10.6 EQUATION OF FAMILY OF LINES PASSING THROUGH THE INTERSECTION OF TWO LINES

Consider two intersecting lines l_1 and l_2 given by $l_1 : A_1x + B_1y + C_1 = 0$

$l_2 : A_2x + B_2y + C_2 = 0$

Let P be the intersecting point. Through P we can draw infinitely many lines. All these lines are called ***the family of straight lines*** passing through P and the equation of the family of lines through P is given by

$A_1x+B_1y+C_1+k(A_2x+B_2y+C_2)=0$, where k is an arbitrary scalar called ***the parameter***. For different value of k, we get different straight lines. Any particular member of this family is obtained for some particular value of k.

Example 56

Find the equation of line parallel to the y axis and drawn through the intersection of the lines $x-7y+5=0$ and $3x+y-7=0$ **(March 2015 NCERT)**

Solution

The equation of line through the intersection of lines

$x - 7y + 5 = 0$ and $3x + y - 7 = 0$ is

$x - 7y + 5 + k(3x + y - 7) = 0$

i.e., $(1 + 3k)x + (k - 7)y + 5 - 7k = 0$ ......(1)

Since line (1) is parallel to y-axis, its y coordinate is zero.

i.e., $k - 7 = 0 \Rightarrow k = 7$

$\therefore (1) \rightarrow (1 + 3 \times 7)x + (7 - 7)y + 5 - 7 \times 7 = 0$

$22x - 44 = 0$

or $x = 2$, is the required equation.

Example 57

Find the equation of the line passing through the intersection of $x+y+2=0$ and $2x+3y-1=0$ and passing through the point (1, 1).

Solution

The equation of the line passing through the intersection of $x + y + 2 = 0$ and $2x+3y-1=0$ is $x+y+2+k(2x+3y-1)=0$

i.e., $(1 + 2k)x + (1 + 3k)y + 2 - k = 0$ .....(i)

Since line (1) passes through (1, 1) we get

$(1 + 2k)1 + (1 + 3k)1 + 2 - k = 0$

$4k + 4 = 0$

or $k = -1$

$\therefore (1) \rightarrow (1 + 2(-1))x + (1 + 3(-1))y + 2 - (-1) = 0$

$-x - 2y + 3 = 0$

$x + 2y - 3 = 0$, is the required equation

SOLUTIONS TO NCERT TEXT BOOK EXERCISE 10.4

1. Find the equation of the line through the intersection of the lines $3x + 4y = 7$ and $x - y + 2 = 0$ and whose slope is 5.

Solution

The equation of the line through the intersection of lines $3x + 4y = 7$ and $x - y + 2 = 0$ is

$3x + 4y - 7 + k(x - y + 2) = 0$

$(3 + k)x + (4 - k)y - (7 - 2k) = 0$ ......(1)

Given slope of (1) is 5.

i.e., $\dfrac{-(3+k)}{4-k} = 5$

$-3 - k = 20 - 5k$

$4k = 23 \Rightarrow k = \dfrac{23}{4}$

$\therefore (1) \rightarrow \left(3 + \dfrac{23}{4}\right)x + \left(4 - \dfrac{23}{4}\right)y - \left(7 - 2\left(\dfrac{23}{4}\right)\right) = 0$

$\dfrac{35x}{4} - \dfrac{7y}{4} - \left(\dfrac{-18}{4}\right) = 0$

$35x - 7y + 18 = 0$, is the required equation

2. Find the equation of the line through the intersection of lines $x + 2y - 3 = 0$ and $4x - y + 7 = 0$ and parallel to $5x + 4y - 20 = 0$

Solution

The equation of the line through the intersection of $x + 2y - 3 = 0$ and $4x - y + 7 = 0$ is

$x + 2y - 3 + k(4x - y + 7) = 0$

$(1 + 4k)x + (2 - k)y - 3 + 7k = 0$

$(1 + 4k)x + (2 - k)y - (3 - 7k) = 0$ .....(1)

The line $5x + 4y - 20 = 0$ is parallel to (1)

Then $\dfrac{A_1}{A_2} = \dfrac{B_1}{B_2} \Rightarrow \dfrac{1+4k}{5} = \dfrac{2-k}{4}$

$\Rightarrow 4 + 16k = 10 - 5k$

$\Rightarrow 21k = 6$

$\Rightarrow k = \frac{6}{21} = \frac{2}{7}$

$\therefore (1) \rightarrow \left(1+4\left(\frac{2}{7}\right)\right)x + \left(2-\frac{2}{7}\right)y - \left(3-7\left(\frac{2}{7}\right)\right) = 0$

$\frac{15}{7}x + \frac{12}{7}y - \frac{7}{7} = 0$

or $15x + 12y - 7 = 0$ is the required equation

3. Find the equation of the line through the intersection of lines $2x + 3y - 4 = 0$ and $x - 5y = 7$, that has its x-intercept equal to -4

Solution

The equation of the line through the intersection of lines $2x + 3y - 4 = 0$ and $x - 5y - 7 = 0$ is

$2x + 3y - 4 + k(x - 5y - 7) = 0$

i.e., $(2 + k)x + (3 - 5k)y - (4 + 7k) = 0$ ......(1)

Given x-intercept $= -4$

i.e., $\frac{-C}{A} = -4$

$\frac{4+7k}{2+k} = -4 \Rightarrow 4 + 7k = -8 - 4k$

$\Rightarrow 11k = -12$

$\Rightarrow k = \frac{-12}{11}$

$(1) \rightarrow \left(2+\frac{-12}{11}\right)x + \left(3-5\left(\frac{-12}{11}\right)\right)y - \left(4+7\left(\frac{-12}{11}\right)\right) = 0$

$\frac{10}{11}x + \frac{93}{11}y + \frac{40}{11} = 0$

i.e., $10x + 93y + 40 = 0$, is the required equation

4. Find the equation of the line through the intersection of $5x - 3y = 1$ and $2x + 3y - 23 = 0$ and perpendicular to the line $5x - 3y - 1 = 0$

Solution

The equation of the line through the intersection of lines $5x - 3y - 1 = 0$ and $2x + 3y - 23 = 0$ is

$(5x - 3y - 1) + k(2x + 3y - 23) = 0$

i.e., $(5 + 2k)x + (-3 + 3k)y - (1 + 23k) = 0$ ...(1)

Since (1) is perpendicular to $5x - 3y - 1 = 0$,

we get $A_1A_2 + B_1B_2 = 0$

i.e., $5(5 + 2k) + (-3)(-3 + 3k) = 0$

$$25 + 10k + 9 - 9k = 0$$

$$k + 34 = 0 \Rightarrow k = -34$$

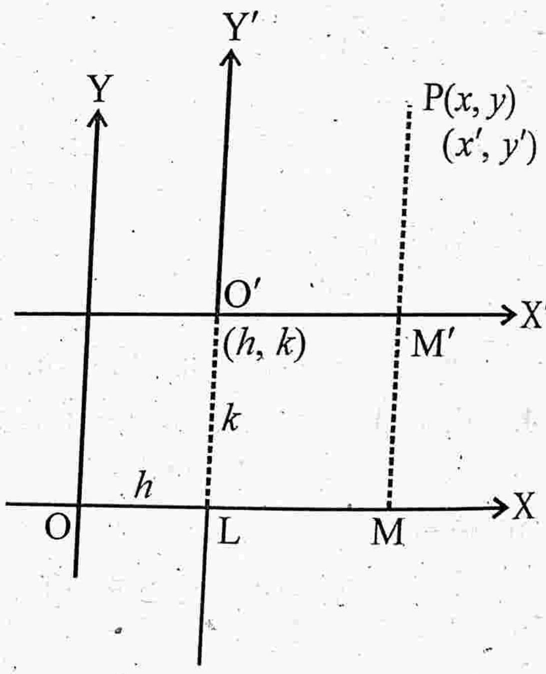
$\therefore (1) \rightarrow (5 + 2(-34))x + (-3 + 3(-34))y - (1 + 23(-34)) = 0$

$-63x - 105y - (-781) = 0$

or $63x + 105y - 781 = 0$, is the required equation

10.7 Shifting of origin (Translation of axes)

A transformation in which the origin of a coordinate system is shifted to a new point and the new axes are parallel to the old axes is known as *translation of axes*. The coordinate of each point in the plane are changed under a translation of axis. The geometric properties of the curves remain unchanged under translation of axes.



Let $P(x, y)$ be the coordinate of the point P *w.r.t.* the axes OX & OY.

Let O' be the point (h, k) *w.r.t.* the axes OX & OY.

Then $OL = h$, $LO' = k$, $OM = x$, $PM = y$

Shift the origin to O' by the translation of axes. Then the new axes are $O'X'$ and $O'Y'$.

Let $P(x', y')$ be the new coordinate of P *w.r.t.* the origin O'.

Now $x' = x$ coordinate of P *w.r.t.* O'

$= O'M' = LM = OM - OL = x - h$

and $y' = y$ coordinate of P *w.r.t.* O'

$= PM' = PM - M'M = PM - O'L = y - k$

Thus $x' = x - h$ and $y' = y - k$ or $x = x' + h$ and $y = y' + k$ gives the relation between old and new coordinates.

STUDY TIP

New coordinate = Old coordinate − coordinate of new origin

$$(x', y') = (x, y) - (h, k)$$

WORKING RULE

How to get the equation of a curve after shifting the origin.

Let (h, k) be the new origin.

Step 1

Replace x by $x' + h$ and y by $y' + k$ and simplify the equation.

Step 2

Replace x' by x and y' by y.

Example 58

Find the new coordinates of the point $(3, -4)$ if the origin is shifted to $(1, 2)$ by a translation **(NCERT)**

Solution

Old coordinate $(x, y) = (3, -4)$

New origin $(h, k) = (1, 2)$

Let the new coordinate be (x', y')

$\therefore (x', y') = (x, y) - (h, k)$

$= (3, -4) - (1, 2)$

$= (3 - 1, -4 - 2) = (2, -6)$

$\therefore$ Coordinate of $(3, -4)$ in the new system is $(2, -6)$.

Example 59

Find the transformed equation of the straight line $2x - 3y + 5 = 0$, when the origin is shifted to the point $(3, -1)$ after translation of axes. **(NCERT)**

Solution

The new origin $(h, k) = (3, -1)$

Let the coordinate (x, y) is changed to (x', y').

Then $x = x' + h = x' + 3$

$y = y' + k = y' - 1$

Substituting for x and y in the equation

$2x - 3y + 5 = 0$,

we get $2(x' + 3) - 3(y' - 1) + 5 = 0$

$\Rightarrow 2x' + 6 - 3y' + 3 + 5 = 0$

$\Rightarrow 2x' - 3y' + 14 = 0$

$\therefore$ The equation in the new system is $2x - 3y + 14 = 0$

Example 60

Find the transformed equation of $x^2 + y^2 = 1$ when the origin is shifted to $(1, -1)$

Solution

New origin $(h, k) = (1, -1)$

Let the coordinate (x, y) is changed to (x', y')

Then $x = x' + h = x' + 1$

$y = y' + k = y' - 1$

Substituting for x and y in the equation $x^2 + y^2 = 1$,

we get $(x' + 1)^2 + (y' - 1)^2 = 1$

$\Rightarrow (x')^2 + 2x' + 1 + (y')^2 - 2y' + 1 = 1$

$\Rightarrow (x')^2 + (y')^2 + 2x' - 2y' + 1 = 0$

$\therefore$ The equation in the new system is

$x^2 + y^2 + 2x - 2y + 1 = 0$

SOLUTIONS TO NCERT EXERCISE 10.5

1. Find the new coordinates of the points in each of the following cases, if the origin is shifted to the point $(-3, -2)$ by a translation of axes.

   i. $(1, 1)$

Solution

Old coordinate $(x, y) = (1, 1)$

New origin $(h, k) = (-3, -2)$

New coordinate $(x', y') = (x, y) - (h, k) = (1, 1) - (-3, -2) = (4, 3)$

ii. $(0, 1)$

Solution

Old coordinate $(x, y) = (0, 1)$

New origin $(h, k) = (-3, -2)$

New coordinate $(x', y') = (x, y) - (h, k) = (0, 1) - (-3, -2) = (3, 3)$

iii. $(5, 0)$

Solution

Old coordinate $(x, y) = (5, 0)$

New origin $(h, k) = (-3, -2)$

New coordinate $(x', y') = (x, y) - (h, k) = (5, 0) - (-3, -2) = (8, 2)$

iv. $(-1, -2)$

Solution

Old coordinate $(x, y) = (-1, -2)$

New origin $(h, k) = (-3, -2)$

New coordinate $(x', y') = (x, y) - (h, k) = (-1, -2) - (-3, -2) = (2, 0)$

v. $(3, -5)$

Solution

Old coordinate $(x, y) = (3, -5)$

New origin $(h, k) = (-3, -2)$

New coordinate $(x', y') = (x, y) - (h, k) = (3, -5) - (-3, -2) = (6, -3)$

2. Find what the following equations become when the origin is shifted to the point $(1, 1)$

i. $x^2 + xy - 3y^2 - y + 2 = 0$

Solution

Let the coordinate (x, y) is changed to (x', y') when origin $(h, k) = (1, 1)$

Then $x = x' + h = x' + 1$

$y = y' + k = y' + 1$

Substituting for x and y in the equation

$x^2 + xy - 3y^2 - y + 2 = 0$, we get

$(x'+1)^2 + (x'+1)(y'+1) - 3(y'+1)^2 - (y'+1) + 2 = 0$

$(x')^2 + 2x' + 1 + x'y' + x' + y' + 1 - 3(y')^2 - 6y' - 3 - y' - 1 + 2 = 0$

i.e., $(x')^2 - 3(y')^2 + x'y' + 3x' - 6y' = 0$

$\therefore$ The equation in the new system is $x^2 - 3y^2 + xy + 3x - 6y = 0$

ii. $xy - y^2 - x + y = 0$

Solution

New origin $(h, k) = (1, 1)$

Let the coordinate (x, y) is changed to (x', y')

Then $x = x' + h = x' + 1$

$y = y' + k = y' + 1$

Substituting for x and y in the equation

$xy - y^2 - x + y = 0$, we get

$$(x'+1)(y'+1) - (y'+1)^2 - (x'+1) + (y'+1) = 0$$

i.e., $x'y' + x' + y' + 1 - (y')^2 - 2y' - 1 - x' - 1 + y' + 1 = 0$

$x'y' - (y')^2 = 0$

$\therefore$ The equation in the new system is

$xy - y^2 = 0$

iii. $xy - x - y + 1 = 0$

Solution

New origin $(h, k) = (1, 1)$

Let the coordinate (x, y) is changed to (x', y')

Then $x = x' + h = x' + 1$

$y = y' + k = y' + 1$

Substituting for x and y in the equation

$xy - x - y + 1 = 0$, we get

$$(x'+1)(y'+1) - (x'+1) - (y'+1) + 1 = 0$$

$x'y' + x' + y' + 1 - x' - 1 - y' - 1 + 1 = 0$

i.e., $x'y' = 0$

$\therefore$ The equation in the new system $xy = 0$