## Two chords

* Any two diameters of a circle intersect at the centre, and the length of the four pieces are equal.


$$
\mathbf{P A}=\mathbf{P B}=\mathbf{P C}=\mathbf{P D}
$$

(Radii of a circle are equal)

* When two chords which are not diameters intersect within the circle we get four pieces which are not equal.

 when the chords AB \& CD intersect within the circle at $P$. Consider two chords AB \& CD which are not diameters of the circle.
The chords AB \& CD intersect at $P$.

Draw AC \& BD


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We can see,

$$
\angle \mathrm{A}=\angle \mathrm{D} \quad\left[\mathrm{C}=\angle \mathrm{B} \quad\left(\begin{array}{l}
\text { All angles made by an are o } \\
\text { its alternate arc are equal }
\end{array}\right.\right.
$$

Consider $\triangle$ APC \& $\triangle$ DPB
Since two angles of both triangles are equal, third angles are also equal.
So $\triangle$ APC \& $\triangle$ DPB are similar triangles.
Since sides opposite to equal angles of similar triangles are in proportion.

$$
\frac{P C}{P B}=\frac{P A}{P D}
$$

Cross multiplying we get,

$$
\mathbf{P A} \times \mathbf{P B}=\mathbf{P C} \times \mathbf{P D}
$$

Here PA, PB are parts of the chord AB and $P C, P D$ are parts of the chord CD.

So we can say,

If two chords of a circle intersect within the circle, then the products of the parts of the two chords are equal.


$$
P A \times P B=P C \times P D
$$

Q ) In the figure two chords $A B$ and $C D$ intersect at a point $P$. $P B=2 \mathrm{~cm}, P C=3 \mathrm{~cm}, P D=4 \mathrm{~cm}$. Find the length of $A B$.
Ans)

$$
\begin{aligned}
& \mathbf{P A} \times P B=P C \times P D \\
& P A \times 2=3 \times 4 \\
& P A=\frac{12}{2} \\
& P A=6 \mathrm{~cm} \\
& A B=P A+P B \\
&=6+2=8 \mathrm{~cm}
\end{aligned}
$$



Geometrical interpretation
We can interpret the product of two lengths as an area. So,
$P A \times P B=$ Area of the rectangle with sides $P A$ and $P B$
$P C \times P D=$ Area of the rectangle with sides $P C$ and $P D$
So the relation $\mathbf{P A} \times \mathbf{P B}=\mathbf{P C} \times \mathbf{P D}$ can be put in geometric language as below:

If two chords of a circle intersect within a circle, then the rectangles formed by the parts of the same chord have equal area.


$$
\mathbb{P A} \times \mathbb{P B}=\mathbb{P C} \times \mathbb{P D}
$$

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## Assimmment

The chords $A B$ and $C D$ of a circle intersect at a point $P$. If $P A=9 \mathrm{~cm}, P D=12 \mathrm{~cm}, A B=13 \mathrm{~cm}$, find the lengths of $P B, P C$ and $C D$ ?


