

12.1 INTRODUCTION

In earlier sections we studied about two dimensional geometry in which we have only two mutually perpendicular axis (the x and y axes) meeting a point O (origin). These axes divide a plane into four equal parts, each known as a quadrant. In this chapter, we shall study the basic concepts of geometry in three dimensional space.

12.2 COORDINATE AXES AND COORDINATE PLANES IN THREE DIMENSIONAL SPACE

In three dimensional geometry, there are three mutually perpendicular axes (XOX' , YOY' , ZOZ') intersecting at O . They are x , y and z axes. If we hold our fingers as in Fig.12.1, we get the idea about the axes.

Consider three planes intersecting at a point O such that they are mutually perpen-

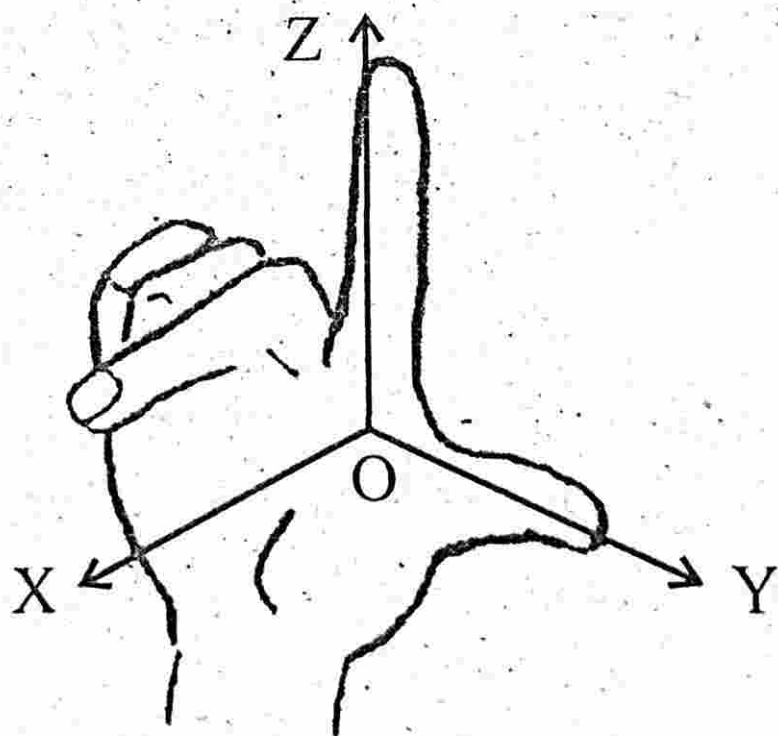


Fig. 12.1

pendicular to each other (Fig. 12.2). In three dimensional geometry, there are three planes XY , YZ and ZX (also called XOY , YOZ , ZOX planes) which are mutually perpendicular to each other. The point O is called the origin. These three planes divide the space into 8 equal parts each known as an octant (octa = 8). Fig. 12.2 and Fig. 12.3 give the idea of the 3-axes along with the 3 planes.

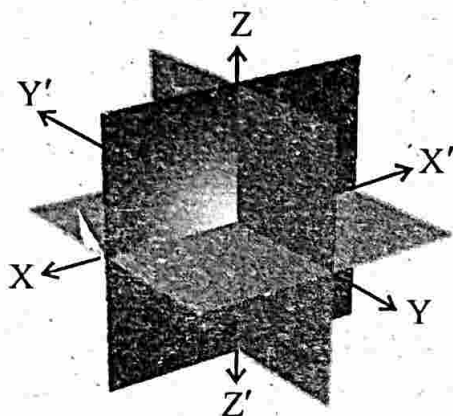


Fig. 12.2

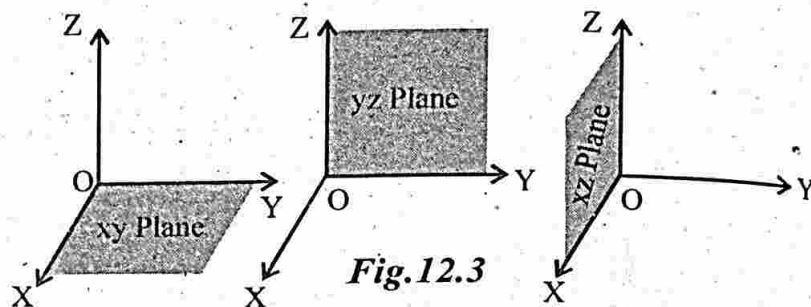


Fig. 12.3

12.3 COORDINATES OF A POINT IN SPACE

Let P be a point in space. Through P draw three planes parallel to the coordinate planes to meet the axes in A , B , C respectively. Let $OA = x$, $OB = y$ and $OC = z$. Then the ordered triplet (x, y, z) is taken as the coordinates of P .

x, y, z are positive or negative according as they are measured along the positive or negative directions of the coordinate axes.

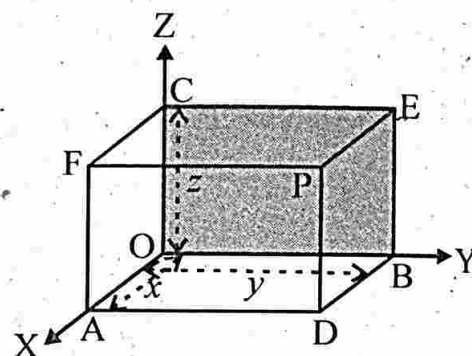


Fig. 12.4

If we are given an ordered triplet (x, y, z) of real numbers, we can find a point P whose coordinates are (x, y, z) .

For this, first find the point A on x -axis corresponding to the real number x , then locate the point N in XY -plane such that (x, y) are the coordinates of the point N . Draw a perpendicular through N to the XY -plane and locate on it the point P corresponding to the real number z . The point P so obtained has the coordinates (x, y, z) .

OX, OY, OZ directions are taken as positive and OX', OY', OZ' are taken as negative. Each

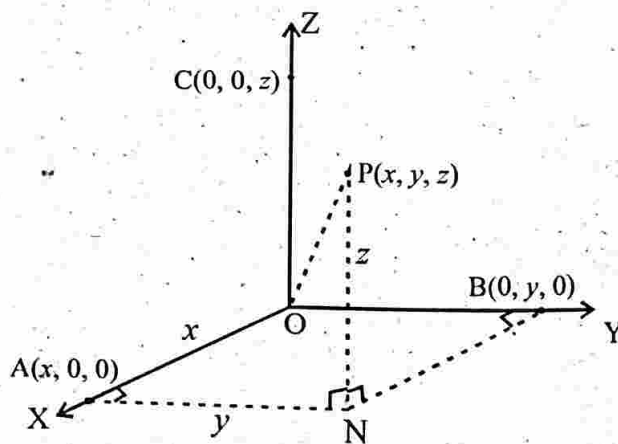


Fig. 12.5

octant is formed by any three of the edges specified here.

If the edges are OX, OY and OZ, we name the octant as XOYZ. Similarly for the edges OX', OY, OZ we name it as X'OYZ and so on. The remaining octants are X'OYZ, X'OY'Z, XOY'Z, XOYZ', X'OYZ', X'OY'Z', XOY'Z'

The following table will provide the idea of signs of the coordinates of points in different octants.

Name	XOYZ	X'OYZ	X'OY'Z	XOY'Z	XOYZ'	X'OYZ'	X'OY'Z'	XOY'Z'
Octant	I	II	III	IV	V	VI	VII	VIII
Sign	+++	--+	---	+-+	++-	-+-	---	+--
e.g.	(3, 2, 1)	(-1, 2, 4)	(-2, -3, 1)	(4, -1, 2)	(2, 1, -3)	(-1, 3, -1)	(-1, -2, -3)	(3, -2, -1)

Example 1

(NCERT)

- A point is on the x-axis. Write the y-coordinate and z-coordinate.
- A point is on the XY-plane. What can you say about its z-coordinate?
- If P is a point on YZ plane, then its x coordinate is

(September 2013, March 2013)

Solution

- Since the point is on x-axis, y and z-coordinates are zero.
- In the XY-plane, z-coordinate of any point is zero.
- In YZ plane, x-coordinate of any point is zero.

∴ The x-coordinate of P is zero.

Example 2

(NCERT)

Fill in the blanks.

- The x-axis and y-axis taken together determine a plane known as
- The coordinates of point in the XY-plane are of the form
- Coordinate planes divide the space into octants.

(March 2010)

Solution

- XY-plane
- (x, y, 0)
- 8

Example 3

Which of the following point is on XZ plane?

- (1, 1, 1)
- (2, 0, 3)
- (2, 3, 0)
- (-1, 2, 3)

(March 2015)

Solution

The y-coordinate of any point on XZ plane is zero. ∴ (2, 0, 3) is a point on XZ plane.

Example 7

(NCERT)

Name the octants in which the following points lie:

- (1, 2, 3), (4, -2, 3), (4, -2, -5), (4, 2, -5), (-4, 2, -5), (-4, 2, 5), (-3, -1, 6), (-2, -4, -7), (-3, 1, 2), (-3, 1, -2)

(March 2010)

Solution

- (1, 2, 3) lies in octant I
 (4, -2, 3) lies in octant IV
 (4, -2, -5) lies in octant VIII
 (4, 2, -5) lies in octant V
 (-4, 2, -5) lies in octant VI
 (-4, 2, 5) lies in octant II
 (-3, -1, 6) lies in octant III
 (-2, -4, -7) lies in octant VII
 (-3, 1, 2) lies in octant II
 (-3, 1, -2) lies in octant VI

XOYZ
 XOY'Z
 XOY'Z'
 XOYZ'
 X'OYZ'
 X'OYZ
 X'OY'Z
 X'OY'Z'
 X'OYZ
 X'OYZ'

STUDY TIPS

The coordinates of any point on

x-axis : (x, 0, 0) (March 2012)

y-axis : (0, y, 0) (March 2015)

z-axis : (0, 0, z)

XY-plane : (x, y, 0)

XZ-plane : (x, 0, z)

YZ-plane : (0, y, z)

**SOLUTIONS TO NCERT TEXT BOOK EXERCISE 12.1**

1. A point is on the *x*-axis. What are its *y*-coordinate and *z*-coordinates?

Solution Refer Example 1(i)

2. A point is in the *XZ*-plane. What can you say about its *y*-coordinate?

Solution

y - coordinate = 0

3. Name the octants in which the following points lie:

(1, 2, 3), (4, -2, 3), (4, -2, -5), (4, 2, -5), (-4, 2, -5), (-4, 2, 5), (-3, -1, 6), (2, -4, -7).

Solution

I, IV, VIII, V, VI, II, III, VIII

4. Fill in the blanks:

- The *x*-axis and *y*-axis taken together determine a plane known as
- The coordinates of points in the *XY*-plane are of the form
- Coordinate planes divide the space into octants.

Solution Refer Example 2