Lab 18

Functions

Aim:

- To visualise one to one and onto function geometrically
- To restrict the domain and codomain of functions so as to make it a bijection
- To visualise composition of two functions geometrically

Concepts:

- One to one and onto functions
- Composition of functions

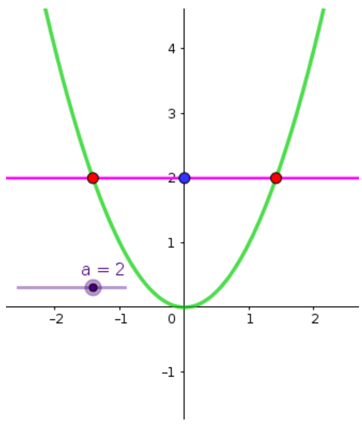
Discussion :
We discuss the properties of graphs of One to One and onto functions.Discuss about how to determine whether a given function is one to one and onto geometrically,how to restrict the domain and codomain of functions so as to make them bijections.We plot the graph of $g \circ f$ from the graphs of the functions f and g

Activity 18.1 One - One and Onto Functions

Procedure

- Draw the graph of $f(x) = x^2$
- Create a slider a and plot the point $(0, a)$
- Draw a line through the point and perpendicular to the y axis.

  If for any value of a, the line meets the graph of a function at more than one point, can we say whether the function one to one or not. Why ?



If for all values of a, the line meets the graph of a function at at least one point, can we say whether the function onto or not. Why ?

Using above applet, say whether the function $f : \mathbf{R} \to \mathbf{R}$ defined above is one to one or onto.

- Keeping above ideas in mind, one can say whether a function is one - one or onto, by observing its graph, even without drawing the horizontal line.
- Save the file as Activity 18.1

Activity 18.2

Draw the graphs of the following functions defined from $\mathbb{R}$ to $\mathbb{R}$ and say whether they are one to one or onto. Find the range of the function in each case.

	Function	One-One	Onto	Range
i	$x^2 + 2$			
ii	$x^3 - 3x^2 + 3$			
iii	$3\sin(x)$			

The following functions are defined from a subset of the set of real numbers. Say whether they are one-one or onto. Find their range.

i) $f : \mathbb{R} - \{3\} \to \mathbb{R} - \{1\}$ defined by $f(x) = \dfrac{x-2}{x-3}$

ii) $f : [-1, 1] \to \left[-1, \dfrac{1}{3}\right]$ defined by $f(x) = \dfrac{x}{x+2}$

iii) $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \dfrac{x}{1+|x|}$

What is the peculiarity of the range of an onto function ?

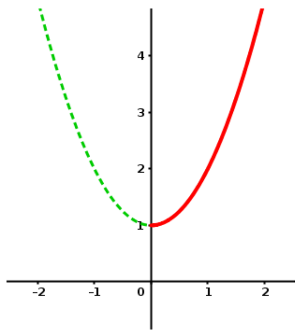
Activity 18.3 Bijective Functions

We can make a function one to one by restricting its domain and onto by restricting its co-domain. If we define a function to its range, it becomes onto. From the graph of a function we can easily select a suitable subset of the domain to make it one to one

Procedure

- Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2 + 1$. Is it one-one or onto ?
- Draw the graph of above function

  Redefine the function by restricting its co-domain so that it becomes onto
- From the graph of the function we can see that if we restrict the domain to $[0, \infty)$ the function becomes one to one.
- So the function $f : [0, \infty) \to [1, \infty)$ defined by $f(x) = x^2 + 1$ is a bijection.



- Draw the graph of above function. (Use input command `if(x>=0,x^2+1)` or `if(x>=0,f)` or `Function(f,0,infinity))`
- If we define $f : (-\infty, 0] \to [1, \infty)$ then also it becomes a bijection. Draw the graph of this function.

Restrict domain and co-domain of the following functions so that they becomes bijections

	Function	Domain	Co-domain
i	$\|x-2\|$	$[2,\infty)$	
ii	x^2-3x+3		
iii	$\|\sin x\|$		

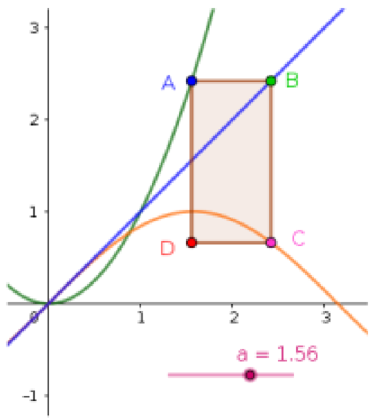
For each of above functions find one more domain which make them one to one. Draw the corresponding graphs.

Activity 18.4 Composition of functions

We create an applet to describe the composition of two functions geometrically

Procedure

- Draw the graph of $f(x)=x^2$ and $g(x)=\sin x$
- Draw the line $y=x$
- Create a slider a
- Plot the point $A(a,f(a))$
- Draw the line passing through A and parallel to x axis and plot its point of intersection B with the line $y=x$
- Draw the line passing through B and parallel to the y axis and plot its point of intersection C with the graph of the function g
- Draw the following lines and take their point of intersection D
  - Passing through A and parallel to the y axis
  - Passing through C and parallel to the x axis



- Complete the rectangle $ABCD$ and hide the lines

Write the co ordinates of the points B,C and D in terms of a

Trace the point D and animate the slider.What does the graph represents?Use locus tool to get the clear path.

- Create input boxes for the functions f and g.

Following functions are of the form $f\circ g(x)$. Identify f and g and use above applet to trace the path of the given functions

i)$\sin|x|$ ii) $e^{log x}$

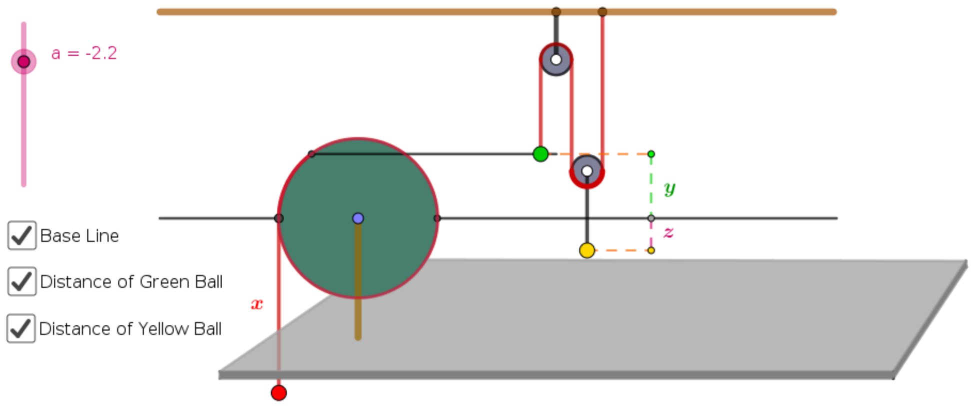
Additional Activities

Activity 18.A Composition - Physical situation

We discuss a physical situation in which the concept of composition of functions included.

Use Applet ML18.A

About the applet



A rope is wound on a disc of radius 1 meter . A red ball is attached at the end of the rope. A green ball is attached to a point on the circumference of the disc by means of a rod which always remains horizontal. A yellow ball is attached to the green ball using two pulleys. We can wind/unwind the rope using the slider **a**. When **a**=0 all the balls lie on the base line, the horizontal line passing through the centre of the disc.

If we unwind the disc so that the red ball moves x meters down wards, let the green ball moves y meters and yellow ball moves z meters from the base line. We take the movement of a balls as positive, if it is above the base line and negative if it is below the base line.

Write y as a function of x.

Write z as a function of y.

Write z as a function of x.

Suppose we pull down the red ball at the rate of 2m/s. Write x, y and z as functions of the time t.

Suppose that the red ball freely falls under gravitational force. Write x, y and z as functions of the time t.

Activity 18.B One-one and onto property of functions

We have seen how to check the one to one and onto properties of functions.Now let us check these properties of composition of such functions.Consider the following functions in an applet

f	one-one	onto	g	one-one	onto	$g \circ f$	one-one	0nto
$2x+3$			$x-4$					
x^2+1			$x-4$					
$x^{\frac{3}{2}}$			x^2					
$\frac{1}{x}$			$x-[x]$					
$\frac{1}{x}, x>0$			x^2					
x^3			x^2					
$\|x\|$			$x-[x]$					
$3x-1$			x^2					
$\sin x$			x^2					
$\cos x$			x^3					

and verify the properties

- f is one to one and g is one to one implies $g \circ f$ is one to one
- f is onto and g is onto implies $g \circ f$ is onto
- $g \circ f$ is one to one implies f is one to one and g need not be one to one
- $g \circ f$ is onto implies g is onto and f need not be onto