11.9 HEAT TRANSFER

So far we have discussed about the transfer of heat energy from one system to another. Now our problem is how this heat transfer takes place? There are three ways by which heat energy is transferred from one place to another or one end to another end of a substance. These three modes are conduction, convection and radiation. For conduction and convection modes a material medium is necessary. But for radiation mode material medium is not necessary. The heat energy coming from the sun to the earth is by radiation mode. There will be no heat transfer between two systems, say a body and its surroundings, if both are at the same temperature, which we have already seen.

11.9.1 Conduction

Conduction is possible in solids, liquids and gases. But it is best in solids. Substances which conduct heat are called good conductors and which do not are called poor conductors or insulators. All metals are good conductors. Gases and non-metals are poor conductors. Asbestos, rubber, fibre glass etc are some examples of non-metals. Silver is the best conductor.

Transfer of heat from one place to another or one end (hot end) to the other end (cold end) of the substance, by the assistance of its particles, but without their actual movement is called ***conduction.***

When a substance is heated, the atoms of the substance begin to vibrate with greater amplitudes. These increased vibrational amplitudes and thus the energy associated with them are passed from atom to atom (or molecule to molecule). Then there is an increase in temperature from point to point. Once the temperature of the regions or points becomes constant, we say that the steady - state is reached.

The quantity of heat (Q) flowing through a substance depends on

a. its area of cross section (A),

b. temperature gradient $\left(\frac{dT}{dx}\right)$,

c. time (t) for which the heat flows and

d. the nature of the material of the substance

Mathematically, $Q \propto A.\left(\frac{dT}{dx}\right)t$

$$\boxed{Q = KA.\left(\frac{dT}{dx}\right)t}$$

where the constant of proportionality, K, is called the Thermal Conductivity (T.C) of the material.

If dQ is the quantity of heat flowing through the substance in a time, 'dt', then $\frac{dQ}{dt}$ is called *heat transfer rate or rate of flow of heat or heat current* and represented by the symbol, H.

$$\therefore \quad H = \frac{dQ}{dt} = KA.\left(\frac{dT}{dx}\right)$$

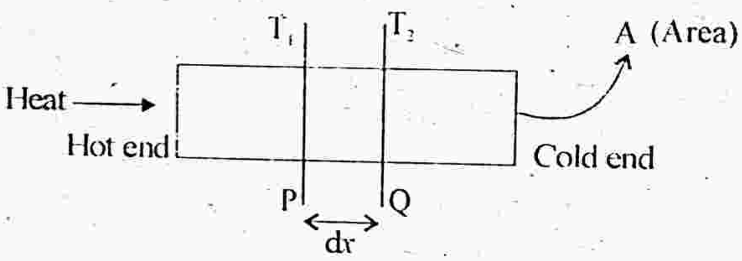


Fig. 9

Consider a rod or a bar of area of cross section A. P and Q are two layers or regions, separated by a length *dx*. Let T_1 and T_2 be the steady state temperatures at P and Q respectively and $T_1 > T_2$. Then the ratio of the temperature difference, $T_1 - T_2 = dT$, to *dx* is called temperature gradient and is represented as $\frac{dT}{dx}$.

Definition of thermal conductivity (K)

We have, $K = \frac{Q}{A\left(\frac{dT}{dx}\right)t}$.

If $A = 1$, $\frac{dT}{dx} = 1$ and $t = 1$, then $K = Q$

Hence ***Thermal Conductivity*** *of a material is defined as the quantity of heat flowing through unit area of cross section of the material in unit time under unit tempera-*

ture gradient.

Units

In S.I ⟶ $Js^{-1}m^{-1}K^{-1}$ or $W.m^{-1}K^{-1}$.

In C G S ⟶ $erg/sec/cm^2$/unit temperature gradient.

Dimensional formula

$$K = \frac{Q}{A\left(\frac{dT}{dx}\right)t} = \frac{[ML^2T^{-2}]}{[L^2]\left[\frac{K}{L}\right][T]} = [MLT^{-3}K^{-1}]$$

'K' is large for metals and K is small for gases and non-metals. T.C. of some substances are given in the following table.

TABLE 6

Substance	TC - $Wm^{-1}K^{-1}$	Substance	TC - $Wm^{-1}K^{-1}$	Substance	TC - $Wm^{-1}K^{-1}$
Metals		**Gases**		**Non - metals**	
Silver	427	Air (dry)	0.0234	Ice	2
Copper	397	Hydrogen	0.172	Concrete	0.8
Gold	314	Helium	0.138	Glass	0.8
Aluminium	238	Oxygen	0.0238	Water	0.6
Iron	79.5	Nitrogen	0.0234	Rubber	0.2
Lead	34.7			Bricks	0.15
Stainless steel	14			Wood	0.08
Mercury	8.3				

11.9.2 Determination of Thermal Conductivity - Searle's Method

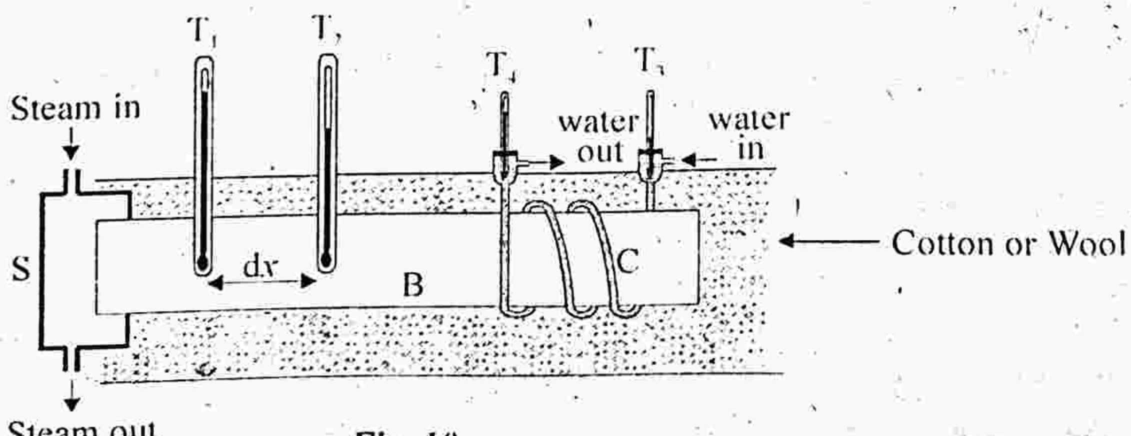


Fig. 10

S - Steam chamber

B - Cylindrical rod of area of cross section A and whose TC (λ) is to be found.

T_1, T_2, T_3, T_4 - Thermometers.

C - Spiral tube made of copper

The above figure shows the schematic diagram of the Searle's apparatus. The whole arrangement is placed inside a wooden box containing cotton to prevent loss of heat from the sides.

The material whose thermal conductivity is to be studied is taken in the form of a cylinder (AB). It's area of cross section is 'A'. One end of the cylinder is put in a steam chamber (S). A copper spiral tube (C), with inlet and outlet water circulation arrangement is wound at the other end. Two small holes are drilled (and filled with mercury for good thermal contact) separated by a distance 'dx' and two thermometers T_1 and T_2 are placed in them. Thermometers T_3 and T_4 are also placed at the ends of the spiral tube (C) to record the temperature of inflow and outflow water.

Steam is passed and the cylinder is heated to attain its steady state. Cold water is circulated through 'C' and its rate is adjusted so as to get fixed temperature difference between T_3 and T_4. Under steady state condition, for convenience, we take the readings of the thermometers as T_1, T_2, T_3 and T_4. When steady state is reached heat flowing through the rod is equal to the heat absorbed by the amount of water flowing through the spiral tube. Let 't' be the time for which the steam flows. Then,

$$KA\left(\frac{T_1 - T_2}{dx}\right)t = ms_w\,(T_4 - T_3)$$

where m - mass of water collected, s_w - Specific heat of water

$$K = \frac{ms_w(T_4 - T_3)dx}{A(T_1 - T_2)t}$$

Using the above formula, TC of the rod, K, can be determined.

11.9.3 Thermal Resistance

In electricity, by Ohm's law, electic current, $I = \dfrac{\text{Potential difference}}{\text{Resistance}}$

In the same manner here heat current, $H = \dfrac{Q}{t} = KA\left(\dfrac{dT}{dx}\right) = \dfrac{dT}{\left(\frac{dx}{KA}\right)}$

If 'dx' is replaced with 'L', the length, then, $H = \dfrac{dT}{\left(\frac{L}{KA}\right)}$

The term $\dfrac{L}{KA}$ is known as thermal resistance of the material and is represented as R.

Hence, thermal resistance, $R = \dfrac{L}{KA}$

It is also referred to as R value of the material.

Lower the value of K of the material, then higher its R value. A substance having a higher R value is called a poor thermal conductor or a good thermal insulator.

The unit of R is *foot*^{2}. *°F. hour per Btu* (British thermal unit)

The insulating properties of materials used in buildings are commonly expressed in R values by the engineers.

11.9.4 Practical Applications

a. Cooking utensils are made of metals and their handles are made of wood. This is because metals are good conductors of heat, while wood is a bad conductor of heat.

b. When we take the metal ice tray and a package of frozen food from the freezer of the refrigerator; metal tray feels colder than the package. This is because metal is a good conductor of heat and it removes heat from our hand much faster.

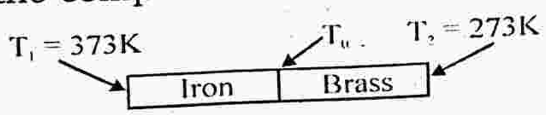
c. Houses made of hollow brick walls are cooler than concrete walls.

d. During winter birds swell their feathers. In doing so the air trapped between the feathers prevent the loss of heat from their body. This is because air is a poor conductor of heat.

e. Ice is packed in saw dust or gunny bags. This is because air trapped in them prevents loss of heat and so ice does not melt. Air is a poor conductor of heat.

Solved Examples

15. An iron bar (L_1 = 0.1m, A_1 = 0.02 m^2, K_1 = 79W m^{-1}K^{-1}) and a brass bar (L_2 = 0.1 m, A_2 = 0.02 m^2, K_2 = 109W m^{-1}K^{-1}) are soldered end to end as shown in figure. The free ends of the iron bar and brass bar are maintained at 373 K and 273 K respectively. Obtain expressions for and hence compute (i) the temperature of the junction of the two bars, (ii) the equivalent thermal conductivity of the compound bar, and (iii) the heat current through the compound bar.



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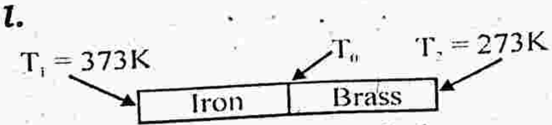


Given, $L_1 = L_2 = L = 0.1$ m, $A_1 = A_2 = A = 0.2\text{m}^2$, $K_1 = 79\text{W m}^{-1}\text{ K}^{-1}$, $K_2 = 109\text{ W m}^{-1}\text{ K}^{-1}$, $T_1 = 373\text{K}$, and $T_2 = 273\text{K}$.

Under steady state condition, the heat current (H_1) through iron bar is equal to the heat current (H_2) through brass bar.

So, $H = H_1 = H_2$

$$= \frac{K_1A_1(T_1 - T_0)}{L_1} = \frac{K_2A_2(T_0 - T_2)}{L_2}$$

For $A_1 = A_2 = A$ and $L_1 = L_2 = L$, this equation leads to $K_1(T_1 - T_0)$ $= K_2(T_0 - T_2)$

Thus the junction temperature T_0 of the two bars is $T_0 = \frac{(K_1T_1 + K_2T_2)}{(K_1 + K_2)}$

Using this equation, the heat current H through either bar is

$$H = \frac{K_1A(T_1 - T_0)}{L} = \frac{K_2A(T_0 - T_2)}{L}$$

$$= \left(\frac{K_1K_2}{K_1+K_2}\right)\frac{A(T_1-T_0)}{L} = \frac{A(T_1-T_2)}{L\left(\frac{1}{K_1}+\frac{1}{K_2}\right)}$$

Using these equations, the heat current H' through the compound bar of length $L_1 + L_2 = 2L$ and the equivalent thermal conductivity K', of the compound bar are given by

$$H' = \frac{K'A(T_1-T_2)}{2L} = H$$

$$K = \frac{2K_1K_2}{K_1+K_2}$$

i. $T_0 = \frac{(K_1T_1+K_2T_2)}{(K_1+K_2)}$

$$= \frac{(79 Wm^{-1}K^{-1})(373K) + (109 Wm^{-1}K^{-1})(273K)}{79 Wm^{-1}K^{-1} + 109 Wm^{-1}K^{-1}}$$

$= 315K$

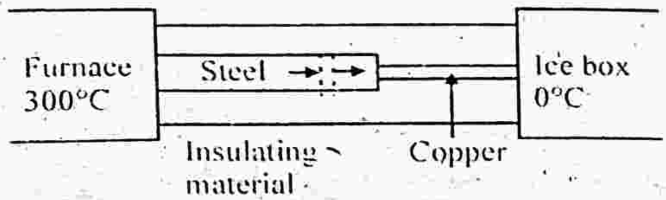
ii. $K' = \frac{2K_1K_2}{K_1+K_2}$

$$= \frac{2\times(79 Wm^{-1}K^{-1})\times(109 Wm^{-1}K^{-1})}{79 Wm^{-1}K^{-1}+109 Wm^{-1}K^{-1}}$$

$= 91.6\ Wm^{-1}K^{-1}$

iii. $H = \frac{K'A(T_1-T_2)}{2L} = \frac{91.6\times0.02\times100}{2\times0.01} = 916\ J$

16 What is the temperature of the steel-copper junction in the steady state of the system shown in the figure given below? Length of the steel rod = 15.0 cm, length of the copper rod = 10.0 cm, temperature of the furnace = 300 °C, temperature of the other end = 0 °C. The area of cross section of the steel rod is twice that of the copper rod. (Thermal conductivity of steel = 50.2 $Js^{-1}\ m^{-1}K^{-1}$; and of copper = 385 $Js^{-1}m^{-1}K^{-1}$).



Sol.

The insulating material around the rods reduces heat loss from the sides of the rods. Therefore, heat flows only along the length of the rods. Consider any cross section of the rod. In the steady state, heat flowing into the element must equal the heat flowing out of it; otherwise there would be a net gain or loss of heat by the element and its temperature would not be steady. Thus in the steady state, rate of heat flowing across a cross section of the rod is the same at every point along the length of the combined steel-copper rod. Let T be the temperature of the steel-copper junction in the steady state. Then,

$$\frac{K_1A_1(300-T)}{L_1} = \frac{K_2A_2(T-0)}{L_2}$$

where 1 and 2 refer to the steel and copper rod respectively. For $A_1 = 2A_2$, $L_1 = 15.0$ cm, $L_2 = 10.0$ cm, $K_1 = 50.2\ J\ s^{-1}\ m^{-1}\ K^{-1}$, $K_2 = 385\ J\ s^{-1}\ m^{-1}\ K^{-1}$, we have

$$\frac{50.2\times2(300-T)}{15} = \frac{385T}{10}$$

which gives T = 44.4 °C

17. A slab made of copper has a thickness 25 cm and area 90 cm^2. If the temperatures on either side of the slab are 125°C and 10°C, calculate the conduction rate through the slab. TC for copper is 400 $Wm^{-1}K^{-1}$.

Sol.

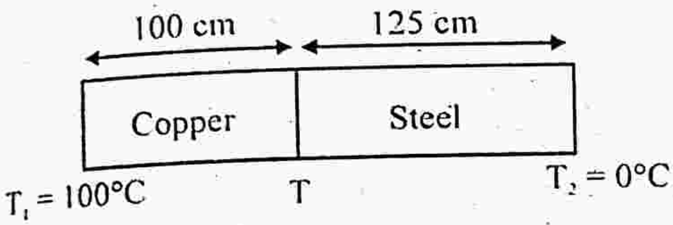
Given $K = 400\ Wm^{-1}K^{-1}$,
$A = 90\times10^{-4}\ m^2$, $dT = 115°C$,
$dx = 25\times10^{-2} m$

$$\frac{dQ}{dt} = KA\left(\frac{dT}{dx}\right)$$

$$= 400\times90\times10^{-4}\times\frac{115}{25\times10^{-2}} = 1656\ Js^{-1}$$

18. A copper rod of 100 cm and a steel rod of 125 cm are joined end to end. Both have same diameter of 2 cm. The free ends of copper and steel are kept at 100°C and 0°C. Calculate temperature at the copper - steel junction and rate of flow of heat through the combination. T.C. of copper is 400 $Wm^{-1}K^{-1}$ and that of steel is 50 $Wm^{-1}K^{-1}$.

Sol.



Given, For copper, $K_1 = 400\ Wm^{-1}K^{-1}$,

For steel, $K_2 = 50\ Wm^{-1}K^{-1}$

$A = \pi r^2$, $r = 1\ cm = 1 \times 10^{-2}\ m$,

$dx_1 = 100\ cm = 1\ m$, $dx_2 = 1.25\ m$

$$\frac{dQ}{dt} = \frac{K_1 A (T_1 - T)}{dx_1} = \frac{K_2 A (T - T_2)}{dx_2}$$

$$\frac{400 \times (100 - T)}{1} = \frac{50 (T - 0)}{1.25}$$

$$1.25 \times 400\ (100 - T) = 50T$$

$$500 \times 100 - 500T = 50T$$

$$\therefore \quad T = \frac{500 \times 100}{550} = 90.9°C \approx 91°C$$

$$\therefore \quad \frac{dQ}{dt} = \frac{50 \times \pi \times 10^{-4} \times 91}{1.25} = \frac{1.4287}{1.25}$$

$$= 1.14\ J/sec$$

11.9.5 Convection

Transfer of heat by convection mode takes place in liquids and gases.

Convection *is the phenomenon in which heat is transferred from one place to another by the actual movement of the particles of the heated substance.*

If we boil a kettle of water, the hot water molecules become less dense and move to the top surface. At the same time dense water molecules at the top move to the bottom. Thus a convectional current gets established and the entire water gets heated. Thus it is clear that if there were no convectional currents, it would be difficult to boil water.

The same procedure is taking place in air movement also. As dense cool air enters a room through doors and windows, the less dense hot air gets exhausted through the ventilators in the room. The reason for convectional current is due to the unequal heating of a fluid. This is the basic principle for the formation of winds. The summer and winter monsoons on the earth are due to the transfer of heat by convection and radiation, rotation of earth and earth's gravity.

The rate at which heat is transferred from the surface of a fluid is given by the relation,

$$\boxed{\frac{\Delta Q}{\Delta t} = h\, A\, \Delta T}$$

where, A - surface area of fluid.

ΔT - temperature difference between the fluid at the upper and lower surfaces.

h - a constant, known convection coefficient.

The value of 'h' depends on

i. the shape of the surface and ii. whether the surface is horizontal or vertical

11.9.6 Radiation

Thermal radiation is the third mode of heat transfer, which does not require a material medium. All bodies radiate energy in the form of electro-magnetic waves. The energy transferred in this mode is often called thermal radiation, just to distinguish it from electromagnetic signals and nuclear radiation. The type of radiation associated with the transfer of heat energy from one location to another location is often known as infrared radiations. This is because the wavelength range of thermal radiations is from 800 nm to 400 μm, which belongs to the infrared region.

Some basic properties of thermal radiations are given below.

i. They travel in straight lines with the speed of light (3×10^8 m/s).

ii. A material medium is not necessary for the propagation.

iii. They do not heat the medium through which they are travelling.

iv. They can be reflected and refracted just as light.

v. They also exhibit the phenomena like interference, diffraction and polarisation.

vi. They obey the inverse square law (i.e, *Intensity of thermal radiation at a point is inversely proportional to the square of the distance of the point from the radiating source*).

vii. Thermal radiations are of longer wavelength than that of visible light.

The rate at which energy radiated depends on

a. temperature of the body and

b. nature of the radiating surface of the body

Any object whose temperature is above 0 K, emits and absorbs thermal radiation.