 - 9x^3 + 5x^2 + 3x - 8$ by $x^2 - 4x + 1$ and verify the division algorithm. **[2010 (T-I)]**

9. Divide $30x^4 + 11x^3 - 82x^2 - 12x + 48$ by $(3x^2 + 2x - 4)$ and verify the result by division algorithm. **[2010 (T-I)]**

10. Find all zeroes of the polynomial $4x^4 - 20x^3 + 23x^2 + 5x - 6$, if two of its zeroes are 2 and 3. **[2010 (T-I)]**

11. Find all the zeroes of the polynomial $2x^4 - 10x^3 + 5x^2 + 15x - 12$, if it is given that two of its zeroes are $\sqrt{\frac{3}{2}}$ and $-\sqrt{\frac{3}{2}}$. **[2010 (T-I)]**

12. Find all the zeroes of the polynomial $2x^4 - 3x^3 - 5x^2 + 9x - 3$, it being given that two of its zeros are $\sqrt{3}$ and $-\sqrt{3}$. **[2010 (T-I)]**

13. Obtain all the zeroes of $x^4 - 7x^3 + 17x^2 - 17x + 6$, if two of its zeroes are 1 and 2. **[2010 (T-I)]**

14. Find all other zeroes of the polynomial $p(x) = 2x^3 + 3x^2 - 11x - 6$, if one of its zero is -3. **[2010 (T-I)]**

15. What must be added to the polynomial $P(x) = 5x^4 + 6x^3 - 13x^2 - 44x + 7$ so that the resulting polynomial is exactly divisible by the polynomial $Q(x) = x^2 + 4x + 3$ and the degree of the polynomial to be added must be less than degree of the polynomial $Q(x)$. **[2010 (T-I)]**

16. Find all the zeroes of the polynomial $x^4 + x^3 - 34x^2 - 4x + 120$, if two of its zeroes are 2 and -2. **[2009]**

17. If the polynomial $6x^4 + 8x^3 - 5x^2 + ax + b$ is exactly divisible by the polynomial $2x^2 - 5$, then find the values of a and b. **[2009]**

FORMATIVE ASSESSMENT

Activity

Objective : To understand the geometrical meaning of the zeroes of a polynomial.

Materials Required : Graphs of different polynomials, paper etc.

Procedure :

1. Let us consider a linear equation $y = 5x - 10$.
   Fig.1 shows graph of this equation. We will find zero/zeroes of linear polynomial $5x - 10$.

   $$5x - 10 = 0 \Rightarrow x = \frac{10}{5} = 2 \Rightarrow x = 2 \text{ is a zero of } 5x - 10.$$

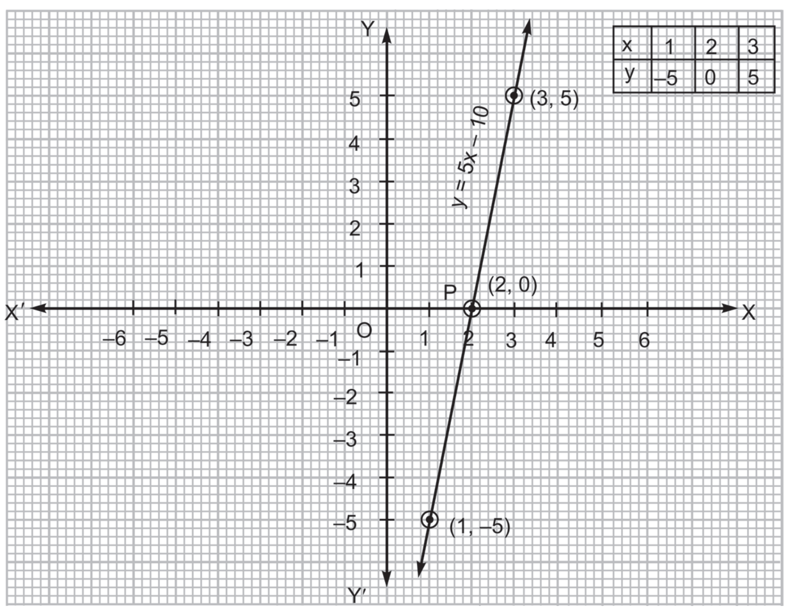


Figure 1

2. From graph in Fig.1, the line intersects the x-axis at one point, whose coordinates are (2, 0)
3. Also, the zero of the polynomial $5x - 10$ is 2. Thus, we can say that the zero of the polynomial $5x - 10$ is the x coordinate (abscissa) of the point where the line $y = 5x - 10$ cuts the x-axis.
4. Let us consider a quadratic equation $y = x^2 - 5x + 6$. Fig. 2 shows graph of this equation.
5. From graph in Fig. 2, the curve intersects the x-axis at two points P and Q, coordinates of P and Q are (2, 0) and (3, 0) respectively.
6. $x^2 - 5x + 6 = 0 \Rightarrow (x - 3)(x - 2) = 0 \Rightarrow x = 3$ and $x = 2 \Rightarrow x = 2$ and 3 are zeroes of the polynomial $x^2 - 5x + 6$.

Thus, we can say that the zeroes of the polynomial $x^2 - 5x + 6$ are the x-coordinates (abscissa) of the points where the graph of $y = x^2 - 5x + 6$ cuts the x-axis.

7. Complete the following table by observing graphs shown in Fig. 3 (*a*), 3 (*b*) and 3 (*c*).

Fig. No.	**No. of zeroes**	***x*-coordinates**
3 (*a*)		
3 (*b*)		
3 (*c*)		

Result : A polynomial of degree *n* has atmost *n*-zeroes.

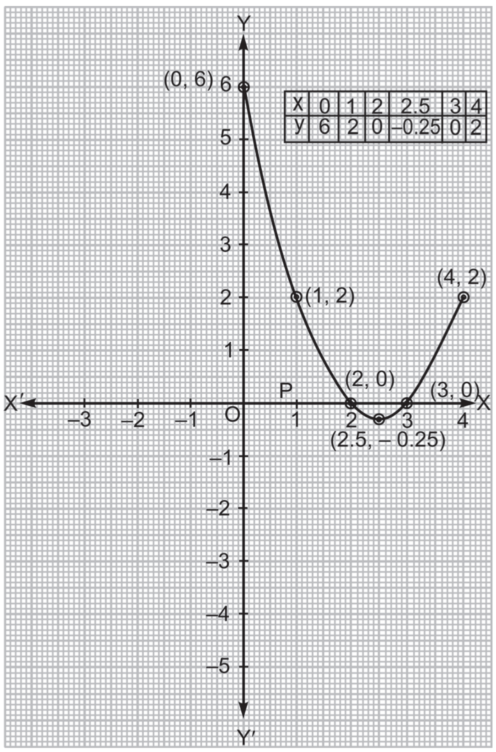


Figure 2

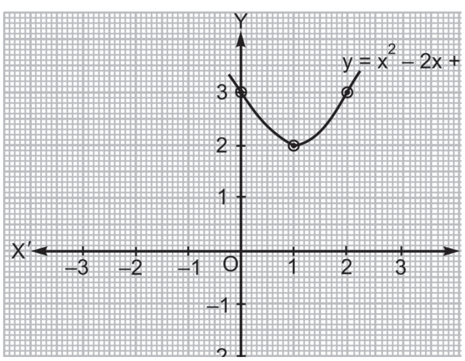


Figure 3(a)

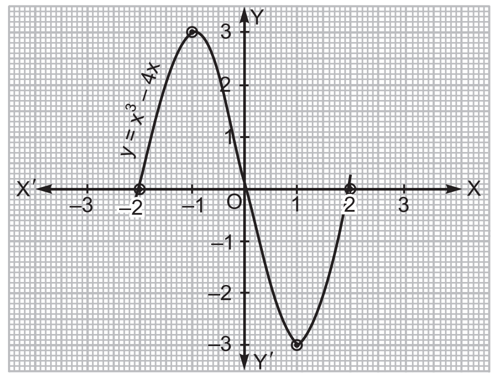


Figure 3(b)

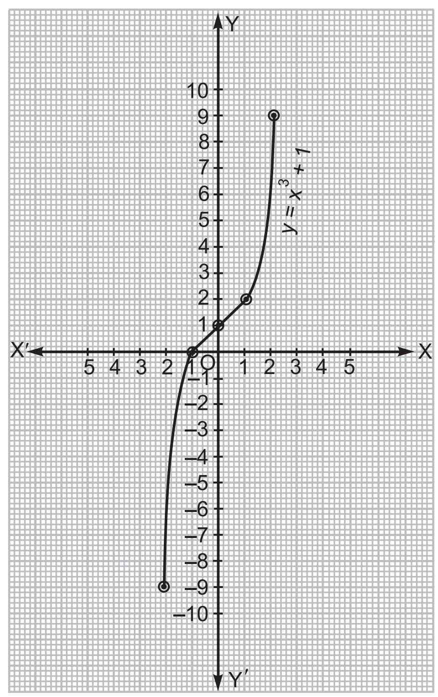


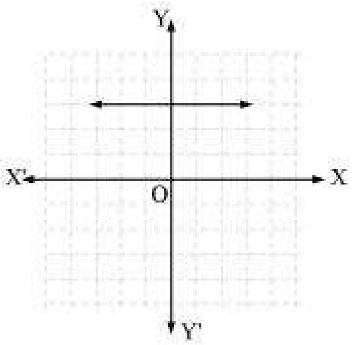
Figure 3(c)

Exercise 2.1

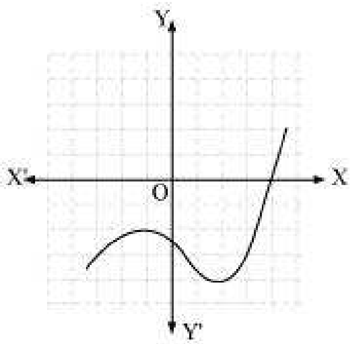
Question 1:

The graphs of $y = p(x)$ are given in following figure, for some polynomials $p(x)$. Find the number of zeroes of $p(x)$, in each case.

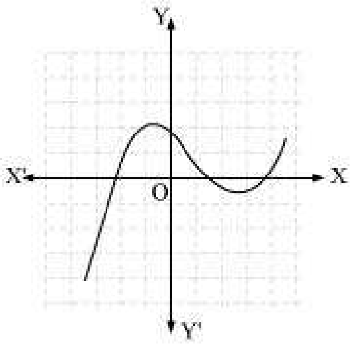
(i)



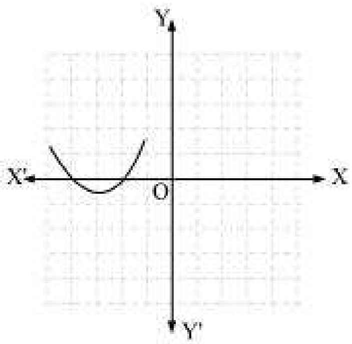
(ii)



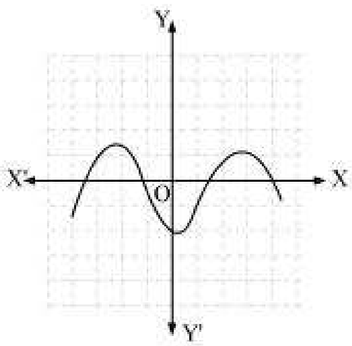
(iii)



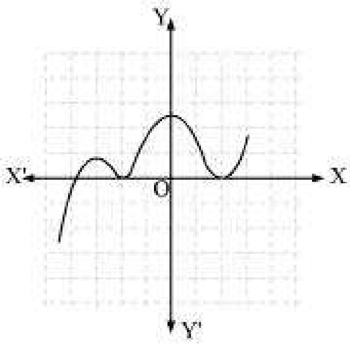
(iv)



(v)



(v)



Answer:

(i) The number of zeroes is 0 as the graph does not cut the *x*-axis at any point.

(ii) The number of zeroes is 1 as the graph intersects the *x*-axis at only 1 point.

(iii) The number of zeroes is 3 as the graph intersects the *x*-axis at 3 points.

(iv) The number of zeroes is 2 as the graph intersects the *x*-axis at 2 points.

(v) The number of zeroes is 4 as the graph intersects the *x*-axis at 4 points.

(vi) The number of zeroes is 3 as the graph intersects the *x*-axis at 3 points.

Exercise 2.2

Question 1:

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

$(\text{i})\, x^2-2x-8$ $(\text{ii})\, 4s^2-4s+1$ $(\text{iii})\, 6x^2-3-7x$

$(\text{iv})\, 4u^2+8u$ $(\text{v})\, t^2-15$ $(\text{vi})\, 3x^2-x-4$

Answer:

(i) $x^2-2x-8=(x-4)(x+2)$

The value of x^2-2x-8 is zero when x − 4 = 0 or x + 2 = 0, i.e., when x = 4 or x = −2

Therefore, the zeroes of x^2-2x-8 are 4 and −2.

Sum of zeroes = $4-2=2=\frac{-(-2)}{1}=\frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes $=4\times(-2)=-8=\frac{(-8)}{1}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

(ii) $4s^2-4s+1=(2s-1)^2$

The value of $4s^2 - 4s + 1$ is zero when $2s - 1 = 0$, i.e., $s=\frac{1}{2}$

Therefore, the zeroes of $4s^2 - 4s + 1$ are $\frac{1}{2}$ and $\frac{1}{2}$.

Sum of zeroes = $\frac{1}{2}+\frac{1}{2}=1=\frac{-(-4)}{4}=\frac{-(\text{Coefficient of } s)}{(\text{Coefficient of } s^2)}$

Product of zeroes $= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \frac{\text{Constant term}}{\text{Coefficient of } s^2}$

(iii) $6x^2 - 3 - 7x = 6x^2 - 7x - 3 = (3x+1)(2x-3)$

The value of $6x^2 - 3 - 7x$ is zero when $3x + 1 = 0$ or $2x - 3 = 0$, i.e.,

$x = \frac{-1}{3}$ or $x = \frac{3}{2}$

Therefore, the zeroes of $6x^2 - 3 - 7x$ are $\frac{-1}{3}$ and $\frac{3}{2}$.

Sum of zeroes = $\frac{-1}{3} + \frac{3}{2} = \frac{7}{6} = \frac{-(-7)}{6} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes = $\frac{-1}{3} \times \frac{3}{2} = \frac{-1}{2} = \frac{-3}{6} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

(iv) $4u^2 + 8u = 4u^2 + 8u + 0$

$= 4u(u+2)$

The value of $4u^2 + 8u$ is zero when $4u = 0$ or $u + 2 = 0$, i.e., $u = 0$ or $u = -2$

Therefore, the zeroes of $4u^2 + 8u$ are 0 and -2.

Sum of zeroes = $0 + (-2) = -2 = \frac{-(8)}{4} = \frac{-(\text{Coefficient of } u)}{\text{Coefficient of } u^2}$

Product of zeroes = $0 \times (-2) = 0 = \frac{0}{4} = \frac{\text{Constant term}}{\text{Coefficient of } u^2}$

(v) $t^2 - 15$

$= t^2 - 0t - 15$

$= (t - \sqrt{15})(t + \sqrt{15})$

The value of $t^2 - 15$ is zero when $t-\sqrt{15}=0$ or $t+\sqrt{15}=0$, i.e., when $t=\sqrt{15}$ or $t=-\sqrt{15}$

Therefore, the zeroes of $t^2 - 15$ are $\sqrt{15}$ and $-\sqrt{15}$.

Sum of zeroes = $\sqrt{15}+\left(-\sqrt{15}\right)=0=\frac{-0}{1}=\frac{-(\text{Coefficient of } t)}{\left(\text{Coefficient of } t^2\right)}$

Product of zeroes = $\left(\sqrt{15}\right)\left(-\sqrt{15}\right)=-15=\frac{-15}{1}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

(vi) $3x^2-x-4$

$=(3x-4)(x+1)$

The value of $3x^2 - x - 4$ is zero when $3x - 4 = 0$ or $x + 1 = 0$, i.e., when $x=\frac{4}{3}$ or $x = -1$

Therefore, the zeroes of $3x^2 - x - 4$ are $\frac{4}{3}$ and -1.

Sum of zeroes = $\frac{4}{3}+(-1)=\frac{1}{3}=\frac{-(-1)}{3}=\frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes $=\frac{4}{3}(-1)=\frac{-4}{3}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Question 2:

Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i) $\frac{1}{4},-1$ (ii) $\sqrt{2},\frac{1}{3}$ (iii) $0,\sqrt{5}$

(iv) $1,1$ (v) $-\frac{1}{4},\frac{1}{4}$ (vi) $4,1$

Answer:

(i) $\frac{1}{4}, -1$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α and β.

$$\alpha + \beta = \frac{1}{4} = \frac{-b}{a}$$
$$\alpha\beta = -1 = \frac{-4}{4} = \frac{c}{a}$$
If $a = 4$, then $b = -1$, $c = -4$

Therefore, the quadratic polynomial is $4x^2 - x - 4$.

(ii) $\sqrt{2}, \frac{1}{3}$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α and β.

$$\alpha + \beta = \sqrt{2} = \frac{3\sqrt{2}}{3} = \frac{-b}{a}$$
$$\alpha\beta = \frac{1}{3} = \frac{c}{a}$$
If $a = 3$, then $b = -3\sqrt{2}$, $c = 1$

Therefore, the quadratic polynomial is $3x^2 - 3\sqrt{2}x + 1$.

(iii) $0, \sqrt{5}$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α and β.

$$\alpha + \beta = 0 = \frac{0}{1} = \frac{-b}{a}$$
$$\alpha \times \beta = \sqrt{5} = \frac{\sqrt{5}}{1} = \frac{c}{a}$$
If $a = 1$, then $b = 0$, $c = \sqrt{5}$

Therefore, the quadratic polynomial is $x^2+\sqrt{5}$.

(iv) 1, 1

Let the polynomial be ax^2+bx+c, and its zeroes be α and β.

$$\alpha+\beta=1=\frac{1}{1}=\frac{-b}{a}$$
$$\alpha\times\beta=1=\frac{1}{1}=\frac{c}{a}$$
If $a=1$, then $b=-1$, $c=1$

Therefore, the quadratic polynomial is x^2-x+1.

(v) $-\frac{1}{4},\frac{1}{4}$

Let the polynomial be ax^2+bx+c, and its zeroes be α and β.

$$\alpha+\beta=\frac{-1}{4}=\frac{-b}{a}$$
$$\alpha\times\beta=\frac{1}{4}=\frac{c}{a}$$
If $a=4$, then $b=1$, $c=1$

Therefore, the quadratic polynomial is $4x^2+x+1$.

(vi) 4, 1

Let the polynomial be ax^2+bx+c.

$$\alpha+\beta=4=\frac{4}{1}=\frac{-b}{a}$$
$$\alpha\times\beta=1=\frac{1}{1}=\frac{c}{a}$$
If $a=1$, then $b=-4$, $c=1$

Therefore, the quadratic polynomial is x^2-4x+1.

Exercise 2.3

Question 1:

Divide the polynomial $p(x)$ by the polynomial $g(x)$ and find the quotient and remainder in each of the following:

(i) $p(x)=x^3-3x^2+5x-3, \qquad g(x)=x^2-2$

(ii) $p(x)=x^4-3x^2+4x+5, \qquad g(x)=x^2+1-x$

(iii) $p(x)=x^4-5x+6, \qquad g(x)=2-x^2$

Answer:

(i) $p(x)=x^3-3x^2+5x-3$

$q(x)=x^2-2$

$$
\begin{array}{r|l}
 & x-3 \\
\hline
x^2-2 & x^3-3x^2+5x-3 \\
 & x^3 \qquad\quad -2x \\
 & - \qquad\quad\;\; + \\
\hline
 & -3x^2+7x-3 \\
 & -3x^2 \qquad +6 \\
 & + \qquad\qquad - \\
\hline
 & 7x-9 \\
\hline
\end{array}
$$

Quotient = $x - 3$

Remainder = $7x - 9$

(ii) $p(x)=x^4-3x^2+4x+5=x^4+0.x^3-3x^2+4x+5$

$q(x)=x^2+1-x=x^2-x+1$

$$\begin{array}{r|l} & x^2+x-3 \\ \hline x^2-x+1 & x^4+0.x^3-3x^2+4x+5 \\ & x^4-x^3+x^2 \\ & -\quad+\quad- \\ \hline & x^3-4x^2+4x+5 \\ & x^3-x^2+x \\ & -\quad+\quad- \\ \hline & -3x^2+3x+5 \\ & -3x^2+3x-3 \\ & +\quad-\quad+ \\ \hline & 8 \\ \hline \end{array}$$

Quotient = $x^2 + x - 3$

Remainder = 8

(iii) $p(x) = x^4 - 5x + 6 = x^4 + 0.x^2 - 5x + 6$

$q(x) = 2 - x^2 = -x^2 + 2$

$$\begin{array}{r|l} & -x^2-2 \\ \hline -x^2+2 & x^4+0.x^2-5x+6 \\ & x^4-2x^2 \\ & -\quad+ \\ \hline & 2x^2-5x+6 \\ & 2x^2\qquad-4 \\ & -\qquad+ \\ \hline & -5x+10 \\ \hline \end{array}$$

Quotient = $-x^2 - 2$

Remainder = $-5x + 10$

Question 2:

Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

(i) $t^2-3, 2t^4+3t^3-2t^2-9t-12$

(ii) $x^2+3x+1, 3x^4+5x^3-7x^2+2x+2$

(iii) $x^3-3x+1, x^5-4x^3+x^2+3x+1$

Answer:

(i) $t^2-3,\ 2t^4+3t^3-2t^2-9t-12$

$t^2-3 = t^2+0.t-3$

$$\begin{array}{r|l}
 & 2t^2+3t+4 \\
\hline
t^2+0.t-3 & 2t^4+3t^3-2t^2-9t-12 \\
 & 2t^4+0.t^3-6t^2 \\
 & - \quad - \quad + \\
\hline
 & 3t^3+4t^2-9t-12 \\
 & 3t^3+0.t^2-9t \\
 & - \quad - \quad + \\
\hline
 & 4t^2+0.t-12 \\
 & 4t^2+0.t-12 \\
 & - \quad - \quad + \\
\hline
 & 0 \\
\hline
\end{array}$$

Since the remainder is 0,

Hence, t^2-3 is a factor of $2t^4+3t^3-2t^2-9t-12$.

(ii) $x^2+3x+1,\ 3x^4+5x^3-7x^2+2x+2$

$$
\begin{array}{r|l}
 & 3x^2-4x+2 \\
\hline
x^2+3x+1 & 3x^4+5x^3-7x^2+2x+2 \\
 & 3x^4+9x^3+3x^2 \\
 & - \quad - \quad - \\
\hline
 & -4x^3-10x^2+2x+2 \\
 & -4x^3-12x^2-4x \\
 & + \quad + \quad + \\
\hline
 & 2x^2+6x+2 \\
 & 2x^2+6x+2 \\
\hline
 & 0 \\
\hline
\end{array}
$$

Since the remainder is 0,

Hence, x^2+3x+1 is a factor of $3x^4+5x^3-7x^2+2x+2$.

(iii) $x^3-3x+1,\ x^5-4x^3+x^2+3x+1$

$$
\begin{array}{r|l}
 & x^2-1 \\
\hline
x^3-3x+1 & x^5-4x^3+x^2+3x+1 \\
 & x^5-3x^3+x^2 \\
 & - \quad + \quad - \\
\hline
 & -x^3 \qquad +3x+1 \\
 & -x^3 \qquad +3x-1 \\
 & + \qquad - \quad + \\
\hline
 & 2 \\
\hline
\end{array}
$$

Since the remainder $\neq 0$,

Hence, x^3-3x+1 is not a factor of $x^5-4x^3+x^2+3x+1$.

Question 3:

Obtain all other zeroes of $3x^4+6x^3-2x^2-10x-5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Answer:

$p(x)=3x^4+6x^3-2x^2-10x-5$

Since the two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$,

$\therefore\left(x-\sqrt{\frac{5}{3}}\right)\left(x+\sqrt{\frac{5}{3}}\right)=\left(x^2-\frac{5}{3}\right)$ is a factor of $3x^4+6x^3-2x^2-10x-5$.

Therefore, we divide the given polynomial by $x^2-\frac{5}{3}$.

$$\begin{array}{r|l} x^2+0.x-\frac{5}{3} & \begin{array}{l} 3x^2+6x+3 \\ \hline 3x^4+6x^3-2x^2-10x-5 \\ 3x^4+0x^3-5x^2 \\ - \quad - \quad + \\ \hline 6x^3+3x^2-10x-5 \\ 6x^3+0x^2-10x \\ - \quad - \quad + \\ \hline 3x^2+0x-5 \\ 3x^2+0x-5 \\ - \quad - \quad + \\ \hline 0 \\ \hline \end{array} \end{array}$$

$$3x^4+6x^3-2x^2-10x-5=\left(x^2-\frac{5}{3}\right)\left(3x^2+6x+3\right)$$

$$=3\left(x^2-\frac{5}{3}\right)\left(x^2+2x+1\right)$$

We factorize x^2+2x+1

$=(x+1)^2$

Therefore, its zero is given by $x + 1 = 0$

$x = -1$

As it has the term $(x+1)^2$, therefore, there will be 2 zeroes at $x = -1$.

Hence, the zeroes of the given polynomial are $\sqrt{\frac{5}{3}}$, $-\sqrt{\frac{5}{3}}$, −1 and −1.

Question 4:

On dividing x^3-3x^2+x+2 by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find $g(x)$.

Answer:

$p(x) = x^3 - 3x^2 + x + 2 \qquad (\text{Dividend})$

$g(x)$ = ? (Divisor)

Quotient = $(x - 2)$

Remainder = $(-2x + 4)$

Dividend = Divisor × Quotient + Remainder

$x^3 - 3x^2 + x + 2 = g(x) \times (x - 2) + (-2x + 4)$

$x^3 - 3x^2 + x + 2 + 2x - 4 = g(x)(x - 2)$

$x^3 - 3x^2 + 3x - 2 = g(x)(x - 2)$

$g(x)$ is the quotient when we divide $(x^3 - 3x^2 + 3x - 2)$ by $(x - 2)$

$$
\begin{array}{r|l}
 & x^2 - x + 1 \\
\hline
x - 2 & x^3 - 3x^2 + 3x - 2 \\
 & x^3 - 2x^2 \\
 & - \quad + \\
\hline
 & \quad -x^2 + 3x - 2 \\
 & \quad -x^2 + 2x \\
 & \quad + \quad - \\
\hline
 & \qquad\quad x - 2 \\
 & \qquad\quad x - 2 \\
 & \qquad\quad - \quad + \\
\hline
 & \qquad\qquad 0 \\
\hline
\end{array}
$$

$\therefore g(x) = (x^2 - x + 1)$

Question 5:

Give examples of polynomial $p(x)$, $g(x)$, $q(x)$ and $r(x)$, which satisfy the division algorithm and

(i) deg $p(x)$ = deg $q(x)$

(ii) deg $q(x)$ = deg $r(x)$

(iii) deg $r(x)$ = 0

Answer:

According to the division algorithm, if $p(x)$ and $g(x)$ are two polynomials with

$g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$ such that

$p(x) = g(x) \times q(x) + r(x)$,

where $r(x) = 0$ or degree of $r(x)$ < degree of $g(x)$

Degree of a polynomial is the highest power of the variable in the polynomial.

(i) deg $p(x)$ = deg $q(x)$

Degree of quotient will be equal to degree of dividend when divisor is constant (i.e., when any polynomial is divided by a constant).

Let us assume the division of $6x^2+2x+2$ by 2.

Here, $p(x) = 6x^2+2x+2$

$g(x) = 2$

$q(x) = 3x^2+x+1$ and $r(x) = 0$

Degree of $p(x)$ and $q(x)$ is the same i.e., 2.

Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$6x^2+2x+2 = 2(3x^2+x+1)$

$= 6x^2+2x+2$

Thus, the division algorithm is satisfied.

(ii) deg $q(x)$ = deg $r(x)$

Let us assume the division of $x^3 + x$ by x^2,

Here, $p(x) = x^3 + x$

$g(x) = x^2$

$q(x) = x$ and $r(x) = x$

Clearly, the degree of $q(x)$ and $r(x)$ is the same i.e., 1.

Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$x^3 + x = (x^2) \times x + x$

$x^3 + x = x^3 + x$

Thus, the division algorithm is satisfied.

(iii)deg $r(x) = 0$

Degree of remainder will be 0 when remainder comes to a constant.

Let us assume the division of $x^3 + 1$by x^2.

Here, $p(x) = x^3 + 1$

$g(x) = x^2$

$q(x) = x$ and $r(x) = 1$

Clearly, the degree of $r(x)$ is 0.

Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$x^3 + 1 = (x^2) \times x + 1$

$x^3 + 1 = x^3 + 1$

Thus, the division algorithm is satisfied.

Exercise 2.4

Question 1:

Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

(i) $2x^3+x^2-5x+2;\quad \frac{1}{2},1,-2$

(ii) $x^3-4x^2+5x-2;\quad 2,1,1$

Answer:

(i) $p(x)=2x^3+x^2-5x+2.$

Zeroes for this polynomial are $\frac{1}{2},1,-2$

$$p\left(\frac{1}{2}\right)=2\left(\frac{1}{2}\right)^3+\left(\frac{1}{2}\right)^2-5\left(\frac{1}{2}\right)+2$$
$$=\frac{1}{4}+\frac{1}{4}-\frac{5}{2}+2$$
$$=0$$
$$p(1)=2\times1^3+1^2-5\times1+2$$
$$=0$$
$$p(-2)=2(-2)^3+(-2)^2-5(-2)+2$$
$$=-16+4+10+2=0$$

Therefore, $\frac{1}{2}$, 1, and −2 are the zeroes of the given polynomial.

Comparing the given polynomial with ax^3+bx^2+cx+d, we obtain a = 2, b = 1, c = −5, d = 2

We can take $\alpha = \frac{1}{2}, \beta = 1, \gamma = -2$

$$\alpha + \beta + \gamma = \frac{1}{2} + 1 + (-2) = -\frac{1}{2} = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{1}{2} \times 1 + 1(-2) + \frac{1}{2}(-2) = \frac{-5}{2} = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{1}{2} \times 1 \times (-2) = \frac{-1}{1} = \frac{-(2)}{2} = \frac{-d}{a}$$

Therefore, the relationship between the zeroes and the coefficients is verified.

(ii) $p(x) = x^3 - 4x^2 + 5x - 2$

Zeroes for this polynomial are 2, 1, 1.

$$p(2) = 2^3 - 4(2^2) + 5(2) - 2$$
$$= 8 - 16 + 10 - 2 = 0$$
$$p(1) = 1^3 - 4(1)^2 + 5(1) - 2$$
$$= 1 - 4 + 5 - 2 = 0$$

Therefore, 2, 1, 1 are the zeroes of the given polynomial.

Comparing the given polynomial with $ax^3 + bx^2 + cx + d$, we obtain *a* = 1, *b* = −4, *c* = 5, *d* = −2.

Verification of the relationship between zeroes and coefficient of the given polynomial

$$\text{Sum of zeroes} = 2 + 1 + 1 = 4 = \frac{-(-4)}{1} = \frac{-b}{a}$$

Multiplication of zeroes taking two at a time = (2)(1) + (1)(1) + (2)(1)

=2 + 1 + 2 = 5 $= \frac{(5)}{1} = \frac{c}{a}$

Multiplication of zeroes = 2 × 1 × 1 = 2 $=\frac{-(-2)}{1}=\frac{-d}{a}$

Hence, the relationship between the zeroes and the coefficients is verified.

Question 2:

Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, − 7, − 14 respectively.

Answer:

Let the polynomial be ax^3+bx^2+cx+d and the zeroes be $\alpha, \beta, \text{ and } \gamma$.

It is given that

$$\alpha+\beta+\gamma=\frac{2}{1}=\frac{-b}{a}$$

$$\alpha\beta+\beta\gamma+\alpha\gamma=\frac{-7}{1}=\frac{c}{a}$$

$$\alpha\beta\gamma=\frac{-14}{1}=\frac{-d}{a}$$

If a = 1, then b = −2, c = −7, d = 14

Hence, the polynomial is $x^3-2x^2-7x+14$.

Question 3:

If the zeroes of polynomial x^3-3x^2+x+1 are $a-b, a, a+b$, find a and b.

Answer:

$$p(x)=x^3-3x^2+x+1$$

Zeroes are a − b, a + a + b

Comparing the given polynomial with px^3+qx^2+rx+t, we obtain

$p = 1$, $q = -3$, $r = 1$, $t = 1$

$\text{Sum of zeroes} = a - b + a + a + b$

$\frac{-q}{p} = 3a$

$\frac{-(-3)}{1} = 3a$

$3 = 3a$

$a = 1$

The zeroes are $1-b, 1, 1+b$.

$\text{Multiplication of zeroes} = 1(1-b)(1+b)$

$\frac{-t}{p} = 1 - b^2$

$\frac{-1}{1} = 1 - b^2$

$1 - b^2 = -1$

$1 + 1 = b^2$

$b = \pm\sqrt{2}$

Hence, $a = 1$ and $b = \sqrt{2}$ or $-\sqrt{2}$.

Question 4:

]It two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$, find other zeroes.

Answer:

Given that $2 + \sqrt{3}$ and $2 - \sqrt{3}$ are zeroes of the given polynomial.

Therefore, $\left(x - 2 - \sqrt{3}\right)\left(x - 2 + \sqrt{3}\right) = x^2 + 4 - 4x - 3$

$= x^2 - 4x + 1$ is a factor of the given polynomial

For finding the remaining zeroes of the given polynomial, we will find the quotient by dividing $x^4 - 6x^3 - 26x^2 + 138x - 35$ by $x^2 - 4x + 1$.

$$
\begin{array}{r|l}
 & x^2 - 2x - 35 \\
\hline
x^2 - 4x + 1 & x^4 - 6x^3 - 26x^2 + 138x - 35 \\
 & x^4 - 4x^3 + x^2 \\
 & - \quad + \quad - \\
\hline
 & -2x^3 - 27x^2 + 138x - 35 \\
 & -2x^3 + 8x^2 - 2x \\
 & + \quad - \quad + \\
\hline
 & -35x^2 + 140x - 35 \\
 & -35x^2 + 140x - 35 \\
 & + \quad - \quad + \\
\hline
 & 0
\end{array}
$$

Clearly, $x^4 - 6x^3 - 26x^2 + 138x - 35 = (x^2 - 4x + 1)(x^2 - 2x - 35)$

It can be observed that $(x^2 - 2x - 35)$ is also a factor of the given polynomial.

And $(x^2 - 2x - 35) = (x - 7)(x + 5)$

Therefore, the value of the polynomial is also zero when $x - 7 = 0$ or $x + 5 = 0$

Or $x = 7$ or -5

Hence, 7 and -5 are also zeroes of this polynomial.

Question 5:

If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be *x* + *a*, find *k* and *a*.

Answer:

By division algorithm,

Dividend = Divisor × Quotient + Remainder

Dividend − Remainder = Divisor × Quotient

$x^4 - 6x^3 + 16x^2 - 25x + 10 - x - a = x^4 - 6x^3 + 16x^2 - 26x + 10 - a$ will be perfectly divisible by $x^2 - 2x + k$.

Let us divide $x^4 - 6x^3 + 16x^2 - 26x + 10 - a$ by $x^2 - 2x + k$

$$\begin{array}{r|l}
 & x^2 - 4x + (8-k) \\
\hline
x^2 - 2x + k & x^4 - 6x^3 + 16x^2 - 26x + 10 - a \\
 & x^4 - 2x^3 + kx^2 \\
 & - \quad + \quad - \\
\hline
 & -4x^3 + (16-k)x^2 - 26x \\
 & -4x^3 + 8x^2 - 4kx \\
 & + \quad - \quad + \\
\hline
 & (8-k)x^2 - (26-4k)x + 10 - a \\
 & (8-k)x^2 - (16-2k)x + (8k - k^2) \\
 & - \quad + \quad - \\
\hline
 & (-10+2k)x + (10 - a - 8k + k^2)
\end{array}$$

It can be observed that $(-10+2k)x + (10 - a - 8k + k^2)$ will be 0.

Therefore, $(-10+2k)$ = 0 and $(10-a-8k+k^2)$ = 0

For $(-10+2k)$ = 0,

2 k =10

And thus, k = 5

For $(10-a-8k+k^2)$ = 0

10 − a − 8 × 5 + 25 = 0

10 − a − 40 + 25 = 0

− 5 − a = 0

Therefore, a = −5

Hence, k = 5 and a = −5