Worksheet 13

1) First term of an arithmetic sequence is 3 and the common difference $4,$

    a) Write the algebraic form of this sequence

    b) Calculate the sum of first 14 terms of this sequence

    c) Can the sum of any 25 terms of this sequence $2020.$ How can we realize it

    d) Write the sum of first n terms of this sequence

2) The algbraic form of the sum of first n terms of a sequence is $n^2 + n.$

    a) Write the sequence

    b) Write the algebraic form of the sequence

    c) Can the sum of any number of terms $2021?$How can we realize it

    d) How many terms are there below 100 in this sequence?

    d) Calculate the sum of all terms below 100 in this sequence

3) Consider an arithmeic sequence of algebraic form $3n + 2$

    a) What is the common difference of this sequence ?

    b) What is its first term?

    c) Find the sum of first n terms of this sequence

    d) Calculate the sum of first 10 terms of this sequence?

    e) Calculate the sum of the terms from 10 th to 20 th of this sequence

4) Look at the pattern given below

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                    2
            4    6    8
    10    12   14   16   18
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    a) Write the number of numbers in each line as a sequence

    b) How many numbers are there in 30 th line ?

    c) Which number comes in the right end of 30 th line?

    d) Which number comes in the left end of 30 th line?

    e) Calculate the sum of all numbers in 30 lines of this pattern

5) You have already studied the calculation of the sum of first n natural numbers . Look at the pattern given below

$$1^3 = 1$$
$$1^3 + 2^3 = 9 = 3^2 = (1+2)^2$$
$$1^3 + 2^3 + 3^3 = 36 = 6^2 = (1+2+3)^2$$

We can see an order in this calculaion. This will help us to write more lines below . Answer the following questions

a) How many cubic numbers are there from 1 to 8000?

b) Find $1^3 + 2^3 + 3^3 + 4^3$

c) Find the sum of the cubes of all natural numbers from 1 to 6

d) We know that the sum of all natural numbers from 1 to 10 is 55. Calculate $1^3 + 2^3 + 3^3 \cdots 10^3$

e) Write a formula to find the sum $1^3 + 2^3 + 3^3 \cdots + n^3$

Answers and Explanation

1)
  a) $x_n = 4n - 1$

  b) $x_{14} = 4 \times 14 - 1 = 55$
Sum $= (x_1 + x_n) \times \frac{n}{2} = (3 + 55) \times 7 = 406$

  c) All terms are odd numbers. Sum of 25 odd numbers cannot be an even number.

  d) Sum $= (x_1 + x_n) \times \frac{n}{2} = (3 + 4n - 1) \times \frac{n}{2} = (4n + 2) \times \frac{n}{2} = n(2n + 1) = 2n^2 + n$

2)
  a) $x_1 = 1^2 + 1 = 2, x_1 + x_2 = 2^2 + 2 = 6$
$x_2 = 6 - 2 = 4, d = x_2 - x_1 = 4 - 2 = 2$
Sequence :$2, 4, 6 \cdots$

  b) $x_n = 2n$

  c) All terms are even numbers . Sum of even numbers cannot be the odd . 2021 cannot be the sum

  d) $2n = 98, n = 49$There are 49 terms below 100

  e) Sum $= 2(1 + 2 + 3 + \cdots 49) = 2 \times (49 + 1) \times \frac{49}{2} = 49 \times 50 = 2450$

3)
  a) 3

  b) $3 \times 1 + 2 = 5$

  c) $3 \times (n + 1) \times \frac{n}{2} + 2n = \frac{3}{2}n^2 + \frac{7}{2}n$

  d) $\frac{3}{2} \times 10^2 + \frac{7}{2} \times 10 = 185$

  e) Sum of the terms from 10 th 20 th $=$ sum of first 20 terms $-$ sum of first 9 terms
$s_{20} = 670, s_9 = 153$sum $= 670 - 153 = 517$

4)
  a) $1, 3, 5, 7 \cdots$

  b) $x_n = 2n - 1, x_{30} = 2 \times 30 - 1 = 59$

  c) Sequence of numbers in the right end : $2, 8, 18, 32 \cdots$
$x_n = 2n^2, x_{30} = 2 \times 30^2 = 1800$

  d) 29മത്തെ വരിയിലെ അവസാന സംഖ്യയേക്കാൾ 2കൂട്ടലാണ് മൂപ്പതമത്തെ വരിയിലെ ആദ്യസംഖ്യ.
It is $2 \times 29^2 + 2 = 2 \times 841 + 2 = 1684$

  e) Sum $= 2 + 4 + 6 + \cdots 1800 = 2(1 + 2 + 3 + \cdots 900) = 2 \times (900 + 1) \times \frac{900}{2} = 810900$

5)
  a) $20^3 = 8000$. There are 20 cubic numbers upto 8000

  b) $(1 + 2 + 3 + 4)^2 = 10^2 = 100$

  c) $(1 + 2 + 3 + 4 + 5 + 6)^2 = 21^2 = 441$

  d) $55^2 = 3025$

  e) $s_n = \left[\frac{n(n+1)}{2}\right]^2$