Answer key

First Year Higher Secondary Model Examination – 2021

Mathematics (Science)

1. (i) A = { 2, 3, 5, 7 }

(ii) (a) 16 since , $2^4=16$

2. (i) $n(A\cup B)=n(A)+n(B)-n(A\cap B)$

= 100 + 150 – 50 = 200

(ii) $n(A-B)=n(A)-n(A\cap B)$

= 100 – 50 = 50

3. (i) $p(n):7^n-3^n$ is divisible by 4

For n = 1

$p(1):7^1-3^1=7-3=4$

Which is divisible by 4.

Therefore p(1) is true.

(ii) Assume that p(k) is true.

$p(k):7^k-3^k$ is divisible by 4

so $7^k-3^k=4M$

and $7^k=4M+3^k$

Now p(k+1) = $7^{(k+1)}-3^{(k+1)}$

= $7^k.7-3^k.3$

= $(4M+3^k).7-3^k.3$

= 28 M + 7. 3^k - 3. 3^k

= 28 M + (7 – 3) 3^k

= 28 M + 4. 3^k

= 4 (7 M + 3^k)

Which is divisible by 4.

Therefore p(k+1) is true .

4.

$$(a+b)^n={}^nC_0a^n+{}^nC_1a^{(n-1)}b+{}^nC_2a^{(n-2)}b^2+\ldots+{}^nC_nb^n$$

$$\left(x^2+\frac{3}{x}\right)^5={}^5C_0(x^2)^5+{}^5C_1(x^2)^4\left(\frac{3}{x}\right)+{}^5C_2(x^2)^3\left(\frac{3}{x}\right)^2$$

$$+\ {}^5C_3(x^2)^2\left(\frac{3}{x}\right)^3+{}^5C_4x^2\left(\frac{3}{x}\right)^4+{}^5C_5\left(\frac{3}{x}\right)^5$$

$$=\ 1.x^{10}+5.x^8.\frac{3}{x}+10.x^6.\frac{9}{x^2}+10.x^4.\frac{27}{x^3}$$

$$+\ 5.x^2.\frac{81}{x^4}+1.\frac{243}{x^5}$$

$$=\ x^{10}+15x^7+90x^4+270x+\frac{405}{x^2}+\frac{243}{x^5}$$

5. n = 10, even number.

General term , $T_{r+1}={}^nC_ra^{(n-r)}b^r$

$$T_{r+1}={}^{10}C_r\left(\frac{x}{3}\right)^{(10-r)}(9y)^r$$

Middle term = $\left(\frac{n}{2}+1\right)^{th}$ term

= $\left(\frac{10}{2}+1\right)^{th}$ term

= 6^{th} term

$$T_6=T_{5+1}={}^{10}C_5\left(\frac{x}{3}\right)^{(10-5)}(9y)^5$$

$$=\ {}^{10}C_5\left(\frac{x}{3}\right)^5.9^5.y^5$$

$$=\ 252.\frac{x^5}{243}.59049.y^5$$

= 61236

6. (i) $a_8=16\Rightarrow a+7d=16$

$a_{16}=48\Rightarrow a+15d=48$

subtracting them,

8d = 32 , d = 4, common difference = 4

(ii) Also a = 16 -7d = 16 – 28 = - 12

$a_{25}=a+24d$

= -12 + 24 . 4

= - 12 + 96 = 84

7. (i) slope = $\frac{y_2-y_1}{x_2-x_1}$ = $\frac{8-6}{4--2}$ = $\frac{2}{6}$

= $\frac{1}{3}$

option (b) $\frac{1}{3}$

(ii) If lines are perpendicular then

$m_1 m_2=-1$

so , $m_2=-3$

$m_2=\frac{y_2-y_1}{x_2-x_1}$ = $\frac{24-12}{x-8}$ = $\frac{12}{x-8}$

so $\frac{12}{x-8}=-3$

- 3 (x – 8) = 12

- 3x + 24 = 12

3x = 12

x = 4

8. (i) (c) z = 0

(ii) distance

= $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

= $\sqrt{(1--2)^2+(2-3)^2+(3-5)^2}$

= $\sqrt{(3)^2+(-1)^2+(-2)^2}$

= $\sqrt{9+1+4}=\sqrt{14}$

9. By section formula,

$\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n},\frac{mz_2+nz_1}{m+n}\right)$

here m : n is the ratio

$\left(\frac{6m+4n}{m+n},\frac{10m+8n}{m+n},\frac{-8m+10n}{m+n}\right)$

the point lies in the YZ plane, so x – coordinate is zero.

$\frac{6m+4n}{m+n}=0$

6 m + 4 n = 0

6 m = - 4 n

$\frac{m}{n}=\frac{-4}{6}$

$\frac{m}{n}=\frac{-2}{3}$

Therefore ratio is 2 : 3

10. (i) $y^2=4ax$

4 a = 16

a = 4

Length of the latus rectum = 4 a = 16

option (b) 16

(ii) coordinates of focus ; (a , 0) = (4, 0)

Equation of directrix is x = - 4

11. $\lim_{x\to 2}\left(\frac{x^5-32}{x^3-8}\right)=\lim_{x\to 2}\left(\frac{x^5-2^5}{x^3-2^3}\right)$

$$= \lim_{x\to 2}\left(\frac{\left(\frac{x^5-2^5}{x-2}\right)}{\left(\frac{x^3-2^3}{x-2}\right)}\right)$$

$$= \frac{\lim_{x\to 2}\left(\frac{x^5-2^5}{x-2}\right)}{\lim_{x\to 2}\left(\frac{x^3-2^3}{x-2}\right)}$$

$$= \frac{5.2^4}{3.2^2}=\frac{20}{3}$$

12. (i) It is false that if a number is divisible by 10, then it is divisible by 5.

(ii) If a number is divisible by 5 , then it is divisible by 10.

(iii) If a number is not divisible by 5, then it is not divisible by 10.

13. (i) A^l = { 5, 6 }

B^l = { 1, 2, 6 }

(ii) A – B = { 1, 2 }

$A\cap B^l$ = { 1, 2 }

A – B = $A\cap B^l$

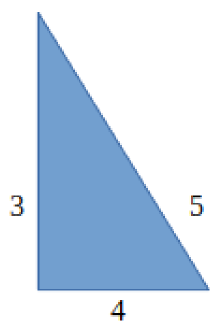
14. (i) R = { (- 1, 4), (0, 3), (1, 2), (2, 1) }

(ii) Domain = { -1, 0, 1, 2 }

Range={4,3,2,1}

15. (i) (a) $\frac{-1}{2}$

(ii) $\sin x=\frac{3}{5}$



$\cos x=\frac{-4}{5}$

$\tan x=\frac{-3}{4}$

16. $p(n):1+2+3+...+n=\frac{n(n+1)}{2}$

For n = 1

LHS = 1

RHS = $\frac{1(1+1)}{2}=1$

p(1) is true.

Assume that p(k) is true.

$p(k):1+2+3+...+k=\frac{k(k+1)}{2}$

Now , p(k+1) = 1 + 2 + 3+ ... + k + (k+1)

$$= \frac{k(k+1)}{2}+(k+1)$$

$$= \frac{k(k+1)+2(k+1)}{2}$$

$$= \frac{(k+1)(k+2)}{2}$$

Therefore p(k+1) is true.

p(n) is true by the principle of mathematical induction.

17. (i) (a) 1

(ii) $z=\frac{2+3i}{1+2i}$

$$= \frac{(2+3i)(1-2i)}{(1+2i)(1-2i)}$$

$$= \frac{2-4i+3i-6i^2}{1^2-(2i)^2}$$

$$= \frac{2-i+6}{1--4}$$

$= \frac{8-i}{5}$

$= \frac{8}{5} - i\frac{1}{5}$

18. (i) (b) $\sqrt{2}$

(ii) z = r (cos θ + i sin θ)

z = 1 + i , x = 1 and y = 1

|z| = $\sqrt{1+1}$ = $\sqrt{2}$

$\tan\theta = \frac{y}{x}$ = $\frac{1}{1}$

$\theta = \frac{\pi}{4}$

$z = \sqrt{2}(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$

19. (i) number of ways = $5^5 = 625$

(ii) number of ways = 5! = 120

20. (i) ${}^nC_r = {}^nC_{n-r}$

therefore ${}^{20}C_{12} = {}^{20}C_8$

option (b) 8

(ii) 3 red balls selected from 7 red balls in ${}^7C_3$ ways and 2 white balls selected from 5 white balls in ${}^5C_2$ ways.

Number of ways = ${}^7C_3$ x ${}^5C_2$

= 35 x 10 = 350

21. Assume that $\sqrt{2}$ is rational.

$\sqrt{2} = \frac{a}{b}$ where a and b are integers

having no common factors.

Squaring, we get $2 = \frac{a^2}{b^2}$

$a^2 = 2b^2$

ie, 2 divides a

therefore there exist an integer c such that a = 2 c

squaring, we get $a^2 = 4c^2$

But $2b^2 = 4c^2$

$b^2 = 2c^2$

ie, 2 divides b

which means 2 divides both a and b, which is a contradiction to our assumption a and b have no common factors.

Therefore $\sqrt{2}$ is rational.

22. (i) intercept form of the equation is

$\frac{x}{a} + \frac{y}{b} = 1$

But the intercepts are same,

so $\frac{x}{a} + \frac{y}{a} = 1$

ie x + y = a

But the line passes through (2,3)

2 + 3 = a

ie, a = 5

equation of the line is x + y = 5

(ii) y = -x +5

slope = -1

23.(i) here a = 5 and c = 4

but $a^2 = b^2 + c^2$

$25 = b^2 + 16$

$b^2 = 9$

b = 3

Equation of the ellipse is, $\frac{x^2}{25}+\frac{y^2}{9}=1$

(ii) Length of latus rectum = $\frac{2b^2}{a}$ = $\frac{18}{5}$

24. $y=\frac{x^2+1}{x+1}$

$$\frac{dy}{dx}=\frac{(x+1)\frac{d}{dx}(x^2+1)-(x^2+1)\frac{d}{dx}(x+1)}{(x+1)^2}$$

$$=\frac{(x+1)(2x)-(x^2+1).1}{(x+1)^2}$$

$$=\frac{2x^2+2x-x^2-1}{(x+1)^2}$$

$$=\frac{x^2+2x-1}{(x+1)^2}$$

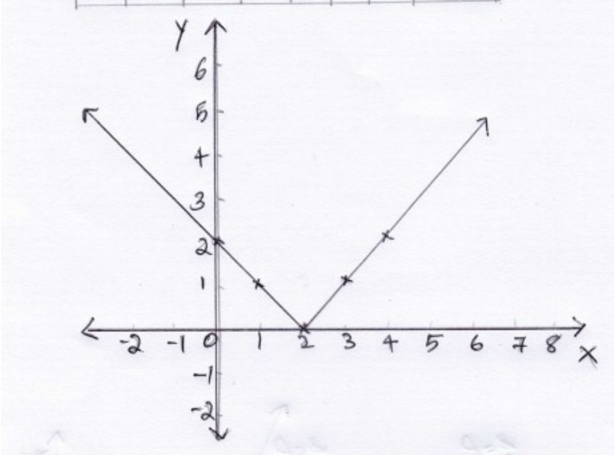
25. (i) f(0) = | 0 – 2 | = | -2 | = 2

option (b) 2

(ii)

(ii)

x	-3	-2	-1	0	1	2	3	4
f(x)	5	4	3	2	1	0	1	2



(iii) Range = set of all positive real numbers including zero, **or**

or [0, ∞), **or R+**

26. (i) sin 75 = sin (45 + 30)

= sin 45 cos 30 + cos 45 sin 30

$$=\frac{1}{\sqrt{2}}x\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}x\frac{1}{2}$$

$$=\frac{\sqrt{3}+1}{2\sqrt{2}}$$

(ii)

$$\frac{\sin 5x+\sin 3x}{\cos 5x+\cos 3x}=\frac{2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)}{2\cos\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)}$$

$$=\frac{\sin\left(\frac{5x+3x}{2}\right)}{\cos\left(\frac{5x+3x}{2}\right)}$$

$$=\frac{\sin 4x}{\cos 4x}$$

= tan 4x

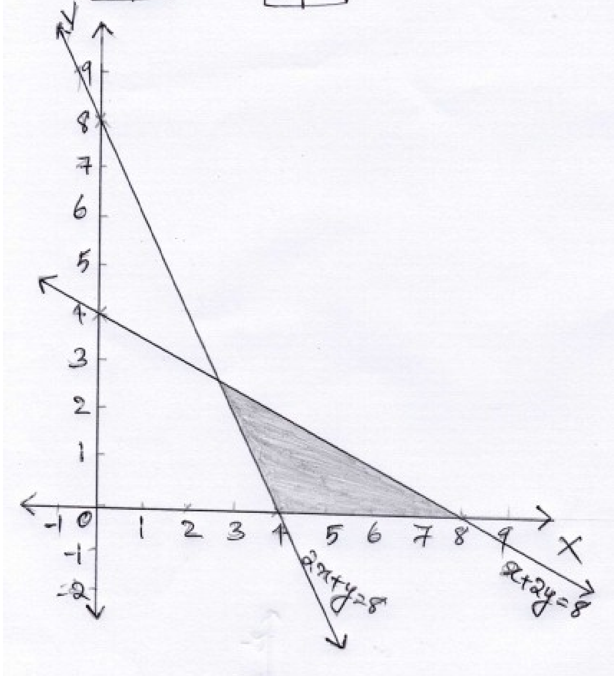
27.

x+2y=8

x	y
0	4
8	0

2x+y=8

x	y
0	8
4	0



28. (i) common ratio

$$\frac{x}{(\frac{1}{4})}=\frac{4}{x}$$

$$x^2=1$$

$$x=\pm 1$$

(ii) $5+55+555+\ldots$ up to n terms

= 5(1 + 11+ 111 + ... + up to n terms)

$$= \frac{5}{9}(9+99+999+\ldots+n\,terms)$$

$$= \frac{5}{9}((10-1)+(10^2-1)+\ldots+n\,terms)$$

$$= \frac{5}{9}(10+10^2\ldots+n\,terms-n)$$

$$= \frac{5}{9}\left[\frac{10(10^n-1)}{10-1}-n\right]$$

$$= \frac{5}{9}\left[\frac{10(10^n-1)}{9}-n\right]$$

29.

x_i	f_i	$x_i f_i$	x_i^2	$x_i^2 f_i$
3	7	21	9	63
8	10	80	64	640
13	15	195	169	2535
18	10	180	324	3240
23	6	138	529	3174
	N=48	614		9652

Mean , $\bar{x}=\frac{\sum f_i x_i}{N} = \frac{614}{48}=12.79$

Variance, $\sigma^2=\frac{\sum f_i x_i^2}{N}-(\bar{x})^2$

$$= \frac{9652}{48}-(12.79)^2=37.499\approx 37.5$$

S. D , $\sigma=\sqrt{Varience}$

$$= \sqrt{37.5}=6.12$$

30. (i) n(NCC) = 30

n(NSS) = 32

$n(NCC\cap NSS)$ = 24

P(NCC) = $\frac{30}{60}=\frac{1}{2}$

(ii) $n(NCC\cup NSS)$

$$= n(NCC)+n(NSS)-n(NCC\cap NSS)$$

= 30 + 32 – 24 = 38

$$P(NCC\cup NSS)=\frac{38}{60}=\frac{19}{30}$$

(iii) P(neither NCC nor NSS)

= 1 - $P(NCC\cup NSS)$

= 1 - $\frac{19}{30}$

= $\frac{11}{30}$

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HSST Mathematics