MATHEMATICS ONLINE CLASS X ON 04-08-2021

CIRCLES

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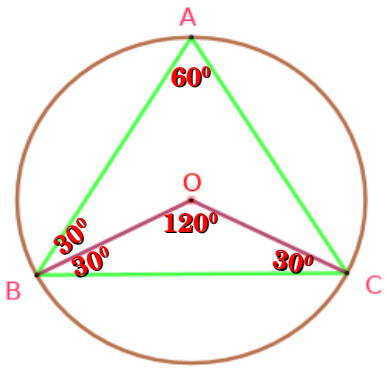
Answers to assignments of previous class

1) In the figure, O is the centre of the circle and ABC is an equilateral triangle. Find∠BAC and ∠ABO



Since △ABC is an equilateral triangle

$\angle A = \angle B = \angle C = 60^0$

$\angle BAC = 60^0$

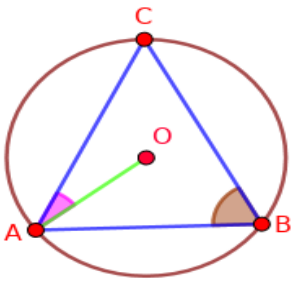
$\angle BOC = 2\times\angle BAC = 2\times60^0 = 120^0$

In △OBC , OB = OC

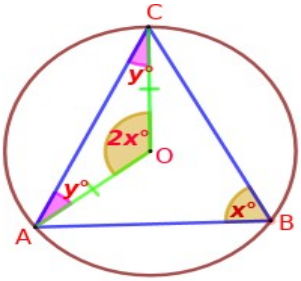
$$\therefore \angle OBC = \angle OCB = \frac{180^0 - 120^0}{2} = \frac{60^0}{2} = 30^0$$

$\angle ABO = \angle ABC - \angle OBC = 60^0 - 30^0 = 30^0$

2) In the picture O is the centre of the circle and A , B , C are points on it . Prove that $\angle OAC + \angle ABC = 90^0$



In the figure, O is the centre of the circle and A, B, C are points on the circle.
Let∠ABC = x and ∠OAC = y
Angle made by an arc at its alternate arc is half the central angle of the arc.
Therefore, ∠AOC = 2 ×∠ABC = 2x.
Join OC.
OA and OC are radii of the circle . ∴ OA = OC



ΔOAC is an isosceles triangle.

$\therefore \angle OAC = \angle OCA = y$

In any triangle, sum of all angles = 180°

$$2x + y + y = 180°$$

$$2x + 2y = 180°$$

$$2(x+y) = 180°$$

$$x+y = \frac{180^0}{2} = 90°$$

$\therefore$ we get $\angle ABC + \angle OAC = 90°$

Constructions

I. Constructions of angles which cannot be directly measured using protractor .

1. Draw an angle of measure $\left(22\frac{1}{2}\right)^0$

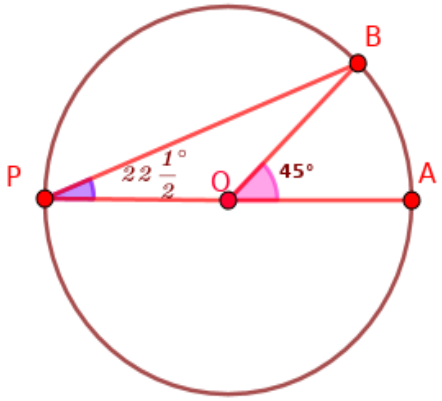
Here we can apply the ideas "Angle made by an arc at its alternate arc is half the central angle of the arc."

Steps:

First draw a circle of any radius.

Draw a radius of the circle.

Measure 45^0 at the centre of the circle based on this radius using protractor. Mark any point P on the larger part of the circle other than A and B. Join AP and BP.



$$\angle APB = \frac{1}{2}\angle AOB = \frac{1}{2} \times 45^0 = \left(22\frac{1}{2}\right)^0$$

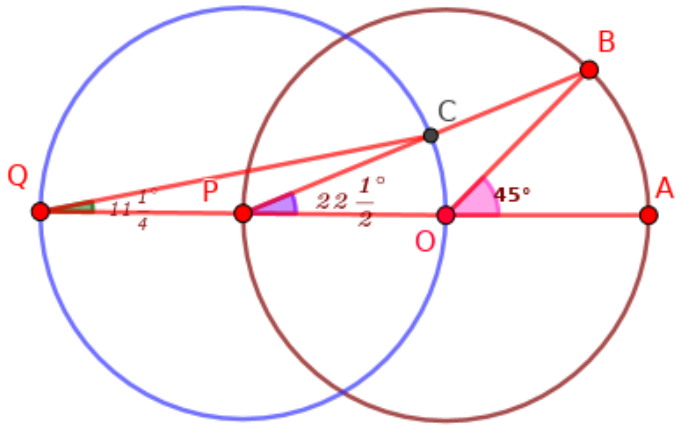
2. Draw an angle of measure $\left(11\frac{1}{4}\right)^0$

Draw the figure as same in the previous construction.

Draw another circle with centre at P as shown in the figure. Complete the construction as given in the figure.

$$\angle APB = \angle OPC = \left(22\frac{1}{2}\right)^0$$

$$\angle OQC = \frac{1}{2}\angle OPC = \frac{1}{2} \times \left(22\frac{1}{2}\right)^0 = \left(11\frac{1}{4}\right)^0$$



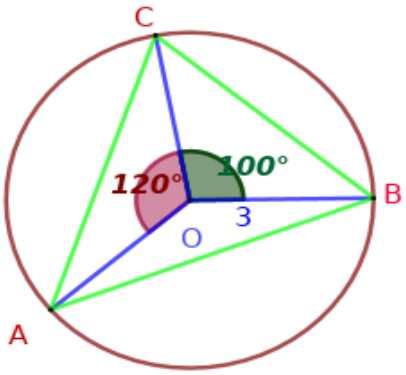
II. Drawing a triangle in which circumradius and two of its angles are given

1. Draw a triangle of circumradius 3cm and two of angles 50°and 60°

Answer

Two angles of triangle are 50° and 60°. ∴ Central angles of two arcs of circle are 2×50° = 100° and 2×60° = 120°

First draw a circle of radius 3cm. Draw a radius of the circle. Measure 100° based on this radius using protractor on the centre of the circle. At that point draw the radius. Again measure 120° based on any radius drawn and draw the radius. we get 3 end points of radii. Join these points to get the triangle.



$\angle A = \frac{1}{2}\angle BOC = \frac{1}{2}100° = 50°$

$\angle B = \frac{1}{2}\angle AOC = \frac{1}{2}120° = 60°$

$\angle C = 180° - (50° + 60°) = 180° - 110° = 70°$

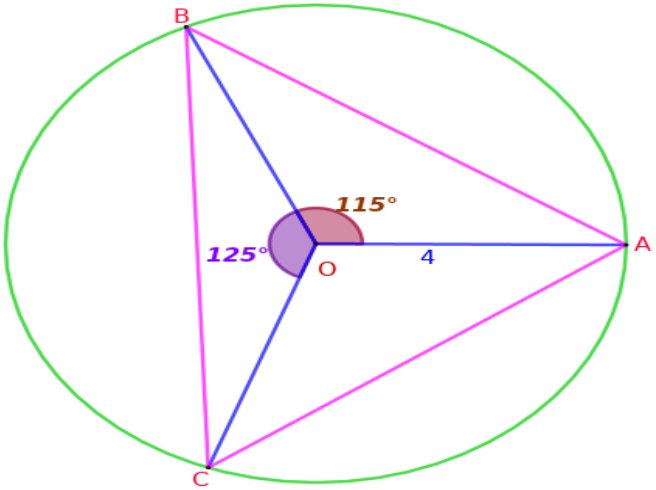
2. Draw a triangle of circumradius 4cm and two of its angles are $\left(57\frac{1}{2}\right)^0$ and $\left(62\frac{1}{2}\right)^0$

Answer

Draw this figure as done in the previous question.

Here, Central angles of two arcs of circle are 2 × $\left(57\frac{1}{2}\right)^0$ = 115°

and $2 \times \left(62\frac{1}{2}\right)^0 = 125°$



$\angle A = \frac{1}{2}\angle BOC = \frac{1}{2}125° = 62\frac{1}{2}°$

$\angle C = \frac{1}{2}\angle AOB = \frac{1}{2}115° = 57\frac{1}{2}°$

$\angle B = 180° - (62\frac{1}{2}° + 57\frac{1}{2}°) = 180 - 120° = 60°$

3. In each problem below, draw a circle and a chord to divide it into two parts such that the parts are as specified;

i) All angles on one part 80 °

ii) All angles on one part 110 °

iii) All angles on one part half of all angles on the other.

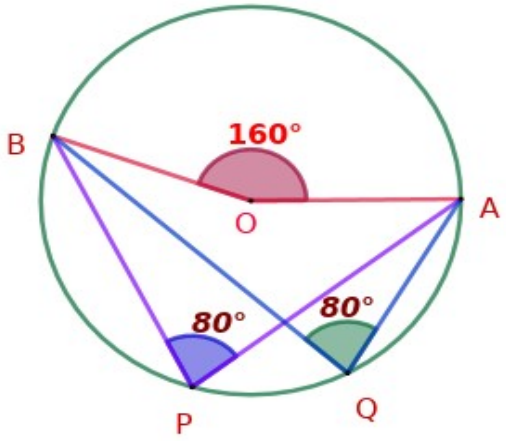
iv) All angles on one part, one and a half times the angles on the other.

Answer

Draw a circle of any radius.

i) If all angles on one part 80°,

Central angle of the other part = 2 x 80^0 = 160°



$\angle AOB = 160°$

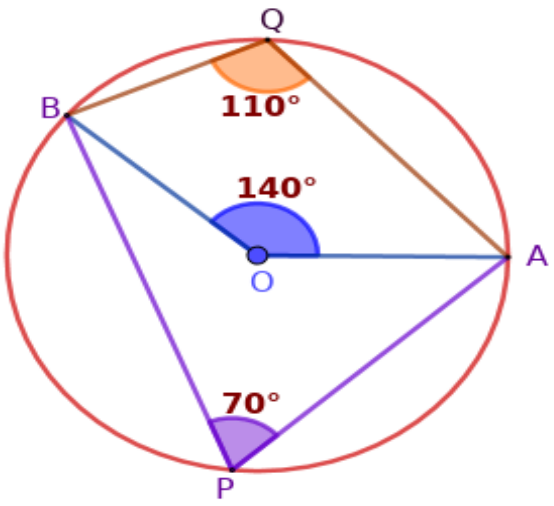
$\angle APB = \angle AQB = \frac{1}{2}\angle AOB = \frac{1}{2}160° = 80°$

ii) If all angles on one part 110°,

∴ Angle of the other part = $180^0 - 110^0 = 70°$

Then,

Central angle of one part = 2 × 70° =140°



$\angle AOB=140°$

$$\angle APB = \frac{1}{2}\angle AOB = \frac{1}{2}140° = 70°$$

$\angle AQB=180°-\angle APB=180°-70°=110°$

iii) Let, All angles on one part = x, then

All angles on the other part = 2x

Now $x + 2x = 180°$

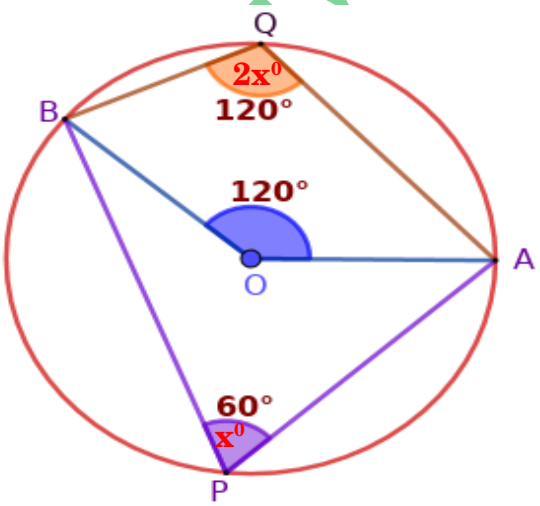
$3x = 180°$

$x = \frac{180^0}{3} = 60^0$

That is, All angles on one part = 60 °

All angles on the other part = 120 °

∴ Central angle of one part = 2 × 60° = 120 °



$\angle AOB=120°$

$$\angle APB = \frac{1}{2}\angle AOB = \frac{1}{2}120° = 60°$$

$\angle AQB=180°-\angle APB=180°-60°=120°$

iv) Let all angles on one part = x then

All angles on the other part = 1.5x = $\frac{3}{2}$ x^0

$$x + \frac{3}{2}x^0 = 180°$$

$$\frac{5}{2}x = 180°$$

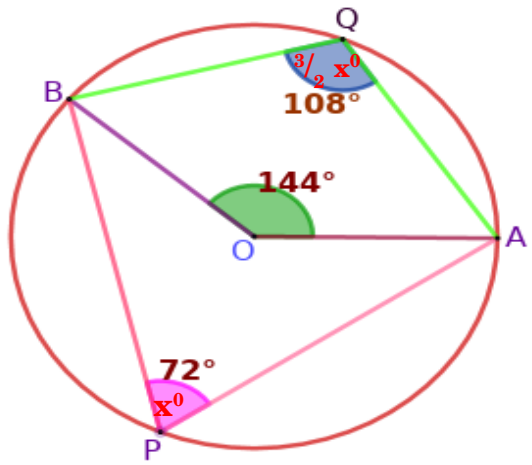
$$x = \frac{180 \times 2}{5} = 72^0$$

That is,

All angles on one part = 72°

All angles on the other part = $\frac{3}{2}$ x = $\frac{3}{2}$ × 72° =108°

∴ Central angle of one part = 144°



$\angle AOB=144°$

$\angle APB = \frac{1}{2}\angle AOB = \frac{1}{2}144° = 72°$

$\angle AQB=180°-\angle APB=180°-72°=108°$

ASSIGNMENT

Draw a triangle of circumradius 3cm and two of its angles are $(32\frac{1}{2})^0$ and $(37\frac{1}{2})^0$