11.1 INTRODUCTION

We have already studied various forms of the equations of a line. In this chapter, we shall discuss about other curves like circle, ellipse, parabola, hyperbola etc. These curves are the intersection of a plane with a double napped right circular cone

11.2 SECTIONS OF A CONE

Let L be a fixed vertical line and l be a nonvertical line intersecting L at V, inclined at an angle θ (Fig 11.1).

Let us rotate the line l around L in such a way that the angle θ remains the same. Then the surface generated by l is called a double - napped right circular hollow cone or cone.

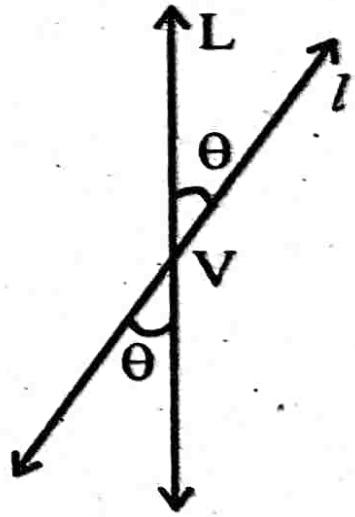


Fig. 11.1

Vertex of the cone	: The point V
Axis of the cone	: The fixed line L
Generator of the cone	: The rotating line l
Nappes	: The vertex separates the cone into two parts called nappes.
Semivertical angle θ	: The angle between axis L and generator l

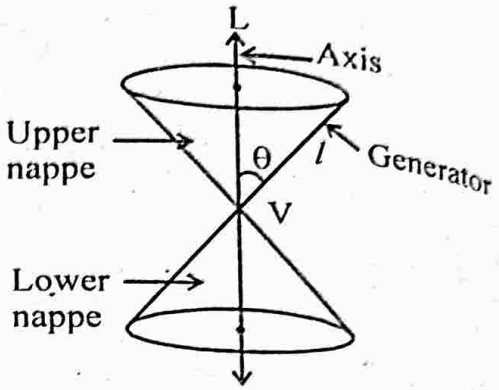


Fig. 11.2

On intersecting a right circular cone by a plane in different positions, different sections so obtained are called **conic sections** or **conics**. The shape of the section depends on the position of the intersecting plane with respect to the cone and by the angle made by it with the axis of the cone. Let α be the angle between the axis and the plane.

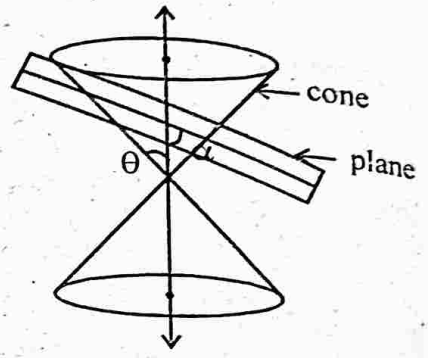


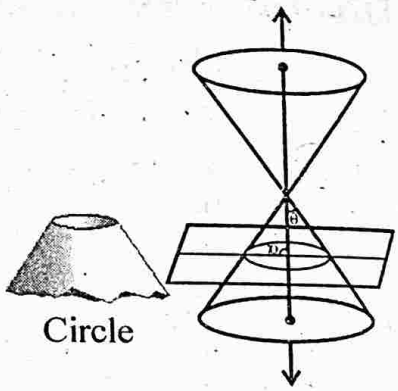
Fig. 11.3

11.2.1 Circle, Ellipse, Parabola, Hyperbola

When a plane cuts the nappe other than the vertex of the cone, there are four possibilities.

a. Circle

When the plane does not pass through the vertex V and angle between the axis and the plane, $\alpha = 90°$, the section is *a circle.*



b. Ellipse

When the plane does not pass through the vertex V and $\theta < \alpha < 90°$, the section is an *ellipse.*

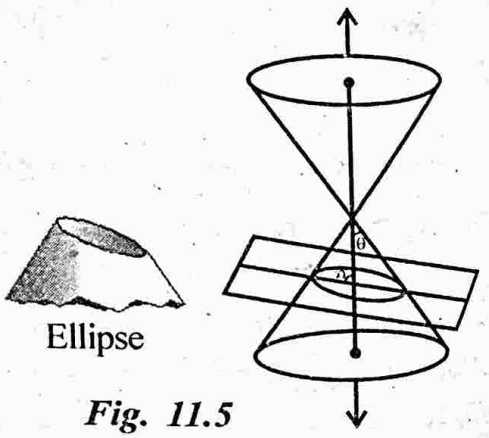
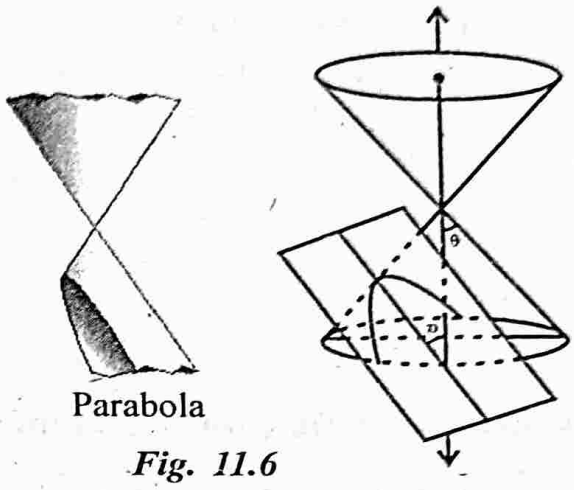


Fig. 11.5

c. Parabola

When the plane does not pass through the vertex V and $\alpha = \theta$, the section is ***a parabola.***

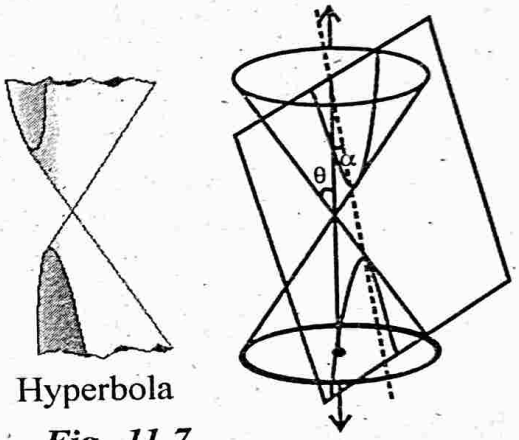


Parabola

Fig. 11.6

d. Hyperbola

When the plane intersects both the nappes and $0° < \theta < \alpha$, the section is ***a hyperbola.***



Hyperbola

Fig. 11.7

11.2.2 Degenerated conic sections

When the plane cuts the nappe at the vertex V of the cone, there are three possibilities.

a. Point

When the plane cuts the nappe at the vertex V and $\theta < \alpha \leq 90°$, the section is ***a point.*** This is a degenerated Conic Section.

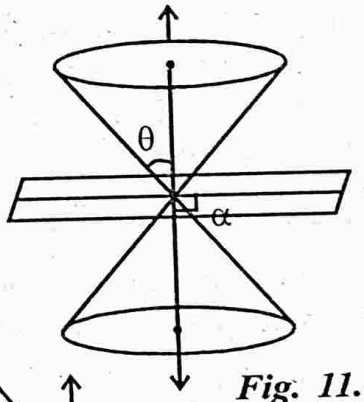


Fig. 11.8

b. Straight line

When the plane cuts the nappe at vertex V and $\theta = \alpha$, the section is a ***straight line.*** This is a degenerated case of the parabola.

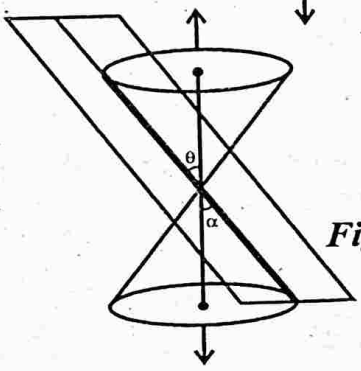


Fig. 11.9

c. Pair of straight lines

When $0 \leq \alpha < \theta$, the section is a pair of intersecting straight lines. This is a degenerated case of the hyperbola.

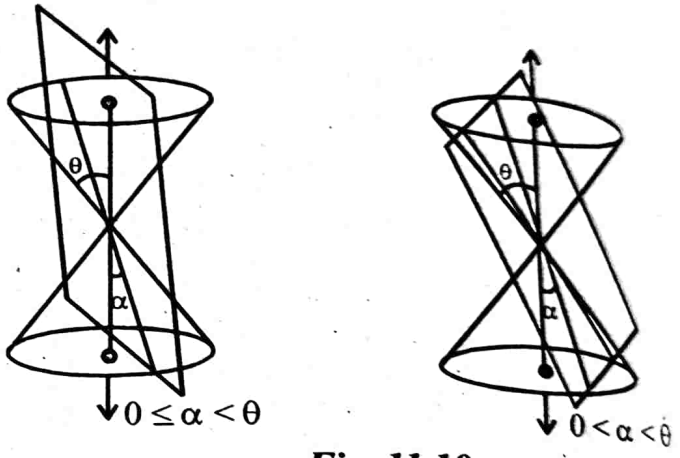


Fig. 11.10

Analytical definition of a conic

Conic is the set of all points in a plane which moves so that the distance from a fixed point is in a constant ratio to its distance from a fixed straight line. The fixed point is called the **focus**. The constant ratio is called the ***eccentricity*** denoted by '*e*'.

The fixed straight line is called the ***directrix***

If $e = 1$, the conic is a parabola

If $e < 1$, the conic is an ellipse

If $e > 1$, the conic is a hyperbola

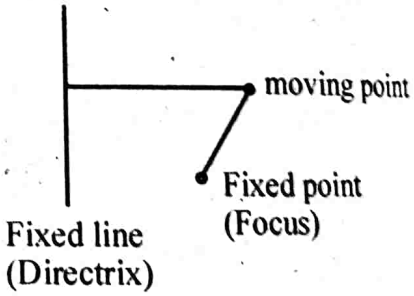


Fig 11.11