

11.5.4 Standard equation of an ellipse

In this section, we consider the equation of the ellipse with centre at the origin and foci along the coordinate axes.

First we derive the equation of the ellipse with foci along the x -axis. Let F_1 and F_2 be the foci and O be the midpoint of the line segment F_1F_2 . Choose O as the origin. Draw the horizontal and vertical lines through O as x and y axes.

$$\text{Take } F_1F_2 = 2c$$

Then F_1 is the point $(-c, 0)$ and F_2 is $(c, 0)$.

Let $P(x, y)$ be any point on the ellipse such that the sum of the distances from P to the two foci be $2a$.

$$PF_1 + PF_2 = 2a$$

By using distance formula we get,

$$\sqrt{(x+c)^2 + (y-0)^2} + \sqrt{(x-c)^2 + (y-0)^2} = 2a$$

$$\sqrt{(x+c)^2 + y^2} = 2a - \sqrt{(x-c)^2 + y^2}$$

$$\text{Squaring both sides, we get, } (x+c)^2 + y^2 = 4a^2 - 4a\sqrt{(x-c)^2 + y^2} + (x-c)^2 + y^2$$

$$\text{i.e., } x^2 + 2cx + c^2 + y^2 = 4a^2 - 4a\sqrt{(x-c)^2 + y^2} + x^2 - 2cx + c^2 + y^2$$

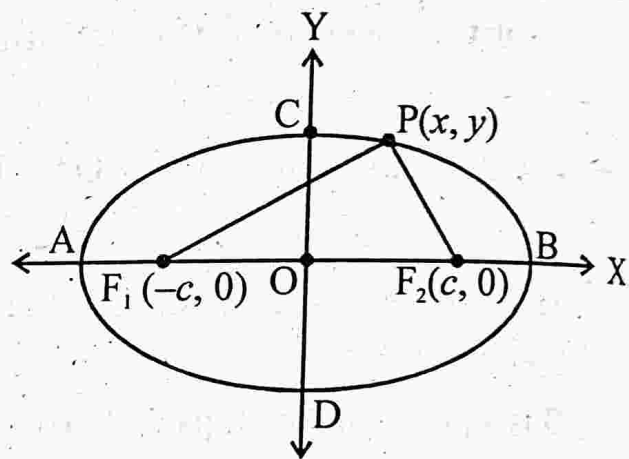


Fig 11.23

$$\text{i.e., } 4cx = 4a^2 - 4a\sqrt{(x-c)^2 + y^2}$$

$$\text{i.e., } \frac{cx}{a} = a - \sqrt{(x-c)^2 + y^2} \Rightarrow \sqrt{(x-c)^2 + y^2} = a - \frac{cx}{a}$$

$$\text{Squaring we get, } (x-c)^2 + y^2 = \left(a - \frac{cx}{a}\right)^2$$

$$\text{i.e., } x^2 - 2cx + c^2 + y^2 = a^2 - 2cx + \frac{c^2}{a^2}x^2$$

$$\text{i.e., } x^2 \left(1 - \frac{c^2}{a^2}\right) + y^2 = a^2 - c^2$$

$$\text{i.e., } x^2 \left(\frac{a^2 - c^2}{a^2}\right) + y^2 = (a^2 - c^2)$$

$$\text{i.e., } \frac{x^2}{a^2} + \frac{y^2}{a^2 - c^2} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \text{ where } c^2 = a^2 - b^2$$

Any point (x, y) on the ellipse satisfies the equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ and conversely if any point satisfies } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

then the point (x, y) lies on the ellipse.

Thus the standard equation of the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

- Centre of the ellipse is $(0, 0)$
- Foci of the ellipse are $(-c, 0)$ and $(c, 0)$.
- Vertices are $(-a, 0)$ and $(a, 0)$
- Equation of the major axis is $y = 0$
- Length of the major axis is $= 2a$
- Equation of the minor axis is $x = 0$
- Length of the minor axis is $2b$.
- $c^2 = a^2 - b^2$

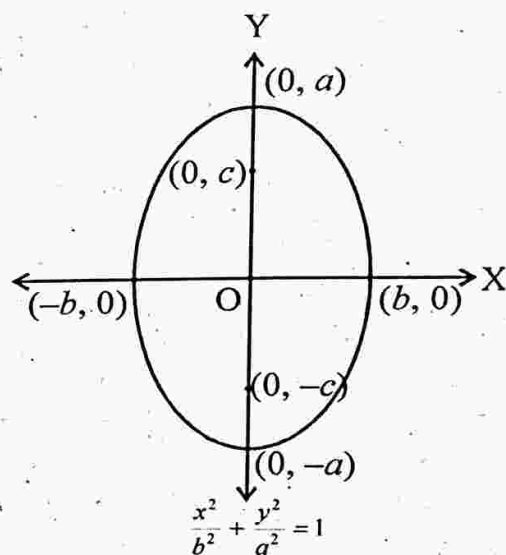


Fig 11.24

In the equation of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ where $a > b$, the major axis is the x -axis.

If we choose the major axis as y -axis, then the equation of the ellipse takes the form $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, which is given in Fig 11.24

OBSERVATIONS

- For any point $P(x, y)$ on the ellipse, $-a \leq x \leq a$.
- Ellipse is symmetric with respect to both the coordinate axes.
- The foci always lie on the major axis.
- The major axis is along the x -axis if x^2 has the larger denominator.
The major axis is along the y -axis if y^2 has the larger denominator.

11.5.5 Latus rectum

The eccentricity e of an ellipse is given by $e = \frac{c}{a} \therefore c = ae$

Hence the foci are $(-ae, 0)$ and $(ae, 0)$.

The equation of the latus rectum are $x = -ae$ and $x = ae$

Let the length of AF_2 be l .

Then the coordinates of A are (c, l) , i.e., (ae, l)

Since A lies on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we have

$$\frac{(ae)^2}{a^2} + \frac{l^2}{b^2} = 1 \Rightarrow \frac{l^2}{b^2} = 1 - e^2 \Rightarrow l^2 = b^2(1 - e^2)$$

$$\text{But } e^2 = \frac{c^2}{a^2} = \frac{a^2 - b^2}{a^2} = 1 - \frac{b^2}{a^2}$$

$$\text{Hence } 1 - e^2 = \frac{b^2}{a^2}$$

$$\text{Therefore } l^2 = \frac{b^4}{a^2}, \text{ i.e., } l = \frac{b^2}{a}$$

Since the ellipse is symmetric with respect to y -axis, $AF_2 = F_2B$.

\therefore Length of the latus rectum is $\frac{2b^2}{a}$.

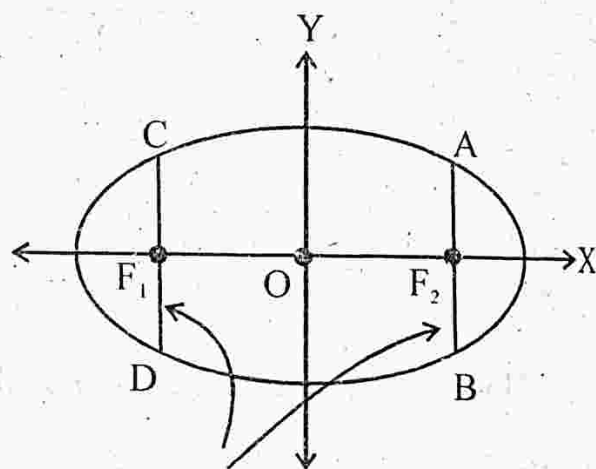


Fig 11.25

Example 23

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and latus rectum of the following ellipses (NCERT)

i. $\frac{x^2}{25} + \frac{y^2}{9} = 1$ (March 2013, 2014)

ii. $\frac{x^2}{25} + \frac{y^2}{100} = 1$ (March 2009)

iii. $4x^2 + 9y^2 = 36$ (March 2011)

Solution

- i. Since the denominator of x^2 is greater than the denominator of y^2 , the equation of the ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1. \therefore \text{The equation of the ellipse is } \frac{x^2}{25} + \frac{y^2}{9} = 1$$

$$\therefore a^2 = 25 \text{ and } b^2 = 9 \quad \therefore a = 5 \text{ and } b = 3$$

$$c = \sqrt{a^2 - b^2} = \sqrt{25 - 9} = \sqrt{16} = 4$$

$$\therefore \text{Foci } (\pm c, 0) = (\pm 4, 0) \text{ or } (4, 0) \text{ and } (-4, 0)$$

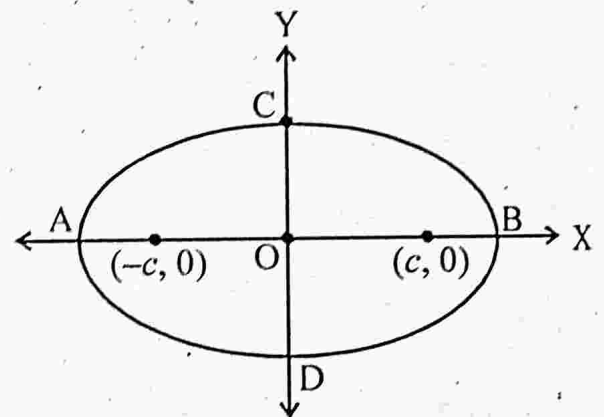
$$\text{Vertices } (\pm a, 0) = (\pm 5, 0) \text{ or } (5, 0) \text{ and } (-5, 0)$$

$$\text{Length of the major axis } 2a = 2 \times 5 = 10$$

$$\text{Length of the minor axis } 2b = 2 \times 3 = 6$$

$$\text{Eccentricity } e = \frac{c}{a} = \frac{4}{5}$$

$$\text{Length of the latus rectum } \frac{2b^2}{a} = \frac{2 \times 9}{5} = \frac{18}{5}$$



ii. Equation of the ellipse is $\frac{x^2}{25} + \frac{y^2}{100} = 1$

Since the denominator of y^2 is greater than the denominator of x^2 , the equation of the ellipse is

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1.$$

$$a^2 = 100 \text{ and } b^2 = 25 \quad \therefore a = 10 \text{ and } b = 5$$

$$c = \sqrt{a^2 - b^2} = \sqrt{100 - 25} = \sqrt{75}$$

Foci are $(0, \pm\sqrt{75})$

Vertices are $(0, \pm 10)$

Length of the major axis $= 2a = 20$

Length of the minor axis $= 2b = 10$

$$\text{Eccentricity } e = \frac{c}{a} = \frac{\sqrt{75}}{10} = \frac{5\sqrt{3}}{10} = \frac{\sqrt{3}}{2}$$

$$\text{Length of the latus rectum} = \frac{2b^2}{a} = \frac{2 \times 25}{10} = 5$$

iii. The equation of the ellipse is $4x^2 + 9y^2 = 36$

Dividing by 36, we get $\frac{x^2}{9} + \frac{y^2}{4} = 1$. Since the denominator of x^2 is greater than that of

y^2 , the equation of the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$a^2 = 9 \text{ and } b^2 = 4 \therefore a = 3 \text{ and } b = 2$$

$$c = \sqrt{a^2 - b^2} = \sqrt{9 - 4} = \sqrt{5}$$

Foci are $(\pm\sqrt{5}, 0)$ and the vertices are $(\pm 3, 0)$

The length of the major axis $= 2a = 6$

The length of the minor axis $= 2b = 4$

$$\text{Eccentricity } e = \frac{c}{a} = \frac{\sqrt{5}}{3}$$

$$\text{The length of the latus rectum} = \frac{2b^2}{a} = \frac{2 \times 4}{3} = \frac{8}{3}$$

Example 24

Find the equation of the ellipse whose

- i. Vertices $(\pm 5, 0)$, foci $(\pm 4, 0)$ ii. Vertices $(0, \pm 13)$, foci $(0, \pm 5)$

(NCERT)

Solution

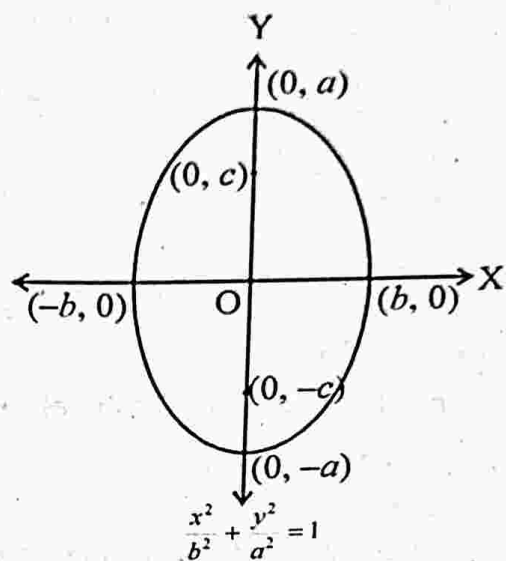
The vertices are $(\pm 5, 0)$ and foci are $(\pm 4, 0)$

Since the vertices are on x-axis, the equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\therefore a = 5 \text{ and } c = 4$$

$$\text{Hence } c^2 = a^2 - b^2 \Rightarrow 16 = 25 - b^2 \therefore b^2 = 9$$

$$\therefore \text{Equation of the ellipse is } \frac{x^2}{25} + \frac{y^2}{9} = 1$$



Hence eccentricity, $e = \frac{c}{a} = \frac{3}{5}$

Example 30

An ellipse whose major axis as x -axis and the centre $(0, 0)$ passes through $(4, 3)$ and $(-1, 4)$

i. Find the equation of the ellipse.

ii. Find its eccentricity.

(March 2010)

Solution

Since the major axis is along x -axis, the equation of the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (1)

Since $(4, 3)$ is a point on the ellipse, we get $\frac{4^2}{a^2} + \frac{3^2}{b^2} = 1$ or $\frac{16}{a^2} + \frac{9}{b^2} = 1$ (2)

Since $(-1, 4)$ is a point on the ellipse, we get $\frac{(-1)^2}{a^2} + \frac{4^2}{b^2} = 1$ or $\frac{1}{a^2} + \frac{16}{b^2} = 1$ (3)

Multiplying (3) by 16 and subtracting (2), we get $\frac{256}{b^2} - \frac{9}{b^2} = 16 - 1$

i.e., $\frac{247}{b^2} = 15$ or $b^2 = \frac{247}{15}$

(3) $\rightarrow \frac{1}{a^2} = 1 - \frac{16}{b^2} = 1 - \frac{16}{\frac{247}{15}} = 1 - \frac{16 \times 15}{247} = \frac{247 - 240}{247} = \frac{7}{247}$

i.e., $\frac{1}{a^2} = \frac{7}{247}$ or $a^2 = \frac{247}{7}$

Hence (1) $\rightarrow \frac{x^2}{\frac{247}{7}} + \frac{y^2}{\frac{247}{15}} = 1$ or $7x^2 + 15y^2 = 247$

ii. $c^2 = \frac{247}{7} - \frac{247}{15} = \frac{8(247)}{15 \times 7} \therefore c = \sqrt{\frac{8 \times 247}{15 \times 7}}$

Eccentricity $e = \frac{c}{a} = \frac{\sqrt{\frac{8 \times 247}{15 \times 7}}}{\sqrt{\frac{247}{7}}} = \sqrt{\frac{8}{15}}$

Example 31

A rod AB of length 15 cm rests in between two coordinate axes in such a way that the end point A lies on x -axis and end point B lies on y -axis. A point $P(x, y)$ is taken on the rod in such

a way that $AP = 6$ cm. Show that the locus of P is an ellipse.

(NCERT)

Solution

Let AB be the rod making an angle θ with OX as shown in figure and $P(x, y)$ the point on it such that $AP = 6$ cm.

Since $AB = 15$ cm, we have $PB = 9$ cm.

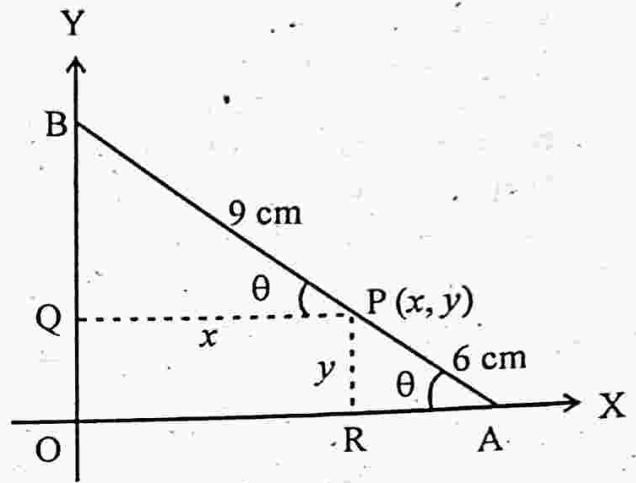
From P draw PQ and PR perpendicular on y -axis and x -axis, respectively.

From $\triangle PBQ$, $\cos \theta = \frac{x}{9}$

From $\triangle PRA$, $\sin \theta = \frac{y}{6}$

Since $\cos^2 \theta + \sin^2 \theta = 1$ we get $\left(\frac{x}{9}\right)^2 + \left(\frac{y}{6}\right)^2 = 1$ or $\frac{x^2}{81} + \frac{y^2}{36} = 1$

Thus the locus of P is an ellipse.



Example 32

An arch on a road is in the shape of semi-ellipse. The breadth of the road is 30 feet. A man 6 feet tall just touches the arch when he stands 2 feet from the side.

- Assuming the road level as x -axis (major axis). Find the point C .
- What is the maximum height of arch (minor axis)?

Solution

i. Take O as the origin, Since $OA = 15$, C is the point $(13, 6)$

ii. Let the equation of the ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

The length of the major axis $2a = 30$

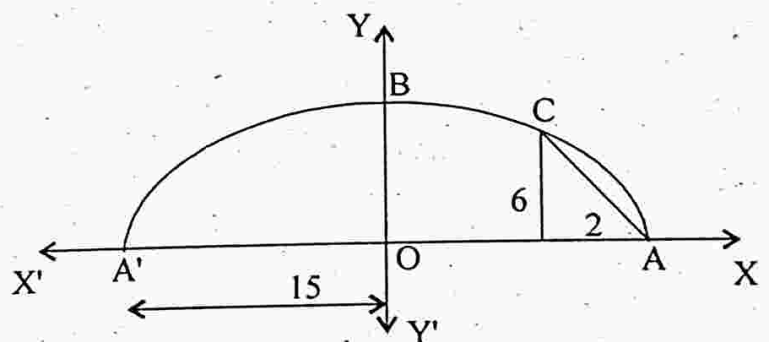
$$\therefore a = 15$$

Hence the equation of the ellipse is

$$\frac{x^2}{15^2} + \frac{y^2}{b^2} = 1 \quad \dots\dots\dots(1)$$

Since $(13, 6)$ is a point on (1), we get $\frac{13^2}{15^2} + \frac{6^2}{b^2} = 1$

$$\frac{6^2}{b^2} = 1 - \frac{13^2}{15^2} = \frac{225 - 169}{225} = \frac{56}{225} \Rightarrow b^2 = \frac{36}{56} \times 225 \Rightarrow b = \frac{6 \times 15}{2\sqrt{14}} = \frac{45}{\sqrt{14}} \text{ feet}$$



SOLUTIONS TO NCERT TEXT BOOK EXERCISE 11.3

In each of the Questions 1 to 9, find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse.

1. $\frac{x^2}{36} + \frac{y^2}{16} = 1$

Solution

The equation of the ellipse is $\frac{x^2}{36} + \frac{y^2}{16} = 1$

Since the denominator of x^2 is greater than the denominator of y^2 , the equation of the

ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

$$a^2 = 36 \text{ and } b^2 = 16 \therefore a = 6 \text{ and } b = 4$$

$$c = \sqrt{a^2 - b^2} = \sqrt{36 - 16} = \sqrt{20}$$

\therefore The foci are $(-\sqrt{20}, 0)$ and $(\sqrt{20}, 0)$

Vertices are $(-6, 0)$ and $(6, 0)$

Length of major axis is $2a = 2 \times 6 = 12$

Length of minor axis is $2b = 2 \times 4 = 8$

$$\text{The eccentricity } e = \frac{c}{a} = \frac{\sqrt{20}}{6}$$

$$\begin{aligned} \text{Length of the latus rectum} &= \frac{2b^2}{a} \\ &= \frac{2 \times 16}{6} = \frac{16}{3} \end{aligned}$$

2. $\frac{x^2}{4} + \frac{y^2}{25} = 1$

Solution

The equation of the ellipse is $\frac{x^2}{4} + \frac{y^2}{25} = 1$

Since the denominator of y^2 is greater than denominator of x^2 , the equation of the

$$\text{ellipse is } \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

$$b^2 = 4 \text{ and } a^2 = 25 \therefore b = 2 \text{ and } a = 5$$

$$\therefore c = \sqrt{a^2 - b^2} = \sqrt{25 - 4} = \sqrt{21}$$

Foci are $(0, \pm c)$ i.e., $(0, \pm \sqrt{21})$

Vertices are $(0, \pm a)$ i.e., $(0, \pm 5)$

Length of major axis is $2a = 10$

Length of minor axis is $2b = 4$

$$\text{The eccentricity } e = \frac{c}{a} = \frac{\sqrt{21}}{5}$$

$$\begin{aligned} \text{Length of the latus rectum} &= \frac{2b^2}{a} \\ &= \frac{2 \times 4}{5} = \frac{8}{5} \end{aligned}$$

3. $\frac{x^2}{16} + \frac{y^2}{9} = 1$

Solution

The equation of the ellipse is $\frac{x^2}{16} + \frac{y^2}{9} = 1$

Since the denominator of x^2 is greater than denominator of y^2 , the equation of the

ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$a^2 = 16, b^2 = 9 \therefore a = 4, b = 3$$

$$c = \sqrt{a^2 - b^2} = \sqrt{16 - 9} = \sqrt{7}$$

Foci are $(\pm c, 0)$ i.e., $(\pm \sqrt{7}, 0)$

Vertices are $(\pm a, 0)$ i.e., $(\pm 4, 0)$

Length of major axis is $2a = 8$

Length of minor axis is $2b = 6$

$$\text{Eccentricity } e = \frac{c}{a} = \frac{\sqrt{7}}{4}$$

$$\begin{aligned}\text{Length of the latus rectum} &= \frac{2b^2}{a} \\ &= \frac{2 \times 9}{4} = \frac{9}{2}\end{aligned}$$

$$4. \quad \frac{x^2}{25} + \frac{y^2}{100} = 1$$

Solution

Refer Example 23(ii)

$$5. \quad \frac{x^2}{49} + \frac{y^2}{36} = 1$$

Solution

The equation of the ellipse is $\frac{x^2}{49} + \frac{y^2}{36} = 1$.

Since the denominator of x^2 is greater than denominator of y^2 , the equation of the

ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$a^2 = 49 \text{ and } b^2 = 36$$

$$\therefore a = 7 \text{ and } b = 6$$

$$c = \sqrt{a^2 - b^2} = \sqrt{49 - 36} = \sqrt{13}$$

Foci are $(\pm\sqrt{13}, 0)$

Vertices are $(\pm 7, 0)$

Length of the major axis $= 2a = 14$

Length of the minor axis $= 2b = 12$

$$\text{Eccentricity } e = \frac{c}{a} = \frac{\sqrt{13}}{7}$$

$$\begin{aligned}\text{Length of the latus rectum} &= \frac{2b^2}{a} \\ &= \frac{2 \times 36}{7} = \frac{72}{7}\end{aligned}$$

$$6. \quad \frac{x^2}{100} + \frac{y^2}{400} = 1$$

Solution

The equation of the ellipse is

$\frac{x^2}{100} + \frac{y^2}{400} = 1$. Since the denominator of y^2 is greater than denominator of x^2 ,

the equation of the ellipse is $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

$$a^2 = 400 \text{ and } b^2 = 100$$

$$\therefore a = 20 \text{ and } b = 10$$

$$c = \sqrt{a^2 - b^2} = \sqrt{400 - 100} = \sqrt{300}$$

$$\therefore c = 10\sqrt{3}$$

Foci are $(0, \pm 10\sqrt{3})$

Vertices are $(0, \pm 20)$

Length of the major axis $= 2a = 40$

Length of the minor axis $= 2b = 20$

$$\text{Eccentricity } e = \frac{c}{a} = \frac{10\sqrt{3}}{20} = \frac{\sqrt{3}}{2}$$

$$\begin{aligned}\text{Length of the latus rectum} &= \frac{2b^2}{a} \\ &= \frac{2 \times 100}{20} = 10\end{aligned}$$

$$7. \quad 36x^2 + 4y^2 = 144$$

Solution

The equation of the ellipse is

$$36x^2 + 4y^2 = 144$$

Dividing by 144, we get

$$\frac{x^2}{4} + \frac{y^2}{36} = 1. \text{ Since the denominator of } y^2$$

is greater than denominator of x^2 , the

equation of the ellipse is $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

$$a^2 = 36 \text{ and } b^2 = 4$$

$$\therefore a = 6 \text{ and } b = 2$$

$$c = \sqrt{a^2 - b^2} = \sqrt{36 - 4} = \sqrt{32}$$

Foci are $(0, \pm \sqrt{32})$

Vertices are $(0, \pm 6)$

Length of the major axis $= 2a = 12$

Length of the minor axis $= 2b = 4$

$$\text{Eccentricity } e = \frac{c}{a} = \frac{\sqrt{32}}{6} = \frac{4\sqrt{2}}{6} \\ = \frac{2\sqrt{2}}{3}$$

$$\text{Length of the latus rectum} = \frac{2b^2}{a} \\ = \frac{2 \times 4}{6} = \frac{4}{3}$$

8. $16x^2 + y^2 = 16$

Solution

The equation of the ellipse is

$$16x^2 + y^2 = 16$$

Dividing by 16, we get $\frac{x^2}{1} + \frac{y^2}{16} = 1$

Since the denominator of y^2 is greater than denominator of x^2 , the equation of the

ellipse is $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$.

$$a^2 = 16 \text{ and } b^2 = 1 \therefore a = 4 \text{ and } b = 1$$

$$c = \sqrt{a^2 - b^2} \\ = \sqrt{16 - 1} = \sqrt{15}$$

Foci are $(0, \pm \sqrt{15})$

Vertices are $(0, \pm 4)$

Length of the major axis $= 2a = 8$

Length of the minor axis $= 2b = 2$

$$\text{Eccentricity } e = \frac{c}{a} = \frac{\sqrt{15}}{4}$$

$$\text{Length of the latus rectum} = \frac{2b^2}{a} \\ = \frac{2 \times 1}{4} = \frac{1}{2}$$

9. $4x^2 + 9y^2 = 36$

Solution

Refer Example 23(iii)

In each of the following Questions 10 to 20, find the equation for the ellipse that satisfies the given conditions:

10. Vertices $(\pm 5, 0)$, foci $(\pm 4, 0)$

Solution

Refer Example 24(i)

11. Vertices $(0, \pm 13)$, foci $(0, \pm 5)$.

Solution

Refer Example 24(ii)

12. Vertices $(\pm 6, 0)$, foci $(\pm 4, 0)$

Solution

Vertices are $(\pm 6, 0)$ and foci are $(\pm 4, 0)$.

The vertices lie on x-axis.

The equation of the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\therefore a = 6 \text{ and } c = 4$$

$$c^2 = a^2 - b^2 \Rightarrow 16 = 36 - b^2$$

$$\therefore b^2 = 20$$

$$\therefore \text{The equation of the ellipse is } \frac{x^2}{36} + \frac{y^2}{20} = 1$$

13. Ends of major axis $(\pm 3, 0)$,
ends of minor axis $(0, \pm 2)$.

Solution

Refer Example 25

14. Ends of major axis $(0, \pm \sqrt{5})$,
ends of minor axis $(\pm 1, 0)$

Solution

End points of major axis are $(0, \pm \sqrt{5})$

End points of minor axis are $(\pm 1, 0)$
 Since the major axis is y -axis, the equation of the ellipse is $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

Here $a = \sqrt{5}$ and $b = 1$

\therefore The equation of the ellipse is $\frac{x^2}{1} + \frac{y^2}{5} = 1$

15. Length of major axis 26, foci $(\pm 5, 0)$

Solution

Since the foci lie on x -axis, the equation of the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

The length of the major axis $2a = 26$

$\therefore a = 13$

The foci are $(\pm c, 0) \therefore c = 5$

$$c^2 = a^2 - b^2 \Rightarrow b^2 = a^2 - c^2$$

$$= 169 - 25 = 144$$

\therefore The equation of the ellipse is $\frac{x^2}{169} + \frac{y^2}{144} = 1$

16. Length of minor axis 16, foci $(0, \pm 6)$.

Solution

Refer Example 27

17. Foci $(\pm 3, 0)$, $a = 4$

Solution

Since the foci are on x -axis, the equation

of the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Foci are $(\pm 3, 0) \therefore c = 3, a = 4$

$$\therefore c^2 = a^2 - b^2 \Rightarrow b^2 = a^2 - c^2$$

$$= 16 - 9 = 7$$

\therefore The equation of the ellipse is $\frac{x^2}{16} + \frac{y^2}{7} = 1$

18. $b = 3, c = 4$, centre at the origin; foci on the x axis.

Solution

Since the foci are on x -axis, the equation

of the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Given $b = 3$ and $c = 4$

$$c^2 = a^2 - b^2$$

$$\Rightarrow a^2 = b^2 + c^2 = 16 + 9 = 25$$

\therefore The equation of the ellipse is $\frac{x^2}{25} + \frac{y^2}{9} = 1$

19. Centre at $(0, 0)$, major axis on the y -axis and passes through the points $(3, 2)$ and $(1, 6)$.

Solution

Refer Example 28

20. Major axis on the x -axis and passes through the points $(4, 3)$ and $(6, 2)$.

Solution

Since major axis is on the x -axis, the

equation of the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Since the ellipse passes through $(4, 3)$ and $(6, 2)$ we get

$$\frac{4^2}{a^2} + \frac{3^2}{b^2} = 1 \dots\dots (1)$$

$$\frac{6^2}{a^2} + \frac{2^2}{b^2} = 1 \dots\dots (2)$$

Solving (1) and (2) we get

$$a^2 = 52 \text{ and } b^2 = 13$$

\therefore The equation of the ellipse is $\frac{x^2}{52} + \frac{y^2}{13} = 1$